UNIVERSITY OF CAMBRIDGE
Registrary's Office

Dr James Knapton
Information Compliance Officer

Peter Alexander
By email

Reference: FOI-2018-272

1 May 2018

Dear Mr Alexander,

Your request was received on 12 April 2018 and I am dealing with it under the terms of the Freedom of Information Act 2000 ('the Act').

You asked:

> *Could you please release the exam papers for the core and optional modules offered in MPhil Economics and MPhil Economic Research, for the academic years 2015-2016 and 2016-2017?*

The requested information is attached. Please note that the attached documentation should not be copied, reproduced or used except in accordance with the law of copyright.

If you are unhappy with the service you have received in relation to your request and wish to make a complaint or request an internal review of this decision, you should write to Dr Kirsty Allen, Head of the Registrary's Office, quoting the reference above, at The Old Schools, Trinity Lane, Cambridge, CB2 1TN or send an email marked for her attention to foi@admin.cam.ac.uk. The University would normally expect to receive your request for an internal review within 40 working days of the date of this letter and reserves the right not to review a decision where there has been undue delay in raising a complaint. If you are not content with the outcome of your review, you may apply directly to the Information Commissioner for a decision. Generally, the Information Commissioner cannot make a decision unless you have exhausted the complaints procedure provided by the University. The Information Commissioner may be contacted at: The Information Commissioner's Office, Wycliffe House, Water Lane, Wilmslow, Cheshire, SK9 5AF (https://ico.org.uk/).

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UNIVERSITY OF CAMBRIDGE
Registrary's Office

Yours sincerely,

James Knapton

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil Finance

Wednesday 11 January 2017 10:00-12:00

M100
MICROECONOMICS I

Candidates are required to answer **three** out of **four** questions

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Consider an agent with initial wealth w who plans to bid in a first price sealed bid auction (i.e., an auction in which everyone submits a bid simultaneously and the highest bid wins). The value she assigns to the item being auctioned is v. She believes that if she submits a bid b, she will win with probability $G(b)$, where G is continuous, differentiable, and strictly increasing function. The agent's wealth will be either $w + v - b$ if she wins or w if she loses. Her (Bernouli) utility over final wealth is $u(x)$, where u is continuous and differentiable with $u'(0) > 0$ and $u''(0) < 0$. You can assume that $\log G(b)$ is concave in b and that optimal bids are always interior.

   (i) Write down the agent's expected utility.

   (ii) Set up the agent's problem and show that optimal bid b^* is characterized by:

   $$\frac{u'(w + v - b^*)}{u(w + v - b^*) - u(w)} = \frac{G'(b^*)}{G(b^*)}.$$

   (iii) Using implicit differentiation and the log concavity of G, show that the agent bids strictly more for an object she values more (higher v). [Note: This is the hardest part of this question. You do *not* need to answer this part to be able to answer the remaining parts of the question].

   (iv) Suppose $G(b) = b^{(0.5)}$ and the agent is risk neutral. Find an expression for the amount the agent optimally bids in terms of just v.

   (v) Continuing to let $G(b) = b^{(0.5)}$ suppose the agent is risk averse. Does the agent bid more or less than when risk neutral?

2. Consider an economy with two agents and two goods: a numeraire good and houses. Agents $i = 1, 2$ are endowed with e_i units of the numeraire. We will assume that $e_1 \geq e_2$. There is a single house and the agents initially own equal shares of it. However, the house must be owned in full by an agent for an agent to consume it (and derive any utility from it). The agents have identical preferences. Agent i's utility from consuming c_i units of the numeraire and $s_i = 1, 0$ units of houses is $u(c_i, s_i)$, where $\frac{du(c_i,s_i)}{dc_i} > 0$, $\frac{d^2u(c_i,s_i)}{dc_i^2} < 0$, $u(c, 1) - u(c, 0) > 0$ and $\frac{du(c,1)}{dc} > \frac{du(c,0)}{dc}$. Negative consumption of the numeraire good is permitted and you can normalise its price to 1. Let p be the price of a house.

   (i) Write down agent i's constrained optimisation problem.

   (ii) Given the house can only be consumed as a whole, what does it mean for houses to be a normal good?

   (iii) Show that houses are a normal good.

   (iv) Hence argue that if $e_1 > e_2$, agent 1 will own the house in a Walrasian equilibrium.

   (v) Find a Walrasian equilibrium of the economy.

   (vi) Are there are multiple Walrasian equilibria? If so, show that there exist multiple Walrasian equilibria, if not, show that there is a unique Walrasian equilibrium. Then provide some intuition for your answer.

3. Consider a two-good economy and a consumer with complete, transitive, continuous, and monotonic preferences. Prices in this economy are always strictly positive. Suppose the consumer has wealth $w = 9$. You know that at prices $p = (2, 1)$, the consumer's unique preferred bundle is $x = (3, 3)$ and at prices $p' = (1, 2)$, the consumer's unique preferred bundle is $x' = (5, 2)$.

   (i) In the context of the two-good economy define (a) complete preferences, (b) transitive preferences, (c) continuous preferences and (d) monotonic preferences.

   (ii) Identify graphically the set of consumption bundles that the consumer must weakly prefer to x.

   (iii) If the consumer's preferences are also convex, how does your answer to (ii) change?

   (iv) Suppose that in addition to the two marketed goods, there is also a third good whose level is fixed in the short run. If the consumer's choices between the first two goods are not affected by the fixed level x_3 of the third good, what can be inferred about the utility function $u(x_1, x_2, x_3)$.

4. There are three goods labeled 1, 2 and 3. Consider the following three possible consumption bundles $x_A = (1, 3, 2)$, $x_B = (2, 2, 2)$ and $x_C = (2, 2, 1)$.

   (i) Suppose Consumer A is rational. At prices $p_A = (2, 1, 1)$, you observe him choose x_A. At prices $p_B = (1, 2, 1)$, you observe him choose x_B. Now he has wealth $w = 7$ and faces prices $p = (1, 1.5, 1)$. Can you say for certain if any of the bundles will or will not be chosen? What if A's preferences are also locally non-satiated?

   (ii) Suppose Consumer B has the following indirect utility function

$$v(p_1, p_2, w) = \left(\frac{w}{p_1}\right)^{\frac{1}{4}} \left(\frac{w}{p_2}\right)^{\frac{3}{4}}.$$

   (a) Calculate the expenditure function $e(p_1, p_2, u)$.

   (b) Calculate the Hicksian demand functions $h_1(p_1, p_2, u)$ and $h_2(p_1, p_2, u)$.

   (iii) Suppose Consumer C has rational, continuous and locally non-satiated preferences, and that her preferences are also homothetic, (i.e. if $x \sim x'$ then $\alpha x \sim \alpha x'$). What can you conclude about how Consumer C's Marshallian demand $x(p, w)$ will vary with w?

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil Finance

Monday 8 May 2017 02:00pm - 04:00pm

M100
MICROECONOMICS I

Candidates are required to answer **three** out of four questions

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Consider an economy in which consumers face uncertainty over S possible states of the world. The economy has I consumers, and each consumer i maximizes her expected utility given her own subjective assessment of the probability of each state s being realized. Let q_{is} be the probability i places on state s being realized. (Assume that these subjective probabilities are NOT caused by different information and so consumers would not update them when learning each others' beliefs directly or indirectly, e.g. through their market behavior.) The economy has one consumption good, of which each consumer i has an endowment $e_i > 0$, which does NOT depend on the state. Before state s is realized, consumers can buy and sell S different contracts, that respectively each pay out 1 in a different state s and 0 otherwise. This question asks you to study the Walrasian equilibrium of this economy, with the prices $p_1, \ldots, p_S$ of the contracts. You may assume that agents have constant relative risk aversion utility functions given by $u(c) = \frac{1}{1-r}c^{1-r}$, where $r \neq 1$ is a parameter.

   (i) Write down an expression for r in terms of just the shape of then Bernoulli utility function ($u'(c)$ and $u''(c)$) and level of consumption c.

   (ii) Letting p_e be the price of the endowment good, argue that in any Walrasian equilibrium $p_e = \sum_{s=1}^{S} p_s$.

   (iii) Letting $x_{is} \in \mathbb{R}$ be the quantity that consumer i buys of the contract that pays out in state s, argue that it is without loss of generality to look for Walrasian equilibria in which $c_{is} = x_{is}$ for all s.

   (iv) Formulate the expected utility maximization problem faced by each price-taking consumer in this economy assuming that $c_{is} = x_{is}$ for all i and all s.

   (v) Solve this problem to obtain an expression for the demands x_{is} in terms of the prices, subjective probabilities and i's endowment.

   (vi) Write down any additional conditions required for a Walrasian equilibrium in this economy.

   (vii) When is there a unique Walrasian equilibrium of this economy?

2. This question asks you to apply the concept of rationalizability to production choices. A price-taking firm can choose any nonnegative input vector $z \in \mathbb{R}_+^m$ and a nonnegative output vector $y \in \mathbb{R}_+^n$ subject to the constraint that the vector $(-z, y)$ lies in the firm's production possibility set $Y \subseteq \mathbb{R}_+^{m+n}$. Suppose that we can observe the firm's input choices z for different nonnegative input price vectors $w \in \mathbb{R}_+^m$, but cannot observe the output choices y or output prices p (and may not even know how many outputs n it has). We do know that the output prices are non-negative, such that $p \in \mathbb{R}_+^n$, and that they do not change between observations.

   (i) Suppose that $m = 2$, and that at input prices $\widehat{w} = (1, 1)$ the firm bought inputs $\widehat{z} = (10, 15)$ and at prices $\widetilde{w} = (2, 3)$ it bought inputs $\widetilde{z} = (13, 14)$. Show that this pair of observations is not rationalizable (i.e. consistent with profit-maximizing behavior for some production set and output prices)? (*Hint:* What does the firm's choices of $\widehat{z}$ instead of $\widetilde{z}$ at prices $\widehat{w}$ imply?)

   (ii) Find a necessary and sufficient condition for two input price-demand observations $(\widehat{z}, \widehat{w})$ and $(\widetilde{z}, \widetilde{w})$ to be rationalizable. Show that your condition is necessary and sufficient. (*Hint:* Consider how your approach to answering part (i) can be generalized.)

   (iii) Suppose we now have the following three observations:

   - At prices $\widehat{w} = (1, 1)$ the firm bought inputs $\widehat{z} = (10, 15)$.

- At prices $\widetilde{w} = (2, 3)$ the firm bought inputs $\widetilde{z} = (13, 13)$.
- At prices $\bar{w} = (4, 1)$ the firm bought inputs $\bar{z} = (8, 9)$.

Are these three observations jointly rationalizable?

3. A risk neutral student wants to sell her car. Her value for keeping the car is 0 but dealers have value v for it which they privately observe. From the student's perspective, v is drawn from a cumulative distribution function F, with associated probability density function f, on the unit interval. Suppose the student offers to sell to the dealer at a price p, and the dealer buys if and only if $p < v$.

   (i) When does this market yield a Pareto efficient outcome? Do inefficiencies vary with p?

   (ii) Find an expression for the price p that maximizes the student's expected utility. You may assume that F is differentiable.

   (iii) Suppose instead that the value is $\widehat{v}$ is drawn from $\widehat{F}$ and that $\widehat{F}$ first order stochastically dominates F. Show that under $\widehat{F}$ the student's expected utility is higher when she chooses an optimal p.

   (iv) Suppose now that instead of quoting an acceptable price to the dealer the student can ask 2 dealers to offer her a price for her car. From the student's perspective, the price each dealer offers (p) is drawn independently from the same cumulative distribution function $G(p)$. The cost of obtaining each quote is k where $0 < k < \mathbb{E}[p]$. The student can only visit one dealer at a time and when an offer is made it must be accepted or else it is withdrawn and the dealer will not make another offer. If the student wishes to maximize the expected price she receives net of the cost of obtaining quotes what is her optimal decision rule?

   (v) Suppose instead that the distribution quotes are drawn form is $\widehat{G}(p)$ instead of $G(p)$. How will the student's decision rule change if $\widehat{G}(p)$ first order stochastically dominates $G(p)$?

   (vi) Suppose $\widetilde{G}(p)$ and $G(p)$ have the same mean, but $\widetilde{G}(p)$ second order stochastically dominates $G(p)$. Does the student prefer the dealers' quotes to be drawn from $\widetilde{G}(p)$ instead of $G(p)$? Discuss.

4. Consider an economy with two agents indexed $1, 2$ and two goods indexed a, b. Let the agents have the utility functions:

$$u^1(a^1, b^1) = a^1 + b^1 + \alpha \max\{a^1, b^1\} \quad (1)$$
$$u^2(a^2, b^2) = a^2 + b^2 + \alpha \max\{a^2, b^2\} \quad (2)$$

   Individual endowments are given by $e^1 = (\frac{1}{5}, \frac{4}{5})$ and $e^2 = (\frac{4}{5}, \frac{1}{5})$. Except where otherwise stated, **assume that** $-1 < \alpha < 0$.

   (i) Define a Walrasian equilibrium for this exchange economy.

   (ii) Illustrate the rough shape of the indifference curves and the contract curve in an Edgeworth box diagram.

   (iii) Provide a formal argument showing that $a^1 = b^1$ on the contract curve.

   (iv) Using your Edgeworth box diagram from part (ii) consider an arbitrary point on the contract curve, and argue that there are prices for which this allocation is a Walrasian equilibrium.

   (v) Does the contract curve coincide with the set of Walrasian equilibria?

   (vi) Find the set of allocations that form the contract curve.

   (vii) Find the range of prices that form part of a Walrasian equilibrium.

   (viii) Does a Walrasian equilibrium exist if $\alpha = 1$? Provide a formal argument to support your claim.

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Tuesday 23 May 2017 9:00am - 11:00am

M110
MICROECONOMICS II

Candidates are required to answer **three out of four** questions.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

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STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. (a) Consider the following two-player game

1/2	A	B	C
A	a, b	1,1	c, d
B	1,1	0,0	1,1
C	e, f	1,1	g, h

where a, b, c, d, e, f, g, h are arbitrary payoffs such that $a + e > 2$, $c + g > 2, b + 3d > 4$ and $f + 3h > 4$.

i. Show that for each player strategy B is strictly dominated by another strategy (possibly a mixed one).

ii. Show that the above game has a pure or a mixed strategy Nash equilibrium. (Hint: First, consider cases in which at least one player has a dominant strategy. Then consider the case in which no player has a dominant strategy.)

(b) Consider a game in which each player $i = 1, .., n$ chooses a real number s_i in the interval $[0, 1]$ once and simultaneously. Let $\pi_i(s_1, .., s_n)$ be the payoff of player i if the players choose a strategy profile $(s_1, ..., s_n)$. Suppose $\pi_i(s_1, .., s_n)$ is differentiable. Explain why, in general, any non-interior Nash equilibrium of this game is Pareto inefficient.

(c) By means of an example or otherwise show that rational players do not necessarily choose a Nash equilibrium in complete information normal form games.

(d) Discuss the assumptions needed to justify the concept of Nash equilibrium (both pure and mixed) in games of complete information.

2. (a) Consider the following linear Cournot model. Two firms, 1 and 2, simultaneously choose the quantities they will sell on the market. If q_1 and q_2 are respectively the quantities firms 1 and 2 sell, then the price each receives for each unit given these quantities is $P(q_1, q_2) = a - b(q_1 + q_2)$, where a and b are two positive constants. Each firm $i = 1, 2$ has a probability μ of having a cost function given by $c_L q_i^2$ and $(1 - \mu)$ of having a cost finction given by $c_H q_i^2$, where c_H and c_L are two positive constants such that $c_H > c_L$. Solve for the Bayesian Nash Equilibrium.

(b) Suppose that an object is being auctioned and that there are n bidders. Each bidder i has a private valuation for the object v_i. The valuation of each bidder is private and independently drawn from a continuously differentiable distribution F on the interval $[0, w]$ where $0 < w < \infty$. Each bidder knows his own valuation and the distribution from which the valuations of others are drawn. Bidders are risk-neutral.

i. Suppose that the auction is first-price. Show that the following strategy profile is a symmetric Bayesian Nash equilibrium: each bidder i with valuation $v_i \in [0, w]$ bids

$$v_i - \int_0^{v_i} \frac{(F(x))^{n-1}}{(F(v_i))^{n-1}} dx.$$

ii. Suppose that the auction is second-price. Show that the expected revenue of the seller in a symmetric Bayesian Nash equilibrium of this auction is the same as that obtained in the equilibrium of the first-price auction in part i.

(TURN OVER)

3. (a) Show that any game with a finite number of pure strategies has a mixed Nash equilibrium. (You may assume the following result: any normal form game has a Nash equilibrium if the set of strategies for every player is convex and compact and the payoff function for each player i is quasi-concave in $i's$ strategy and continuous in the strategies of all players). Hence, explain briefly why every finite extensive form game has a mixed subgame perfect equilibrium strategy.

(b) Consider a 2-player game G described by the following payoff matrix:

	A	B	C
A	$3,3$	$0,4$	$3,0$
B	$4,0$	$0,0$	c,d
C	$0,3$	a,b	$0,0$

for some paramters a, b, c and d. Assume that the players are restricted to using pure strategies.

i. Assume that $a > c + 1$, $c > 3$, $b > 3$ and $1 > d > 0$. Compute the set of Nash equilibria of G. Suppose that G in part (b) is played twice, the players do not discount the future and there is perfect monitoring. Show that there exists a subgame perfect equilibrium such that both players choose A in period 1. Are there any subgame perfect equilibrium that results in outcome (B, A) or (A, B) in period 1 for these paramater values?

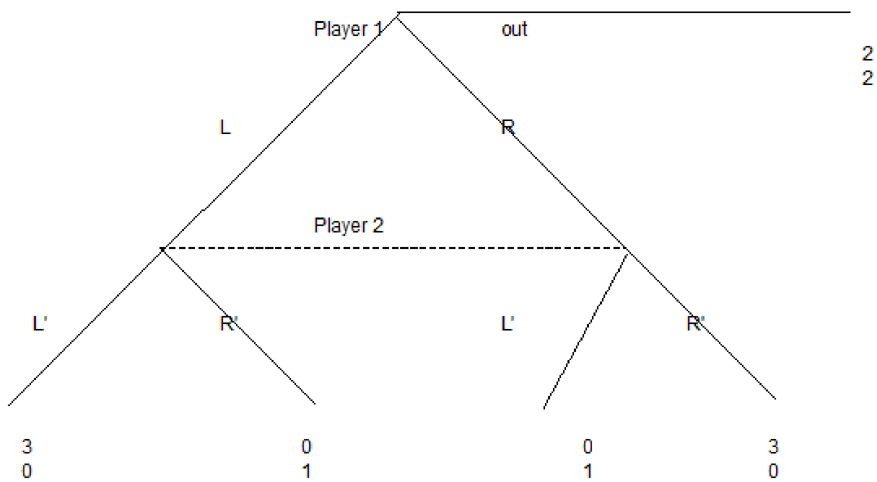
ii. Assume that $a = d = 1$ and $b = c = 4$. Compute the set of Nash equilibrium of G. Suppose that G is infinitely repeated and the players discount the future by a factor $\delta < 1$. Show that there exists a subgame perfect equilibrium that induces the outcome (A, A) at every period on the equilibrium path if δ is sufficiently close to 1. Are there any subgame perfect equilibrium that results in the following cyclical equilibrium path: play (B, A) at odd periods and (A, B) at even periods?

iii. Assume that $a = d = 1$ and $b = c = 2$. Does the game G have a Nash equilibrium? What are the minmax payoffs for each player? Suppose that G is infinitely repeated and the players discount the future by a factor $\delta < 1$. Does there exist a subgame perfect equilibrium that induces the outcome (A, A) at every period on the equilibrium path for δ sufficiently close to 1? Explain your answer.

4. (a) Consider the following ultimatum 2-player bargaining game over a pie of size 2. First, Player 1 makes an offer of 0, 1 or 2. Next player 2 responds to the offer of player 1 by either accepting or rejecting the offer. If an offer $x = 0, 1, 2$ is made and Player 2 accepts then the game ends and the payoffs of 1 and 2 are respectively x and $2 - x$. If Player 2 rejects an offer the game ends and each players payoff will be zero.

Describe both the extensive and the normal form representations of the above game. What are the Nash equilibria and the subgame perfect equilibria of this game? Is there a Nash equilibrium that is not credible? If so, explain why this is the case.

(b) Consider the following extensive form game.



i. Show that there does not exist a pure-strategy perfect Bayesian equilibrium in this extensive form game.

ii. Describe the mixed-strategy perfect Bayesian equilibria of the game.

(c) Explain why perfect Bayesian equilibrium may not be always a reasonable solution concept in dynamic games of imperfect information. Discuss briefly how one may refine the concept of perfect Baysian equilibrium to make it a more reasonable solution concept.

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Thursday 1 June 2017 1:30pm - 3:30pm

M120
TOPICS IN ECONOMIC THEORY

Candidates are required to answer **three out of four** questions

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

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STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. a. Consider the infinite horizon alternating bargaining game with 2 players (Rubinstein model). The size of the pie is 1. The rules are in odd periods player 1 makes an offer of a split $x \in [0, 1]$ to player 2, which player 2 may accept or reject and in even periods player 2 makes an offer of a split to player 1, which player 1 may accept or reject. If at any period a player accepts an offer, the proposed split is immediately implemented and the game ends. If she rejects, nothing happens until the next period. Assume that each player i discounts future payoffs by a common factor $\delta < 1$.

   i. Show there exists a subgame perfect equilibrium with the following outcome: there is agreement at the first date and the players receive a unique payoff described by the following split: player 1 receives $\frac{1}{1+\delta}$ and player 2 receives $\frac{\delta}{1+\delta}$.

   ii. Explain why the equilibrium payoff described in part (a) is the unique subgame perfect equilibrium outcome (A sketch of the arguments will suffice).

   iii. Modify the above bargaining game by giving player 2 the following option: at the end of any even period player 2 can opt out and obtain a payoff of $s \in \mathbb{R}_+$ after rejecting the offer of player 1. Assume that $s \leq \frac{\delta}{1+\delta}$. Describe the subgame perfect equilibrium of this modified game.

   b. Consider the set of 2-person bargaining problems $\mathcal{B}$ where the set of possible utility agreements is some subset of $\mathbb{R}^2_+$ and disagreement results in a payoff vector $d = (d_1, d_2) \in \mathbb{R}^2_+$. Thus $\mathcal{B} = \{(U, d) \mid U \subseteq \mathbb{R}^2_+ \text{ and } d \in U\}$. Let $f : \mathcal{B} \to \mathbb{R}^2$ be the bargaining solution. Thus, $f(U, d) \in U$ describes the utility vector the players will receive if $U \subseteq \mathbb{R}^2_+$ is the set of possible utility agreements and d is the disagreement payoff vector.

   i. Suppose that f is such that, for any positive real numbers a, b and c, $f(U, d) = (\frac{c}{2a}, \frac{c}{2b})$ if $U = \{(u_1, u_2) \in \mathbb{R}^2_+ \mid au_1 + bu_2 \leq c\}$ and if $d = 0$. Assume that f satisfies Independent of irrelvant alternatives. Show that $f(U, d) = \arg\max_{(u_1,u_2)\in U} u_1 u_2$ if U is convex, $d = 0$.

   ii. Suppose that f is egalitarian; that is $f(U, d) = \arg\max_{(u_1,u_2)\in U} \min_{i=1,2} u_i$. Show that f satisfies Pareto and independence of irrelevant alternatives axioms but it violates independent of utility units.

2. a. Suppose that there exists a mechanism that implements the social choice function $f(\cdot)$ in Bayesian Nash equilibrium. Show that $f(\cdot)$ is truthfully implementable in Bayesian Nash equilibrium.

b. Consider the following mechanism design problem of implementing a project. There is a finite number of agents I and a finite set of projects denoted by K. An alternative is given by a vector $x = (k, t_1, ..., t_I)$ where $k \in K$ is the project choice and $t_i \in \mathbb{R}$ is a transfer to agent i. Denote the set of private signals each agent i receives by Θ_i. Assume Θ_i is finite. Agent i's utility is given by

$$u_i(x, \theta_i) = w_i(v_i(k, \theta_i) + t_i)$$

where $v_i : K \times \Theta_i \to \mathbb{R}_+$ and $w_i : \mathbb{R} \to \mathbb{R}$. Assume that $w_i(.)$ is an increasing function. For any $\theta = (\theta_1, ..., \theta_I) \in \times_{i=1}^{I} \Theta_i$, let $k^*(\theta)$ be an expost efficient project; thus

$$\sum_{i=1}^{I} v_i(k^*(\theta), \theta_i) \geq \sum_{i=1}^{I} v_i(k, \theta_i) \text{ for all } k \in K \tag{1}$$

Show that the social choice function $f(\cdot) = (k^*(\cdot), t_1(\cdot), ..., t_I(\cdot))$ is truthfully implementable in dominant strategies if, for all $i = 1, ..., I$ and all $\theta = (\theta_i, \theta_{-i}) \in \times_{i=1}^{I} \Theta_i$,

$$t_i(\theta) = \left[\sum_{j \neq i} v_j(k^*(\theta), \theta_j) \right] + h_i(\theta_{-i}),$$

where $h_i(\cdot)$ is an arbitrary function of θ_{-i}.

c. Consider the mechanism design problem described in part b. Suppose that $w_i(.)$ is the identity function for all i; thus each agent i's utility is given by

$$u_i(x, \theta_i) = v_i(k, \theta_i) + t_i.$$

Also assume that each agent i's signal $\theta_i \in \Theta_i$ is independently distributed according to some probability distribution.Show that for any expost efficient project $k^*(.)$, there exists transfer functions $(t_1(\cdot), ..., t_I(\cdot))$ such that $\sum_{i=1}^{I} t_i(\theta) = 0$ and the social choice function $f(\cdot) = (k^*(\cdot), t_1(\cdot), ..., t_I(\cdot))$ is truthfully implementable in Bayesian Nash equilibrium.

