

THE COLLEGES OF OXFORD UNIVERSITY

MATHEMATICS FOR PHYSICISTS

MONDAY, 10 DECEMBER 2001

Time allowed: 1 hour

For candidates applying for Physics, and Physics and Philosophy

No calculators or tables may be used

Attempt as many questions as you can

1. (a) Evaluate $1 - \frac{1}{1.00001}$ to 1 significant figure.
(b) Evaluate $(1.01)^{10}$ to 4 significant figures.
[4]
2. The difference between two numbers x and y is 4, and the difference of their squares is 8. Determine the two numbers.
[3]
3. Draw sketches of the functions $\sin x$, $\sin^2 x$ and $\sin x/x$ over the range $-2\pi < x < 2\pi$. (Label the axes).
[6]
4. Two dice are thrown, one after the other. Calculate the probabilities that
 - (a) the numbers shown are identical;
 - (b) the number on the second dice is 2 larger than that on the first;
 - (c) one number is even and the other is odd.[5]

5. The coordinates (x, y) of a point are given as functions of time t by the equations

$$\begin{aligned}x &= 2 + \cos t \\ y &= -1 + \sin t\end{aligned}$$

Sketch the path of the point in the (x, y) plane as t varies from 0 to 2π .
[4]

6. A rhombus is a parallelogram with all sides of equal length. Show that its diagonals intersect at right angles.

[5]

7. A is the point with (x, y) coordinates $(1, 3)$, while B is the point $(5, 1)$. What is the equation of the line which bisects AB at right angles? [4]

8. Evaluate

$$\int_{-3}^{+3} 3x^3 \, dx .$$

Sketch the curve $y = x^3$, and hence explain the value of the integral.
[5]

9. What are the largest and the smallest values of $y = x^3 - 12x + 1$ for values of x in the range -3 to $+5$? [3]

10. The fifth term of a geometric series is 1 and the eighth term is $-1/27$. What is the sum of the infinite series? [4]

11. The curve $y = 3(e^{2x} - 1)$ and the line $y = ax$ have the same gradient at the origin. Determine the value of a . [3]

12. Explain why, for $0 < x < \pi/2$, $\sin x$ is smaller than $\tan x$. [4]