EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY.
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN FINANCE FOR THE DEGREE OF MASTER OF PHILOSOPHY

Tuesday 11 January $2011 \quad 10.00$ to 12.00
Module M100
MICROECONOMICS I
Candidates are required to answer three compulsory questions. All questions carry equal weight.

Write your number not your name on the cover sheet of each booklet

## STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS

20 Page booklet x 2
Approved calculators allowed
Rough work pads

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. Let $\mathbf{L}$ be the set of simple lotteries over the finite set of outcomes $\left\{x_{1}, x_{2}, \ldots, x_{n}\right\}$. An agent has a preference ordering $\succeq$ over $\mathbf{L}$, with strict preference denoted by $\succ$. Among degenerate lotteries in $\mathbf{L}$ the agent's most-preferred one, denoted $e_{1}$, puts probability 1 on $x_{1}$, and her least-preferred, denoted $e_{n}$, puts probability 1 on $x_{n}$ (and $e_{1} \succ e_{n}$ ).
(a) State the independence axiom.
(b) Let $L$ be the lottery which gives probability $\alpha$ to $x_{1}$ and probability $(1-\alpha)$ to $x_{n}$, where $\alpha>0$. Show that $L \succ e_{n}$ if the preference ordering $\succeq$ satisfies the independence axiom.
(c) Explain what it means to say that a utility function $U$ defined on $\mathbf{L}$ represents $\succeq$.
(d) Explain what it means to say that $U$ is a von Neumann-Morgenstern (vNM) utility function with underlying (Bernoulli) utilities $u_{i}$.
(e) If $U$, with underlying utilities $u_{i}$, is a vNM utility function, show that if $\tilde{u}_{i}$ is a positive affine transformation of $u_{i}$, then $\tilde{u}_{i}$ are the underlying utilities of another vNM utility function which represents the same preferences.
2. A decision maker $(D)$ has to decide whether or not to undertake a project. The project is equally likely to be good or bad and $D$ does not know which. If $D$ undertakes the project $D$ 's payoff will be 2 if the project is good and -1 if the project is bad. An expert $(E)$ knows whether the project is good or bad. $E$ gives $D$ his advice: either $Y$ (yes) or $N$ (no). $D$ then makes her decision: either $P$ (undertake the project) or $N P$ (don't undertake the project). If the project is undertaken then E's payoff is 1 , regardless of whether it is good or bad. If the project is not undertaken $D$ and $E$ both get zero payoff.
(a) Draw the extensive form of the above game.
(b) Describe the set of pure strategies for $D$. Describe the set of pure strategies for $E$.
(c) Describe informally what is meant by a Weak Perfect Bayesian Equilibrium in pure strategies for this game.
(d) Is there a Weak Perfect Bayesian Equilibrium in pure strategies in which E's action varies with the project's quality? Explain why or why not.
(e) Find all the Weak Perfect Bayesian Equilibria in pure strategies.
3. Consider the following problem of contracting between a firm and a consumer. The firm may produce one unit of a good, which may be of any quality $z \geq 0$. If quality is $z$ and the consumer pays $t$ then the firm's profit is $t-c(z)$ and the consumer's utility is $\theta z-t$, where $c($.$) is a convex cost function and \theta>0$ is a constant. Assume that $c(0)=0, c^{\prime}(0)=0$ and $c^{\prime \prime}(z)>0$ for all $z \geq 0$. The firm offers a contract to the consumer; if the consumer rejects it his utility is zero.
(a) Sketch the consumer's indifference curves and the firm's iso-profit curves.
(b) Characterize the firm's optimal contract algebraically and illustrate it graphically.

Now suppose that the value of $\theta$ is private information to the consumer. With probability $\gamma \in(0,1), \theta=\theta_{h}$ and otherwise $\theta=\theta_{l}<\theta_{h}$.
(c) Write the firm's optimization problem for finding its optimal menu of contracts.
(d) Show graphically why it cannot be, at the optimum, that the participation constraint is slack for the $\theta_{l}$-type of consumer.
(e) Show graphically why it cannot be that the incentive constraint is slack for the $\theta_{h}$-type of consumer.
(f) Characterize algebraically the firm's optimal menu of contracts and illustrate it graphically. You may assume that the firm supplies strictly positive quality to both types.
(g) Show that, for one type, quality is lower than the first-best level.

EXAMINATION IN ECONOMICS
FOR THE DEGREE OF MASTER OF PHILOSOPHY.
EXAMINATION IN ECONOMIC RESEARCH
FOR THE DEGREE OF MASTER OF PHILOSOPHY.
FRIDAY, 13 MAY $2011 \quad 10.00-12.00$
M110
MICROECONOMICS II
This paper is made up of two sections.
Candidates are required to answer three questions, with at least one from each Section.

Each question carries equal weight.

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STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Rough work pads

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## Section A

## Question 1.

(a) What does it mean for a mechanism to implement a social choice function in dominant strategies? [You are asked to provide a formal definition. There is no need for a discussion.]
(b) State and prove the revelation principle for dominant strategy implementation.
(c) Consider a single agent environment. The agent's type is $\theta \in \Theta$, and his utility from a social outcome $x \in X$ is $u(x ; \theta)$. For the social decision rule $f: \Theta \rightarrow X$, there exists $t: \Theta \rightarrow \mathbb{R}$ such that for all $\theta, \hat{\theta} \in \Theta$

$$
u(f(\theta) ; \theta)-t(\theta) \geq u(f(\hat{\theta}) ; \theta)-t(\hat{\theta})
$$

Show that there exists a function $p: X \rightarrow \mathbb{R}$ such that

$$
f(\theta) \in \underset{x \in X}{\operatorname{argmax}}\{u(x ; \theta)-p(x)\} \quad \text { for all } \theta \in \Theta .
$$

Question 2. Consider a bilateral trade setting. The seller (agent 1) has valuation $\theta_{1}$ and the buyer (agent 2) has valuation $\theta_{2}$. These privately known valuations are independently drawn from a uniform distribution on $[0,1]$.
(a) Compute the transfer functions in the expected externality mechanism.
(b) Verify that truth telling is a Bayesian Nash equilibrium.
(c) Write the participation constraints and show that there is a type of buyer or seller who will strictly prefer not to participate.