(TURN OVER)

3. Consider the Crawford-Sobel model of strategic information transmission. An expert privately observes the state $\theta \in \Theta = [0,1]$ and then sends a message $m \in M = [0,1]$ to a decision maker. The decision maker does not observe θ but (possibly) obtains information about it from the expert's message. Before receiving a message from the expert, the decision maker has a prior belief about the state θ that is uniformly distributed on $[0,1]$. The decision maker observes the message and then chooses an action $y \in \mathbb{R}$. The expert's payoff function is $u^S(y,\theta) = -\left(y - (\theta + b)\right)^2$, where b is some positive constant, and that of the decision maker is $u^R(y,\theta) = -\left(y - \theta\right)^2$.
   a. Define a pure strategy perfect Bayesian equilibrium in this setup. Explain why the restriction to pure strategy equilibria is without loss of generality in this context.
   b. Describe a "babbling" equilibrium of this game and explain why this kind of equilibrium always exists.
   c. Find a three-message perfect Bayesian equilibrium for this game in which there is information transmission.
   d. Compute the decision maker's ex ante expected payoff in the three-message perfect Bayesian equilibrium.

4. Two agents $(i = 1, 2)$ are engaged in a productive relationship. There are two physical assets, a_1 and a_2. There are two stages. At stage 1 each simultaneously chooses a level of investment, i's investment being $e_i \in [0, \infty)$. At stage 2 the owner of asset j $(j = 1, 2)$ chooses an action $y_j \in \{h, l\}$. The two actions y_1 and y_2 are chosen simultaneously. The agents can also make money transfers to each other. The payoff for agent i $(i = 1, 2)$ is

$$V_i(e_1, e_2, y_1, y_2, t_i) = (e_1 + e_2)\phi_i(y_1, y_2) + t_i - e_i^2,$$

where t_i is the net money transfer to i and ϕ_i is given by the following matrix, in which the row corresponds to y_1, the column to y_2, and the first entry in a cell gives $\phi_1(y_1, y_2)$ and the second entry gives $\phi_2(y_1, y_2)$

	h	l
h	2,2	-1,4
l	4,-1	0,0

At the outset, before stage 1, the agents contract. The only contracts permitted are ownership contracts. At stage 2 they engage in Nash bargaining and can negotiate a binding contract on (y_1, y_2, t_1, t_2). It is not possible to leave the relationship.

a. Find the first-best investments and the maximum total surplus.

b. Solve for the subgame-perfect equilibrium in the case (i) in which 1 owns both assets, and (ii) in which 1 owns asset 1 and 2 owns asset 2. Which ownership structure is better? Is either structure efficient? If not, discuss the intuition for this.

c. If agent 1 initially owns both assets and initially has all the bargaining power, what would you expect to happen at the initial contracting stage (i) if both are already committed to the relationship, and (ii) if they are not?

d. Do the two assets satisfy Grossman and Hart's definitions of *complementary* and *independent* assets?

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Wednesday 31 May 2017 9:00am - 11:00am

M130
APPLIED MICROECONOMICS

This paper is in two sections. Candidates are required to answer **four out of six** questions in Section A; and the **one** compulsory question in Section B.

Questions within Section A carry equal weight. Each section carries equal weighting.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Section A

1. Consider an economy with only two individuals who differ in terms of their ability. How does cutting the marginal tax rate of the lower ability person affect the possible incentive compatible tax rates for the high ability?

2. Suppose there is one input, labour z, and one output, y, in th economy. The government wants to raise revenue, R, but there are no lump sum taxes. What prices should the producers face? What prices should consumers face?

3. What is the difference between the Marshallian and Hicksian wage elasticity of labour supply? What is the Firsch elasticity? If wages are known to be high this year, but falling next year, how would you evaluate the labour supply response?

4. Assuming preferences are quadratic,

$$u(c) = -\frac{1}{2}(b-c)^2$$

   what would the effect on consumption be of (i) a permanent increase in income and (ii) a transitory increase in income? How would your answer change if preferences had the CRRA form:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

5. The government believes uncertainty about the economic situation after Brexit will cause individuals to spend less for the next two years. It is considering announcing now that VAT will be cut to 15% when Brexit actually happens. Evaluate this policy.

6. An econometrician observes that 30% of recipients of disability insurance are not actually limited in how much work they can do. She also observes that only 43% of those who are work limited are in receipt of disability insurance. What can be infered from these observations about the disability insurance program?

Section B

1. A researcher estimates a demand system by OLS with each budget share regression given by:

$$w_{ih} = \alpha_{i0} + \alpha_{iad} adults_h + \alpha_{ikids} kids_h + \sum_{j=1}^{7} \gamma_{ij} \ln p_{j,h} + \beta_i \ln \frac{x_h}{P_h} + u_{ih}$$

where w_{ih} is the budget share on good i for household h; the price index P_h is the Stone price index for household h. There are seven goods in this demand system: food at home (fath); food in restaurants (rest); transport (tranall); services (serv); recreation (recr); alcohol and tobacco (vice); and clothing (clth). The coefficient estimates from each OLS regression are given in the columns in Figure 1 (overleaf).

(a) What conditions have to hold for the adding-up condition to hold? What would it mean if the adding up condition were violated?

(b) Calculate the Hicksian, Marshallian and Income elasticities for services (serv), clothing (clth) and food in restaurants (rest). The budget share on services is 17%, on clothing is 10.5% and on food in restaurants is 9%.

(c) Explain whether the estimates are consistent with a utility function of the form:

$$u = \left(q_{fath}^{\alpha_1} \cdot q_{rest}^{\alpha_2} \cdot q_{tranall}^{\alpha_3} \cdot q_{serv}^{\alpha_4} \cdot q_{recr}^{\alpha_5} \cdot q_{vice}^{\alpha_6} \cdot q_{clth}^{\alpha_7} \right)$$

where α_i are preference parameters.

(d) Two concepts that can be used to measure the change in welfare assoicated with a price change are the equivalent variation and compensating variation. How do these measures differ? When are the two measures going to give identical answers?

(e) Using either the compensating variation or the equivalent variation, calculate the deadweight loss of imposing a 5% tax on each of the three goods: clothing, food in restaurants, services.

(f) Assume that initially there is no VAT on clothing but food in restaurants and services are both subject to a 20% rate of VAT. Is there a way to change the tax schedule to make it more efficient at raising a given amount of revenue?

TURN OVER

Figure 1: Estimated Demand System

Variable	BS fath	BS rest	BS tranall	BS serv	BS recr	BS vice	BS clth
No of adults	0.079	-0.040	0.022	-0.014	-0.026	-0.019	-0.001
	(0.002)	(0.0015)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)
No of kids	0.053	-0.020	-0.018	0.019	-0.003	-0.015	-0.016
	(0.0008)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
log p(fath)	-0.086	0.075	-0.120	0.064	0.032	-0.039	0.073
	(0.026)	(0.019)	(0.026)	(0.021)	(0.021)	(0.023)	(0.018)
log p(rest)	-0.014	-0.021	0.150	-0.065	0.085	-0.069	-0.066
	(0.022)	(0.012)	(0.022)	(0.018)	(0.018)	(0.020)	(0.015)
log p(tranall)	0.010	0.009	0.006	-0.016	-0.009	-0.004	0.004
	(0.0095)	(0.007)	(0.003)	(0.008)	(0.008)	(0.008)	(0.007)
log p(serv)	0.029	0.002	0.017	-0.028	-0.089	0.089	-0.020
	(0.026)	(0.019)	(0.027)	(0.015)	(0.021)	(0.023)	(0.018)
log p(recr)	0.066	-0.064	-0.013	0.087	0.014	-0.079	-0.010
	(0.031)	(0.023)	(0.031)	(0.025)	(0.025)	(0.028)	(0.021)
log p(vice)	-0.008	0.007	-0.029	-0.019	-0.035	0.069	0.016
	(0.010)	(0.008)	(0.010)	(0.008)	(0.008)	(0.009)	(0.007)
log p(clth)	-0.093	-0.003	0.006	0.070	0.066	0.000	-0.046
	(0.023)	(0.017)	(0.024)	(0.019)	(0.019)	(0.021)	(0.016)
log real expend	-0.160	0.050	0.022	-0.011	0.046	0.017	0.037
	(0.002)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)
Constant	1.624	-0.306	-0.043	0.283	-0.284	-0.030	-0.244
	(0.016)	(0.012)	(0.016)	(0.013)	(0.013)	(0.014)	(0.011)

Standard errors are given in brackets.

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Thursday 25 May 2017   1:30pm - 3:30pm

M150
ECONOMICS OF NETWORKS

Candidates are required to answer **three out of four** questions.

Each question carries equal weight

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

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20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1: Consider a n player network formation game. Suppose that two players i and j can form a link if they both agree, and pay a cost $c > 0$. The network created by this bilateral linking is denoted by $\mathbf{g}$. The payoffs to player i under network $\mathbf{g}$ are

$$\Pi_i(\mathbf{g}) = 1 + \sum_{j \neq i} \delta^{d(i,j;\mathbf{g})} - \eta_i^d(\mathbf{g})c, \tag{1}$$

where $d(i, j; \mathbf{g})$ is the (geodesic) distance between i and j in network $\mathbf{g}$ and $\delta \in (0, 1)$ is the decay factor.

a. Define a pairwise stable network. Provide the range of parameter values, c and δ, for which the empty, the complete and star network are pairwise stable.

b. Now consider a variant of the model in which link creation and benefit flow are both directed (in the spirit of Twitter). So only directed paths matter for flow of benefits. Suppose that there is no decay. Write out the payoffs carefully. Show that the cycle containing all nodes is the unique strict (non-empty) Nash network.

Question 2: Consider a n player network formation game. Link formation is two-sided. Every every pair of players who have a path between them create a total surplus of 1. Assume that each player pays $c > 0$ for every link he forms. Suppose that the surplus is independent of the length of the path. The payoffs are shared with players who are *critical* for the exchange. Define player i to be critical for players j and k if she lies on every path between them in the network. Let $C_{jk}(\mathbf{g})$ be the set of critical players for j and k in network $\mathbf{g}$. The surplus of 1 is equally divided between j and k and all the critical players $C_{jk}(\mathbf{g})$.

a. Consider a game with 5 players and write down all the critical players in a cycle and the star network.

b. Write down the strategies of linking and the payoffs. Discuss the different incentives to create links.

c. Show that a star and a cycle containing all players are both pairwise stable.

d. Discuss the strategic robustness of the cycle and the star network. In particular, examine the possibility of sustaining large intermediation rents if pairs of players are able to coordinate their linking activity.

Question 3: Consider a n player game on a network $\mathbf{g}$. Players simultaneously chooses a network action $x_i \in \{0,1\}$ and a market action $y_i \in \{0,1\}$. Define $a_i = (x_i, y_i)$. Suppose that in a network $\mathbf{g}$ faced with a action profile $a = (a_1, a_2, .., a_n)$, the payoff function for player i is given by

$$\Phi_i(a|\mathbf{g}) = (1 + \theta y_i) x_i \chi_i(a|\mathbf{g}) + y_i - x_i p_x - y_i p_y \tag{2}$$

where $\chi_i(a|\mathbf{g})$ is the number of neighbors in network $\mathbf{g}$ who choose the network action, and $p_x \geq 0$ and $p_y \geq 0$, respectively, are the prices of actions x and y. We say that the actions x and y are substitutes if $\theta \in [-1, 0]$ and complements if $\theta \geq 0$. A Nash equilibrium is said to be maximal if there does not exist another equilibrium that Pareto dominates it.

a. Suppose that $\theta = -1$, $p_x = 4$ and $p_y = 0.5$. Describe how the network shapes behavior in the maximal equilibrium.

b. Suppose that $\theta = 1$, $p_x = 7$ and $p_y = 2$. Describe how the network shapes behavior in the maximal equilibrium.

c. Assess the impact of markets on aggregate welfare (measured as the sum of individual payoffs) and inequality (measured as a ratio of highest versus the lowest income) in the two above settings.

Question 4: Consider a four-region economy, with regional liquidity demands in state S_i, $i = 1, 2$, as specified in the table below. Each state takes place with equal probability.

Regional liquidity demand

	Region			
State	**A**	**B**	**C**	**D**
S_1	0.75	0.25	0.75	0.25
S_2	0.25	0.75	0.25	0.75

There is a liquid asset which represents a storage technology. Investment in a long asset is available at $t = 0$. Per unit invested in the long asset, the yield is of $r = 0.4$ at $t = 1$ (premature liquidation), and of $R > 1$ at $t = 2$. Assume that the period utility function is of the form $u(c_t) = \ln(c_t)$.

a. Denote as y and x the per capita amounts that the social planner invests in the short and long assets, respectively. The feasibility constraint is thus $y + x \leq 1$. Noting that at $t = 0$ the probability of being an early or a late consumer equals 0.5, derive the first-best allocation.

b. Describe the combination of investment decisions and interbank deposits that achieve the first-best allocation when (i) the representative bank of a region holds deposits in the representative banks of all other regions (complete structure); and (ii) the representative bank of region **A** deposits in the representative bank of region **B**, the latter deposits in the representative bank of region **C**, and so on (a cycle network). For each case, explain the sequence of withdrawals if state S_1 takes place.

c. Next suppose that an unanticipated state $\bar{S}$ (assigned zero probability at $t = 0$), takes place. In this state, regions **B**, **C** and **D** have a liquidity demand of 0.5, but region **A** faces a demand of $0.5 + \varepsilon$. Consider the cycle network and set $\varepsilon = 0.1$ and $R = 1.5$. Show that there is no contagion. Would your results change if ε was larger?

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Tuesday 30 May 2017 9:00am - 11:00am

M170
INDUSTRIAL ORGANISATION

Candidates are required to answer 3 compulsory questions

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

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STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1

Consider three firms $i = 1, 2, 3$ located on a circular city with distance 1 between them (so the total length of the city is 3). Consumers with unit demand for a product are uniformly distributed on the circle with density one. Firms produce at constant and symmetric marginal costs $c > 0$ and offer uniform prices to consumers. A consumer's utility is given by

$$u(x,p) = \begin{cases} s - d - p & if \quad p \leq s - d \\ 0 \quad if & p > s - d \end{cases}$$

where $d \geq 0$ denotes the distance of the consumer's location to that of the firm and p is the price charged for the product. Assume throughout that there is full market coverage.

a. Consider the case in which all firms remain independent and compete non-cooperatively in prices. Find the equilibrium prices p_i^*, quantities q_i^* and profits π_i^*, $i = 1, 2, 3$.

b. Now consider a merger between firms 1 and 2 to form. This merged company retains the product varieties (locations) of firms 1 and 2 and competes against the remaining independent firm 3. You may assume that the two firms that merge behave symmetrically, so that $p_1 = p_2 = \hat{p}$, $q_1 = q_2 = \hat{q}$ and $\pi_1 = \pi_2 = \hat{\pi}$. Find the equilibrium prices $\hat{p}^*$ and p_3^*, quantities $\hat{q}^*$ and q_3^* and profits $\hat{\pi}^*$ and π_3^*, noting that quantities and profits for the merged firm are $2\hat{q}^*$ and $2\hat{\pi}^*$, respectively.

c. Compare the equilibrium prices, quantities and profits before and after the merger. Who benefits/loses from the merger, in absolute and in relative term?

Question 2

Consider two downstream firms who produce horizontally differentiated goods and who compete in quantities. To produce one unit of the final good, the downstream firm need one unit of an intermediate good. It can either produce the intermediate good in-house (In), our outsource it to a perfectly competitive upstream firm (Out). In-house production involves marginal cost $c > 0$, whereas the marginal cost of outsourcing depends on how much the upstream firm sells. Assume that the upstream firm has economies of scale such that if only one downstream firm outsources, it can offer the intermediate good at marginal cost $\overline{k} > c$ whereas if both downstream firms outsource, it can offer a reduces marginal cost $\underline{k} < c$. The two firms simultaneously and non-cooperatively make outsourcing decisions.

At the market stage, the firms face inverse demand functions

$$\begin{aligned} p_1 &= a - bq_1 - dq_2 \\ p_2 &= a - bq_2 - dq_1 \end{aligned}$$

a. Illustrate the normal form of the outsourcing game, denoting by $\pi_i(c_i, c_j)$ the profits accruing to firm $i = 1, 2$ when it has cost $c_i \in \{c, \underline{k}, \overline{k}\}$ and the rival has costs $c_j \in \{c, \underline{k}, \overline{k}\}$.

b. Solve for the pure strategy Nash equilibria of the outsourcing game, paying special attention to the parameter restrictions that must be imposed in each case.

c. Interpret your findings, with reference to the restrictions imposed in sub-question 2. Are equilibrium outsourcing decisions always efficient? Does efficient in-house production (measured as a low c) make efficient outsourcing decisions more or less likely?

Question 3

Consider two firms producing horizontally differentiated goods. The firms are located at the end-points of a linear city of unit length. Consumers have unit demand and are uniformly distributed along the city with density one. Firms produce at constant and symmetric marginal costs $c > 0$ and offer uniform prices to consumers. A consumer's utility is given by

$$u(x,p) = \begin{cases} s - \tau d - p & if \quad p \leq s - \tau d \\ 0 & if \quad p > s - \tau d \end{cases}$$

where $d \geq 0$ denotes the distance of the consumer's location to that of the firm and p is the price charged for the product. The constant $\tau > 0$ is the travel cost borne by the consumers. Assume throughout that there is full market coverage.

a. Consider an infinite period game in which the firms play the market game in each period $t = 1, 2, ...$ The future is discounted by a factor $\delta \in (0, 1)$. Carefully define grim trigger strategies in this setting.

b. Derive explicit expressions π^C, π^D and π^N, i.e. for the payoffs to cooperating, deviating and playing according to the (stage-game) Nash equilibrium respectively. Derive a condition on the discount factor δ that determines whether tacit collusion in grim trigger strategies is feasible.

c. Carefully determine how π^C, π^D and π^N vary with the transport cost τ. Explain the tradeoffs involved and discuss how these findings influence the sustainability of tacit collusion via grim trigger strategies.

END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Friday 26 May 2017 1:30pm - 3:30pm

M180
LABOUR: SEARCH, MATCHING AND AGGLOMERATION

Candidates are required to answer **three out of four** questions. Each question carries equal weight.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Consider the Postel-Vinay & Robin (2002) model. We assume that workers and firms are homogeneous. The productivity of a match is p. Parameters are as they were in the lecture notice that is: ρ is the discount rate, b is the flow value of unemployment, λ_0 is the arrival rate of jobs to unemployed, λ_1 is the arrival rate of jobs to employed, δ is the job destruction rate. Let U be the value of unemployment and $W(w)$ the value of employment at a wage w.
   (a) Write the Bellman equation for the value of unemployment.
   (b) Let w_0 be the wage that makes a worker indifferent between remaining in unemployment and beginning work ($U = W(w_0)$). Use this relation to simplify the expression under (a) and provide an intuition for the result.
   (c) What wage offer will an employed worker get when receiving an outside offer from another employer? Is there any indeterminacy about whether he will move/stay?
   (d) Write the Bellman equation for a worker at his maximum wage and simplify this expression to obtain the maximum value a worker can get.
   (e) Show that

$$w_0 = b - \frac{\lambda_1}{\rho + \delta}(p - b)$$

   (f) What happens to w_0 if (i) as $\lambda_1 \to 0$ and (ii) as $\rho \to \infty$? Explain your results.
   (g) What is the distribution of wages amongst employed?
   (h) Derive the mean wage paid in the economy.
   (i) What is the mean wage wage when workers don't discount future earnings? Give intuition for your answer and discuss the implications for firms vacancy posting decisions.

2. Minimum wages and job search models
   (a) Give a short overview of the stylized facts regarding the introduction of a minimum wage.
   (b) David Lee regresses the 10-50% log wage differential on inter-state variation in minimum wages. He finds a strong negative effect: the higher the minimum wage, the smaller the 10-50% differential. Can this result be interpreted as evidence that the wage differentials between workers with different human capital are compressed by an increase in the minimum wage?
   (c) To what extend can these facts be explained by job search models?
   (d) Some studies find that a higher minimum wage might increase employment. What do you have to assume on wage setting, labour supply, vacancy creation, and the matching function to explain this outcome?

3. Public transport in cities

(a) Consider a circular city as in Lucas & Rossi Hansberg (2002). Suppose that the city would develop a public transport railsystem with a stop at the city center that extends from there in all directions with a single stop at all locations at distance R from the center (so every point on the ring R has a stop that connects it to the city center). Assume that the city autorities have chosen a proper location for these stops such that the system is useful for commuting. Furthermore, this city turns out to have a mixed neighbourhood where residential and commercial use of land coincide. Commuting cost by any other mode of transport are κ per unit of distance, commuting by public transport occurs at cost $\psi < \kappa$ per unit of distance. Depict a possible profile of land rents and depict the locations of the various zones (residential, commercial, mixed). Explain your answer, in particular the location of the zones and the wage function that goes with this pattern.

(b) Consider the density of land use in the neighbourhood of the stations, both at the center and at location R. Is this density is lower or higher than at locations further away from the station? Is this efficient?

(c) Public transport is a means of transport with high fixed cost (construction of tracks etc.) and low variable cost per trip. Suppose you are the mayor of the city. Design an efficient system for funding the public transport system. Explain your design. What are the drawbacks of alternative designs?

4. Agglomeration of human capital

(a) The agglomeration of people in cities has played an important role in the evolution of the world economy since the rise of agriculture. Give a short historical overview in at most 10 sentences.

(b) Gennaio et.al. (2013) provide a model for the agglomeration of human capital within countries. Describe the main features of that model in at most 10 sentences. Pay attention to the role of economies of scale, knowledge spill-overs, and entrepreneurship. Are there externalities in their model?

(c) Is the equilibrium efficient in their model? Why or why not? If not, what policies could help to alleviate the problem?

(d) Who captures the benefits of agglomeration in their model?

(e) Teulings & Van Rens (2008) show that an increase in the average level of education in a country reduces the return to human capital. Is this consistent with the analysis of Gennaio et.al.? Explain your answer.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Thursday 12 January 2017 10:00-12:00

M200
MACROECONOMICS I

The paper is in two sections: You should answer all **four** questions in Section A and **one** question from Section B. Each Section carries equal weight.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A.1 **Solow growth model**

Consider the standard Solow growth model with technological progress (g) and population growth (n). Assume that the economy is initially on the balance growth path. Suddenly there is a one-time unexpected and permanent downward jump in the number of workers due to a government repatriation program of illegal immigrants. Explain both the immediate and transitional effect of this jump on capital per effective worker and output per effective worker. Draw the time path of output per capita and compare it to the case without the repatriation program.

A.2 **Investment Theory**

Consider a q-theory model with the following dynamics for the capital stock K_t and the shadow price of capital q_t :

$$\begin{aligned} \Delta K_{t+1} &= \frac{1}{\chi}(q_t - 1) \\ \Delta q_{t+1} &= rq_t - A_{t+1}F'\left(K_t + \frac{1}{\chi}(q_t - 1)\right) \end{aligned} \tag{1}$$

where r denotes the real interest rate, with $1 + r > 0$; the production function is given by $Y_t = A_t F(K_t)$ with $F'(\cdot) > 0$ and $F''(\cdot) < 0$, where A is a positive productivity parameter; and χ is a positive parameter related to the investment adjustment costs. The initial capital stock $K_0 > 0$ is given and the transversality condition is $\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T q_T K_{T+1} = 0$.
Draw the phase diagram from the system of equations (1) and explain the dynamics implied by this model. Suppose that productivity A temporarily decreases at time $t = T_1$ from A to A^*, with $A^* < A$. At $t = T_2$, with $T_2 - T_1 \geq 3$, the productivity reverts back to A. Carefully explain how K_{t+1} and q_t are affected over time using a phase diagram and provide an intuitive explanation for the effect on investment.

