THE COLLEGES OF OXFORD UNIVERSITY

MATHEMATICS FOR PHYSICISTS

MONDAY, 13 DECEMBER 2004

Time allowed: 1 hour

For candidates applying for Physics, and Physics and Philosophy

No calculators or tables may be used

Attempt as many questions as you can

---

1. If $p = \sqrt{3}$ and $q = \sqrt{2}$ evaluate
$$\sqrt{(5p-4q)^2 - (4p-5q)^2}\ .$$
[3]

2. Find the set of real numbers λ for which the quadratic equation
$$x^2 + (\lambda - 3)x + \lambda = 0$$
has distinct, real roots for x. [4]

3. Show that $(3, 4)$, $(-4, 0)$ and $(0, -2)$ are the vertices of a right-angled triangle, and find its area. [5]

4. Find $\dfrac{dy}{dx}$ if
(i) $y = \sin(\sin x)$,
(ii) $\ln y = x \ln x$. [4]

5. Find the value of x for which
(i) $\ln_2 x = 2$,
(ii) $\ln_x 2 = 2$,
(iii) $\ln_2 2 = x$. [3]

6. Evaluate $(2.002)^6$ to 4 decimal places. [4]

7. A ball is dropped from a height h. After the n^{th} bounce it returns to a height $h/(3^n)$. Find the total distance travelled by the ball. [4]

8. The ace of spades is missing from a pack of cards. What is the probability that
(i) a card, drawn at random, will be a spade?
(ii) two cards, drawn at random, will both be hearts? [4]
(A complete pack of cards comprises 13 spades, 13 hearts, 13 diamonds, and 13 clubs.)

9. (i) Sketch the curve $y = 2\,|\,x\,|\,+1$ for $-1 \leq x \leq 1$.
(ii) Find the area between the curve $y = 2\,|\,x\,|\,+1$, the x-axis, and the lines $x = 1$ and $x = -1$. [4]

10. Evaluate

$$\int_0^1 \frac{(1+x)^2}{1+x^2}\ dx\ .$$

[3]

11. (i) Sketch $y = \cos^4\theta$ for $-2\pi \leq \theta \leq 2\pi$.
(ii) Using the equality $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ or otherwise, express $\cos^4\theta$ in terms of $\cos 2\theta$ and $\cos 4\theta$.
(iii) Evaluate

$$\int_0^{2\pi} \cos^4\theta\ d\theta.$$

[7]

12. A geometric progression and an arithmetic progression have the same first term. The second and third terms of the geometric progression (which are distinct) are equal to the third and fourth terms of the arithmetic progression respectively.
(i) Find the common ratio of the geometric progression.
(ii) Show that the fifth term of the arithmetic progression is zero. [5]