Question 3. A seller $s$ of a single indivisible object and a buyer $b$ simultaneously choose prices $p_{s}, p_{b} \in[0,1]$. If $p_{b}<p_{s}$, then there is no trade. If $p_{b} \geq p_{s}$, then they trade at price $p=\frac{p_{b}+p_{s}}{2}$. Their privately known valuations $\theta_{b}$ and $\theta_{s}$ are drawn independently from the uniform distribution on $[0,1]$. Therefore payoffs are given by:

$$
\begin{aligned}
& u_{b}\left(p_{b}, p_{s} ; \theta_{b}\right)= \begin{cases}\theta_{b}-\frac{p_{b}+p_{s}}{2} & \text { if } p_{b} \geq p_{s} \\
0 & \text { otherwise }\end{cases} \\
& u_{s}\left(p_{b}, p_{s} ; \theta_{s}\right)= \begin{cases}\frac{p_{b}+p_{s}}{2}-\theta_{s} & \text { if } p_{b} \geq p_{s} \\
0 & \text { otherwise }\end{cases}
\end{aligned}
$$

(a) Show that there is a BNE where trade never occurs.
(b) Show that for any $c \in[0,1]$, there is a BNE where trade occurs if and only if $\theta_{b} \geq c \geq \theta_{s}$.
(c) Find a BNE equilibrium in linear strategies, i.e.:

$$
\begin{aligned}
& p_{b}\left(\theta_{b}\right)=a_{b}+c_{b} \theta_{b} \\
& p_{s}\left(\theta_{s}\right)=a_{s}+c_{s} \theta_{s} .
\end{aligned}
$$

for some $a_{s}, a_{b}, c_{b}, c_{s}>0$.

## Section B

Question 4. Consider a two-date financial economy, with two states at date 1. There are two agents A and B with consumption space $\mathbb{R}_{+}^{3}$, and endowments $(1,1,0)$ and $(1,0,1)$, respectively.
In this market, there exists only one asset, called $\gamma$, which has a payoff of 1 in both states.

Suppose that A's utility function $U^{A}: \mathbb{R}_{+}^{3} \rightarrow \mathbb{R}$ is given by

$$
U^{A}\left(x_{0}, x_{1}, x_{2}\right)=\ln \left(1+x_{0}\right)+\ln \left(1+x_{1}\right)+\ln \left(2+x_{2}\right),
$$

and B's utility function $U^{B}: \mathbb{R}_{+}^{3} \rightarrow \mathbb{R}$ is given by

$$
U^{B}\left(x_{0}, x_{1}, x_{2}\right)=2 \ln \left(1+x_{0}\right)+\ln \left(2+x_{1}\right)+\ln \left(1+x_{2}\right)
$$

(a) Prove that this economy does not have an equilibrium.
(b) If B 's endowment is $(1, \varepsilon, 1)$ for some $\varepsilon>0$, show that there is an equilibrium. Who sells the asset $\gamma$ ?
(c) Show that, for $\varepsilon>0$ small enough, in equilibrium, one of the agents consumes nothing at the first state of date 1 .

Question 5. Robinson is the only agent in an island economy. He divides his time between leisure and work. If he works $h$ hours, he has $\ell=24-h$ hours of leisure to enjoy, and he can produce $c=\sqrt{2 h}$ units of a consumption good. His utility function is $u(\ell, c)=\ln (\ell)+\ln (c)$.
(a) Model this island economy as if there is the consumer Robinson endowed with 24 hours of leisure time, and none of the consumption good, and there is the profit-maximizing firm Robinson who buys labor from and sells the consumption good to the consumer Robinson. (The firm is owned by the consumer Robinson, and the profits of the firm belong to him.) Define what a market equilibrium means.
(b) Show that there is an equilibrium price $p=\left(p_{\ell}, p_{c}\right)$, where $p_{\ell}$ stands for the price of time (leisure or labour), and $p_{c}$ stands for the price of the consumption good. How much does Robinson work?
(c) Is the equilibrium price $p$ unique (up to scaling of course)? What is the meaning of a price to Robinson?

Question 6. Consider an exchange economy. We say the agent $a$ 's excess demand function $z^{a}$ satisfies the gross substitutes (GS) property if

$$
\left.\begin{array}{l}
p_{j}^{\prime}>p_{j} \\
p_{k}^{\prime}=p_{k}
\end{array} \quad \text { for } k \neq j\right\} \Rightarrow z_{k}^{a}\left(p^{\prime}\right)>z_{k}^{a}(p) \text { for } k \neq j
$$

(a) Show that if $z^{a}$ satisfies GS for every $a \in A$, then the aggregate excess demand function $Z=\sum_{a \in A} z^{a}$ also satisfies GS, i.e.,

$$
\left.\begin{array}{l}
p_{j}^{\prime}>p_{j} \\
p_{k}^{\prime}=p_{k}
\end{array} \quad \text { for } k \neq j, ~\right\} \quad Z_{k}\left(p^{\prime}\right)>Z_{k}(p) \text { for } k \neq j
$$

(b) Suppose that the individual excess demand functions satisfy the GS property for every agent. Prove that there is a unique market equilibrium.
(c) Suppose that every agent $a$ has an initial endowment $w^{a} \gg 0$. If all agents have Cobb-Douglas utilities, show that there must be a unique equilibrium.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY.
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Wednesday 11 May $2011 \quad 2.00-4.00$

M120
TOPICS IN ECONOMIC THEORY

Candidates are required to answer two out of three questions. All questions carry equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Rough work pads

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. Consider, in the dynamic risk sharing game, the following monetary strategy:
(I) The state space is $S=\{0,1,2\}$,
(II) The initial state is $\bar{s}=1$,
(III) The behavior function for player 1 is $B(m)=P$ if $m=0,1$ and $B(2)=N P$,
(IV) The behavior function of player 2 is $B(m)=P$ if $m=1,2$ and $B(0)=N P$,
(V) The transition function on the equilibrium path is $T(1, m, P)=m+1$ if $m=0,1, T(1,2, N P)=2, T(2, m, P)=m-1$ if $m=1,2$ and $T(2,0, N P)=0$ and
(VI) The transition function outside the equilibrium path is $T(1, m, N P)=m$ for all $0 \leq m<2, T(1,2, P)=2, T(2, m, N P)=m$ for all $0<m \leq 2$ and $T(2,0, P)=0$.