A.3 **Equilibrium with Imperfect Competition**

Suppose a representative firm $i \in [0,1]$ maximizes real profits

$$\Pi_i = \frac{P_i}{P} Q_i - \frac{W}{P} L_i$$

where Q_i denotes the production of good i, P_i the price of good i, $P \equiv \int_0^1 P_i di$ the aggregate price level, W the nominal wage, and L_i the firm's labour input. The production technology is described by

$$Q_i = AL_i$$

where A denotes productivity. The firm operates under monopolistic competition and faces demand

$$Q_i = Y \left(\frac{P_i}{P} \right)^{-\eta}$$

where $Y \equiv \int_0^1 Q_i di$ denotes aggregate output and the parameter $\eta > 1$. In the labour market, the firm faces the following real wage function:

$$\frac{W}{P} = BL^{\beta}$$

where $L \equiv \int_0^1 L_i di$ denotes aggregate employment, and B and β are positive parameters. The aggregate demand relation is described by

$$Y = \frac{M}{P}$$

where M denotes aggregate demand.

Derive the level of the real wage $\frac{W}{P}$, aggregate employment L and aggregate output Y in the flexible price equilibrium. Explain how they are affected by an increase in B.

A.4 **Anticipated Positive Aggregate Demand Shock in Dornbusch Model**

Suppose the economy of Americana can be described by a Dornbusch model with the following short-run dynamics for the (log) nominal exchange rate s (defined as the domestic price of foreign currency) and the (log) aggregate price level p:

$$\begin{aligned} \dot{s} &= 4(s - \bar{s}) - 2(p - \bar{p}) \\ \dot{p} &= (s - \bar{s}) - (p - \bar{p}) \end{aligned}$$

where the steady state (indicated by an upper bar) equals $\bar{s} = \bar{q} + \bar{k}$ and $\bar{p} = \bar{p}^* + \bar{k}$, with (log) real exchange rate $\bar{q} = \frac{1}{2}(\bar{y} - \bar{u})$ and macroeconomic fundamentals $\bar{k} \equiv (\bar{m} - \bar{m}^*) - (\bar{y} - \bar{y}^*)$. In addition, $\bar{m}$ denotes the money supply, $\bar{y}$ the natural rate of output and $\bar{u}$ is a measure of exogenous aggregate demand. Foreign variables are indicated by an asterisk.

Suppose the new President of Americana suddenly announces fiscal policy measures that will lead to a permanent increase in $\bar{u}$ in one year. Explain how this affects the nominal exchange rate s and aggregate price level p over time, and carefully illustrate the dynamics in a phase diagram. Does the nominal exchange rate s 'overshoot' the new steady state $\bar{s}'$ at any time?

SECTION B

B.1 **Real Business Cycles**

Consider the following two-period economy. There is a continuum of measure one of identical individuals $i \in [0,1]$. A representative individual maximises the following utility function:

$$U = \sum_{t=0,1} \beta^t \left[\ln(c_t) + \ln(1 - L_t)\right]$$

where c_t denotes consumption, L_t labour supply, subscript $t = 0, 1$ the time period, and $\beta \in (0,1)$ is a constant parameter The time endowment is equal to one in each period. Assume that individuals start with no initial assets ($b_0 = 0$) and cannot die with a debt. Individuals supply labour to the firms at a real wage w_t per unit of labour. The consumption good is the numeraire.

There is also a continuum of measure one of identical, profit maximising firms. The production technology of the representative firm is represented by the following function:

$$Y_t = A_t L_t$$

where Y_t is output, L_t is labour used in production and $A_t > 0$ is an exogenous productivity factor. There is perfect competition in the product and labour market. Assume that the economy is closed and there is no investment. There is no government and the real interest rate is r.

(a) Formulate the optimization problem of the representative firm and take the first-order condition. Give an economic interpretation of the result.

(b) Formulate the optimization problem of the representative individual and take the first order conditions associated with the problem. Give an economic interpretation of the conditions which characterise the trade offs for this problem.

(c) Derive the optimal levels of consumption c_0 and c_1, labour supply L_0 and L_1 and assets b_1, for given w_0, w_1 and r.

(d) Assume that this is a small open economy so that the real interest rate r is given. Explain the effect of a decrease in current productivity A_0 consumption c_0 and c_1, labour supply L_0 and L_1 and assets b_1.

(e) Assume now that the economy is closed. Derive the equilibrium real interest rate r. Explain the effect of a decrease in current productivity A_0 on the real interest rate r, consumption c_0 and c_1, labour supply L_0 and L_1 and asset b_1.

B.2 **Quantitative Easing, Seignorage and Optimal Tax Rates**

Suppose the government sets the income tax τ_t each period to minimize the expected value of the following loss function:

$$L_t = \sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} \frac{1}{2} \kappa \tau_s^2 Y_s$$

where Y_s denotes aggregate output in period s, r is the constant real interest rate, and κ is a positive parameter. The government's budget constraint equals

$$G_s + rB_s = \tau_s Y_s + S_s + D_s$$

where G_s denotes government purchases in period s, B_s government debt at the start of period s, S_s seignorage in period s, and $D_s \equiv B_{s+1} - B_s$ the government's budget deficit. Assume money is superneutral and $Y_s = \bar{Y}$ for $s = t, t+1, \ldots$. In addition, $B_t > 0$ is given and $\lim_{T\to\infty} \frac{1}{(1+r)^{T+1-t}} B_{T+1} \leq 0$.

(a) Give an economic interpretation for the latter inequality, and derive the intertemporal budget constraint of the government.

(b) Assume that initially, $G_s = \bar{G}$ and $S_s = \bar{S}$ for $s = t, t+1, \ldots$. Derive the optimal tax rate τ_t and the government's primary budget deficit $\tilde{D}_t \equiv D_t - rB_t$. Give an intuitive explanation of the results.

(c) Suppose that a rise in the growth rate of the money supply due to 'quantitative easing' leads to a permanent increase in seignorage $\bar{S}$ by ΔS in period t. Carefully explain how this would affect the income tax τ_s, the primary deficit $\tilde{D}_s$ and the level of government debt B_s over time.

(d) Now suppose instead that future 'quantitative easing' leads to a permanent increase in seignorage to $\bar{S} + \Delta S$ from period $t + T$ onwards. Carefully explain how this would affect the income tax τ_s, the primary deficit $\tilde{D}_s$ and the level of government debt B_s over time from period $s = t$ onwards. Compare the results to part (c).

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Wednesday 10 May 2017 2:00pm - 4:00pm

M200
MACROECONOMICS I

The paper is in two sections: You should answer all four questions in Section A and one question from Section B. Each Section carries equal weight.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

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Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

A.1 **Solow Growth Model**

Consider the following continuous time Solow growth model. The production function is:

$$Y(t) = (A_K(t) K(t))^{\alpha} (A_L(t) L(t))^{1-\alpha}, \qquad \alpha \in (0,1),$$

where $Y(t)$ denotes aggregate output, $K(t)$ aggregate capital, and $L(t)$ aggregate labour force; $\dot{A}_K(t) = \gamma_K A_K(t)$ and $\dot{A}_L(t) = \gamma_L A_L(t)$, where γ_K and γ_L are positive and exogenous; t indicates time. The initial values of $A_K(0)$, $A_L(0)$ and $K(0)$ are given and they are positive. The labour force grows at a constant rate n and agents save a fraction $s \in (0,1)$ of output. The economy is closed and capital evolves according to the following equation:

$$\dot{K}(t) = I(t) - \delta K(t),$$

where $I(t)$ denotes investment and δ the depreciation rate with $\delta > 0$. Derive the growth rate of capital, $K(t)$, on the balanced growth path. Is $\frac{K(t)}{A_L(t)L(t)}$ constant on the balanced growth path? Explain how your results depend on γ_K.

A.2 **Investment Theory and Unanticipated Permanent Change in Interest Rates**

Consider a q-theory model of investment with the following dynamics for the capital stock K_t and the shadow price of capital q_t :

$$\begin{aligned} \Delta K_{t+1} &= \frac{1}{\chi}(q_t - 1) \qquad (1) \\ \Delta q_{t+1} &= r q_t - AF'\left(K_t + \frac{1}{\chi}(q_t - 1)\right), \end{aligned}$$

where r denotes the real interest rate, with $1 + r > 0$; the production function is given by $Y_t = AF(K_t)$ with $F'(\cdot) > 0$ and $F''(\cdot) < 0$, where A is a positive productivity parameter; and χ is a positive parameter related to the investment adjustment costs. The initial capital stock $K_0 > 0$ is given and the transversality condition is $\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^T q_T K_{T+1} = 0$.

Draw the phase diagram from the system of equations (1) and explain the dynamics implied by this model. Suppose that the interest rate r unexpectedly and permanently increases from r to r' , with $r < r'$. Carefully explain how K_{t+1} and q_t are affected over time using a phase diagram and provide an intuitive explanation for the effect on investment.

A.3 **Central Bank Contract with Output Growth Bonus**

Suppose the economy of Americana is described by the following Phillips equation:

$$\pi = \pi^e + (g_Y - \bar{g}_Y) + \varepsilon,$$

where π denotes inflation, g_Y the growth rate of real output, $\bar{g}_Y$ the long run growth rate of real output, and ε a white noise cost push shock. Private sector inflation expectations π^e are formed using rational expectations at the beginning of the period, before the shock ε has been realized. Monetary policy is delegated to a central banker who sets real output growth g_Y after the shock ε has been observed. The central banker minimizes the following loss function:

$$L = \frac{1}{2}(\pi - \pi^*)^2 + \frac{1}{2}(g_Y - g_Y^*)^2 - \beta(g_Y - g_Y^*),$$

where π^* denotes the central banker's inflation target, g_Y^* the central banker's real output growth target, and β a non-negative parameter. Suppose that before the beginning of the period, the new president of Americana appoints a central banker with an inflation target of 2% and a real output growth target of 4%, while long run real output growth equals 2%. In addition, the president decides to provide the central banker a personal incentive to achieve the real output growth target through a 'bonus' payment that is increasing in real output growth g_Y, so that $\beta > 0$.
Derive the equilibrium outcome of private sector inflation expectations π^e, real output growth g_Y and inflation π. Explain intuitively how these outcomes are affected by the real output growth bonus.

A.4 **Anticipated Negative Aggregate Demand Shock in Dornbusch Model**

Suppose the economy of Britannia can be described by a Dornbusch model with the following short-run dynamics for the (log) nominal exchange rate s (defined as the domestic price of foreign currency) and the (log) aggregate price level p:

$$\begin{aligned} \dot{s} &= (s - \bar{s}) + (p - \bar{p}), \\ \dot{p} &= (s - \bar{s}) - (p - \bar{p}), \end{aligned}$$

where the steady state (indicated by an upper bar) equals $\bar{s} = \bar{q} + \bar{k}$ and $\bar{p} = \bar{p}^* + \bar{k}$, with macroeconomic fundamentals $\bar{k} \equiv (\bar{m} - \bar{m}^*) - \frac{1}{2}(\bar{y} - \bar{y}^*)$ and (log) real exchange rate $\bar{q} = \bar{y} - \bar{u}$. In addition, $\bar{m}$ denotes the (log) money supply, $\bar{y}$ the (log) natural rate of output, and $\bar{u}$ is a measure of exogenous aggregate demand. Foreign variables are indicated by an asterisk.
Suppose the government of Britannia suddenly announces that it will abandon a single market with Europia in T years, which will increase barriers to international trade and lead to a permanent decrease in $\bar{u}$ in T years. Explain how this affects the nominal exchange rate s and aggregate price level p over time, and carefully illustrate the dynamics in a phase diagram. How do the effects on the nominal exchange rate s and aggregate price level p depend on T?

B.1 **Consumption Theory and a Risky Asset**

Consider a household whose preferences are described by:

$$U = E_t \sum_{s=t}^{\infty} \frac{u(C_s)}{(1+r)^s},$$

where C is consumption and $u'(C) > 0$ and $u''(C) < 0$ and $r > 0$ is the discount rate.

Assume that the household's unique source of income is the dividends, D_t, and capital gains from holding equity A_t which has price P_t. Assume that equity holdings could be sold after dividends have been paid. Moreover assume $A_0 > 0$. The budget constraint of the household is:

$$C_t + P_t A_{t+1} = (P_t + D_t) A_t.$$

(a) Provide an economic interpretation of the household's budget constraint.

(b) Set up the household's optimization problem, derive the intertemporal Euler equation for consumption and interpret it.

For the remainder of the question assume that $u(C_t) = C_t$.

(c) Using the Euler equation obtained in part (b) derive P_t in terms of future dividends. Which condition do you have to impose? Give an economic intuition.

(d) How does an increase of dividends in period $t+s$, D_{t+s}, affect the equity price in period t? How is this price affected as s increases? How does a permanent increase of all future dividends by $\varepsilon > 0$ from period t onwards affect the price in t? Give an economic intuition of your findings.

(e) Assume that $u(C_t) = C_t$ and P_t equals the expression derived in part (c) plus a deterministic bubble, $(1+r)^t b$, and $b > 0$. Carefully show whether the intertemporal Euler equation for consumption derived in part (b) is still satisfied. Could b be negative? Give an economic intuition.

B.2 **Optimal Tax Rates, Seignorage and Inflation Target**

Suppose social welfare losses are described by

$$L_t = \sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} \left\{ \frac{1}{2} \kappa \tau_s^2 Y_s + \frac{1}{2} \pi_s^2 \right\},$$

where τ_s denotes the income tax rate, Y_s aggregate output, π_s inflation, and the subscript s indicates the time period. The real interest rate r is constant and κ is a positive parameter. The government sets the income tax rate τ_t in each period to minimize social welfare losses L_t. The government's intertemporal budget constraint equals

$$(1+r)\,B_t + \sum_{s=t}^{\infty} \frac{G_s}{(1+r)^{s-t}} = \sum_{s=t}^{\infty} \frac{\tau_s Y_s}{(1+r)^{s-t}} + \sum_{s=t}^{\infty} \frac{S_s}{(1+r)^{s-t}},$$

where $B_t > 0$ is government debt at the start of period t and G_s government purchases, which equal $G_s = \bar{G}$ for $s = t, t+1, ...$ In addition, seignorage S_s is determined by $S_s = \mu_s \frac{M_s}{P_s}$, where M_s denotes the money supply, μ_s the (approximate) growth rate of M_s, and P_s the aggregate price level. Real money demand is described by

$$\frac{M_s}{P_s} = e^{-\theta i_s} Y_s,$$

where i_s is the nominal interest rate, which satisfies $i_s = r + \pi_s^e$, where π_s^e denotes inflation expectations and θ is a positive parameter. Assume that money is superneutral, with $Y_s = \bar{Y}$ for $s = t, t+1, ...,$ and that perfect foresight holds, so that $\pi_s^e = \pi_s$. Also assume that the economy is in a steady state with constant real money balances M_s/P_s, so that $\mu_s = \pi_s$. The money growth rate μ_s is set in each period by an independent central bank that minimizes the loss function

$$L_t^{CB} = \sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} \frac{1}{2} (\pi_s - \pi^*)^2,$$

where π^* denotes the central bank's inflation target.

(a) Compute the money growth rate μ_s that the central bank sets to minimize its loss function L_t^{CB}. In addition, compute the resulting inflation expectations π_s^e, nominal interest rate i_s, real money holdings M_s/P_s and seignorage S_s in terms of the central bank's inflation target π^*.

(b) Derive the money growth rate μ that maximizes steady-state seignorage S. Compute the corresponding level of seignorage $\bar{S}_{\max}$. What level of the inflation target π^* would generate the maximum steady-state seignorage $\bar{S}_{\max}$?

(c) For a given level of π_s and $S_s = \bar{S}$ for $s = t, t+1, ...,$ derive the optimal tax rate τ_t set by the government in terms of B_t, $\bar{G}$, $\bar{Y}$ and $\bar{S}$. Give an intuitive explanation of the results.

(d) Use the results from parts (a) and (c) to write social welfare losses L_t in terms of the inflation target π^*, and derive the optimality condition for the socially optimal inflation target π_{SO}^* that minimizes social welfare losses L_t. Determine how π_{SO}^* compares to $\pi^* = 0$ and to the inflation target derived in part (b). Provide an intuitive explanation for the results.

END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 5 June 2017 1:30pm - 3:30pm

M210
MACROECONOMICS II

This paper is in two sections. Section A: Candidates are required to answer **three out of four** questions. Section B: Candidates are required to answer **one** compulsory question.

Each section carries equal weighting.

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SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
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You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A.1 **Neoclassical Growth Model**

Consider an economy with aggregate production function

$$Y(t) = K(t)^{\alpha} E(t)^{\beta} (A(t)N(t))^{1-\alpha-\beta}, \ \alpha, \beta \in (0,1) \ \text{ and } \ \beta + \alpha < 1.$$

Y is output, K is capital, N is labour, E is energy, and A is labour productivity. Technological progress grows at rate g and the initial technology, $A(0) > 0$, is given. Capital evolves according to:

$$\dot{K}(t) = I(t) - \delta K(t),$$

where I corresponds to investment and $\delta > 0$ is the depreciation rate, and $K(0)$ is given. There is a continuum of measure one of households who are infinite lived, with preferences

$$U = \int_0^{\infty} e^{-\rho t} L(t) u(c(t)) dt, \quad \rho > 0, u(c) = \frac{c^{1-\theta} - 1}{1-\theta}, \ \theta > 0,$$

where $c(t)$ denotes consumption of each household member. $L(t)$ is the number of household members, which grows at rate n. Assume that $L(0) = 1$ and each household member has one unit of productive time. Suppose there is a fixed stock of nonrenewable resources, $R(0)$, and that energy use depletes this stock, so that

$$\dot{R}(t) = -s_R(t) R(t),$$

where $s_R(t) \in (0,1)$ is the endogenously determined extraction rate. Set up the Social Planner's problem for this economy and derive the optimal growth rate of output per capita along the balanced growth path equilibrium. Would output per capita be growing along the balanced growth path equilibrium?

A.2 **Search Model with Probability of Dying**

An agent receives every period an offer to work forever at a wage w, where w is drawn from the distribution $F(w)$ with support $[0, B]$ where B is finite. Offers are independently and identically distributed. The objective of the agent is to maximise $E_{t=0}\{\sum_{t=0}^{\infty} (\beta(1-s))^t y_t\}$, where $y_t = w$ if the worker has accepted a job that pays w, and $y_t = b$ if the agent remains unemployed, $\beta \in (0,1)$ is a subjective discount factor, and $s \in (0,1)$ is the probability that the agent will die in each period. Assume that $0 < b < B < \infty$.

(i) Write down Bellman's Functional Equation for an unemployed agent and argue (intuitively) that the optimal policy function is a reservation wage $\bar{w}$ such that the agent will accept a job offer if $w > \bar{w}$.

(ii) Suppose that there is now a continuum 1 of identical individuals sampling jobs from the same stationary distribution $F(w)$. There is a mass of s workers born every period, so that population is constant, and these workers start out as unemployed. What would be the steady-state unemployment rate for this economy?

A3. **Contraction Mapping**

Consider the following cake-eating problem with infinite horizon: an agent has an initial cake of size x_0. She can eat away $c_t \geq 0$ from the cake each period, so $x_{t+1} = x_t - c_t$. The size of the cake can never become negative. The agent has per-period utility $u(c_t) = \sqrt{c_t}$ and discount factor β. Write the Bellman equation for this problem. Is this Bellman equation a Contraction? Explain.

A.4 **Asset Pricing**

Consider an economy populated by a continuum of measure one of identical households, each with preferences given by $\mathbf{E}_0[\sum_{t=0}^{\infty} \beta^t u(c_t)]$ where c_t is consumption at period t, $u(c_t) = \frac{c_t^{1-\theta}-1}{1-\theta}$ with $\theta \geq 0$, and $\beta \in (0,1)$ corresponds to the subjective discount factor. There is a single asset tree which yields perishable dividends d_t (fruits). Households are initially endowed with some number of shares of the tree: s_0. Assume that d_t is a random variable taking values from the set $\mathbf{D} \equiv \{d_1, ..., d_N\}$ and is distributed according to a first-order Markov chain with constant transition probability matrix. Households can trade the share of the tree. The price of the tree when the current realization of d_t is d is given by the function $p : \mathbf{D} \to R_+$. There are no other assets in this economy. State the problem of the representative household in recursive form and assuming that the value function is differentiable, derive the the optimal price of share of the tree as a function of present and future dividends. Does 'good news' about future output (dividends) always produce an increase in stock prices? Explain.

SECTION B

B.1 **Neoclassical Growth Model**

Consider a Dynamic New Keynesian model, with *flexible* prices and *sticky* wages. Each household ι is the monopolistic supplier of a specific variety of labour service and consumes the final good. Households maximize expected lifetime utility, discounted by a factor $0 < \beta < 1$, and their instantaneous utility is:

$$u(C_t) - v(N_t(\iota)) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(\iota)^{1+\eta}}{1+\eta}, \quad \sigma > 0,\ \eta > 0,\ 0 < \beta < 1.$$

Households consume all their income, so that $C_t = Y_t^d$ in equilibrium. They are allowed to set nominal wages optimally in every period with probability $1 - \psi_w$, or they keep their nominal wage from the previous period with probability ψ_w (i.e. there is no indexation), where $0 \leq \psi_w < 1$. The different labour varieties are combined into final labour by an intermediary, according to a Dixit-Stiglitz aggregator with parameter of elasticity of substitution between labour types $\theta_w > 1$. The final labour is then supplied to the intermediate good firms that have a production function which is linear in labour. Intermediate goods firms set their prices optimally, i.e. there are no price rigidities.

Let W_t and $\Delta_{w,t}$ be the aggregate wage index and wage dispersion in period t. Also let $M_{t,t+i} = \beta^i u'(C_{t+i})/u'(C_t)$ be the equilibrium stochastic discount factor, N_t be the aggregate labour supply, $S_t = v'(N_t)/u'(C_t)$ be the average leisure-consumption marginal rate of substitution, $\Omega_t = W_t/P_t$ be the real wage index, W_t^* be the optimally set wage of households that are allowed to reset their wage in period t and $G_t = W_t^*/W_t$. Finally let $\Pi_{w,t} = W_t/W_{t-1}$ be the gross wage inflation in this economy. The equilibrium conditions that describe wage setting are:

$$\begin{aligned}
\frac{\Gamma_t}{G_t} &= \frac{\Omega_t N_t}{\Delta_{w,t}} + \psi_w \mathbb{E}_t\left[M_{t,t+1}\frac{\Gamma_{t+1}}{G_{t+1}}\Pi_{w,t+1}^{\theta_w-1}\right] && (1)\\
\Upsilon_t G_t^{\eta\theta_w} &= \frac{\theta_w}{\theta_w-1}\frac{S_t N_t}{\Delta_{w,t}^{1+\eta}} + \psi_w \mathbb{E}_t\left[M_{t,t+1}\Pi_{w,t+1}^{(1+\eta)\theta_w}\Upsilon_{t+1}G_{t+1}^{\eta\theta_w}\right] && (2)\\
\Gamma_t &= \Upsilon_t && (3)\\
1 &= (1-\psi_w)\,G_t^{1-\theta_w} + \psi_w \Pi_{w,t}^{\theta_w-1} && (4)
\end{aligned}$$

(a) Derive the real wage index Ω_t^f in the case of fully *flexible* wages and interpret the expression.

(b) Consider the steady state at which gross wage inflation is $\Pi_w = 1$ and assume that the dispersion term is very small at the first order and can be safely ignored. Log-linearise the above system of equations around this deterministic steady state and derive the *wage Phillips curve*:

$$\pi_{w,t} = \beta \mathbb{E}_t \pi_{w,t+1} + \kappa\,(s_t - \omega_t)\,,$$

where $\kappa > 0$ depends on the model's parameters and lower case letters denote log-deviations of variables from their steady state. Provide an interpretation of the wage Phillips curve.

(c) Explain how the slope of the wage Phillips curve κ changes as

i. households become more patient,

ii. wages become less sticky,

iii. the labour supply becomes more inelastic and

iv. as different types of labour become more substitutable.

END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Wednesday 24 May 9:00am - 11:00am

M220
MACROECONOMICS III

This paper is in two sections:
Section A - candidates are required to answer **three out of four** questions.
Section B – candidates are required to answer **one** compulsory question.

Each section carries equal weight.

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Write legibly.

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Section A

QUESTION 1: The current account

Suppose the world economy consists of two countries, Europia and Americana (indicated by an asterisk), that are initially completely identical, and only operates for two period ($t = 1, 2$). In each country, the representative agent maximizes lifetime utility

$$U = \ln C_1 + \beta \ln C_2$$

where C_t denotes consumption, the subscript t the time period, and $0 < \beta < 1$. The agent's budget constraint is

$$B_{t+1} = Y_t + (1 + r)\, B_t - I_t - T_t - C_t$$

where Y_t denotes output, I_t investment, T_t lump sum taxes, r the world real interest rate, and B_{t+1} net foreign assets at the end of period t, with $B_1 = B_3 = 0$. Output Y_t is produced according to

$$Y_t = 2\sqrt{K_t}$$

where K_t denotes capital, which satisfies

$$K_{t+1} = K_t + I_t$$

where $I_t \in \mathbb{R}$, $K_t \geq 0$ and $K_1 = 1$. The government's budget constraint is

$$G_t = T_t$$

where government purchases G_t are given by $G_1 = \bar{G}$ and $G_2 = 0$. Assume free trade between Europia and Americana, and no international capital market imperfections.

