

THE COLLEGES OF OXFORD UNIVERSITY

MATHEMATICS FOR PHYSICISTS

MONDAY, 16 DECEMBER 2002

Time allowed: 1 hour

For candidates applying for Physics, and Physics and Philosophy

No calculators or tables may be used

Attempt as many questions as you can

1. Evaluate

(a) $501^2 - 499^2$

(b) $\sqrt{0.4} = \sqrt{0.4444\dots}$

(c) $(1.01)^4$ and $\sqrt[4]{1.04}$ (i.e. the 4th root of 1.04), each to 3 significant figures. [6]

2. Two circles are drawn, one inside a square and just touching its four sides, and the other outside the same square and passing through its four corners. What is the ratio of the areas of the two circles? [3]

3. Find the range of values for which x is greater than x^3 . Draw a graph which explains your result. [5]

4. The points $(5, 0)$ and $(-3, 6)$ are at opposite ends of the diameter of a circle. Determine the equation of the circle. [4]

5. Using a graph of any function $y(x)$ against x , explain the meaning of

(a) the differential $\frac{dy}{dx}$ at $x = 0$, and

(b) the definite integral $\int_1^3 y \, dx$. [4]

[Turn over]

6. Draw labelled sketches over the range -2 to $+2$ in x for the following functions:

$$\begin{aligned} &1/x \\ &1/x^2 \\ &1/(x^2 + 1) \\ &1/((x - 1)^2 + 1) \end{aligned} \tag{6}$$

7. Show that the values of x for which there are maxima or minima of the function $y = e^{-ax} \sin bx$ (where a and b are constants) satisfy the equation $\tan bx = b/a$. [4]

8. A straight line is drawn between the points $(1, -3)$ and $(4, 0)$. Another is drawn between $(3, -3)$ and $(3, 0)$. Calculate the angle at which the lines cross. [4]

9. Explain why $\sin^2 x + \cos^2 x = 1$. [4]

10. (a) An unbiased penny is tossed 4 times. What is the probability that the result will be 3 heads and 1 tail?

(b) An unbiased penny is tossed until tails comes up for the first time. What is the probability that this will take exactly 4 throws? [5]

11. The coordinates (x, y) of a point vary with time t according to

$$\begin{aligned} x &= at \\ y &= t^2 \end{aligned}$$

where a is a positive constant. Sketch the path of the point in the (x, y) plane as t varies. What is the value of $\frac{dy}{dx}$ when $x = 1$? [5]