Denote player 1's value function by $V_{m}(\delta)$ where $m \in\{0,1,2\}$ and $\delta \in(0,1)$. Furthermore, let $\gamma_{2}(\delta)=V_{2}(\delta)-V_{1}(\delta)$ and $\gamma_{1}(\delta)=V_{1}(\delta)-V_{0}(\delta)$.
(a) Using player 1's value function, provide inequalities that are necessary and sufficient for the monetary strategy to be a subgame perfect equilibrium.
(b) For all $\delta \in(0,1)$, compute $\gamma_{2}(\delta)$ and $\gamma_{1}(\delta)$.
(c) Show that the monetary strategy is a subgame perfect equilibrium for all $\delta$ sufficiently large.
2. Consider a multi-person bargaining game. For each $i \in N$ and $s \in S_{i}$, let $R\left(s ; f_{i}\right)=$ $\left|\left\{f_{i}(h, s): h \in H^{\infty}\right\}\right|$ be the number of actions player $i$ plays when he observes the partial history $s$ in the current stage. We say that $f_{i}$ is more complex than $f_{i}^{\prime}$, denoted by $f_{i}>f_{i}^{\prime}$, if $R\left(s ; f_{i}\right) \geq R\left(s ; f_{i}^{\prime}\right)$ for all $s \in S_{i}$ and $R\left(s^{\prime} ; f_{i}\right)>R\left(s^{\prime} ; f_{i}^{\prime}\right)$ for some $s^{\prime} \in S_{i}$.

Let $F_{i}$ denote the strategy set of player $i$ for all $i \in N$. A strategy profile $f=\left(f_{1}, \ldots, f_{n}\right)$ constitutes a NEC with respect to $>$ (henceforth abbreviated to $\mathrm{NEC}(>))$ if, for all $i \in N$, (1) $\pi_{i}(f) \geq \pi_{i}\left(f_{i}^{\prime}, f_{-i}\right)$ for all $f_{i}^{\prime} \in F_{i}$, and (2) there does not exist $f_{i}^{\prime} \in F_{i}$ such that $\pi_{i}(f)=\pi_{i}\left(f_{i}^{\prime}, f_{-i}\right)$ and $f_{i}>f_{i}^{\prime}$.

A strategy profile $f$ is stationary if $f_{i}(h, s)=f_{i}\left(h^{\prime}, s\right)$ for all $i \in N, h, h^{\prime} \in H^{\infty}$ and $s \in S_{i}$.
(a) Show that if $f$ is a stationary Nash equilibrium, then $f$ is a $\mathrm{NEC}(>)$.
(b) Show that if $f$ is a $\operatorname{NEC}(>)$, then $f$ is stationary Nash equilibrium.
3. Consider the infinite repetition of the following prisoners' dilemma game played by players 1 and 2:

| $1 \backslash 2$ | $c$ | $d$ |
| :---: | :---: | :---: |
| $c$ | 3,3 | $-1,4$ |
| $d$ | $4,-1$ | 0,0 |

Assume that players can only play pure strategies. For all $a \in A=\{c, d\} \times\{c, d\}$, the notation $(a ; k)$ stands for action $a$ played for $k$ consecutive periods. Let $D$ denote the convex hull of $u(A)$ and $\mathcal{U}^{0}=\left\{u \in D: u_{i}>v_{i}\right.$ for all $\left.i \in N\right\}$ denote the set of strictly individually rational payoffs. Let $\mathbb{N}$ stand for the set of natural numbers and $\mathbb{N}_{0}=\mathbb{N} \cup\{0\}$.
(a) Show that for all $u \in \mathcal{U}^{0}$ and $\varepsilon>0$, there exists $K \in \mathbb{N}$ and $\left(p_{a}\right)_{a \in A}$ such that (1) $p_{a} \in \mathbb{N}_{0}$ for all $a \in A$, (2) $\sum_{a \in A} p_{a}=K$, (3) $\left\|u-\sum_{a \in A} p_{a} u(a) / K\right\|<\varepsilon$ and (4) $\sum_{a \in A} p_{a} u_{i}(a) / K>0$ for all $i \in\{1,2\}$.

Let $\sigma$ be the outcome path consisting of the repetition of the cycle

$$
\left(\left(c c ; p_{c c}\right),\left(d d ; p_{d d}\right),\left(c d ; p_{c d}\right),\left(d c ; p_{d c}\right)\right),
$$

where $p_{a} \in \mathbb{N}_{0}$ for all $a \in A, K=\sum_{a \in A} p_{a}$ and $\sum_{a \in A} p_{a} u_{i}(a) / K>0$ for all $i \in\{1,2\}$. Define a simple strategy profile by $\left(\sigma^{(0)}, \sigma^{(1)}, \sigma^{(2)}\right)$ where $\sigma^{(0)}=\sigma$ and $\sigma^{(1)}=\sigma^{(2)}=(d d, d d, \ldots)$.
(b) Show that there exists $\delta^{*} \in(0,1)$ such that $\left(\sigma^{(0)}, \sigma^{(1)}, \sigma^{(2)}\right)$ is a subgame perfect equilibrium.
(c) Combine parts (a) and (b) to state and prove a Folk Theorem for the prisoners' dilemma.

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY
Thursday 12 May $2011 \quad 2.00-4.00$

M130
APPLIED MICROECONOMICS
This paper is made up of two sections. Each section carries equal weight. You should answer four questions from Section A. Within Section A, each question carries equal weight. You must answer the compulsory question in Section B.