Solve for optimal investment I_1, consumption C_1, national saving S_1 and the current account CA_1 in terms of β, $\bar{G}$ and r. Explain the effect of an increase in Americana's government purchases $\bar{G}^*$ in period 1 on the equilibrium world real interest rate r_W and on investment I_1, consumption C_1, national saving S_1 and the current account CA_1 in period 1 in Europia and in Americana.

QUESTION 2: Risk sharing without trade in assets

Consider a two-country (H and F) endowment economy. Each country has a stochastic endowment of a country-specific good, Y_H and Y_F. Also, preferences are subject to country-specific taste shocks ϖ and ϖ^*. The social planner problem is:

$$Max \;\; \mu \, \varpi \left(\frac{C_H^{\gamma} C_F^{1-\gamma}}{1-\sigma} \right)^{1-\sigma} + (1-\mu) \, \varpi^* \left(\frac{C_H^{*\gamma} C_F^{*1-\gamma}}{1-\sigma} \right)^{1-\sigma}$$

$$\begin{aligned} s.t. \quad &: \;\; C_H + C_H^* = Y_H \\ &: \;\; C_F + C_F^* = Y_F \end{aligned}$$

where C_H and C_H^* denote the consumption of the H good at Home and in Foreign, C_F and C_F^* denote consumption of the F good at Home and in Foreign, μ denotes the social planner weight attributed to the Home country.

(a) Solve for the social planner allocation.

(b) For the same economy, work out the market equilibrium under financial autarky. Under what conditions is perfect risk sharing achievable without trade in assets across borders?

(c) How does you answer to (b) changes if shocks are anticipated to occur in the future? Explain.

QUESTION 3: Exchange rate pass through

Consider a firm producing a good h under imperfect competition, and supplying to both the Home and the Foreign markets. The firm's marginal cost is linear in unit labor costs $\frac{W_t}{Z_{\mathrm{H},t}}$, where W_t is the wage rate in domestic currency (taken as given by the firm) and $Z_{\mathrm{H},t}$ denotes productivity. The demand for the individual h in either market is

$$C_t(h) = \left(\frac{p_t(h)}{P_{\mathrm{H},t}}\right)^{-\theta} C_{\mathrm{H},t},$$

$$C_t^*(h) = \left(\frac{p_t^*(h)}{P_{\mathrm{H},t}^*}\right)^{-\theta} C_{\mathrm{H},t}^*.$$

where $p_t(h)$ and $p_t^*(h)$ denote the price of h at consumer level, $P_{\mathrm{H},t}$ and $P_{\mathrm{H},t}^*$ denote the price index of the Home country exports (the subscript H refers to the bundle of Home good exports), which are substitute with each other with elasticity θ, and $C_{\mathrm{H},t}$ and $C_{\mathrm{H},t}^*$ denote the scale of consumption of the basket of Home goods at Home and in Foreign. As in the model analyzed in the lectures, bringing the good h to the consumer requires the use of η retail services. However, departing from the lecture notes, posit that the retail services are provided by competitive foreign companies which hire local workers with country-specific productivity Z_R and Z_R^*, but pay a contract wage that is preset in foreign currency only, W^*—the same in either country. The consumer-level price is thus:

$$p_t(h) = \bar{p}_t(h) + \eta\frac{W^*}{Z_R}$$

$$p_t^*(h) = \bar{p}_t^*(h) + \eta\frac{\mathcal{E}_t W^*}{Z_R^*}$$

where $\mathcal{E}_t$ denotes the exchange rate.

(a) Derive the profit maximizing prices. Are the optimal prices identical across markets of destination? Carefully explain.

(b) Assuming that all nominal wages are preset, is the exchange rate pass through complete if a monetary shock causes a home currency devaluation? Explain.

(c) State the sufficient condition for incomplete pass through proposed by Marston (1990). Is this condition satisfied in the problem above?

(d) Suppose retailers in one country have the option to buy the product h directly from retailers from the other country, rather than from the producer. Using the optimal prices you derived under 1, express the range of exchange rate values (as a function of elasticities and productivities) for which the producer is able to charge the optimal markup.

(e) Assume that the exchange rate falls outside the range derived under (d), so that, if firms charged the unconstrained optimal price, retailers could arbitrage across markets. How would firms adjust their prices in the Home and export markets to rule out this arbitrage? Would the law of one price hold at producer level?

QUESTION 4: Empirical tests of risk sharing

Consider a two-country model. Use the following notation: U (U^*) denotes utility, C (C^*) consumption at Home (abroad), $\mathbb{P}_t$ ($\mathbb{P}_t^*$) the price level at Home (abroad), $\zeta_{C,t}$ ($\zeta_{C,t}^*$) country-specific taste shocks, and $\mathcal{E}_t$ is the exchange rate (units of Home currency per unit of Foreign currency).

(a) State and explain the condition for efficient risk sharing.

(b) Show how to derive a regression model to test of "market completeness" at international level. In doing so, posit that the instantaneous utility is

$$U = \zeta_{C,t} \frac{C^{1-\sigma}}{1-\sigma}, \quad \text{and} \quad U^* = \zeta_{C,t}^* \frac{(C^*)^{1-\sigma}}{1-\sigma}$$

Carefully write down the null hypothesis.

(c) Would the regression model be valid if markets are not complete? Explain.

Section B

QUESTION 5: Global monetary stabilization with "dollar pricing"

Consider the model of monetary policy stabilization after Corsetti-Pesenti 2005 discussed in the lecture notes. The Home country welfare function can be written

$$E_{t-1} \ln C_t - \text{constant}{=}E_{t-1} \ln \frac{\mu_t}{P_t} - \text{constant}$$

where μ_t is the monetary stance of the country, controlled by monetary authorities, and the CPI is defined as

$$P_t = 2P_H^{1/2} P_F^{1/2}$$

with P_H denoting the price index of domestically produced goods (each sold at the individual price $p(h)$), P_F the price index of imported goods (each sold at $p(f)$), all denominated in Home currency. Foreign welfare, consumption, monetary stance and prices are similarly defined, with a star referring to foreign variables and foreign-currency denominated prices.

Assume that Home firms preset the price of their product variety h in own currency (according to the hypothesis of Producer Currency Pricing, or PCP):

$$\mathcal{E}_t p_t^* (h) = p_t (h) = mkp \cdot E_{t-1} \left(\frac{W_t}{Z_{Ht}} \right)$$

while Foreign preset their export prices of their good variety f in Home currency (according to Local Currency Pricing, or LCP).

$$p_t (f) = mkp \cdot E_{t-1} \left(\frac{\mathcal{E}_t W_t^*}{Z_{Ft}} \right) \quad \text{and} \quad p_t^* (f) = mkp \cdot E_{t-1} \left(\frac{W_t^*}{Z_{Ft}} \right).$$

Above: W_t and W_t^* denote wages, denominated in Home and Foreign currency, respectively; Z_{Ht} and Z_{Ft} denote productivities, mkp denote constant markup; and $\mathcal{E}_t$ denote the exchange rate (units of Home currency per unit of Foreign currency).

Assume that risk sharing is perfect, such that $\mathcal{E}_t = \frac{\mu_t}{\mu_t^*}$, and that countries chooses μ_t and μ_t^* under commitment, i.e., they solve the optimal policy problem ex ante, at $t-1$.

(a) Derive and discuss the optimal policy rules:
  - (a.1) under Nash, i.e. assuming that each country sets its monetary stance taking the stance abroad as given;
  - (a.2) under policy cooperation.

(b) Contrast the optimal response to a positive productivity shock in the foreign country under Nash and Cooperation.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Tuesday 6 June 2017 9:00am - 11:00am

M230
APPLIED MACROECONOMICS

Candidates are required to answer **three out of seven** questions.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 2
Metric Graph Paper
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1

A representative agent is exogenously endowed with y_t in period $t = 0, 1, \ldots$ The sequence of endowments, $\{y_t\}_{t=0}^{\infty}$ is entirely deterministic and known already at time $t = 0$. The agent may buy and sell zero-coupon bonds of different maturities, and spend the remainder of her resources on consumption, c_t. Denote the price of a bond at time t with redemption date at time $t+n$ as q_t^{t+n}. Thus, a standard one-period bond is denoted q_t^{t+1}. All bonds have face value 1, so that $q_t^t = 1$.

(a) The representative agent's total amount of resources available in period t is given by

$$y_t + \sum_{n=0}^{N-1} q_t^{t+n} \times b_t^{t+n}, \qquad t = 0, 1, 2, \ldots$$

What is her total amount of expenditures in period t? Write down the agent's complete budget constraint in period t.

(b) The consumer faces the following optimisation problem

$$\max_{\{c_t, \{b_{t+1}^{t+n}\}_{n=1}^{N}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to the budget constraint in part (a). Here $u(\cdot)$ denotes the instantaneous utility function satisfying $u' > 0$, $u'' < 0$, c_t denotes consumption in period t, and $\beta \in (0,1)$ represents the discount factor.

Derive the asset prices, $\{q_t^{t+n}\}_{n=1}^{N}$, from the above optimisation problem, and show that they equal

$$q_t^{t+n} = \beta \frac{u'(c_{t+1})}{u'(c_t)} q_{t+1}^{t+n}.$$

(c) The yield to maturity, $r_{t,n}$, is defined as the average return on an asset, such that

$$\frac{1}{(1+r_{t,n})^n} = \frac{1}{\Pi_{s=1}^{n}(1+r_{t+s})},$$

where r_{t+s} denotes the one period return

$$r_{t+s} = \frac{q_{t+s}^{t+s+n}}{q_{t+s-1}^{t+s+n}} - 1.$$

Using the approximation $\ln(1+x) \approx x$ for x close to zero, show that

$$r_{t,n} \approx \frac{1}{n}(r_{t+1} + r_{t+2} \ldots r_{t+n}),$$

and therefore that

$$r_{t,n} \approx -\ln\beta + \frac{\gamma}{n}(g_t^c + g_{t+1}^c \ldots g_{t+n-1}^c),$$

where γ is defined as $u'(c) = c^{-\gamma}$, and $g_t^c = \ln c_{t+1} - \ln c_t$.

Question 2

A representative agent solves the following optimisation problem

$$\max_{\{c_t, b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t),$$

$$\text{subject to} \quad b_t + w_{t-1} - T_t = p_t c_t + q_t b_{t+1}, \qquad t = 0, 1, 2, \ldots$$

where b_t denotes nominal bond holdings, w_{t-1} nominal income, c_t real consumption, p_t the price level, q_t the nominal bond price, T_t nominal lump-sum taxes, and $\beta \in (0, 1)$ represents the discount factor. The utility function, $u(\cdot)$, satisfies $u' > 0$, $u'' < 0$. It is assumed that $w_t = p_t y_t$, where y_t is exogenous.

(a) Derive the Euler equation associated with the above optimisation problem.

(b) The public sector's budget constraint is given by

$$d_t = q_t d_{t+1} + T_t + (m_t - m_{t-1}), \qquad t = 0, 1, 2, \ldots$$

where d_t denotes nominal debt, and m_t denotes the monetary base. Use the fact that $m_t = p_t y_t$, and that $d_t = b_t$ and derive the economy's real resource constraint (i.e. the GDP identity).

(c) Now assume that the central bank targets inflation such that

$$\mu_t = (1 - \rho)\bar{\mu} + \rho \mu_{t-1},$$

with $m_{t+1} = \mu_t m_t$. Assume further that output is constant over time, $y_t = y_{t+1} = \ldots = y$. Derive a similar law of motion for the bond price, q_t.

Question 3

In period t, a representative agent is in possession of x_t units of cash and b_t units of a riskless bond. She receives w_{t-1} units of labour income, and interest payments on bond holdings are denoted i_t. These (nominal) resources may be spent on (real) consumption, c_t; purchases of new bonds, b_{t+1}; and future excess cash holdings, x_{t+1}. The price level is denoted p_t. The budget constraint is therefore

$$x_t + b_t + y_t - T_t = p_t c_t + b_{t+1} + x_{t+1},$$

where y_t denotes total income, $y_t = w_{t-1} + b_t i_t$, and T_t denotes some lump-sum taxes. Given a sequence of prices, $\{i_t, p_t\}$, and taxes, $\{T_t\}$, the agent's optimisation problem is given as

$$\max_{\{c_t, b_{t+1}, x_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to the budget constraint above. The utility function, $u(\cdot)$, satisfies $u' > 0$, $u'' < 0$, and $\beta \in (0, 1)$ represents the discount factor.

The government's sequence of (nominal) debt, d_t, taxes, T_t, and real government spending, g_t, satisfies

$$d_{t+1} + T_t = d_t(1 + i_t) + p_t g_t, \qquad t = 0, 1, 2, \ldots$$

Money supply is constant and equals m. The sequence of aggregate real output, $\{Y_t\}$, is treated as given.

(a) Derive the Euler equations associated with the household's problem. Given an intuitive explanation.

(b) Define a competitive equilibrium and derive the equation of exchange. Do prices, p_t, succumb to the monetarist view?

(c) Now suppose that a fraction of the *contemporaneous* labour income, $\gamma_t w_t$, is received already in period t, but that the reminder, $(1 - \gamma_t) w_t$, is still only received in period $t + 1$. The sequence $\{\gamma_t\}$ is treated as given.

Under these new conditions derive the household's budget constraint, and derive the equation of exchange. How is the velocity of money related to γ? Explain intuitively.

Question 4

A government's period-by-period budget constraint is given by

$$p_t g_t + d_t = q_t d_{t+1} + T_t + (m_t - m_{t-1}), \qquad t = 0, 1, 2, \ldots$$

where g_t denotes real government spending, d_t nominal debt, T_t nominal lump-sum taxes, and m_t the stock of money. Here q_t denotes the price of a one period riskless bond which pays out one unit of nominal resources in period $t+1$. The implied nominal interest rate is given by $q_t = \frac{1}{1+i_{t+1}}$. Denote real taxes as $\tau_t = T_t/p_t$, and the real interest rate as $1 + r_{t+1} = (1 + i_{t+1})\frac{p_t}{p_{t+1}}$.

(a) Derive the government's budget constraint in *real* terms.

(b) The equation of exchange states that $p_t y_t = v_t m_t$. Assume that $y_t = y$, and $v_t = 1$, and $r_t = r$, for $t = 0, 1, \ldots$ Show that real government debt can be written as a function of the net present value of real government spending, real taxes, real output, and inflation, $\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}}$. (Hint: Impose the "no-Ponzi condition" $\lim_{s\to\infty} \frac{1}{(1+r)^s} d_{t+s} = 0$)

(c) In addition to the assumption imposed in part (c) above, assume that taxes follow the autoregressive law of motion

$$\tau_{t+1} = (1-\rho)\bar{\tau} + \rho\tau_t,$$

and that real government debt as well as spending are constant over time, $\hat{d}_t = d_t/p_t = d$ and $g_t = g$, $t = 0, 1, \ldots$

Derive an expression for inflation from the government budget constraint and show that inflation must follow an autoregressive law of motion. What is the long-run inflation rate in this economy? Why did Sargent and Wallace call this "some unpleasant monetarist arithmetics"?

(TURN OVER)

Question 5

Consider the following two-period cash in advance model

$$\max_{c_0, c_1, b_1, x_1} \{u(c_0) + \beta u(c_1)\},$$

$$\begin{aligned} \text{subject to} \quad p_0 c_0 + b_1 + x_1 &= b_0(1+i_0) + w_{-1} + x_0 - T_0, \\ p_1 c_1 &= b_1(1+i_1) + w_0 + x_1 - T_1, \\ x_1 &\geq 0, \\ x_0, w_{-1}, b_0, i_0, \quad \text{are given, and} \quad w_{-1} &+ x_0 = m_{-1}, \end{aligned}$$

where c_t denotes consumption of the output good, b_t nominal government bonds, x_t excess cash holdings, w_{t-1} nominal income, i_t the nominal interest rate, and T_t are nominal taxes, $t = 0, 1$. The utility function, $u(\cdot)$, is given by $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ with $\gamma > 1$, and $\beta \in (0, 1)$.

(a) The government's budget constraint is

$$p_t g_t = T_t + (m_t - m_{t-1})$$

where g_t denotes real government expenditures, and m_t money supply. The equilibrium conditions are $b_t = 0$, and $y_t = c_t + g_t$, $t = 0, 1$, where the latter condition defines nominal GDP. The cash in advance structure implies that $w_0 = y_0$.
Derive the equation of exchange in both period 0 and 1.

(b) Potential output in period 0 is equal to one. Find the value of real output in period 1, y', that brings the economy to the cusp of a liquidity trap. If $y_1 < y'$ and $g_1 = 0$, what is the fiscal multiplier?

(c) Suppose that $y_1 < y'$ and $p_0 = m_0$, such that $y_0 < 1$. Consider implementing two different policies in period 0: (i), to increase government spending, g_0 such that $y_0 = 1$, or (ii) to increase m_1 such that $y_0 = 1$. Which policy yields the largest increase in private consumption, c_0? What is the intuition for this result?

Question 6

Consider the cash-in-advance model,

$$\max_{\{c_t, b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t),$$

$$\text{s.t.} \quad p_t c_t + b_{t+1} + x_{t+1} = (1 + i_t) b_t + w_{t-1} + x_t - T_t,$$

where $\{y_t\}$ is a given sequence of endowments, c_t denotes real consumption, b_t nominal government bonds, x_t excess cash holdings, w_{t-1} nominal income, i_t the nominal interest rate, and T_t are nominal taxes. The utility function, $u(\cdot)$, is given by $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ with $\gamma > 1$, and $\beta \in (0, 1)$.

The government's budget constraint is given by

$$p_t g_t + i_t d_t = (d_{t+1} - d_t) + T_t + (m_t - m_{t-1}),$$

where g_t denotes real government expenditures, d_t nominal debt, and m_t money supply.

Assume that $m_t = m \; \forall t$, and $g_t = 0 \; \forall t$. Potential output in period t is equal to one.

Suppose that agents unexpectedly receive news in period t that output in period $t+1$ will decline and equal $y' < 1$. In addition, assume throughout this exercise that the economy is not in a liquidity trap in period $t + 1$.

(a) If the fall in period $t + 1$ output is *not* sufficient to bring the economy to a liquidity trap in period t, how do these news affect p_{t+1}, p_t, and i_{t+1}? How is the real interest rate, r_{t+1}, affected? What is the intuition for these price movements?

(b) How big must the fall in output be for the economy to fall into a liquidity trap in period t? Suppose that y_t is still equal to 1, what is the effect on period t prices, velocity of money, v_t, and excess cash holdings, x_{t+1}?

(c) Assume that the price level in period t, p_t, is pre-set and equal to m.
How is output in period t related to output in period $t + 1$? Illustrate this relationship both mathematically and graphically.

Question 7

A government's budget constraint in nominal terms is given by

$$p_t g_t + i_t d_t = (d_{t+1} - d_t) + T_t + (m_t - m_{t-1}),$$

where g_t denotes real government expenditures, d_t nominal debt, i_t the nominal interest rate, T_t nominal tax revenue, and m_t money supply.

(a) What is the government's budget constraint in real terms?

(b) Explain intuitively why the real debt is defined as d_t/p_{t-1} instead of d_t/p_t.

(c) Government budgets often report the *nominal* gross of interest deficit defined as

$$p_t g_t - T_t + i_t d_t$$

and also the ratio to nominal GDP

$$\frac{p_t g_t - T_t + i_t d_t}{p_t y_t}. \tag{1}$$

However, the gross of interest deficit in *real* terms as a fraction of real GDP is instead given by

$$\frac{g_t - \tau_t + r_t \hat{d}_t}{y_t},$$

where $\hat{d}_t$ denotes real government debt.

How come the definition in equation (1) may overstate the government's "deficit problem"? How does this bias change with the rate of inflation, π_t?

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil Finance

Friday 13 January 2017 10:00-12:00

M300
ECONOMETRIC METHODS

You should answer eight questions in total. Answer all **six** questions from **Section A** and **two** questions from **Section B**. Part A is 60% of total marks.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads
New Cambridge Elementary Statistical Tables

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A1 You have data on the height Y_i, $i = 1, ..., n$, of n individuals, n_1 of which are men and the rest are women. You run the following regression

$$Y_i = \beta_1 D_{mi} + \beta_2 D_{wi} + \beta_3 X_i + \varepsilon_i,$$

where D_{mi} is the dummy variable for men, D_{wi} is the dummy variable for women, and X_i is the weight of individual i. Note that the constant is not included in the regression. Show that the sum of the OLS residuals, $\sum_{i=1}^{n} \hat{\varepsilon}_i$, equals zero.

A2 Consider the following regression

$$lwage_i = \beta_0 + \beta_1 educ_i + \beta_2 tenure_i + \varepsilon_i, \tag{1}$$

where $lwage_i$ is the log wage of individual i, $educ_i$ is the education, and $tenure_i$ is the number of years with current employer. The following are the results of the OLS regression of the squared residuals from the above regression on $educ$ and $tenure_i$. Use these results to conduct the Breusch-Pagan test the homoskedasticity of the original regression (1).

```
. reg resid2 educ tenure

      Source |       SS       df       MS              Number of obs =     526
-------------+------------------------------           F(2, 523)     =    4.83
       Model |  1.10887555     2  .554437776           Prob > F      =  0.0083
    Residual |  60.0010537   523  .114724768           R-squared     =  0.0181
-------------+------------------------------           Adj R-squared =  0.0144
       Total |  61.1099293   525  .116399865           Root MSE      =  .33871

------------------------------------------------------------------------------
      resid2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   .0021394    .005347     0.40   0.689    -.0083648    .0126436
      tenure |   .0063546   .0020494     3.10   0.002     .0023285    .0103807
       _cons |   .1356805   .0701332     1.93   0.054    -.0020969     .273458
------------------------------------------------------------------------------
```

A3 You would like to run a regression of log wage ($lwage_i$), on constant, eductation ($educ_i$), and experience ($exper_i$). However, you suspect that $educ_i$ might be endogenous. You have two instruments: mother's education ($meduc_i$), and father's education, ($feduc_i$). Let $resid_i$ be the OLS residuals from the regression of $educ_i$ on constant, $meduc_i$, $feduc_i$, and $exper_i$. The OLS regression of $lwage_i$, on constant, $educ_i$, $exper_i$, and $resid_i$ gives the following results. Using these results, test the endogeneity of $educ_i$. Argue that the reported coefficients on $educ_i$ and $exper_i$ are numerically equal to the 2SLS estimates.

```
. reg lwage educ exper resid
```

Source	SS				
					722
				F(3, 718)	44.87
Model	20.022858	3	6.67428601	Prob > F	0.0000
Residual	106.789058	718	.148731278	R-squared	0.1579
				Adj R-squared	0.1544
Total	126.811916	721	.175883378	Root MSE	.38566

lwage	Coef.					
educ	.142298	.018011	7.90	0.000	.1069375	.1776585
exper	.0376059	.005374	7.00	0.000	.0270552	.0481565
resid	-.0769213	.0196251	-3.92	0.000	-.1154509	-.0383918
_cons	4.429426	.2962819	14.95	0.000	3.847744	5.011108

A4 Consider a panel regression

$$y_{it} = c_{it} + \beta x_{it} + \varepsilon_{it},$$

where c_{it} is unobserved, but it is known that $c_{it} = c_{i,t+2}$ for any i and t. Propose a consistent estimator of β. Show its consistency, stating any additional assumptions that you need.

A5 A process y_t satisfies the following regression

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t,$$

where x_t and ε_t are two independent AR(1) processes

$$\begin{aligned} x_t &= 0.5x_{t-1} + u_t, \text{ and} \\ \varepsilon_t &= 0.1\varepsilon_{t-1} + w_t \end{aligned}$$

with i.i.d. standard normal u_t and w_t. Find the autocorrelation function of y_t.

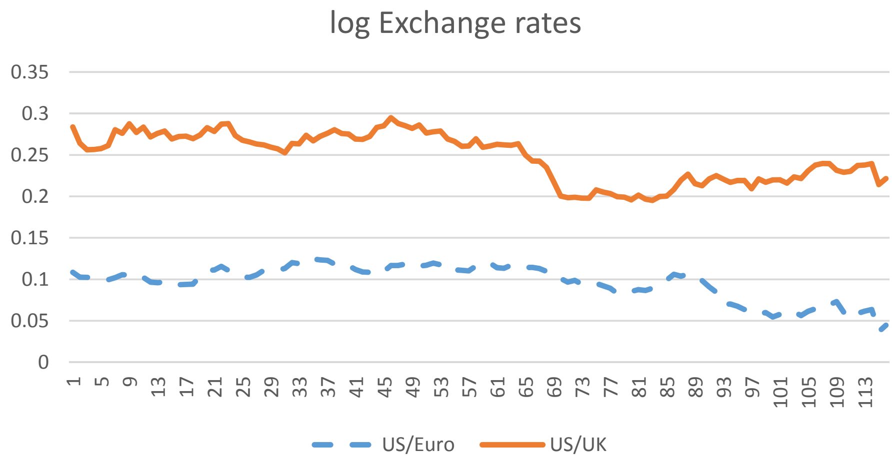
A6 You have n i.i.d. observations y_i, $i = 1, ..., n$, with density function

$$f(y, \theta) = \frac{1}{\sqrt{2\pi\theta}} \exp\left\{-\frac{y^2}{2\theta}\right\}.$$

Find the maximum likelihood estimator of θ. Approximate its variance for large n.

