THE COLLEGES OF OXFORD UNIVERSITY

MATHEMATICS FOR PHYSICISTS

MONDAY, 15 DECEMBER 2003

Time allowed: 1 hour

For candidates applying for Physics, and Physics and Philosophy

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No calculators or tables may be used

Attempt as many questions as you can

1. $y = \mathrm{e}^{x \sin x}$. Calculate $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

2. Find the range of values of x for which $(2x+1)^2 \leq 9$. [3]

3. The first three terms of a geometric series are $q-8$, q and $2q+12$ respectively. Calulate the possible values of q. [4]

4. Find the co-ordinates of the point where the line through $(-3, 13)$ and $(6, 10)$ cuts the line through $(1, 5)$ with gradient 3. [4]

5. Given that $2\log_2 x = y$ and $\log_2(2x) = y + 4$ find the values of x and y. [4]

6. Use the binomial theorem to find the value of $(0{\cdot}999)^9$ to 3 significant figures. [3]

7. Simplify

$$\frac{\sin^4\theta - \sin^4(90^\circ - \theta)}{\sin^2\theta - \sin^2(270^\circ + \theta)}$$

[3]

[Turn over]

8. Integrate

(a) $\displaystyle\int \frac{x}{1+x^2}\,\mathrm{d}x$,

(b) $\displaystyle\int_0^1 \frac{x^2}{(1-x^2)^{1/2}}\,\mathrm{d}x$, using the substitution $x = \sin y$. [6]

9. Draw sketches of the following functions:

(a) $y = 1 + |\, x - 2\,|$,

(b) $x = 3\cos t,\ y = \sin t, \quad \text{for } 0 < t < 2\pi$. [6]

10. A dice is biased so that the numbers 2, 4 and 6 are thrown twice as often as 1, 3 and 5. Calculate the probability that

(a) a two is thrown.

(b) two consecutive throws give a total ≤ 3. [4]

11. Show that the length of the body diagonal of a cube of side a is $\sqrt{3}\,a$. [5]

12.

$$I_n = \int_0^\infty x^n e^{-x}\,\mathrm{d}x$$

where n is an integer. By integration by parts or otherwise show that for $n > 0$

$$I_n = nI_{n-1}.$$

Hence calculate I_8. [5]