Write your candidate number not your name on the cover sheet of each booklet

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
| :--- | :--- |
| 20 Page booklet x 2 | Approved calculators allowed |
| Rough work pads |  |

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

## Question 1

The following Engel curve for vices was estimated on household budget data drawn from a single time period and region. The left hand side variable (lxvices) is the logarithm of expenditure on alcohol and tobacco. The household types in the data are couples without children, and couples with 1 child. The variable dkid is a dummy variable equal to 1 if there is a child present. The variable lxtot is the logarithm of total expenditure.

```
. regress lxvices lxtot dkid;
```

| Source | SS | $d f$ | MS |
| ---: | :---: | ---: | :---: |
| Mode1 | 45.8162472 | 2 | 22.9081236 |
| Residual | 345.190885 | 191 | 1.80728212 |
| Total | 391.007132 | 193 | 2.02594369 |


| Number of obs | $=194$ |
| :--- | ---: |
| F( $2, ~ 191)$ | $=12.68$ |
| Prob $>$ F | $=0.0000$ |
| R-squared | $=0.1172$ |
| Adj R-squared | $=0.1079$ |
| Root MSE | $=1.3444$ |


| 7xvices | coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{t\mid}$ | [95\% Conf. Interval] |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1xtot | 1.307077 | .266235 | 4.91 | 0.000 | .7819382 | 1.832215 |
| dkid | -.2378989 | .2044945 | -1.16 | 0.246 | -.6412565 | .1654588 |
| _cons | -5.849039 | 2.622979 | -2.23 | 0.027 | -11.02276 | -.6753135 |

a) Use these estimates, and the Robarth method, to calculate the cost of a child.
b) Briefly discuss possible problems with this approach to estimating the cost of a child.

## Question 2

Consider a collective household model of 2 person households. Imagine there are two private goods, but that only household (not individual) demands are observed. Imagine through that there are two distribution factors.
a) What is meant by a distribution factor?
b) Show how this collective model can be tested given data on household expenditures on the two goods, total household budget (or income) and the distribution factors.

## Question 3

Suppose the household's cost function is defined implicitly by:

$$
\ln C=\alpha_{0}+\sum_{i=1}^{N} \alpha_{i} \ln \left(p_{i}\right)+\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{i j} \ln \left(p_{j}\right) \ln \left(p_{i}\right)+\ln U
$$

$N$ is the number of goods, and goods are indexed by $i$ and $j . C$ is cost, $U$ is utility, $p_{i}$ is the price of good $i$ and $\alpha_{i}$ and $\gamma_{i j}$ are parameters.
a) Derive demand equations from this cost function that you could estimate from data.
b) What restriction(s) does this cost function impose on the nature of demand?

## Question 4

The government is considering extending the VAT (currently 17.5\%) to food. Assume there is full pass through.
a) Write down a second-order approximation to the percentage change in costs associated with maintaining the utility of a given household at its initial level, as a function of the percentage change in the price of food. Interpret each term in the approximation.
a) Suppose that households have very limited scope to substitute for food. Would you expect the change in the retail price index (RPI) reported by the ONS at the time of this change in the VAT to be greater or smaller than the increase in cost of living for a household with average income? Explain.

## Question 5

Consider a non-profit organization that uses two inputs, $q_{1}$ and $q_{2}$, to produce a service, $y$. The two inputs have prices $w_{1}$ and $w_{2}$ respectively. The organization maximizes the amount of the service it provides subject to the budget it is assigned, $B$ (that is, subject to $w_{1} q_{1}+w_{2} q_{2} \leq B$ ). What will be the properties of this organizations demand's for $q_{1}$ and $q_{2}$, given by $q_{i}^{*}=g^{i}\left(w_{1}, w_{2}, B\right)$ ? Explain.

## Question 6

Here are estimates of a food demand equation on household level budget data.
. reg bsfath `list';

| Source | SS | $d f$ | MS |
| ---: | ---: | ---: | ---: |
| Mode1 | 113.3918 | 9 | 12.5990889 |
| Residual | 123.147118 | 14984 | .008218574 |
| Total | 236.538918 | 14993 | .015776624 |


| Number of obs | $=14994$ |
| :--- | ---: |
| F( 9,14984$)$ | $=1533.00$ |
| Prob $>$ | $=0.0000$ |
| R-squared | $=0.4794$ |
| Adj R-squared | $=0.4791$ |
| Root MSE | $=.09066$ |


| bsfath | Coef. | Std. Err. | t | $\mathrm{P}>\mid \mathrm{tl}$ | [95\% Conf. Interva1] |  |
| ---: | ---: | :---: | :---: | :---: | ---: | ---: |
| 1hhsize | .1404119 | .0015026 | 93.45 | 0.000 | .1374667 | .1433572 |
| ppfath | -.0792139 | .0255883 | -3.10 | 0.002 | -.12937 | -.0290577 |
| lprest | -.008085 | .0216953 | -0.37 | 0.709 | -.0506105 | .0344404 |
| pptran | .0096601 | .0093916 | 1.03 | 0.304 | -.0087486 | .0280687 |
| 1pserv | .0277738 | .0257975 | 1.08 | 0.282 | -.0227926 | .0783401 |
| precr | .0569671 | .0305359 | 1.85 | 0.064 | -.0033869 | .1163212 |
| 1pvice | -.0095315 | .0100788 | -0.95 | 0.344 | -.0292873 | .0102242 |
| 1pclth | -.0945545 | .0229933 | -4.11 | 0.000 | -.1396242 | -.0494847 |
| 1rxtot | -.1612637 | .0016537 | -97.52 | 0.000 | -.1645052 | -.1580222 |
| cons | 1.697317 | .0154508 | 109.85 | 0.000 | 1.667031 | 1.727602 |

The demand equation is in share form and bsfath is the share of food at home in total expenditure. The mean food share in the sample is $26.5 \%$. The variable Irxtot is the logarithm of total expenditure deflated by a Stone price index and lhhsize is the logarithm of household size. The variables Ipfath through Ipclth are logarithms of the prices of the commodities that make up total expenditure; Ipfath is the logarithm of the price of food at home.
a) Calculate the elasticity of expenditure on food at home with respect to total expenditure
b) How would you test if this demand equation satisfied homogeneity? Be precise.

## SECTION B

## Question B1

In the UK many benefits (income transfers) are indexed: From this year indexation will be to the Current Price Index (CPI).
a. What shortcomings does the CPI have as a cost-of-living index?
b. Suppose that the government asks you to investigate whether the CPI is a good measure of changes in the cost-of living households of different types, and hence whether changes in the benefits they received have maintained their living standards. You have at your disposal microdata on the dis-aggregated expenditures, income and demographic composition of Canadian households. The data are derived from household surveys, and covers a 30 year period. In addition, you have a source of price data, which captures temporal variation in prices. How would you go about answering the question? Describe how you would use the data at hand and be as specific as possible.