SECTION B

B1 Suppose that you would like to forecast US/UK and US/Euro exchange rates. You have 116 daily observations of log of dollars per 1 pound and 1 euro since July 1, 2016 (shortly after the Brexit vote). The graph of this time series is shown in figure below.



(i) Let us call the log US/UK and US/Euro exchange rates at time t as y_t and z_t, respectively. The regression of $\Delta y_t = y_t - y_{t-1}$ on constant, y_{t-1} and Δy_{t-1} and a similar regression for z give the following results

$$\begin{aligned} \widehat{\Delta y_t} &= \underset{(0.0056)}{0.0058} - \underset{(0.0224)}{0.025}\, y_{t-1} - \underset{(0.0924)}{0.025}\, \Delta y_{t-1}, \\ \widehat{\Delta z_t} &= \underset{(0.0023)}{-0.0006} + \underset{(0.0233)}{0.001}\, z_{t-1} - \underset{(0.099)}{0.024} \Delta z_{t-1}, \end{aligned}$$

where the standard errors are given in the parentheses. Test the hypothesis of the unit roots in y_t and z_t at 5% significance level. Given these results, what would be your forecasts for y and z for January 1 of 2018?

(ii) The OLS regression of y_t on z_{t-1} gives the following results

$$\widehat{y_t} = \underset{(0.011)}{0.167} + \underset{(0.11)}{0.83}\, z_{t-1}.$$

On December 21, 2016, $z_t = 0.039$. Would your prediction for y_t on December 22, 2016 equal

$$0.167 + 0.83 * 0.039 = 0.19937$$

Explain your answer.

(iii) The OLS regression of Δy_t on $\Delta z_{t-1}, \Delta z_{t-2}$, and Δz_{t-3} gives the following results

```
. reg dy L1.dz L2.dz L3.dz
```

Source	SS					
						112
				F(3, 108)		0.30
Model	.000045452	3	.000015151	Prob > F	=	0.8240
Residual	.005420629	108	.000050191	R-squared	=	0.0083
				Adj R-squared	=	-0.0192
Total	.005466081	111	.000049244	Root MSE	=	.00708

dy	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
dz						
L1.	-.039442	.1389438	-0.28	0.777	-.3148527	.2359688
L2.	.1110407	.161019	0.69	0.492	-.2081268	.4302083
L3.	-.1026289	.1610944	-0.64	0.525	-.421946	.2166882
_cons	-.0003422	.000678	-0.50	0.615	-.0016862	.0010018

What is the estimate of the long run multiplier in the effect of Δz_t on Δy_t? Test the hypothesis that the coefficients on $\Delta z_{t-1}, \Delta z_{t-2}$, and Δz_{t-3} are equal to zero. Discuss the validity of your test.

(iv) How would you test for no serial correlation in the errors of the regression from (iii)?

B2 Consider a regression model

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i,$$

where (x_i, ε_i), $i = 1, ..., n$, are i.i.d. with $Ex_i = E\varepsilon_i = 0$, but $Cov(x_i, \varepsilon_i) \neq 0$. Suppose that v_i is a valid instrument for x_i. Assume that v_i, $i = 1, ..., n$, are i.i.d., and $Ev_i = 0$. A friend suggests that you can use an extra instrument $w_i = v_i + \eta_i$, where η_i, $i = 1, ..., n$, are i.i.d., independent from v_j, x_j and ε_j, for all $j = 1, ..., n$, and have zero mean.

(i) Let $\mathbf{z}_i = (1, v_i, w_i)'$. Show that the rank condition holds for $\mathbf{z}_i$.

(ii) Recall that under homoskedasticity,

$$\sqrt{n}\left(\hat{\boldsymbol{\beta}}_{2SLS} - \boldsymbol{\beta}\right) \xrightarrow{d} N\left(0, \sigma^2\left(E\mathbf{x}_i\mathbf{z}_i'\left(E\mathbf{z}_i\mathbf{z}_i'\right)^{-1}E\mathbf{z}_i\mathbf{x}_i'\right)^{-1}\right)$$

where $\hat{\boldsymbol{\beta}}_{2SLS}$ is the $2SLS$ estimator of $\boldsymbol{\beta} = (\beta_0, \beta_1)'$ that uses $\mathbf{z}_i$ as instruments. Verify that the homoskedasticity holds, assuming that $E\left(\varepsilon_i^2|v_i\right) = \sigma^2$.

(iii) Compute the variance covariance matrix of the asymptotic distribution of $\sqrt{n}\left(\hat{\boldsymbol{\beta}}_{2SLS} - \boldsymbol{\beta}\right)$, assuming that $\sigma^2 = Ev_i^2 = E\eta_i^2 = Ev_ix_i = 1$.

(iv) Compare the variance covariance matrix computed in (iii) with the variance covariance matrix of the asymptotic distribution of $\sqrt{n}\left(\tilde{\boldsymbol{\beta}}_{2SLS} - \boldsymbol{\beta}\right)$, where $\tilde{\boldsymbol{\beta}}_{2SLS}$ is the 2SLS estimator of $\boldsymbol{\beta}$ that uses only $(1, v_i)$ as instruments. Briefly comment on your comparison.

B3 A researcher studies the effect of water pollution on the mortality rate in a fish population. He runs extensive laboratory experiments and determines that the 0.1% and 0.05% concentrations of a pollutant in water lead to 50% and 23% mortality rate, respectively.

(i) Assuming that the probability of death is described by a probit model, determine the mortality rate of fish in absolutely clean water.

(ii) What would your answer to (i) be had you assumed that the probability of death is described by a linear probability model? Comment.

(iii) In fact, to establish the 50% and 23% mortality rates, the researcher simply computed the proportion of fish in two large tanks of water with the concentration of a pollutant equal to 0.1% and 0.05%, respectively. Argue that the researcher's method is equivalent to a maximum likelihood procedure.

(iv) Assume that each of the tanks of water had 1000 fish. Find the standard errors of the 50% and 23% estimates.

B4 Consider the following regression model

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i,$$

where (y_i, x_i), $i = 1, ..., n$ are i.i.d. and $\varepsilon_i = x_i \times u_i$ with u_i distributed as a standard normal conditionally on x_i.

(i) Show that ε_i, $i = 1, ..., n$ are i.i.d.

(ii) Which Gauss-Markov assumptions hold for this model and which fail? What does this imply for the OLS estimator of β_1?

(iii) Suppose that $n = 3$, and

i	1	2	3
y_i	1	0	-1
x_i	1	2	4

Find the GLS estimate $\hat{\beta}_{1,GLS}$.

(iv) What is the finite sample distribution of $\hat{\boldsymbol{\beta}}_{GLS} = \left(\hat{\beta}_{0,GLS}, \hat{\beta}_{1,GLS}\right)'$ conditional on x_1, x_2, and x_3?

END OF PAPER

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil Finance

Friday 12th, May, 2017 2:00pm - 4:00pm

M300
ECONOMETRIC METHODS

Answer all **six** questions from Section A and **two** questions from Section B. Part A is 60% of total marks.

All questions in Section A carry equal weight.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly. Credit will be given for clear presentation of relevant statistics.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator
Dickey-Fuller Tables
New Cambridge Statistical Tables
Engle-Granger Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Section A

A1 You have data on the height Y_i, $i = 1, ..., n$, of n individuals. You run the following regression

$$Y_i = \beta_0 + \beta_1 D_{<30,i} + \beta_2 D_{>20,i} + \varepsilon_i,$$

where $D_{<30,i}$ is the dummy variable for individuals who are younger than 30, and $D_{>20,i}$ is the dummy variable for individuals who are older than 20. Interpret the coefficient β_2.

A2 You would like to test the hypothesis that $\beta_0 = \beta_1 = \beta_2$ in the regression

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \varepsilon_i.$$

Represent this hypothesis in the standard form $R\beta = a$, where $\beta = (\beta_0, \beta_1, \beta_2)'$. Suppose that you know that $R\hat{\beta} - a$ is distributed as $N(0, I_2)$ under the null, and that the value of $R\hat{\beta} - a$ is $(2, -1)'$. Test the null hypothesis. (Here $\hat{\beta}$ is the OLS estimate of β.)

A3 Consider regression

$$lwage_i = \beta_0 + \beta_1 educ_i + \beta_2 exper_i + \varepsilon_i,$$

where $lwage_i$ is log wage, $educ_i$ is education, and $exper_i$ is experience. Suppose that $educ_i$ is endogenous and that you have two instruments: mother's education ($meduc_i$), and father's education, ($feduc_i$). Let

$$resid_i = lwage_i - \hat{\beta}_{0,2SLS} + \hat{\beta}_{1,2SLS} educ_i + \hat{\beta}_{2,2SLS} exper_i$$

be the 2SLS residuals. The OLS regression of $resid_i$, on constant, $exper_i$, $meduc_i$ and $feduc_i$ gives the following results. Using these results, test the overidentifying restrictions. Interpret the outcome of your test.

```
. reg resid exper meduc feduc
```

Source	SS	df	MS
Model	.007060894	3	.002353631
Residual	121.267793	718	.168896647
Total	121.274854	721	.168203681

Number of obs	=	722
F(3, 718)	=	0.01
Prob > F	=	0.9978
R-squared	=	0.0001
Adj R-squared	=	-0.0041
Root MSE	=	.41097

resid	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
exper	-.000047	.0037216	-0.01	0.990	-.0073536	.0072596
meduc	.0012458	.0066271	0.19	0.851	-.011765	.0142566
feduc	-.0009902	.0057649	-0.17	0.864	-.0123084	.0103279
_cons	-.0027745	.0840162	-0.03	0.974	-.1677212	.1621723

A4 Consider a very simple fixed effects regression

$$y_{it} = c_i + \varepsilon_{it}, \qquad i = 1, ..., n; t = 1, ..., T, \tag{1}$$

where ε_{it} are i.i.d. $N(0, \sigma^2)$ variables and there are no explanatory variables. A colleague proposes to estimate σ^2 as $\frac{1}{nT}$ times the SSR from the dummy variables regression. Find the bias of such an estimate.

A5 A time series y_t satisfies the following AR(3) equation

$$y_t = \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \rho_3 y_{t-3} + \varepsilon_t, \ t = 1, ..., T.$$

For which values of ρ_1, ρ_2 and ρ_3 does y_t have a unit root? How would you test for the unit root in y_t?

A6 You have n i.i.d. observations y_i, $i = 1, ..., n$ that equal -1 with probability p, $+1$ with probability q, and 0 with probability $1 - p - q$. Find the maximum likelihood estimator of $\theta = (p, q)'$ and the Fisher information matrix.

(TURN OVER)

Section B

B1 A researcher has data on the logarithms of UK households disposable income, $\log Y_t$, and consumption, $\log C_t$, from 1955 to 2015 (61 years in total).

(i) To test the hypotheses of unit roots in $\log Y_t$ and $\log C_t$, the researcher runs the OLS regression of $\Delta \log Y_t = \log Y_t - \log Y_{t-1}$ on constant, time trend t, $\log Y_{t-1}$, and $\Delta \log Y_{t-1}$ and a similar regression for $\log C_t$. The results of these regressions are reported below

$$\begin{aligned}
\Delta \widehat{\log Y_t} &= \underset{(0.052)}{0.077} + \underset{(0.0009)}{0.0008t} - \underset{(0.010)}{0.012} \log Y_{t-1} + \underset{(0.074)}{0.881} \Delta \log Y_{t-1}, \\
\Delta \widehat{\log C_t} &= \underset{(0.045)}{0.070} + \underset{(0.0008)}{0.0007t} - \underset{(0.009)}{0.011} \log C_{t-1} + \underset{(0.065)}{0.905} \Delta \log C_{t-1},
\end{aligned}$$

where the standard errors are given in the parentheses. Test the hypotheses of unit roots in $\log Y_t$ and $\log C_t$ at 5% significance level.

(ii) Explain why the researcher included the time trend, and why did she include the lagged changes in the logarithms in the regressions from (i).

(iii) The permanent income theory of consumption predicts that, in the long-run equilibrium, the average propensity to consume C_t/Y_t is constant. The researcher argues that this prediction can be tested using the following OLS results

$$\Delta \log \widehat{(C_t/Y_t)} = \underset{(0.006)}{0.020} - \underset{(0.076)}{0.280} \log (C_{t-1}/Y_{t-1}) + \underset{(0.119)}{0.353} \Delta \log (C_{t-1}/Y_{t-1}).$$

Explain the researcher's argument, and perform the test.

(iv) To forecast households consumption, the researcher runs the regression of $\Delta \log C_t$ on $\Delta \log C_{t-1}$, $\Delta \log Y_{t-1}$ and $\log (C_{t-1}/Y_{t-1})$. She gets the following OLS results

$$\Delta \widehat{\log C_t} = \underset{(0.008)}{0.001} + \underset{(0.176)}{0.760} \Delta \log C_{t-1} + \underset{(0.174)}{0.153} \log Y_{t-1} + \underset{(0.113)}{0.063} \log (C_{t-1}/Y_{t-1}).$$

What is a reason to include $\log (C_{t-1}/Y_{t-1})$ in this regression? How would you test for no serial correlation in the errors of this regression?

B2 Consider a regression model

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i,$$

where $\varepsilon_i = x_i(1+\eta_i)$. Assume that (x_i, η_i), $i = 1, ..., n$, are i.i.d. with $Ex_i = E\eta_i = 0$, $Var(x_i) = \sigma_x^2$, $Var(\eta_i) = \sigma_\eta^2$ and η_i independent from x_i.

(i) What is the value of $Cov(x_i, \varepsilon_i)$? What does such a value imply for the results of the OLS regression of y_i on x_i?

(ii) Show that, if z_i is a valid instrument for x_i, it cannot be independent from η_i.

(iii) Usually, econometricians do not know the structure of regression errors. However, suppose that you know that $\varepsilon_i = x_i(1+\eta_i)$. How would you consistently estimate β_0 and β_1 without using the instrumental variables technique? Discuss the efficiency of the estimators that you propose.

(iv) Now suppose that you do not know that $\varepsilon_i = x_i(1+\eta_i)$, but you suspect that x_i is an endogenous regressor. Further, suppose that you observe z_i, $i = 1, ..., n$, which is a valid instrument for x_i. How would you test for exogeneity of x_i?

B3 A researcher studies the relationship between life satisfaction and gender using a random sample (S_i, G_i), $i = 1, ..., n$, where $S_i = 1$ if person i is satisfied with life and $S_i = 0$ otherwise, and $G_i = 1$ for women and 0 for men. He considers the following probit model

$$\Pr(S_i = 1|G_i) = \Phi(\alpha + \beta G_i),$$

where Φ is the cdf of a standard normal distribution.

(i) Suppose that the observed numbers of life-satisfied men, life-satisfied women, life-dissatisfied men, and life-dissatisfied women are the same and equal $n/4$. Derive an explicit expression for the conditional log likelihood function.

(ii) Find the values of the maximum likelihood estimates of α and β.

(iii) How would the predicted probabilities of life satisfaction be different from those obtained from the linear probability model? Does your answer depend on the assumption about the available sample made in (i)?

(iv) Now suppose that the researcher obtains data on another explanatory variable M_i, equal to 1 if person i is married and 0 otherwise. He considers a new model

$$\Pr(S_i = 1|G_i, M_i) = \Phi(\alpha + \beta G_i + \gamma M_i).$$

Suppose that it is known that the expected value of S_i for not married men is above 1/2. What does this imply for the coefficients of the above model? For which values of α, β, and γ will the predicted effect of marriage on life satisfaction be positive and larger for women than for men? Will the linear probability model have similar predictions?

B4 Consider the following two-period panel regression model

$$y_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it},\ i = 1, ..., n,\ t = 1, 2.$$

where $(y_{i1}, y_{i2}, x_{i1}, x_{i2}, \alpha_i)$, $i = 1, ..., n$ are i.i.d., $\varepsilon_{i1} = 0$, $i = 1, ..., n$, that is, in the first period ε_{i1} is identically zero, and ε_{i2} is independent from $(x_{i1}, x_{i2}, \alpha_i)$, $E\varepsilon_{i2} = 0$, and $Var(\varepsilon_{i2}) = 1$.

(i) Which Gauss-Markov assumptions hold for this model and which may fail?

(ii) Will the fixed effects estimator of β be unbiased? Will it be consistent? Explain your answers.

(iii) Propose a consistent estimator of the fixed effects α_i and show its consistency.

(iv) Suppose that $\alpha_i = \eta_i$, where η_i is independent from (x_{i1}, x_{i2}) , $E\eta_i = 0$ and $Var(\eta_i) = 1$. Let $w_{it} = \eta_i + \varepsilon_{it}$ and

$$w = (w_{11}, w_{12}, w_{21}, w_{22}, ..., w_{n1}, w_{n2})'.$$

Find $\Omega = Var(w)$ and its inverse Ω^{-1}. Using your findings, describe the GLS estimator of β in the regression $y_{it} = \beta x_{it} + w_{it}$.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Friday 2nd June 2017 1:30pm to 3:30pm

M310
TIME SERIES

Candidates are required to answer **five** questions from Section A and **one** question from Section B.

Section A is weighted as 60% and Section B as 40%.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A.1. Consider the random walk plus noise:

$$y_t = \mu_t + \varepsilon_t, \qquad \mu_t = \mu_{t-1} + \eta_t, \quad t = 1, ..., T,$$

where ε_t and η_t are serially and mutually uncorrelated Gaussian disturbances with mean zero and variances σ_ε^2 and σ_η^2 respectively. Find the distribution of $y_{T+\ell}$, where ℓ is a positive integer, conditional on the information that at time T, μ_T is normal with mean m_T and variance p_T.

A.2. Find the autocorrelation function of the airline model,

$$(1-L)(1-L^s)y_t = (1+\theta L)(1+\Theta L^s)\varepsilon_t,$$

where L is the lag operator and ε_t is $NID(0, \sigma^2)$, that is normally and independently distributed with mean zero and constant variance σ^2. If the first five autocorrelations in a quarterly series take the values -0.4, 0, 0.2, -0.5, 0.2, and are zero thereafter, find the implied values of the parameters θ and Θ.

A.3. A detrending moving average has the property that the weights sum to zero. Write such a filter in terms of the lag operator, L, and hence show that when it is applied to a stationary process, the spectrum of the detrended series is zero at frequency zero. What is the effect on the spectrum at the frequency corresponding to a period of two?

A.4. In the time-varying $AR(2)$ model,

$$y_t = \phi_{1t} y_{t-1} + \phi_{2t} y_{t-2} + \varepsilon_t, \qquad t = 1, .., T,$$

where $y_0 = y_{-1} = 0$, ε_t is $NID(0, \sigma^2)$ and ϕ_{1t} and ϕ_{2t} are assumed to be generated by independent $AR(1)$ processes. How would you estimate this model?

A.5 In the models

$$y_t = \phi y_{t-1} + \delta_0 w_t + \delta_1 w_{t-1} + \varepsilon_t, \quad t = 1, .., T, \tag{1}$$

and

$$\begin{aligned} y_t &= \lambda_0 w_t + \lambda_1 w_{t-1} + u_t, \quad t = 1, .., T, \\ u_t &= \phi u_{t-1} + \epsilon_t \end{aligned} \tag{2}$$

where ε_t and ϵ_t are normally and independently distributed with mean zero and constant variance, the intervention variable is defined as

$$w_t = \begin{cases} 0 & \text{for} \quad t < \tau, \\ 1 & \text{for} \quad t \geq \tau \end{cases}, \qquad 1 < \tau < T.$$

Compare and contrast the dynamics of the intervention in the two models. Can the models ever be the same?

A.6. Consider the model

$$y_t = \varepsilon_t + \beta\varepsilon_{t-1}\varepsilon_{t-2}, \quad \varepsilon_t \sim NID(0, \sigma^2), \quad t = 1, ..., T,$$

where $NID(0, \sigma^2)$ denotes normally and independently distributed with mean zero and constant variance σ^2, and ε_0 and ε_{-1} are fixed and known. Find the minimum mean square error estimator (MMSE) of y_{T+1} and compare its MSE with that of the best linear predictor. How would you estimate β?

A.7. The Clayton copula is defined as

$$C(u_1, u_2) = \begin{cases} (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}, & \theta > 0 \\ u_1 u_2, & \theta = 0. \end{cases}$$

Explain what is meant by tail dependence and obtain expressions for tail dependence in a Clayton copula when $\theta > 0$ and when $\theta = 0$. Define the coefficient of lower tail dependence (the lower tail index) and derive this coefficient for the Clayton copula.

SECTION B

B.1. (a) Consider the $VARMA(1,1)$ model

$$\mathbf{y}_t = \mathbf{\Phi y}_{t-1} + \boldsymbol{\varepsilon}_\mathbf{t} + \mathbf{\Theta}\boldsymbol{\varepsilon}_{t-1}, \qquad t = 1, ..., T,$$

where $\mathbf{y}_t$ is an $N \times 1$ vector, $\mathbf{\Phi}$ and $\mathbf{\Theta}$ are $N \times N$ matrices and $\boldsymbol{\varepsilon}_\mathbf{t}$ is multivariate white noise with mean vector zero and covariance matrix $\mathbf{\Omega}$. Derive an expression for the lag one autocovariance matrix, $\mathbf{\Gamma}(1)$, in terms of $\mathbf{\Phi}$, $\mathbf{\Theta}$, $\mathbf{\Omega}$ and $\mathbf{\Gamma}(0)$, where $\mathbf{\Gamma}(0)$ is the covariance matrix of $\mathbf{y}_t$. Given $\mathbf{\Gamma}(1)$, show how the remaining autocovariance matrices can be computed recursively. Write down a set of equations that could, in principle, be solved to yield $\mathbf{\Gamma}(0)$. In the special case $\mathbf{\Phi} = \phi\mathbf{I}$, where ϕ is a scalar such that $|\phi| < 1$, find an expression for $\mathbf{\Gamma}(0)$ in terms of $\phi, \mathbf{\Theta}$ and $\mathbf{\Omega}$.

(b) Consider the model

$$\mathbf{y}_t = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim NID(\mathbf{0}, \mathbf{\Sigma}_\varepsilon), \qquad t = 1, \ldots, T,$$

$$\begin{aligned} \boldsymbol{\mu}_t &= \boldsymbol{\mu}_{t-1} + \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t, \qquad && \eta_t \sim NID(\mathbf{0}, \mathbf{\Sigma}_\eta), \\ \boldsymbol{\beta}_t &= \mathbf{\Phi}\boldsymbol{\beta}_{t-1} + \boldsymbol{\varsigma}_t, && \boldsymbol{\varsigma}_t \sim NID(\mathbf{0}, \mathbf{\Sigma}_\varsigma) \end{aligned}$$

where $\mathbf{y}_t$, $\boldsymbol{\mu}_t$, $\boldsymbol{\varepsilon}_t$, $\boldsymbol{\eta}_t$ and $\boldsymbol{\varsigma}_t$, are all $N \times 1$ vectors and NID denotes multivariate normal. If $\mathbf{\Phi} = \phi\mathbf{I}$, where ϕ is a scalar such that $|\phi| < 1$, explain how you would estimate ϕ and the covariance matrices $\mathbf{\Sigma}_\varepsilon$, $\mathbf{\Sigma}_\eta$ and $\mathbf{\Sigma}_\varsigma$.

(c) Now suppose that $\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \mathbf{i}\beta_{t-1} + \boldsymbol{\eta}_t$, where $\mathbf{i}$ is an $N \times 1$ vector of ones and β_t is a scalar. How is this model to be interpreted and what can be said about co-integration when (i) $\mathbf{\Sigma}_\eta$ is positive definite and (ii) $\mathbf{\Sigma}_\eta = \mathbf{0}$?

B.2. (a) The probability density function of a general error distribution (GED) with mean zero is

$$f(y; \varphi, \upsilon) = \left[2^{1+1/\upsilon}\varphi\Gamma(1 + 1/\upsilon)\right]^{-1} \exp(-|y/\varphi|^\upsilon /2), \qquad -\infty < y < \infty, \qquad \varphi, \upsilon > 0,$$

where φ is a scale parameter, $\Gamma(.)$ is the gamma function and υ is a tail-thickness parameter. Derive the EGARCH model based on the conditional score (the Gamma-GED-EGARCH model). Compare the impact curves given by the score for $\upsilon = 2$ and $\upsilon = 1$. How would you estimate the unknown parameters?