## End of Paper

Philosophy in economics for the degree of master of EXAMINATION IN ECONOMIC MASTER OF PHIL ECONOMIC DEGREE OF MASTER | Thursday 12 May 2011 |
| :--- |
| M130 |
| APPLIED MICROECONOMH |
| This $p$ aper is made | This paper is made up of two sections. Each section carries equal weight. You questions from Section A. Within section carries equal weight Write your candidate wer the compulsory question Section $A$, each question. You should candidate number not your name on the cover suestion in Section B. question carries equal not your name on the cover sheet of each booklet

STATIONERY REQUIREMENTS
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$$
\begin{aligned}
& \text { SPECIAL REQUIREMENTS } \\
& \text { Approved calculators allowed }
\end{aligned}
$$



EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Friday 20 May $2011 \quad 10.00-12.00$
Module140
BEHAVIOURAL ECONOMICS
Candidates are required to answer four compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS

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Tags

1. Two players simultaneously pick a number from the set $\{1,2,3,4,5\}$. The player whose choice is closest to $\frac{4}{5}$ of the average of the two choices receives a prize of $£ 100$. The other player receives nothing. In the event of a tie, the prize is shared.
(a) How would you model sophistication in this game?
(b) How would you expect the outcome to depend on the degree of sophistication of the players?
(c) How would your answer change if there were ten players instead of two?
2. (a) Set out a model of consumer behaviour over the lifecycle.
(b) In the context of this model, which assumptions about time preferences best fit the data?
(c) How would you change your model from part (a) in order to obtain a better understanding of time preferences?
3. Becker, Grossman \& Murphy (1994) assert that "wars on drugs" will often be ineffective.
(a) Set out their model carefully.
(b) How plausible are the assumptions underlying their conclusion?
(c) Would other models of addiction lead to the same policy conclusion? If so, what are the policy alternatives? If not, why not?
4. (a) Set out clearly the learning models of Cheung \& Friedman (1997) and Erev \& Roth (1998).
(b) Which, if either, of these models does the empirical evidence favour? Justify your answer.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RERSEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Friday 13 May $2011 \quad 2.00-4.00$
Module 150
ECONOMICS OF NETWORKS
Candidates are required to answer three out of four questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
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20 Page booklet x 2 Approved calculators allowed
Rough work pads
Tags

1. Consider a game in which $n$ players are located on nodes of an undirected network $g$. Players simultaneously choose an action $x_{i} \in \mathcal{R}_{+}$. The cost of choosing $x_{i}$ is $c x_{i}$, where $c>0$. The gross payoff to a player depends on the sum of own action and the actions of his neighbors. In network $g$, let $N_{i}(g)$ be the set of neighbors of player $i$. Let $y_{i}=x_{i}+\sum_{j \in N_{i}(g)} x_{j}$ denote the sum of efforts of player $i$ and his neighbors in network $g$. The benefit to player $i$ is given by $f\left(y_{i}\right)$, where $f(0)=0, f^{\prime}\left(y_{i}\right)>0 f^{\prime \prime}\left(y_{i}\right)<0$. Define $\hat{y}$ to be such that $f^{\prime}(\hat{y})=c$.
(a) Show that there exists a pure strategy Nash equilibrium in every network. Give an example of one equilibrium each in the star network, the empty network and the complete network.
(b) What are effects of adding new links in a network upon the actions and utility of individuals?
(c) Aggregate social welfare is the sum of individual utilities. A profile of actions $x=\left(x_{1}, \ldots, x_{n}\right)$ is efficient if it maximizes aggregate social welfare. Show that every equilibrium in a non-empty network is inefficient.
2. There are $n$ individuals, each of whom starts with a belief at date 0 , a number $p_{i}^{0}$. In period 1 each individual $i$ updates his belief, by taking an average of the beliefs of others and her own belief. She assigns weight $w_{i j} \in[0,1]$ to each $j \in N$, with $\sum_{j \in N} w_{i j}=1$. This yields $p_{i}^{1}$, for $i \in N$. Define for any $t \geq 0$, the vector of beliefs at start of that period as $\mathbf{p}^{t}=\left(p_{1}^{t}, p_{2}^{t}, p_{3}^{t}, . . p_{n}^{t}\right)$. Let $\mathbf{W}$ be the social influence stochastic matrix defined by the weights which individuals assign to each other's opinions. The belief revision process repeats itself over time. A connected network is defined as one in which between every pair of nodes there is a path, with positive weighted links. In a connected network with $w_{i i}>0$, for all $i \in N$, there exists a unique social influence vector $\mathbf{w}=\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)$, which is the solution to the system: $\mathrm{w} \cdot \mathbf{W}=\mathbf{w}$.
(a) Suppose there are $n=4$ individuals. Individuals 1,2 and 3 , each assign equal weight to everyone (including themselves), while individual 4 assigns weight only to person 1 . Compute the social influence of every individual.
(b) Next consider a society with $n$ individuals. Suppose that individual 1 assigns weight $\delta \in(0,1)$ to himself and weight $(1-\delta) /(n-1)$ to the $n-1$ other persons. Every other individual, $i=2, \ldots, n$ assigns (an equal) weight $\epsilon$, where $\epsilon \in(0,1)$, to himself and $1-\epsilon$ to person 1. Compute the social influence of every individual. What happens to social influence of different individuals as $n$ grows?
(c) Show that in a connected network where $w_{i i}>0$, for all $i \in N$, and $w_{i j}=w_{j i}$, for all $i, j \in N$, every individual has equal social influence.
3. Consider the SIS (susceptible-infected-susceptible) model. There is a fraction of population which is infected with a disease. They recover and become susceptible at some rate. In the meanwhile, they are infectious, and they may infect susceptible others. Suppose that this population of individuals is located in a large social network, and that $d_{i}$, the degree of person $i$, reflects the frequency of interaction of person $i$. Define the probability of contacting someone with degree $d$ as:

$$
\begin{equation*}
\frac{P(d) d}{\langle d\rangle}, \tag{1}
\end{equation*}
$$

where $\langle d\rangle=\sum P(d) d$ is expected degree in the population. Define $\rho(d)$ to be the fraction of $d$ degree nodes who are infected. Then average infection rate is $\sum \rho(d) P(d)$. Define the probability that a contact is infected as

$$
\begin{equation*}
\theta=\sum_{d} \frac{P(d) d}{\langle d\rangle} \rho(d) \tag{2}
\end{equation*}
$$

Suppose that the rate of infection is linear in degree: $\nu \theta d$, where $\nu \in(0,1)$. Assume $\nu d \leq 1$ so that this is a probability. Finally, let recovery rate - from infection to susceptible - be $\delta>0$. Define $\lambda=\nu / \delta$ as the net infection rate.
(a) Show that steady state infection is implicitly defined by the following equation:

$$
\begin{equation*}
\theta=\sum_{d} \frac{P(d) \lambda \theta d^{2}}{(\lambda \theta d+1)\langle d\rangle} \tag{3}
\end{equation*}
$$

(b) Consider scale free networks and suppose $P(d)=2 d^{-3}$. What is the threshold of net infection rate $\lambda^{*}$ - the rate at which the steady state infection level is positive - in such networks?
(c) Suppose next that that the probability of infection for someone with degree $d$ is simply $v \theta$. Show that steady state $\rho(d)$ is independent of $d$ and that $\rho=(\lambda-1) / \lambda$ if $\lambda>1$ and it is equal to 0 , otherwise. What does this tell us about the relationship between thresholds and networks?
4. Consider network formation with $n$ individuals. Suppose that a player $i$ can create a link with player $j$ unilaterally by paying a cost $c>0$. The set of player $i$ links is denoted by $g_{i}$; the profile of linking decisions of all players is given by $g=\left(g_{1}, \ldots . g_{n}\right)$. A strategy profile induces a (directed) network $n$ nodes which is also denoted by $g$. The payoffs to player $i$ under a strategy profile $g=\left(g_{1}, g_{2}, . . g_{n}\right)$ are

$$
\begin{equation*}
\Pi_{i}(g)=n_{i}(g)-\eta_{i}^{d}(g) c, \tag{4}
\end{equation*}
$$

where $n_{i}(\hat{g})$ is the number of individuals to whom $i$ has a path in (the undirected version of) network $g$ plus himself, and $\eta_{i}^{d}(g)$ is the number of links which player $i$ forms in network $g$.
(a) Show that a Nash equilibrium network is either minimally connected or it is empty.
(b) Define aggregate welfare in network $g$ as the sum of individual utilities. A network is efficient if it maximizes aggregate welfare across all networks. Show that the unique efficient network is minimally connected if $c<n$ and it is empty if $c>n$.
(c) For any given $n$, and $c$, denote by $P(n, c)$ the ratio of maximum attainable aggregate welfare divided by the minimum aggregate welfare attained across all Nash equilibria. Define the Price of Anarchy as $\mathcal{P}=\max _{c, n} P(n, c)$. Show that $\mathcal{P}$ is unbounded in the above network formation model.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY
Wednesday 18 May $2011 \quad 2.00-4.00$

Module 160
POLITICAL ECONOMY
Candidates are required to answer three out of the four questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
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## M160 (Political Economy) - Final Exam - Lent 2011

(1) Consider a community composed of 7000 citizens. An election is to be held between two candidates, labeled $A$ and $B$. Candidates compete by making credible policy announcements $p_{j} \in \mathbb{R}$. Each citizen is characterized by an ideal point $p_{i}^{*} \in \mathbb{R}$, corresponding to her most preferred policy. Citizen $i$ 's utility from any policy $p \in \mathbb{R}$ is given by $u_{i}(p)=-\left|p-p_{i}^{*}\right|$. The distribution of voter ideal points is depicted in Figure 1. (For example, 2000 citizens prefer policy $-1 / 2$.) The rules of the election state that whichever candidate receives a majority of votes wins. In case both candidates receive the same number of votes, a coin is flipped to determine the winner. Both candidates choose $p_{j}$ to maximize their probability of winning.


Figure 1
(a) Suppose each citizen votes for the candidate who offers her the higher level of utility. If a voter is indifferent between two candidates, she flips a coin (assume that in this case exactly half of the indifferent voters will vote for each candidate). Show that both candidates choosing $p_{j}=0$ is an equilibrium of this model.
(b) Consider the following modification: Citizen $i$ votes for her preferred candidate (denoted $j$ ) if and only if $1+u_{i}\left(p_{j}\right)>\epsilon_{i}$, where $\epsilon_{i} \sim U[0,1]$.
(I.e. the probability that $\epsilon_{i} \leq x$ is $x$ for $x \in[0,1]$.) Explain intuitively what this modification means.
(c) Suppose both candidates choose $p_{j}=0$. How many votes does each candidate receive? (Hint: If the probability that a citizen with ideal point $p$ votes is $q$, you may assume that a fraction $q$ of all citizens with ideal point $p$ vote.)
(d) Show that both candidates choosing $p_{j}=0$ is not an equilibrium of the modified model.
(e) Find an equilibrium policy pair and show that it is an equilibrium.
(2) Consider a community of $N$ individuals $I=\{1, \ldots, N\}$ who have rational preferences over $L$ alternatives $X=\left\{x_{1}, x_{2}, \ldots, x_{L}\right\}, L \geq 3$. Show that if any alternative $x$ loses to all others in a pairwise vote, the social preference ranking chosen according to Condorcet's rule must rank it last.
(3) Tiebout (1956) proposes a model of local public goods provision. His paper was a direct response to Samuelson's (1954) theory of public expenditure.
(a) According to Samuelson, what defines the "optimal" level of public goods provision? Explain. (It is not enough to provide a mathematical condition. Please explain verbally what the condition says.)
(b) Samuelson argues that the optimal level of public goods provision is difficult to identify in practice. Explain. How is this different in the case of private goods?
(c) Tiebout argues that local public goods can be efficiently provided under certain conditions. Briefly outline his argument.
(d) The costs of moving from one area of the United States to another have steadily fallen over the past century. What prediction does the Tiebout model make concerning the differences between citizens living in a typical community, as well as the difference between communities in terms of the public goods provided?
(4) Consider an economy with a single representative agent, a firm and a government. The agent has utility function depending on consumption at time $t, c_{t}$, and labor supplied, $L_{t}$, up to a maximum of $L$. The total utility of the agent over infinitely many periods is:

$$
\begin{equation*}
\sum_{t=1}^{T} \beta^{t}\left[u\left(c_{t}\right)-L_{t}\right] \tag{1}
\end{equation*}
$$

where $\beta \in(0,1)$ is a discount factor, $u^{\prime}>0$, and $u^{\prime \prime}<0$. The time horizon $T$ may be finite or infinite.