(b) Explain briefly what is meant by the term 'leverage' when applied to a volatility model and adapt the EGARCH model of (a) to handle leverage. Compare your model with the classic EGARCH model of Nelson.

(c) The survival function of a Type II Pareto distribution with scale α is $\overline{F}(y) = [1 + y/\alpha]^{-\eta}$ for $y \geq 0$. What is the relevance of the tail index parameter η for the existence of moments of the distribution? Derive the quantile function and hence obtain the median when $\eta = 1$. Show that the score for the logarithm of the scale is bounded and comment.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research
MPhil in Finance

Wednesday 7 May 2017 9:00am-11:00am

M320
CROSS SECTION AND PANEL DATA ECONOMETRICS

Candidates are required to answer **three** questions from Section A and **one** question from Section B.

Section A is weighted as 60% and Section B as 40%

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Durbin-Watson and Dickey-Fuller Tables
New Cambridge Elementary Statistical Tables
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A1 Consider the following structural equation

$$y_i = \alpha + \beta e_i + \varepsilon_i, \quad i = 1, ..., N. \tag{1}$$

It is assumed that e is endogenous. We write the first stage regression as

$$e_i = z_i \rho + \zeta_{1i}, \tag{2}$$

where z is a single instrument. The reduced form is obtained by substituting (2) into (1) to give

$$y_i = \alpha + \tau z_i + \varepsilon_{ri}. \tag{3}$$

The second stage regression is given by

$$y_i = \alpha + \beta \widehat{e_i} + \zeta_{2i}.$$

(a) Show that the IV estimator is biased in finite samples.

(b) It can be shown that bias of the IV estimator is approximately

$$\text{BIAS}_{\text{2SLS}} = E[\widehat{\beta}_{\text{2SLS}} - \beta] \approx \frac{\sigma_{\varepsilon\zeta_1}}{\sigma^2_{\zeta_1}} \frac{1}{F+1}, \tag{4}$$

where $\sigma^2_\varepsilon = Var(\varepsilon)$, $\sigma^2_{\zeta_1} = Var(\zeta_1)$, and $\sigma_{\varepsilon\zeta_1} = Cov(\varepsilon, \zeta_1)$. F is the population F-statistic for the joint significance of all regressors in the first stage regression.

Using (4) evaluate the statement that the weak instrument bias gets worse as we add more (weak) instruments.

A2 Consider the following latent variable model

$$y_i^* = \psi_0 + \psi_1 x_i + u_i, \quad i = 1, ..., N. \tag{5}$$

The residual u_i is heteroskedastic such that $u_i | x_i \sim$ Normal $(0, \sigma(x_i)^2)$, where $\sigma(x_i)^2 = x_i^2$.

We observe $y_i = \mathbf{1}(y_i^* > 0)$, where $\mathbf{1}(.)$ is the indicator function.

(a) Show that $\Pr(y_i = 1|x_i) = \Phi\left(\psi_0 \frac{1}{x_i} + \psi_1\right)$, where $\Phi(.)$ is the standard normal cumulative density function.

(b) Show that if ψ_0 and ψ_1 are both positive, the marginal effect of x_i on $\Pr(y_i = 1|x_i)$ has the opposite sign to the marginal effect of x_i on the latent dependent variable y_i^*.

(c) Now suppose that $\Pr(y_i = 1|x_i) = \Phi(\psi_0 + x_i\psi_1 + \gamma c_i)$. $u_i \sim$ Normal $(0, \sigma_u^2)$, and $c_i \sim$ Normal $(0, \tau^2)$. Given this model specification evaluate the following statement.

In probit models neglected heterogeneity is a more serious problem than in linear models. That is, even if

$$E(\gamma c_i + u_i|x_i) = E(\gamma c_i + u_i) = 0,$$

the probit coefficients are inconsistent.

A3 Potential outcomes in the following binary model are given by

$$y(x) = \mathbf{1}(\alpha + \beta x + u \geq 0),$$

where x is a continuous variable, $y(x)$ is a zero-one variate which varies with x, and u is an error term. α and β are unknown parameters. $\mathbf{1}(.)$ denotes the indicator function.

(a) Find an expression for the effect of a change from x to x' for an *individual* with error u.

(b) Find an expression for the average effect $E[y(x') - y(x)]$.

(c) Demonstrate that in binary response models with non-additive errors the partial effects are heterogenous.

A.4 Consider the following dynamic panel data model

$$y_{it} = \alpha y_{it-1} + \mathbf{x}_{it}'\zeta + f_i + u_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T,$$

where the composite error is $\varepsilon_{it} = f_i + u_{it}$. f_i denotes the unobserved individual effect.

(a) Let $T = 5$. Assuming that the source of endogeneity is the lagged dependent variable, write down the orthogonality conditions which underly the first difference GMM estimator.

(b) Derive the associated instrument matrix for the i^{th} individual.

(c) In what sense does each orthogonality condition represent a different estimator?

A.5 Elaborate upon the following statements

(a) Relative to the linear probability model, the advantages of the threshold nonlinear binary response model are less when the design matrix is sparse, and/or when there are endogenous binary regressors.

(b) The problem of using a control function estimator where endogenous variables are discrete only applies to cases where the structural equation is nonlinear.

[TURN OVER]

(c) The error components representation of a mixed logit model and the random effects panel data estimator both generate non-iid covariance matrices which may be exploited to solve different problems.

(d) Explain how to construct an estimator from a set of population moment conditions using the generalised method of moments.

SECTION B

B.1 The relationship between female labour force participation (y) and fertility (d_1) is written as

$$y_{it} = \mathbf{1}(\alpha + \delta_1 d_{1,it} + \delta_2 d_{2,it} + \boldsymbol{x}_i' \boldsymbol{\beta} + \xi_{it} > 0), \quad i = 1, ..., N \text{ and } t = 1, ..., T,$$

where $\mathbf{1}(.)$ denotes the indicator function, y_{it} is a binary indicator of participation in year t (defined as positive annual hours of work), fertility ($d_{1,it}$) is a dummy variable which takes the value one if the age of the youngest child at time $t+1$ is 1, and $d_{2,it}$ is a dummy indicator equal to one if the woman has a child between the ages of 2 and 6. $\boldsymbol{x}_i$ is vector of additional controls.

In addition $\xi_{it} = \alpha_i + u_{it}$ is a standard composite panel data error term in which α_i represents an individual-specific effect and u_{it} denotes time varying errors including idiosyncratic labour market shocks.

a) Briefly discuss why fertility is most likely an endogenous variable. Provide justification for the use of an indicator of whether a woman has two children of the same sex as an external instrument for fertility.

b) In Table 1 we report results based on the application of a number of *linear* estimators. Columns 2, 3, and 4, report, respectively, results based upon the use of an OLS, 2SLS and within-group (WG) estimator.

   Columns 5 and 6 present results for two first-difference GMM estimators, that exploit, respectively, strict and weak exogeneity assumptions.

   i) For each estimator outline the identification assumptions and in particular discuss how each estimator accounts for the endogeneity of fertility.

   ii) In what sense does the use of the WG estimator represent a tradeoff between accounting for unobserved heterogeneity and bias?

c) Write down the moment equations which underly the results for the GMM-1 and GMM-2 estimators.

   For each estimator state the degree of overidentification.

d) Using the results in columns 5 and 6 test for strict exogeneity. What do you conclude?

[TURN OVER]

Table 1: Female Labour Force Participation

Linear Probability Models (N=1442, 1986-1989)					
Variable	OLS	2SLS[1]	WG	GMM-1[2]	GMM-2[3]
Fertility	−0.15	−1.01	−0.06	−0.08	−0.13
	(8.2)	(2.1)	(3.8)	(2.8)	(2.2)
Kids2-6	−0.08	−0.24	0.001	−0.005	−0.09
	(5.2)	(2.6)	(0.04)	(0.4)	(2.7)
Sargan				48.0	18.0
(d.f.)				(22)	(10)

Notes
- Heteroskedasticity robust *t*-ratios shown in parentheses
- [1] External instrument: whether a woman has two children of the same sex.
- The variable Kids2-6 is equal to one if a woman has a child between 2 and 6.
- [2] Treats fertility as endogenous and uses all lags and leads of Kids2-6 and the external instrument same sex.
- [3] Treats fertility as endogenous and uses lags of Kids 2-6 and same sex up to t-1.
- Estimators which do not account for individual effects also include a constant, age, race, and education dummies (not reported).
- Sargan denotes the Sargan test for overidentifying restrictions.

The sample consists of 1442 women aged between 18-55 in 1986.

B.2 Denote by U_{ijt} the unobserved utility of individual i who chooses alternative j at time t. We write U_{ijt} as

$$U_{ijt} = \boldsymbol{x}'_{ijt}\boldsymbol{\beta}_i + \epsilon_{ijt},$$

where $i = 1, ..., n$, $j = 1, ..., J$, $t = 1, ..., T$ index, respectively, individuals, alternatives and time periods. $\boldsymbol{x}_{ijt}$ is a k-dimensional vector of explanatory variables, $\boldsymbol{\beta}_i$ is a k-dimensional parameter vector and ϵ_{ijt} is a random shock known to individual i, that has a Type 1 extreme value distribution.

Variables $\boldsymbol{x}_{ijt}$ are denoted collectively (for all time periods and alternatives) by $\boldsymbol{x}_i$. $\boldsymbol{\beta}_i \sim g(\boldsymbol{\beta}|\boldsymbol{\theta})$ in the population, where $g(.)$ is a mixing distribution with $\boldsymbol{\theta}$ a vector of hyperparameters.

We observe $\boldsymbol{y}_i = (y_{i1}, ..., y_{iT})'$ where $y_{it} = j$ if individual i chooses alternative j at time t.

(a) Write down an expression for the conditional and unconditional probability of individual i's sequence of choices, respectively, $P(\boldsymbol{y}_i|\boldsymbol{x}_i, \boldsymbol{\beta}_i)$ and $P(\boldsymbol{y}_i|\boldsymbol{x}_i, \boldsymbol{\theta})$.

(b) An analyst observes that estimating the parameters of distribution $g(\boldsymbol{\beta}|\theta)$ represents a significant constraint on learning about preference heterogeneity in the *population*. We want to determine, insofar as possible, where each customer's $\boldsymbol{\beta}$ lies in this distribution.

What distribution should the analyst try to estimate to resolve this and how does this relate to $g(\boldsymbol{\beta}|\theta)$?

(c) In Table 2 we present parameter estimates based on a stated preference experiment designed to elicit consumer preferences over energy supplier. The suppliers were differentiated based on price, the length of the contract in years, whether the supplier was local, whether the supplier was well known, whether the supplier charged time-of-day (TOD) rates, and whether the supplier charged seasonal rates.

A mixed logit model was estimated under the assumption that the parameters are independently normally distributed in the population.

i. Interpret the results.
ii. Demonstrate how these results can be used to determine preference heterogeneity for the length of contract
iii. Test whether a mixed logit model is to be preferred to a standard logit specification.

[TURN OVER]

Table 2: Mixed logit model of choice among energy suppliers

	$\widehat{\theta}$	$\widehat{\sigma}_{\theta}$
Price coeff.	-0.8827 (0.050)	–
Contract	-0.2125 (0.026)	0.3865 (0.028)
Local	2.2277 (0.127)	1.7514 (.137)
Well-known	1.5906 (0.100)	0.9621 (0.098)
TOD	2.1328 (0.054)	0.4113 (0.040)
Seasonal	2.1577 (0.051)	0.2812 (0.022)
Log Likelihood Logit:	-4959	
Log Likelihood Mixed Logit	-3619	

Standard errors in parenthesis

$\widehat{\theta}$ denotes the mean and $\widehat{\sigma}_{\theta}$ the standard deviation of each parameter estimated.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 22 May 2017 09:00 - 11:0000am - 11:00am

M330
APPLIED ECONOMETRICS

The paper is in two sections: Candidate are require to answer from Section A **two** compulsory questions and section B **two out of four** questions.

Each question carries equal weight.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Section A

1. Using cross-country estimates of physical and human capital stocks, Benhabib and Spiegel (1994) ran the following cross-country growth accounting regression implied by a Cobb-Douglas aggregate production function:

$$\Delta y_i = \alpha + \beta_1 \Delta K_i + \beta_2 \Delta H_i + \beta_2 \Delta L_i + \varepsilon_i$$

The dependent variable, Δy_i, is the log difference of output between 1965 and 1985. The explanatory variables are the log differences of physical capital, ΔK_i, log differences in average years of schooling, ΔH_i, and log differences in labour force, ΔL_i, all measured between 1965 and 1985. The data consists of a cross-section of 78 countries. Column (1) in the following Table gives their baselines estimates.

	(1)	(2)	(3)
Constant	0.269 (2.98)	0.166 (2.86)	0.100 (1.95)
ΔK	0.457 (5.36)	0.507 (8.41)	0.553 (13.16)
ΔH	0.063 (0.80)	0.111 (1.66)	0.165 (4.00)
ΔL	0.209 (1.01)	0.217 (1.39)	0.241 (2.15)
R^2	0.52	0.77	0.83
N	78	72	64
Jarque-Bera p-value	0.00	0.13	0.55

Table: Cross-Country Growth Regressions
Heteroskedasticity-consistent t-ratios in parentheses

(a) Benhabib and Spiegel's baseline results in column (1) suggest that human capital (growth) enters insignificantly in explaining income growth rates. Please interpret the result of the Jarque-Bera test in column (1) and comment on whether this cautions against or supports Benhabib and Spiegel's main result. Describe an alternative graphical procedure which would supplement the Jarques-Bera test.

(b) Columns (2) and (3) are Least Trimmed Squares estimates of the same regression reported by Jonathan Temple (2001). Briefly (i) describe why Temple (2001) may have chosen to deploy robust regression techniques and (ii) describe the Least Trimmed Squares technique.

(c) In column (3) the Least Trimmed Squares procedure is dropping 14 observations. Upon inspection, we learn that these observations stand out from the rest of the sample in that they experienced very little (or even negative) human capital growth relative to the rest of the world during this twenty year period. Does this alone justify the change in point estimates and statistical significance observed when we go from column (1) to column (3)? Explain your answer.

(TURN OVER)

2. Besley and Burgess (2004) consider the following model for outcomes in Indian states (indexed by s) at time t,

$$y_{st} = \alpha_s + \beta_t + \mu r_{st} + \xi x_{st} + \epsilon_{st}, \tag{1}$$

where α_s and β_t are fixed effects, r_{st} is a "labor reform" variable, and x_{st} are other exogenous variables.[1]

(a) Explain how μ defines a "difference-in-difference" (DD) effect. Outline the necessary conditions for the identification of DD effects on the specification (1).

(b) Assume that the econometrician Rob is told by a political scientist friend that there are some states where the labour reforms would be a pro-forma reform only – that is, even though there formally is a reform (and it is recorded in the data as such), it could never be enforced in practice.

- (i) Explain the danger of just recoding these observations as if there were no reforms to begin with.
- (ii) Outline how you could use the information on these problematic states to better formulate a regression model to estimate the effect of labour reforms.

(c) Explain what the DD effects are using Table 5.2.3 of Angrist and Pischke (2009). Discuss what assumption underlying the specification in equation (1) seems to be violated. Explain your reasoning. The Table of Angrist and Pischke is reproduced at the end of the exam (see Figure 1).

[1] In their paper, labour reforms are measured with a time lag, but this is not central.

Section B

3. A researcher is interested in testing the theory of Purchasing Power Parity (PPP) using UK and US quarterly data. Let e_t, p_t^* and p_t denote the logarithms of the US Dollar-Pound Sterling exchange rate, the US price level and the UK's price level in quarter t. Define $f_t = e_t + p_t^*$, such that f_t gives the pound sterling value of the US price level. The researcher considers testing for two distinct versions of PPP theory. The *strong* version of PPP theory requires $f_t - p_t$ to be a stationary, zero mean, process. The *weak* version of PPP requires $f_t - \beta p_t$ to be a stationary process for some positive constant, β. Based on her empirical work, the researcher supplies you with three results:

- First, based on unit-root tests, she has concluded that f_t is an integrated stochastic process of order 1. p_t is also known to be non-stationary.
- Second, a least squares regression of f_t on p_t and a constant yields the following estimates:

  $$f_t = \underset{(0.041)}{0.012} + \underset{(0.026)}{0.988} p_t + \widehat{u}_t$$

  where $\widehat{u}_t$ denotes least square residuals. The figures in parenthesis give the associated standard errors.
- Third, she has additionally estimated the following equation:

  $$\Delta f_t = \underset{(0.004)}{0.019} - \underset{(0.032)}{0.106} \widehat{u}_{t-1} + \widehat{\varepsilon}_t,$$

  where Δf_t is the first difference of f_t and $\widehat{\varepsilon}_t$ denotes least squares residuals. $\widehat{u}_{t\ \ 1}$ is a regressor formed by the lagged residuals of the least squares regression of f_t on p_t and a constant.

(a) The researcher is concerned that the regression of f_t on p_t might be spurious. What is the spurious regression problem? What information would you need regarding $\widehat{u}_t$ in order to dispel this problem? What does this imply for f_t and p_t and the *weak* version of PPP theory?

(b) Suppose the researcher was only interested in testing the *strong* version of PPP theory. Suggest an alternative testing strategy that allows the researcher to test whether the strong version of PPP holds in the data.

(c) Assume that the weak version of PPP theory holds. Give an economic interpretation to the slope parameter in the last equation estimated by the researcher.

(TURN OVER)

4. A widely publicized study in Australia found that highly educated unionized workers received lower wages than highly educated non-union workers. Researchers arrived to this conclusion by estimating the following Mincer equation on a sub-sample of unionized workers:

$$\ln Wage_i = \alpha + \beta_1 Education_i + \beta_2 Age_i + \beta_3 Gender_i + \beta_4 Experience_i + \varepsilon_i$$

under the assumption that the covariates on the right hand side of the equation are independent of the disturbance ε_i. Further it is assumed that ε is normally distributed with mean zero and variance σ_ε^2. The study found that OLS estimates of $\widehat{\beta}_1$ in this sample was smaller than what was typically estimated by other researchers using data on non-unionized workers.

(a) Assume union membership is compulsory and based on the following simple rule: all workers with more than one year of *Experience* are required to join a union by law. Are the estimates of this study on wages of union-workers likely to suffer from selection bias?

(b) Now assume that union membership is voluntary and researchers know that experienced workers with lower levels of motivation are more likely to join a union. In particular, they know that whenever

$$\delta + \gamma_1 Experience_i + \gamma_2 Motivation_i + \nu_i > 0$$

the worker decides to become a member of the union. ν_i is an independent and normally distributed random disturbance with mean zero and variance σ_ν^2. Further, the researchers know that motivation is also a determinant of compensation. That is, they know that the true model for wages is given by:

$$\ln Wage_i = \alpha + \beta_1 Education_i + \beta_2 Age_i + \beta_3 Gender_i + \beta_4 Experience_i + \beta_5 Motivation_i + \epsilon_i$$

where ϵ_i is a normally distributed random variable which is independent of ν_i (and all other covariates). Unfortunately, researchers did not have access to any proxy for motivation and therefore proceeded to estimate the Mincer equation presented above. Prove that the estimates presented by the researchers are biased.

(c) Under the voluntary union membership case, assume further that in the population (i.e. across all workers, unionized and non-unionized), experience and motivation are uncorrelated. In the sample of union workers, do you expect these to remain uncorrelated, positively correlated or negatively correlated? Why is this related to the presence of selection bias?

(d) The arguments in (b) and (c) were noted to the researchers, who were advised to address them with Heckman's two-step procedure. In their reply they indicate that, in the absence of exclusion restrictions, they are skeptical the procedure would be able to deliver unbiased estimates. Discuss the researchers' reply.

5. In many countries, disability insurance (DI) is granted to people with physical or mental conditions limiting their ability to work. Not all applicants to DI are approved, however. The approval of a DI application is based on health conditions and other characteristics of the applicant that are often unobserved to the econometrician. Suppose you are interested in estimating the effect of disability insurance approval on hours of work, for a sample of people who applied for DI. Let y_1 be the hours of work if an applicant receives the DI, and y_0 be the hours of work if the applicant does not receive the DI. Let $DI = 1$ if the applicant's DI application is approved, and $DI = 0$ if the applicant's DI application is denied.

(a) Explain, in words, the economic meaning of $E(y_0|DI = 1), E(y_0|DI = 0)$. Which of these two is easily estimated given your data? Which one of them is more likely to be larger in our context? Explain your answers.

(b) Suppose we are interested in estimating the average treatment effect on the treated: $\alpha = E(y_1|DI = 1) - E(y_0|DI = 1)$. Show the bias in using $E(y_1|DI = 1) - E(y_0|DI = 0)$ to estimate α.

(c) Suppose now you have a panel data of 2 years. In the first year, no one is in the DI program. In the second year, some people are granted the DI and the others' DI applications are rejected. Hours of work are observed in each year. Discuss how the panel structure of the data may help you reduce the bias you showed in (b). Be specific about the assumptions you make.

(d) Suppose you only have data on people who applied for DI in a single year. You are given one additional variable: for each applicant, you observe the type of case worker assigned to the applicant. Case workers are of one of the following two types: some are lenient and some are strict, and the assignment of case workers to the applicant is random (determined by a lottery). Discuss how the additional variable in the data may help you reduce the bias you showed in (b). Be specific about the assumptions you make.

6. An econometrician, Robert, collects data containing total employment (L) and average wages (W) for all the counties in the UK. Robert considers the following model of supply and demand for labor:

$$D_i = \alpha_0 + \alpha_1 W_i + \epsilon_i, \quad (2)$$
$$S_i = \beta_0 + \beta_1 W_i + \mu_i \quad (3)$$

where D_i, S_i and W_i are labor demand, labor supply and wages in county i. The parameters of interest are α_1. Robert assumes that the labor market in each county is independent and competitive, so labor market clears ($D_i = S_i$) for any county i and $\{\epsilon_i, \mu_i\}$ are i.i.d..

(a) Draw a simple labor demand and supply diagram for a given county i and derive its equilibrium wage, W_i.

(b) To estimate labor demand (equation (2)), Robert regresses L_i on W_i and obtains the least square estimator ($\widehat{\alpha}_1$) for α_1. Is $\widehat{\alpha}_1$ unbiased? If so, provide a proof. If not, derive the bias of the least square estimator for α_1.

(c) Suppose that, in addition to variables on employment and wages, Robert observes the number of new migrants in a county (M_i) in his data. Assume that $E(\mu_i|M_i) = E(\epsilon_i|M_i) = 0$. Can Robert use the additional variable on migrants to improve the previous estimates of α_1, and if so, how? Please explain your answers carefully.

(d) Is it possible to identify β_1 with Robert's dataset containing information on M_i, L_i, and W_i? Why? How would you advise Robert to estimate β_1? Be clear on any assumptions you are making.

Figure 1:

Table 5.2.3: Effect of labor regulation on the performance of firms in Indian states

	(1)	(2)	(3)	(4)
Labor regulation (lagged)	-0.186	-0.185	-0.104	0.0002
	(.0641)	(.0507)	(.039)	(.02)
Log development		0.240	0.184	0.241
expenditure per capita		(.1277)	(.1187)	(.1057)
Log installed electricity		0.089	0.082	0.023
capacity per capita		(.0605)	(.0543)	(.0333)
Log state population		0.720	0.310	-1.419
		(.96)	(1.1923)	(2.3262)
Congress majority			-0.0009	0.020
			(.01)	(.0096)
Hard left majority			-0.050	-0.007
			(.0168)	(.0091)
Janata majority			0.008	-0.020
			(.0235)	(.0333)
Regional majority			0.006	0.026
			(.0086)	(.0234)
State-specific trends	NO	NO	NO	YES
Adjusted R-squared	0.93	0.93	0.94	0.95

Notes: Adapted from Besley and Burgess (2004), Table IV. The table reports regression-DD estimates of the effects of labor regulation on productivity. The dependent variable is log manufacturing output per capita. All models include state and year effects. Robust standard errors clustered at the state level are reported in parentheses. State amendments to the Industrial Disputes Act are coded 1=pro-worker, 0 = neutral, -1 = pro-employer and then cumulated over the period to generate the labor regulation measure. Log of installed electrical capacity is measured in kilowatts, and log development expenditure is real per capita state spending on social and economic services. Congress, hard left, Janata, and regional majority are counts of the number of years for which these political groupings held a majority of the seats in the state legislatures. The data are for the sixteen main states for the period 1958-1992. There are 552 observations.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Thursday 8 June 2017 1.30pm - 3.30pm

M500
DEVELOPMENT ECONOMICS

Answer **two out of three** questions. Each question carries equal weight.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1(a) Ethnic migrant networks drawn from different European origin countries historically competed with each other in the U.S. labor market. There is a positive relationship between ethnic-network competition in 1850 and participation in ethnic churches in 1860 at the county level. The relationship between ethnic-network competition in 1850 and church participation grows stronger as we move forward in time, over the course of the twentieth century, long after the ethnic networks themselves had ceased to be relevant. Provide an explanation for this observation.