The firm has a technology

$$
\begin{equation*}
Y_{t}=A K_{t}^{\alpha} L_{t}^{1-\alpha} \tag{2}
\end{equation*}
$$

where $\alpha \in(0,1)$ and $A>0$ is the level of technology. The firm maximizes profits given wages and rental capital. Capital depreciates in every period at rate $\delta$.

The government can raise taxes at the rate $\tau_{t}^{w}$ on wages and $\tau_{t}^{\pi}$ on capital's rental, and has to finance in every period a total expenditure $G$. A tax policy is a given schedule of tax rates in every period.
(a) Write the equations characterizing the feasible allocations in every period, and the budget sets for agent and government
(b) Write the equations characterizing the competitive equilibrium for every given tax policy
(c) For the case $T=1$ write the optimal tax policy for a government that tries to maximize the utility of the representative agent, given that allocations will be determined at the competitive equilibrium given the tax policy.
(d) Suppose that the government can issue bonds, what is the answer to the questions above?

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY

EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Wednesday 12 January 2011 2.00-4.00

Module M200
MACROECONOMICS I
Answer four compulsory questions from Section $A$ and one question from Section B.

Each section carries equal weight.
Write your number not your name on the cover sheet of each booklet

## STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS

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Rough work pads
Tags

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## SECTION A

## A1. Solow Growth Model

Consider a continuous time Solow growth model. Suppose that the production function is:

$$
Y(t)=K(t)^{\alpha} L(t)^{1-\alpha}, \alpha \in(0,1)
$$

where $Y(t)$ corresponds to output, $K(t)$ is the capital stock, and $L(t)$ is labour. Population grows at constant rate $n$ and agents save a fraction $s \in(0,1)$ of income. The economy is closed and capital evolves according to the following equation of motion:

$$
\dot{K}(t)=q(t) I(t)-\delta K(t), \delta>0
$$

where $I(t)$ denotes investment. $q(t)$ corresponds to the inverse of the relative price of machinery to output. Assume that this relative price is declining over time, such that $\frac{\dot{q}(t)}{q(t)}=\gamma$. The empirical evidence supports this assumption. In a balanced growth path, when all variables are growing at a constant rate, what is the growth rate of the aggregate capital stock? What is the growth rate of aggregate output?

## A2. $q$-model of investment

Suppose that a firm facing a market interest rate $1+r>0$ has a production function given by $Y_{t}=A F\left(K_{t}, L_{t}\right)$, where $A>0$ is a productivity parameter. Function $F\left(K_{t}, L_{t}\right)$ is continuous, differentiable, increasing and strictly concave in both arguments. The firm's objective function is to maximise the present discount value of profits. The firm also faces adjustment costs to changing its capital stock. Specifically, it must pay $\frac{\chi I_{t}^{2}}{2 K_{t}}$ with $\chi>0$ in adjustment costs in any period where it invests (or disinvests) at rate $I_{t}$. Capital accumulates according to the following equation:

$$
K_{t+1}=K_{t}+I_{t}, K_{0}>0 \text { is given. }
$$

Set up the firm's problem and find the first-order conditions characterising efficient investment. Interpret the first-order conditions. What are the boundary conditions of this problem? Explain.

## A3. Menu costs

Do small menu costs suffice to explain why prices are sticky?

## A4. Dornbusch model

Consider the following Dornbusch model with short run dynamics for the nominal exchange rate $s$ (defined as the domestic price of foreign currency) and aggregate price level $p$ :

$$
\begin{aligned}
\dot{s} & =\kappa \delta \frac{1}{\theta}(s-\bar{s})+(1-\kappa \delta) \frac{1}{\theta}(p-\bar{p}) \\
\dot{p} & =\psi \delta(s-\bar{s})-\psi \delta(p-\bar{p})
\end{aligned}
$$

where the steady state equals $\bar{s}=\bar{q}+\bar{k}$ and $\bar{p}=\bar{p}^{*}+\bar{k}$, with real exchange rate $\bar{q}=\frac{1}{\delta}(\bar{y}-\bar{u})$ and macroeconomic fundamentals $\bar{k}=\left(\bar{m}-\bar{m}^{*}\right)-\kappa\left(\bar{y}-\bar{y}^{*}\right)$. In addition, $\bar{m}$ denotes the money supply, $\bar{y}$ the natural rate of output and $\bar{u}$ is a measure of exogenous aggregate demand. Foreign variables are indicated by an asterisk. The parameters are given by $\kappa=\delta=\frac{1}{2}, \theta=1$ and $\psi=2$.
Illustrate the short-run dynamics of this model using a phase diagram and explain the effect of a permanent unanticipated decrease in aggregate demand $\bar{u}$. Does it lead to exchange rate overshooting?

## SECTION B

## B1. Consumption

Consider a household whose preferences are described by the following function:

$$
E_{t}\left[\sum_{s=t}^{\infty} \beta^{s} u\left(c_{s}\right)\right], \beta \in(0,1), \quad \text { with } u^{\prime}(c)>0 \text { and } u^{\prime \prime}(c)
$$