(b) A cohort of high school seniors (aged 18) in a representative sample of U.S. counties were surveyed in 1980 and subsequently followed over their working lives. What relationship would you expect between historical ethnic competition in their birth county and their occupational and migration choices? What are the consequences of these choices for economic efficiency and inequality?

2(a) There is a positive relationship between the population density of rural counties in the Punjab in 1931 and the number of individuals that migrated from those counties to the U.K. in the 1950's (when there were no governmental restrictions on immigration). Social connectedness is increasing in spatial proximity, which is increasing in the population density of a rural population. Can we conclude from the observed relationship between population density and migration that migrant networks are active?

(b) Closer inspection of the data indicates that the positive relationship between population density in the origin county and migration is only obtained for counties with population densities that exceed a threshold level. Below the threshold, there is no relationship between population density and migration. Explain this observation with the help of a repeated-game model of migrant networks. How would you use the observed discontinuity to provide additional support for the claim that migrant networks are active?

3(a) Individuals born in rural Chinese counties can either choose a wage occupation in their birth county or move to one of two available urban destinations and set up a business. It is observed that individuals born in higher population density counties are more likely to select into business. Population density is positively associated with spatial proximity, which, in turn, is positively associated with social connectedness. One explanation for the observed occupational choice is thus that higher population density gives rise to stronger business networks. Develop a test based on the relationship between population density in the birth county and the distribution of businesses across the two urban destinations to provide additional support for this claim, being careful to make precise the assumptions underlying the test that you propose.

(b) Hsieh and Klenow in a series of well known papers provide empirical evidence that there is greater dispersion in firms size within industries in China than in the United States. They also show that in the same industry, firms are smaller in China than in the United States. They argue that this evidence is indicative of a misallocation of resources in China and that output would increase if these distortions were eliminated. Comment on this argument in light of the fact that business networks, organized around different birth counties, are active in China.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM11
MPhil in Finance and Economics

Tuesday 10 January 2017 10:00-12:00

F100/F200
FINANCE I/II

You should answer all **four** questions

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

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SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1. The common stock of General Dynamics is currently selling for \$94.50 per share. The stock's current dividend is \$2.40 per share (assume that this will be paid in one year). The firm's Price-to-Earnings (P/E) Ratio is 8. Assume that the appropriate discount rate, or opportunity cost of capital, for General Dynamics is 12%.

1. Using the Gordon Growth model, what is the growth rate, g, implied by General Dynamic's stock and the above information?
2. What is the return on equity (ROE) implied by this value of g and the above information?

Question 2. The standard deviation of returns on the market portfolio is $\sigma_m = 20\%$. Consider a portfolio p which is (i) well-diversified and (ii) has a beta equal to $\beta_p = 1.5$. Calculate the standard deviation of returns on this portfolio σ_p, explaining how you derive your result.

Question 3. A firm has no debt in its capital structure and the expected return on its equity is 10%. The rate of return on government bonds is 4% and the expected market risk premium is 8%. Ignore corporate taxes.

The firm is considering a new capital structure with 60 percent debt. The debt would become risky so that the beta of the debt would equal 0.15. What would be the beta of the equity?

Question 4. The United Construction Co. (UCC) currently has a capital structure of 20% debt to total assets, based on current market values. The current debt is riskless and more debt can be taken on, up to a limit of 35% debt, without making the debt risky and losing the firm's ability to borrow at 7%, the risk free rate. The expected return on the market is 15% and the beta of UCC's equity is 0.8 (at the current debt level).

1. According to the security market line, what is the expected rate of return on UCC equity?
2. What is the beta of UCC assets?
3. What will be the expected return on UCC equity if the company increases its leverage so that debt represents 35% of total assets?

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM11
MPhil in Finance & Economics

Tuesday 9 May 2017 2:00pm - 4:00pm

F100
FINANCE I

Candidates are required to answer **all four** questions.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1. A 4 percent coupon paying bond with maturity of 10 years has a price of \$855 whereas a 8 percent coupon paying bond with same maturity has a price of \$1,150. Coupon payments are semi-annual. What is the semi-annually compounded 10 year spot rate?

Question 2. Blast-Rock Corporation has a capital structure consisting (only) of common stock and corporate debt. The market value of Blast-Rock Corporation's common stock is \$40 million and the market value of its debt is \$60 million. The probability of default on debt obligations is essentially zero. The beta of the company's common stock is 0.8, the expected market risk premium is 10 percent and the Treasury bill rate is 6 percent. Ignoring corporate taxes, what is the firm's cost of capital?

Question 3. Today is Fabio's 5^{th} birthday. His father intends to start saving a fixed amount every month for his university. He will start saving at the end of this month. He will each time place these savings in a bank account that pays an interest of 4 percent per annum with monthly compounding. Will there be 200 times as much as one monthly payment accumulated on the bank account by Fabio's 18^{th} birthday?

Question 4. The CAPM provides the expected return on an asset:

$$E\left[\tilde{r}_i\right] \;=\; r_f \;+\; \beta_{i,m}\left(E\left[\tilde{r}_m\right] \;-\; r_f\right)\;.$$

Formally derive that the expression of $\beta_{i,m}$ is

$$\beta_{i,m} \;=\; \frac{\sigma_{i,m}}{\sigma_m^2}\;.$$

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM11
MPhil in Finance & Economics

Wednesday 24 May 2017 1:30pm - 3:30pm

F200
FINANCE II

Candidates are required to answer **all four** questions..

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1. A stock price is currently 50. Over each of the next two three-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per annum with continuous compounding.

(a) What is the value of a six-month European call option with a strike price of 51?

(b) What is the value of a six-month European put option with a strike price of 51?

(c) Verify that the European call and European put satisfy put-call parity.

(d) If the put option were American, would it ever be optimal to exercise it early at any of the nodes on the tree.

Question 2. The standard deviation of annual returns on the market portfolio is $\sigma_m = 0.25$. Consider a rather risky asset i with a beta equal to $\beta_p = 1.3$ and a standard deviation of annual returns which is double that of the market portfolio, i.e. $\sigma_i = 0.5$. What percentage of the total risk of the risky asset i is diversifiable?

Question 3. Consider two firms, U and L:
– The unlevered firm U is financed by equity only. Denote E_U the market value of firm U's equity. The market value of firm U is $V_U = E_U$.
– The levered firm L is financed by debt and equity. It maintains a debt level D_L and pays a perpetual expected interest rate r on this debt. Denote D_L and E_L the market value of firm L's debt and equity. The market value of firm L is $V_L = D_L + E_L$.
– At date $t = 0$, the two firms only differ in their capital structure, and are expected to generate an identical series of cash flows C_t in year t, for $t \geq 1$.

Provide a formal arbitrage proof of the Miller-Modigliani Proposition, which states that $V_U = V_L$.

Question 4. Consider a firm whose balance sheet is as follows:

Assets	**Liabilities**
Liquid Assets: X_0 Fixed Assets: A	Bank Debt: B_0 due at $t = 0$ Public Debt: λP due at $t = 0$ Public Debt: $(1 - \lambda)P$ due at $t = 1$

A fraction ϕ_0 of the public debt is as senior as the bank. The other $(1 - \phi_0)$ is junior to the bank.

The firm has the following investment opportunity:
– At $t = 0$, investment outlay of F required.
– At $t = 1$, project generates $X \in [0, +\infty)$ with c.d.f. $H(X)$.
– Value (possibly negative): $V = \mathrm{E}[X] - F$.

The firm is however (i) in financial distress, i.e. $X_0 < B_0 + \lambda P + F$, and (ii) insolvent (assets < liabilities), i.e. $X_0 + A < B_0 + P$. As a consequence, in liquidation, public debtholders would receive

$$L_0^P = \begin{cases} \left(\frac{\phi_0 P}{B_0 + \phi_0 P}\right)(A + X_0) & \text{if } A + X_0 < B_0 + \phi_0 P \\ A + X_0 - B_0 & \text{otherwise.} \end{cases}$$

We are going to examine restructuring the bank debt, considering (i) that public debtholders are passive in restructuring and (ii) frictionless bargaining between the bank and equityholders:

In continuation, the bank issues an amount of new debt due at $t = 1$

$$B_1 = F - X_0 + B_0 + \lambda P.$$

A fraction ϕ_1 of the public debt is as senior as the new bank debt. The firm can undertake the investment (investing liquid assets X_0), therefore, total debt due at $t = 1$ has face value

$$D_1 = B_1 + (1 - \lambda)P = F - X_0 + B_0 + P,$$

and the face value of senior debt is

$$S_1 = B_1 + \phi_1(1 - \lambda)P = D_1 - (1 - \phi_1)(1 - \lambda)P.$$

Assuming that the new debt is risky, i.e., $A < D_1$, public debtholders therefore receive (at date $t = 0$ plus at date $t = 1$):

$$\lambda P + \begin{cases} (1 - \lambda)P & \text{if } X + A \geq D_1 \\ L_1^P(X) & \text{if } X + A < D_1 \end{cases}$$

where

$$L_1^P(X) = \begin{cases} \left(\frac{\phi_1(1-\lambda)P}{S_1}\right)(A + X) & \text{if } A + X < S_1 \\ A + X - B_1 & \text{otherwise.} \end{cases}$$

(a) Determine the condition under which restructuring will occur.

(b) Describe the possible investment distortions it would lead to.

(c) Establish the impact of the priority of existing bank debt on investment incentives.

END OF PAPER

UNIVERSITY OF
CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Thursday 25 May 2017 9:00am -11:00am

F300
CORPORATE FINANCE

Candidates are required to answer **all four** questions.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1

A company maintains a constant debt to value ratio of 1/2 and faces equity cost of capital $r_E = 10\%$, debt cost of capital of $r_D = 6\%$ and pays corporate taxes at rate $\tau_C = 40\%$. The company is considering an investment that requires an initial outlay of \$100 million. The investment will increase the company's free cash flow by \$5 million the first year, and this contribution is expected to grow at a rate of 3% every year thereafter. The company will maintain a constant debt to value ratio for this investment.

a. Using the WACC method, calculate the net present value of this investment.

b. How much additional debt must the company issue to finance the investment and maintain a debt to value ratio of 1/2 and how much equity must it issue? Explain your reasoning.

c. Assume that to finance the investment, the company issues new debt of \$65.8 million. Using the APV method, calculate the net present value of this investment.

Question 2

An entrepreneur has a project that can either succeed or fail, with each outcome being equally likely. If it succeeds, the project is worth \$150m while if it fails, it is worth only \$80m. The project's cash flows are unrelated to those of the economy (so the risk is diversifiable) and the risk-free rate is 5%. The company has 10m shares outstanding.

a. Calculate the value of the firm's securities at the start of the year if it is (i) all equity financed and (ii) financed by \$100m in debt. Compare the values of the firm under these capital structures and briefly comment.

b. Now assume that in case of default, financial distress costs reduce the value of the firm by \$20m. Compute the value of the firm when it is (i) all equity financed and (ii) financed by \$100m in debt. Compare the values of the firm under these capital structures and briefly comment.

c. Briefly discuss who bears the costs of financial distress and what the implications for the optimal capital structure are if there are no other market imperfections.

Question 3

Consider two companies with identical assets and cash flows. Company U is an all equity firm and has 1m shares outstanding and trades at \$24 per share. Company L has 2m shares outstanding and \$12m in debt. The debt cost of capital is 5%. Assume that capital markets are perfect. In particular, you can borrow and lend at the same rate as the companies.

a. Compute the value of the equity of Company L. What is Company L's share price?

b. You have \$5000 to invest and you plan to buy shares in Company U. Explain how, using homemade leverage, you can create a portfolio that yields the same return as a \$5000 investment in Company L.

c. You have \$5000 to invest and you plan to buy shares in Company L. Explain how, using homemade (un)leverage, you can create a portfolio that yields the same return as a \$5000 investment in Company U.

Question 4

Consider an agent who can choose between three different investment projects. The investment cost of a project is $I > 0$ but the agent only has net worth $A < I$. If a project is successful (which happens with probability p), it yields $R > 0$, while if it is a failure (which happens with probability $1 - p$) it yields 0. The success probability p is project-specific.

The projects differ in success probability and in private benefit for the agent: The *good* project succeeds with probability p_H and yields no private benefit. The *intermediate* project succeeds with probability p_L and yields private benefit b. The *bad* project succeeds with probability p_L and yields private benefit $B > b$.

The agent's choice of project is unobservable (unless it is monitored, as described below). Consider a monitor who can, at private cost $c > 0$, identify and rule out the bad project. The monitoring activity of the monitor is unobservable. Assume that the market for monitors is perfectly competitive.

Consider sharing rules

$$R = R_b + R_l + R_m$$

where R_b, R_l and R_m are the shares of the borrower, the lender and the monitor respectively, in case of success. All parties are protected by limited liability.

Make the following assumptions:

Assumption 1:

$$p_H \frac{b}{\Delta p} + c < p_H \frac{B}{\Delta p}$$

where $\Delta p \equiv p_H - p_L > 0$.

Assumption 2:

$$p_H R > I + c$$

a. (i) Write down the monitor's incentive compatibility constraint. (ii) Write down the agent's incentive compatibility constraint, assuming that the agent is monitored. (iii) Write down the lender's participation constraint, assuming that the two incentive compatibility constraints are satisfied and denoting by I_m the monitor's contribution towards the funding of the project.

b. (i) Show that if $I_m = 0$, then the monitor earns a positive expected rent. (ii) How large should I_m be for the monitor to break even?

c. (i) Compute the agent's net utility under monitoring and compare it to the net utility that it would obtain if it were funded without monitoring. (ii) Under which conditions on net worth would the agent employ a monitor? (iii) Does monitoring make funding easier to obtain?

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research
MPhil in Finance

Tuesday 23 May 2017 1:30pm - 3:30pm

F400
ASSET PRICING

Candidates are required to answer **all four** questions.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Suppose that there are several investors and several assets. Suppose further that investors have constant absolute risk aversion, and that the returns on the assets are jointly normally distributed.

   (a) Find a formula for the asset demands of an individual investor.

   (b) Explain the main features of this formula.

   (c) Taking the formula from part (a) as your starting point, derive the security-market line (SML).

   (d) Explain the main features of the SML.

2. Suppose that there are two risky assets: a safe asset and a risky asset. The initial price of the risky asset is 20. In each period there is a shock $s \in \{d, u\}$. If $s = d$ then the price falls by 10%. If $s = u$ then the price rises by 10%. The initial price of the safe asset is 1. In each period the price rises by 5%.

   (a) What is meant by a put option with strike price 22 and maturity 2 in this setting?

   (b) Assuming that there is no arbitrage, what is the price of this option at time 0?

3. (a) Write down a model of the dynamics of spot rates. Explain carefully the meaning of the various elements of the model.

   (b) In the context of your model, what does it means to say that the term structure of bond prices is affine? Why is it considered to be "affine"?

   (c) Assuming that the term structure of bond prices is indeed affine in this sense, find the stochastic differential equation followed by the price of a T-bond.

4. There are three assets: a safe asset and two risky assets. The prices B, R and S of the safe asset, risky asset 1 and risky asset 2 evolve according to the s.d.e.s

$$
\begin{aligned}
dB &= r\,B\,dt, \\
dR &= \mu_R\,R\,dt + \sigma_R\,R\,dZ, \\
dS &= \mu_S\,S\,dt + \sigma_S\,S\,dZ
\end{aligned}
$$

respectively, where Z is a one-dimensional standard Wiener process. There is also a derivative written on the first asset that pays $\Phi(R_T)$ at maturity $T > 0$. There is no arbitrage.

(a) Find the equation satisfied by the price of the derivative. Be careful to explain any assumptions that you make, and to set out the main steps of your derivation carefully.

(b) What conditions must μ_R, μ_S, σ_R and σ_S satisfy? Explain carefully how your arrived at these conditions.

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Monday 29 May 2017   1:30pm - 3.30pm

F500
EMPIRICAL FINANCE

This paper is in two sections. Section A: Candidates are required to answer **all four** questions. Section B: Candidates are required to answer **two out of three** questions.

Each section carries equal weight.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Section A

1. Suppose that observed closing (log) stock prices follow the random walk

$$p_t = \mu + p_{t-1} + u_t$$

$$u_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

(a) For what values of θ is this model consistent with the Efficient Market Hypothesis?

(b) For what values of θ is this model consistent with the empirical evidence on daily returns as presented in Table 2.4 of Campbell, Lo and Mackinlay, attached?

(c) What are some market microstructure explanations for this finding?

(d) Define the variance ratio

$$VR(k) = \frac{\text{var}(p_{t+k} - p_t)}{k \times \text{var}(p_{t+1} - p_t)}.$$

What is $VR(k)$ as a function of θ?

2. True, False, Explain

(a) A portfolio hedged against factor risk always has zero risk

(b) In the absence of noise traders, bid ask spreads would be narrower, and so markets would be more efficient.

(c) The long run risks model says that at plausible values for the preference parameters (IES and RRA), a reduction in economic uncertainty or better long-run growth prospects leads to a rise in the wealth–consumption and the price–dividend ratios.

(d) Stock returns are more volatile during Wednesdays than Mondays. This is incompatible with market efficiency

3. Assume that fundamental price P^* is a random walk unrelated to order flow

$$P_t^* = P_{t-1}^* + \varepsilon_t.$$

Suppose that buy and sell orders arrive randomly each period, and let I_t denote the trade direction indicator, +1 for buy and -1 if customer is selling. Assume that I_t is iid with equal probability of +1 and -1 and unrelated to P_t^*. Suppose that the bid ask spread is s so that the transaction price each period P_t satisfies

$$P_t = P_t^* + I_t\frac{s}{2}.$$

Then derive an expression for $\text{cov}\left[\Delta P_{t-1}, \Delta P_t\right]$ in terms of s. What is *Bid ask bounce?* How could one obtain an estimate of the bid ask spread based on transactions data only?

4. The linear factor model for assets returns R_i says that

$$R_i = \alpha_i + \beta_i^\intercal f + \varepsilon_i,$$

where $f \in \mathbb{R}^K$ are common factors and ε_i are idiosyncratic shocks with mean zero, variance σ_i^2 and uncorrelated across i, i.e., $\mathrm{cov}(\varepsilon_i, \varepsilon_j) = 0$ for $i \neq j$. Here, $\beta_i^\intercal = (\beta_{i1}, \ldots, \beta_{iK})$. True false, explain:

(a) The covariance between observed returns under the linear factor model is for all $i \neq j$

$$\mathrm{cov}(R_i, R_j) = \beta_i^\intercal \Sigma_f \beta_j,$$

where Σ_f is the covariance matrix of the factors.

(b) The Arbitrage Pricing theory restricts the covariance matrix of observed returns to not depend on the vector of $\alpha_i's$.

(c) The Arbitrage Pricing theory restricts the covariance matrix of observed returns to not depend on the vector of $\sigma_i^{2\prime}s$.

(d) If the factors are unobservable, the APT is untestable

Section B

1. The consumption CAPM (when utility is CRRA) says that

$$E_t \left[(1 + R_{it+1}) \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} - 1 \right] = 0$$

where C_t is aggregate consumption at time t and R_{it} are the returns on asset i at time t, E_t denotes expectation at time t based on all available information. What are the parameters γ, δ? Explain how you might construct a test of the CAPM based on this equation. Under the additional assumption that log consumption growth and log equity market return are jointly normal we have

$$0 = E_t \left[r_{i,t+1} \right] + \log \delta - \gamma E_t \left[\Delta c_{t+1} \right] + \frac{1}{2} \left[\sigma_i^2 + \gamma^2 \sigma_c^2 - 2\gamma \sigma_{ic} \right],$$

where r_{it} are logarithmic returns and $c_t = \log C_t$ and: $\sigma_{ic} = \mathrm{cov}_t(r_{i,t+1}, c_{t+1})$, $\sigma_i^2 = \mathrm{var}_t(r_{i,t+1})$, $\sigma_c^2 = \mathrm{var}_t(c_{t+1})$. Describe what is the Equity premium puzzle. What are the possible explanations for this puzzle.

2. The Sharpe-Lintner CAPM predicts that stock returns obey

$$E(R_i - R_f) = \beta_i E(R_m - R_f)$$

$$\beta_i = \frac{\mathrm{cov}(R_i, R_m)}{\mathrm{var}(R_m)} = \frac{\mathrm{cov}(R_i - R_f, R_m - R_f)}{\mathrm{var}(R_m - R_f)},$$

where R_m is the random return on the observable market portfolio and R_f is the return on an observable risk-free asset. In practice, we have sample of data $\{R_{it}, R_{ft}, R_{mt}, i = 1, \ldots, n, t = 1, \ldots, T\}$. Describe several methods for testing this model. In your account be careful to say whether you are working with individual stocks or portfolios, what the null and alternative hypotheses are, and what are the test statistics. How is this model useful in conducting event studies?

3. Suppose that daily stock returns satisfy

$$r_t = \mu + \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \gamma r_{t-1}^2,$$

where ε_t is iid standard normal and independent of all past information. What properties of daily stock returns does the GARCH model capture? Describe how you would estimate the parameters $\mu, \omega, \beta, \gamma$ given a sample of data $\{r_1, \ldots, r_T\}$. Compare the resulting estimate of volatility σ_t^2 with the realized volatility estimator and with the implied volatility estimator. Describe in each case how the estimates are constructed, their interpretations, and their properties.

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Wednesday 7 June 2017   1:30pm - 3:30pm

F510
INTERNATIONAL FINANCE

Candidates are required to answer **four out of five** questions.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

QUESTION 1 **Uncovered Interest Parity**

(a) Define the "Uncovered Interest Parity" (UIP). State briefly but precisely how it is tested using regression models, and in which way it fails to hold empirically.

(b) The empirical failure of the UIP is often referred to as the "UIP puzzle". Consider the hypothesis that this puzzle can be attributed entirely to omitting a risk-premium from the regression model.

- Under what conditions would the risk premium help explain the UIP puzzle? Discuss the results by Fama 1984.
- If Fama 1984 is right, what other empirical puzzle would arise? Explain carefully.

QUESTION 2 **Covered Interest Parity**

Suppose that international investors attribute a small probability to the event of another large global crisis. They also believe that, if and when this new global crisis occurs, the US would tax interest income on US financial assets owned by foreigners, at a constant tax rate τ.

(a) Define the cross-currency basis of a foreign currency vis-à-vis the dollar. Derive and discuss the implications of the prospective taxation for the cross-currency basis. In answering this question, make use of the asset pricing equation analyzed in class and explain carefully your results.

(b) Would anticipation of solvency problems (credit risk) among US financial intermediaries make a difference to your answer? (Hint: in addressing this question, it is analytically convenient to ignore the prospective tax discussed in (a)).

QUESTION 3 **The Peso problem**

Explain briefly but rigorously the "peso problem" and its applications to the main puzzles in international finance.

QUESTION 4 **Risk sharing and sovereign risk**

Consider a stylized two-period model of a small open economy populated by many individuals i, each with life-time utility and endowments

$$\ln(c_{i,0}) + E\ln(c_{i,1}),$$

$$y_{i,0} = 0.5, \quad \text{and} \quad y_{i,1} = \begin{cases} 1.80 & \text{with probability } 1/2, \\ 1.20 & \text{with probability } 1/2. \end{cases}$$

Individuals are ex-ante identical, but face individual income risk ex-post. In period 0, all individuals have the same income $y_{i,0} = 0.5$; in period 1, an individual i can be 'lucky' and get $y_{i,1} = 1.80$, or 'unlucky' and get $y_{i,1} = 1.20$, each with probability 1/2.

In period 0, individuals are able to issue liabilities $\widetilde{b}_{i,0}$ promising a cash flow that is contingent on the realization of their income. Namely, an individual debt contract stipulates that, in period 1, she will have to pay back either x_i^H if she turns out to be lucky or x_i^L if she turns out to be unlucky. International investors are risk neutral and do not discount the future, so that $\widetilde{b}_{i,0}$ will be equal to the expected payoff:

$$\widetilde{b}_{i,0} = E_0(x_i).$$

Answer the following:

(a) Define the real rate in financial autarky and compare it with the world market rate: will the country be a net importer or a net exporter of financial assets in period 0?

(b) Write down the individual i consumption-saving problem. What debt contracts would you expect to observe in equilibrium if

(i) international contracts are enforceable by some supranational authority, and individual incomes are verifiable?

(ii) international contracts are not enforceable but incomes are verifiable. However, if an individual defaults she/he has to endure costs for $k = 0.5$.

(iii) contracts are not enforceable and income not verifiable, so that international investors are not willing to lend against contingent payment (hint, set $x_i^H = x_i^L$).

For each case (i) (ii) (iii) above, calculate $\widetilde{b}_{i,0}$, the cash flow x_i^H and x_i^L and the allocation of consumption. Explain you answer discussing briefly the efficiency of the consumption allocation.

(d) Suppose individuals residing in the country revise their income expectations in the future—$\overleftrightarrow{y}_{i,1}$ will be a mean preserving spread of $y_{i,1}$

$$\overleftrightarrow{y}_{i,1} = \begin{cases} 2 & \text{with probability } 1/2, \\ 1 & \text{with probability } 1/2. \end{cases}$$

Compare the cases (i) and (iii) above: In which case will agents change the amount of liabilities they issue in period 0? Carefully explain your answer.

(e) How would you answer to b(ii) above change if $k = 0.4$?

QUESTION 5 **US Monetary Tightening, Financial Fragility and Currency Crises**

Suppose a small open emerging economy is described by the following Aghion, Bacchetta and Banerjee (2000) model. Money market equilibrium is given by

$$\frac{M_t}{P_t} = L(Y_t, i_t)$$

where M_t denotes the nominal money supply, P_t the aggregate price level, Y_t the level of aggregate real output, i_t the nominal interest rate, and the subscript t the time period. Real money demand $L(.)$ satisfies $\partial L/\partial Y > 0$, $\partial L/\partial i < 0$ and $L(0, i) > 0$. Equilibrium in the foreign exchange market is determined by uncovered interest parity:

$$1 + i_t = (1 + i^*) \frac{\mathrm{E}_t [S_{t+1}]}{S_t}$$

where i^* is the US nominal interest rate, and S_t the nominal exchange rate, defined as the domestic price of US dollars. The aggregate price level P_t is preset at the end of period $t-1$. Purchasing power parity holds ex ante so that

$$P_t = \mathrm{E}_{t-1} [S_t]$$

where the foreign price level has been normalized to $P^* = 1$.
Output is produced with capital K_t according to the linear production technology

$$Y_t = AK_t$$

where $A > 0$ denotes productivity. Capital accumulation is determined by

$$K_t = B_t + F_t$$

where B_t is the domestic firm's short term debt, and F_t retained cash flow. The firm faces external financing constraints in the form of a borrowing limit

$$B_t = e^{-\beta i_{t-1}} F_t$$

where $\beta > 0$. Furthermore, the size of the domestic credit market is limited to $\bar{B} > 0$ and any additional borrowing has to take place in US dollars. Assume that $B_1 > \bar{B}$. Retained cash flow depends on past real profits and equals

$$F_t = \max\{0, (1 - \alpha) \Pi_{t-1}/P_{t-1}\}$$

where $0 < \alpha < 1$ and Π_t denotes nominal profits after debt repayments:

$$\Pi_t = P_t Y_t - (1 + i_{t-1}) P_{t-1} \bar{B} - (1 + i^*) \frac{S_t}{S_{t-1}} P_{t-1} \left(B_t - \bar{B}\right)$$

The economy starts from a long run equilibrium in period $t = 0$. Assume that initially, $\frac{1+i^*}{1+i_1} \frac{M_2}{L(0,i_2)} < \frac{P_1 AK_1 - (1+i_0) P_0 \bar{B}}{(1+i^*)\left(B_1 - \bar{B}\right)}$.

(a) Show that asset market equilibrium implies

$$S_1 = \frac{1+i^*}{1+i_1} \frac{M_2}{L(Y_2, i_2)} \tag{IPLM}$$

Explain intuitively how the domestic nominal interest i_1, US nominal interest rate i^* and future domestic output Y_2 affect the nominal exchange rate S_1.

(b) Show that future output Y_2 satisfies

$$Y_2 = \max\left\{0, A\left(e^{-\beta i_1}+1\right)(1-\alpha)\left[AK_1 - (1+i_0)\frac{P_0}{P_1}\bar{B} - (1+i^*)\frac{S_1}{P_1}\left(B_1 - \bar{B}\right)\right]\right\} \tag{FC}$$

Give an intuitive explanation for the effect of the domestic nominal interest rate i_1, US nominal interest rate i^* and the nominal exchange rate S_1 on future output Y_2.

(c) Suppose the economy faces a higher US nominal interest rate i^*. Show graphically and explain intuitively how this affects the nominal exchange rate S_1 and future output Y_2. Could the higher US interest rate lead to a currency crisis in the emerging economy?

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Tuesday 30 May 2017 1:30pm - 3:30pm

F520
BEHAVIOURAL FINANCE

Candidates are required to answer **two out of three** questions

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

QUESTION 1

a. Describe differences and similarities of decision making under Expected utility model and Prospect Theory.

b. Explain what are the main theoretical challenges in testing the Market Efficiency Hypothesis? Fully elaborate your answer.

c. Describe the concept of Adaptive Market Hypothesis.

QUESTION 2

a. Can herding behaviour be rational? Discuss. Provide some examples to support your answer.

b. Describe main intuition why herding can lead to market inefficiency. Provide some examples to support your answer.

c. Explain how risk-shifting behavior could cause occurrences and subsequently bursts of financial bubbles. Fully justify your arguments. Provide some examples that support your arguments.

QUESTION 3

a. Large uninformed trades in the stock market typically cause large price impacts (an example of uninformed trades would be fire sales or trades of index tracing fund due to inclusion of a new stock into a stock index). Explain the role of limits to arbitrage in existence of those price impacts. Should we observe the price impacts in markets with active arbitrageurs who face no constraints or risks in executing arbitrage strategies?

b. The closed-end fund discount is typically used as a proxy for noise traders' risk. Provide the rationale and intuition for this argument. Given that we typically observe closed-end fund discounts rather than premiums, does it mean that noise traders are mostly pessimistic?

c. Demonstrate how ambiguity aversion can lead to excess volatility. Consider a case of the model developed in Dow and Werlang (1992) (Excess volatility of stock prices and Knightian uncertainty, European Economic Review). Specifically, consider a two-period model with a single stock traded in the market and risk-free rate is equal to zero. At the end of the second period, the stock pays a liquidating dividends V. There are there are 3 states of nature; the value of V in states 1, 2 and 3 are 1, 0.5 or 0 respectively. All agents are identical and risk neutral and makes the decisions according to the multiple prior model (Choquet Expected utility). In period t = 1 agents receive public information about the value of the asset represented by a partition I = {{1},{2; 3}} of the set of states. Assume that the true objective probability on the space of states is P({1})=0.5 and P({2})=P({3})=0.25. Assume agents do not know the exact probability and their beliefs and attitudes towards uncertainty are represented by the lower envelope of the following three probability measures: P, Q and O, where Q({1})=Q({2})=0.25 and Q({3})=0.5 and O({1})=O({3})=0.25 and O({2})=0.5. Provide necessary analysis.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance & Economics
MPhil in Economic Research

Friday 9 June 2017 1:30pm - 3:30pm

F540
TOPICS IN APPLIED ASSET MANAGEMENT

This paper is in two sections:
Section A: candidates are required to answer **one** compulsory question.
Section B: Candidates are required to answer **two out of four** questions.

Section A is weighted at 40% and section B is weighted at 60%.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

1. General Question

   (a) What is the minimal number of assets you would require in a $\frac{1}{N}$ portfolio in which each asset has identical statistical properties and the average covariance with other assets is 0.1 and the average variance is 0.5, in order to achieve a portfolio vol of 0.10 ? Provide a mathematical justification for your answer.

   (b) Ferson and Siegel(2001) have shown how the optimal weight function is changed when you take into account the fact that you can exploit a predictive model (conditional information) rather than use the unconditional moments in mean variance optimisation. Derive and explain the rationale behind their formula.

   (c) Explain briefly how mean variance portfolio weights can be computed using an Ordinary Least Squares Regression. The computational problems involved with MV weights in practice may be overcome through the use of relaxation methods such as used for resolving multicollinearity in regression; in particular explain what is meant by RIDGE regression, LASSO and ELASTIC NET.

   (d) Show that Long Horizon predictive regressions (with a positive coefficient) where the predictor variable is persistent will necessarily imply the regression coefficient grows with the horizon. Similarly show, even if returns are independent, that the $R^2$and Sharpe Ratio will also necessarily grow with the horizon.

   (e) Examine whether portfolio volatility (with two assets) is sub-additive and translation invariant (or monotonic) and hence decide whether volatility is a coherent risk measure or not.

   (f) The case for regular portfolio rebalancing rather than adopting a buy and hold strategy is complicated in general but one argument for rebalancing back to constant weights is based on what is known as volatility harvesting. Explain this result, the assumptions on which it is based and potential limitations.

SECTION B

2. Mean Variance and Predictability

   (a) Discuss the conditions DeMiguel, Garlappi and Uppal(2009) derived analytically and by simulation to show when Mean Variance could dominate $\frac{1}{N}$? What further condition became apparent in their empirical results?

   (b) What principal critiques (with justification) can you make of their study and hence its relevance?

   (c) Explain *briefly* how Volatility Timing and Parametric Portfolios attempt to resolve the well known problems with implementing Mean Variance in practice

   (d) Forecast combination, Shrinkage (relaxation) methods and factor models have all recently been shown to deliver clear evidence of out of sample predictability unlike the results of Goyal and Welch(2008);

      i. Campbell and Thompson discuss the relationship between the R^2 in a predictive regression and the Sharpe Ratio from the resulting investment strategy- derive this relationship and discuss how it relates to the more general theoretical results we have considered which relate the predictive R^2 to the stochastic discount factor of an asset pricing model.

      ii. Why should Forecast Combination, Shrinkage or Factor methods therefore be expected to deliver greater profitability.

      iii. Describe why the Kitchen Sink regressions of Goyal and Welch failed and why forecast combination may effectively recover predictability and hence profitability in a mean variance certainty equivalent sense.

3. CAPM and APT

   (a) Compare and contrast the relative theoretical and practical advantages and disadvantages of APT and CAPM.

   (b) Outline the derivation of APT using a no-arbitrage argument or in some other way you have studied.

   (c) Compare the three basic types of factor models commonly employed.

   (d) Describe the Partial Least Squares (PLS) approach to factor construction and compare this with Principal Components as a means of constructing factors.

4. Trend Following and Betting against Beta

   (a) Describe the theoretical argument made to explain the Low Beta anomaly by Frazzini and Pedersen(2103) including why the Security Market Line pivots.

   (b) When and why does low beta investing not deliver abnormal returns?

(c) Discriminate between Time Series and Cross Section Momentum and derive the relationship between the expected returns to cross section momentum and time series momentum and hence why cross section momentum might need to be reconsidered as a strategy

(d) Explain the standard form a trend following strategy (Time Series Momentum) takes including how you would ensure a target annualised volatility through appropriate position sizing (Constant Volatility Trend Following); secondly explain why this strategy failed to perform in the period 2009-2013 and describe the steps through which performance was recovered by incorporating risk parity as suggested by Baltas(2015)UBS.

5. Risk Management

(a) Define and explain the relationship and relatives properties of Value at Risk, Expected Shortfall and Expectiles.

(b) Define the Information Ratio, Calmar Ratio, a CARE model and Maximum Drawdown

(c) What are the principle advantages and disadvantages of Historical Simulation when used as a method of computing Value at Risk and describe one method that may be used to overcome its deficiencies.

(d) Consider 100 independent defaultable bonds each with a current price of 100 and a default probability of 2% for each bond. When they don't default they deliver 105 at the end of the year. Now consider two portfolios, the first , portfolio A, is concentrated and made up of 100 units of only the first bond and the second portfolio, portfolio B, is completely diversified containing one unit of each bond. Examine the risk capital required when using a 95% VaR level for each portfolio and discuss the result you find. Explain your results and why they come about? In general, under what situations are you likely to find VaR fails to satisfy the conditions for coherency?

END OF PAPER

Faculty of Economics

MPhil/MRes/MPhil Finance and Economics

Preparatory Course Exam

29 September 2016

You have TWO hours

Answer ALL FOUR questions

Each question will carry equal weight.

Write your candidate number (not your name) on the cover of your answer booklet.

If you identify an error in this paper, please alert the invigilator.

Write legibly.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the invigilator

1.

(a) Let

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

Compute AB, and BA.

(b)

i. Prove that for a square matrix $A_{n\times n}$ the following holds:

$$tr(AA') = tr(A'A) = \sum_{ij} a_{ij}^2$$

ii. Find the eigenvalues of matrix A

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 5 \end{pmatrix}$$

iii. Determine the rank of matrix C below:

$$C = \begin{pmatrix} 3 & 0 & 1 & 2 \\ 6 & 1 & 0 & 0 \\ 12 & 1 & 2 & 4 \\ 6 & 0 & 2 & 4 \\ 9 & 0 & 1 & 2 \end{pmatrix}$$

iv. Prove that rank(A)=rank(A').

(c) Consider the following sequence

$$x_k = \sqrt{2}x_{k-1} + \frac{3}{8}x_{k-2} - \frac{3\sqrt{2}}{8}x_{k-3}$$

check whether the effect of x_0, the initial value of this sequence, becomes arbitrarily small as $k \to \infty$.

2.

Random variable X takes values in $\{-1, 1\}$. Random variable Y takes values in $\{-1, 0, 1\}$. Their joint probability mass function is given partially in the Table below. The marginal distributions of both X and Y are (discrete) uniform.

		X	
		-1	1
Y	-1	$\frac{1}{4}$	
	0		
	1		$\frac{1}{6}$

(a) Calculate the remaining values for the joint probability mass function.

(b) Are X and Y independent?

(c) Define

$$\eta = Y - \alpha - \beta X$$

and where α and β are fixed coefficients. Show that it is possible to choose values for α and β such that

i. $E(\eta) = 0$

ii. $Cov(X, \eta) = 0$

(d) Calculate the required values of α and β for the distribution obtained in (a).

(e) Given a random sample of observations$\{x_i, y_i;\ i = 1, \ldots, N\}$ from this joint distribution denote the ordinary least squares estimators of the intercept and slope in a linear regression of y_i on x_i as $\hat{\alpha}$ and $\hat{\beta}$. Will these estimators be

i. Unbiased

ii. Efficient

for the coefficients α and β calculated in (d)? Justify your answer.

3.

(a) Consider the following difference equation:

$$x_{t+2} - 3x_{t+1} + 2x_t = 3^t$$

Solve the equation with initial condition $x_0 = 0$ and $x_1 = 0$.

(b) Consider the following system of differential equations:

$$\begin{aligned} \dot{x} &= 2x + 3y \\ \dot{y} &= 3x - 6y \end{aligned}$$

i. Sketch the dynamic solution in a phase diagram and explain the dynamics.

ii. Find the analytical solution of the system. If the initial condition of the system is $x(0) = 1$ and $y(0) = -3$ is the system going to converge to the steady state? Justify your answer?

4.

(a) Answer and explain briefly

i. Is the set $A = [0, 2) \cup (2, 4]$ bounded? Is it convex? Is it compact?

ii. Sketch the set of points (x, y) described by the equation $x^2 + y^2 - 25 = 0$. Call this set B. Is B bounded? Is it convex? Is it compact?

iii. State Brouwer's fixed point theorem.

iv. Consider the set A above. Does Brouwer's theorem apply? If not, give an example of a function $f : A \to A$ which does not have a fixed point.

v. Consider the set B above. Does Brouwer's theorem apply? If not, give an example of a function $f : B \to B$ which does not have a fixed point.

(b) Consider the problem of maximising $f(a, b) = a \exp(b)$ subject to the constraint $a + Kb \leq M$, where K and M are positive constant real numbers, whereas a and b are real variables with $b \geq 0$.

i. For which values of K and M, does the optimal solution imply $b = 0$?

ii. What is the solution when f is known to attain its maximum when a and b are positive?

END OF PAPER

Faculty of Economics

MPhil/MRes Pre-Course Exam

September 2016

ANSWERS

1.

(a)

$$AB = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \quad \text{and} \quad BA = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & -2 & 1 \end{pmatrix}$$

(b)

i.

$$tr(AA') = \sum_i (AA')_{ii} = \sum_i \sum_j a_{ij}^2 = \sum_j \sum_i a_{ij}^2 = \sum_j (AA)_{jj} = tr(A'A)$$

ii. $\lambda_1 = 1$ (twice), $\lambda_2 = 5$.

iii. $rank(C) = 3$.

iv. Since the column rank of A is the row rank of A' and since we know that the column and row ranks are equal, the result immediately follows.

(c) The sequence satisfies:

$$u_k = Au_{k-1},$$

where

$$u_k = \begin{bmatrix} x_k \\ x_{k-1} \\ x_{k-2} \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} \sqrt{2} & \frac{3}{8} & -\frac{3\sqrt{2}}{8} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The eigenvalues of A are

$$|A - \lambda I_3| = \begin{vmatrix} \sqrt{2} - \lambda & \frac{3}{8} & -\frac{3\sqrt{2}}{8} \\ 1 & -\lambda & 0 \\ 0 & 1 & -\lambda \end{vmatrix} = (\sqrt{2} - \lambda)(\lambda^2 - \frac{3}{8})$$

the eigenvalues are $\lambda_1 = \sqrt{2} > 1, \lambda_2 = \sqrt{\frac{3}{8}} < 1, \lambda_3 = -\sqrt{\frac{3}{8}} < 1$. For the effect of the initial value to have an arbitrarily small effect as $k \to \infty$, all eigenvalues should be smaller than one in absolute value, which is not the case in our example, therefore the effect of the initial value x_0 does **not** become arbitrarily small.

2. Note in this question it is very important to be clear about population values and sample values. Thus the (population) variance of X is $Var(X) = E\,(X - \mu)^2$ and the sample variance is $\frac{1}{N-1} \sum (x_i - \bar{x})^2$. Confusion between $\bar{x}$ and μ (or between β and $\hat{\beta}$) will lose marks.

(a) The joint pmf is

		X	
		-1	1
Y	-1	$\frac{1}{4}$	$\frac{1}{12}$
	0	$\frac{1}{12}$	$\frac{1}{4}$
	1	$\frac{1}{6}$	$\frac{1}{6}$

(b) They are not independent as eg $P(X=1, Y=-1) = \frac{1}{12}$ but $P(X=1)\,P(Y=-1) = \frac{1}{2} \times \frac{1}{3}$

(c) We have $E(\eta) = E(Y - \alpha - \beta X) = \mu_Y - \alpha - \beta\mu_X$. Also $Cov(\eta, X) = Cov(Y - \alpha - \beta X, X) = Cov(X, Y) - \beta Var(X)$ so choosing $\beta = \frac{Cov(X,Y)}{Var(X)}$ and $\alpha = \mu_Y - \beta\mu_X$ will do.

(d) For the given distribution we have $\mu_Y = \mu_X = 0$. Trivially $Var(X) = 1$ and $Cov(X, Y) = \sum (x - \mu_X)(y - \mu_Y)\, p(x, y)$ which is

$$\left\{(-1).(-1).\frac{1}{4} + (-1).(1).\frac{1}{12} + (0).(-1).\frac{1}{12} + (0).(1).\frac{1}{4} + (1).(-1).\frac{1}{6} + (1).(1).\frac{1}{6}\right\}$$

$$= \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$$

Using these values we get $\alpha = 0$ and $\beta = \frac{1}{6}$.

(e) There are some subtle issues here and I'm not looking for elaborate proofs (or calculation of CRLB) not least because there's a time constraint. For unbiasedness and efficiency we'd need something like Gauss Markov to apply. The errors η are not normally distributed (they take on only six values) so we won't get efficiency. For unbiasedness we need $E(\eta \mid X) = 0$. We have $Cov(\eta, X) = 0$ by construction but conditional independence is different. You can write out the joint pmf of η and X quite easily and verify this is true. So the least squares estimator will be unbiased for α and β but not efficient.

3.

(a) Solution of the homogeneous part is:

$$\lambda^2 - 3\lambda + 2 = 0$$
$$\lambda_1 = 2 \text{ and } \lambda_2 = 1$$

$$x_{c,t} = c_1 2^t + c_2$$

Solution of the non homogeneous part using the method of undetermined coefficients. We try the solution $x_{p,t} = A3^t$

$$\begin{aligned} A3^{t+2} - 3A3^{t+1} + 2A3^t - 3^t &= 0 \\ 3^t\,(9A - 9A - 2A - 1) &= 0 \end{aligned}$$

we set the expression in brackets equal to 0

$$\begin{aligned} 9A - 9A - 2A - 1 &= 0 \\ A &= -\frac{1}{2} \end{aligned}$$

then the general solution is:

$$x_{g,t} = x_{c,t} + x_{p,t} = c_1 2^t + c_2 - \frac{1}{2}3^t.$$

Using the initial condition $x_0 = 0$ and $x_1 = 0$ the constant c_1 and c_2 can be pinned down solving the following system:

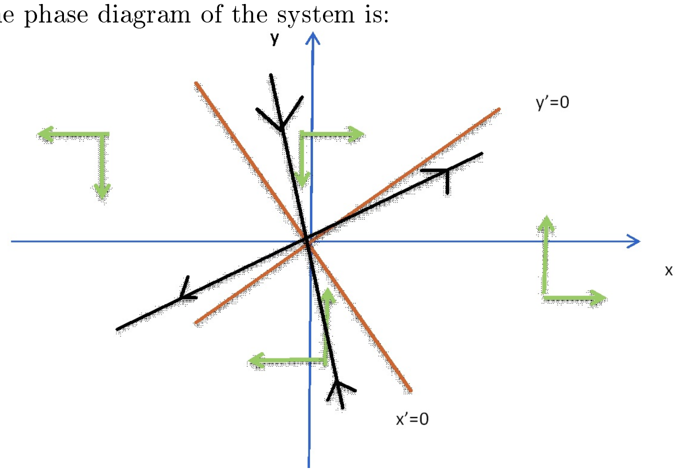
$$\begin{cases} c_1 2^0 + c_2 - \frac{1}{2} = 0 & t = 0 \\ c_1 2^1 + c_2 - \frac{3}{2} = 0 & t = 1 \end{cases}$$

and $c_1 = 1$ and $c_2 = -\frac{1}{2}$. The solution is finally

$$x_{g,t} = 2^t - \frac{1}{2}3^t - \frac{1}{2}$$

(b)

i. The phase diagram of the system is:



where $\dot{x} = 0$ is $y = -\frac{2}{3}x$ and $\dot{y} = 0$ is $y = \frac{1}{2}x$.

ii. The eigenvalues of the matrix $\begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}$ are $r_1 = -7$ and $r_2 = 3$ and the corresponding eigenvectors are $v_1 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

and $v_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. The general solution of the system is:

$$\begin{aligned} x(t) &= -A_1 e^{-7t} + 3A_2 e^{3t} \\ y(t) &= 3A_1 e^{-7t} + A_2 e^{3t} \end{aligned}$$

If the initial condition is $x(0) = 1$ and $y(0) = -3$, we can pin down the constants A_1 and A_2 such that

$$\begin{aligned} x(0) &= 1 = -A_1 + 3A_2 \\ y(0) &= 3 = 3A_1 + A_2 \end{aligned}$$

then $A_1 = -1$ and $A_2 = 0$. Only the stable part of the solution survives. The system will converge.
The solution will converge to the steady state only if the initial condition is on the stable arm. The equation of the stable arm is given by:

$$\frac{x(t) - x^*}{y(t) - y^*} = -\frac{1}{3}$$

however, given $x^* = y^* = 0$

$$\frac{x(0)}{y(0)} = -\frac{1}{3}$$

that is on the stable arm.

4.

(a)

i. Bounded, not convex (a hole in the middle), not compact (because it is not closed, it is missing a limit point, namely 2).

ii. It is a circle with centre at the origin, and radius 5. Bounded, compact, but not convex.

iii. Let S be a non-empty, compact, convex subset of $\mathbb{R}^n$. If $f : S \to S$ is continuous, then f has a fixed point, i.e., there must be a point $c \in S$ such that $f(c) = c$.

iv. No, the theorem doesn't apply as A is not convex (nor compact). A function $f : A \to A$ without a fixed point maps $[0, 2)$ to $(2, 4]$, and maps $(2, 4]$ to $[0, 2)$ by sending x to its mirror image with 2 being the flipping point.

v. No, the theorem doesn't apply as B is not convex. Consider any function with rotates the points in B around the origin by an angle other than a multiple of 360 degrees.

(b) To simplify, let's instead work with the natural log of f, that is

$$\max_{a,b} \ln a + b \quad \text{s.t.} \quad \frac{a}{K} + b \leq \frac{M}{K} \text{ and } -b \leq 0$$

The Kuhn-Tucker Lagrangean is

$$L = \ln a + b + \lambda \left(\frac{M}{K} - \frac{a}{K} - b \right)$$

FOCs are

$$L_a = a^{-1} - \frac{\lambda}{K} = 0$$

and

$$L_b = 1 - \lambda$$

Complementary slackness:

i. $L_b \leq 0$ and $b = 0$. In this case $a = M$. Holds when $\lambda = K/M \geq 1$. That is, when $M \leq K$, the optimal solution implies $b = 0$.

ii. $L_b = 0$ and $b > 0$, in which case $\lambda = 1$ and $a = K; b = M/K - 1$. When $M > K$, we have this solution.