Let $A_{t}$ be household's net wealth at time $t$, which is given at time $t$. Household's budget constraint at time $t$ is:

$$
A_{t+1}=(1+r)\left(A_{t}+y_{t}-c_{t}\right)
$$

where $r$ is the interest rate and it is constant over time. Income of the household, $y_{t}$, is a random variable, which follows a known random process. This household observes $y_{t}$ at period $t$ but does not know the future values of income.
(a) Set up the household's problem and find the first-order conditions associated with this problem. Interpret the optimal consumption decision. Assume that $y_{t}$ is distributed normally and as a result the $\ln \left(c_{t+1}\right)$ is distributed normally. Let $\sigma^{2}$ denote the variance of $\ln \left(c_{t+1}\right)$ conditional on information available at time $t$. Recall that if a variable $x$ is distributed normally with mean $\mu$ and variance $\sigma^{2}$, then $E\left[e^{x}\right]=e^{\mu+\frac{\sigma^{2}}{2}}$.
(b) Suppose that $u(c)=c-\frac{\alpha}{2} c^{2}, \alpha$. Assume that $\alpha$ is sufficiently small such that $c<\frac{1}{\alpha}$ and the marginal utility of consumption is positive. Using the optimal consumption decision show that consumption follows an autoregressive process with drift: $c_{t+1}=a_{1}+a_{2} c_{t}+\epsilon_{t+1}$, where $\epsilon$ is a white noise. Define the values for $a_{1}$ and $a_{2}$.
(c) Suppose that $u(c)=\frac{c^{1-\theta}}{1-\theta}$. Show that the log of consumption follows a random walk with drift: $\ln \left(c_{t+1}\right)=b+\ln \left(c_{t}\right)+u_{t+1}$, where $u$ is a white noise. Define the value for $b$.
(d) How do the variance, $\sigma^{2}$, and the interest rate affect the drift of both consumption processes, $a_{1}$ and $b$ ? Explain intuitively your results.

## B. 2 Shapiro-Stiglitz model with unemployment benefits

Consider an economy with a representative firm that faces perfect competition in the goods market and sets the level of employment $L_{t}$ and the real wage $w_{t}$ to maximize real profits

$$
\Pi_{t}=\left(e_{t} L_{t}\right)^{\alpha}-w_{t} L_{t} \quad 0<\alpha<1
$$

where $e_{t}$ is the employee's effort level.
The representative worker maximizes lifetime utility

$$
U=\sum_{t=0}^{\infty} \frac{1}{1+\rho} v_{t}
$$

where $\rho$ is the intertemporal discount rate $(\rho>0)$ and instantaneous utility is given by

$$
v_{t}= \begin{cases}w_{t}-e_{t} & \text { if employed } \\ b>0 & \text { if unemployed }\end{cases}
$$

where $b$ is the unemployment benefit. The employee's effort level equals

$$
e_{t}= \begin{cases}\bar{e}>0 & \text { if not shirking } \\ 0 & \text { if shirking }\end{cases}
$$

Assume that in each period, $s$ is the probability that an employee loses the job for exogenous reasons, $q$ is the probability that an employee is fired at the end of the period due to shirking, and $f=f(u)$ is the probability that an unemployed worker finds a job, where $u$ is the unemployment rate, which satisfies $u=s /(s+f)$ in equilibrium.
(a) Write down the expected value of lifetime utility for the worker if (i) employed and not shirking, (ii) employed but shirking this period, and (iii) unemployed. Explain briefly.
(b) Derive the no-shirking condition. Give an intuitive explanation for the effect of the unemployment rate $u$ on the optimal wage $w$.
(c) Show graphically and explain intuitively the effect on the equilibrium wage $w$ and employment $L$ if
i. the government reduces the unemployment benefit $b$.
ii. immigration leads to an increase in the labor force $\bar{L}$.
iii. workers reduce their effort level $\bar{e}$.

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Wednesday 11 May $2011 \quad 10.00-12.00$

## M210 <br> MACROECONOMICS II

Answer all four compulsory questions from Section $A$ and one question from Section B.

Each section carries equal weight.
Write your number not your name on the cover sheet of each booklet
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## A1. Dynamic Programming

Consider the following discrete-time optimal growth model

$$
\max _{\left\{c_{t}, k_{t+1}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} u\left(c_{t}\right),
$$

subject to

$$
k_{t+1}=A k_{t}-c_{t}, k_{0}>0, A>0
$$

Assume that $u(\cdot)$ is strictly increasing, strictly concave, and bounded function. Write down the value function for this problem. Is there a solution to this problem? Explain.

## A2. Search Model

An unemployed worker must choose the number of offers $n$ to solicit. At a cost of $k(n)$ the worker receives $n$ offers this period. $k(n+1)>k(n)$ for $n \geq 1$. The number of offers $n$ must be chosen in advance at the beginning of the period and cannot be revised during the period. The worker wants to maximize $\sum_{t=0}^{\infty} \beta^{t} y_{t}$, where $\beta \in(0,1)$ is the subjective discount factor and $y_{t}$ consist of $w$ each period the agent is employed but not searching, $w-k(n)$ the first period the agent is employed but searched for $n$ offers, and $b-k(n)$ each period the agent is unemployed but solicits and rejects $n$ offers. The offers are each independently distributed from $F(w)$ and $w \in[0, B]$, with $B<\infty$. The workers who accept an offer works forever at wage $w$. Let $Q$ be the value of the problem for an unemployed worker who has not yet chosen the number of offers to solicit. Formulate the Bellman equation for this worker. Explain.

## A3. Simple interest rate rule

What is the Taylor Principle? Illustrate it using a first-order difference equation for the inflation rate.

## A4. Inflation shocks in the New Keynesian framework

Consider the following economy derived from a New-Keynesian model:

$$
\begin{align*}
\pi_{t} & =\beta E_{t} \pi_{t+1}+\kappa x_{t}+u_{t}, 0<\beta<1, \kappa>0,  \tag{1}\\
x_{t} & =E_{t} x_{t+1}-\frac{1}{\sigma}\left(i_{t}-E_{t} \pi_{t+1}-\bar{r}\right), \sigma>0,  \tag{2}\\
i_{t} & =\bar{r}+\phi_{\pi} \pi_{t}+\phi_{x} x_{t}, \phi_{\pi}>1, \phi_{x}>0 . \tag{3}
\end{align*}
$$

$\pi_{t}$ and $y_{t}$ are inflation and aggregate output in period $t, x_{t} \equiv y_{t}-\bar{y}$ is the output gap and $\bar{r}$ is the long-run real interest rate, and $i_{t}$ is the short-run nominal interest rate. $u_{t}$ is an exogenous inflation shock that follows the process

$$
u_{t}=\rho_{u} u_{t-1}+\varepsilon_{t}, \rho_{u} \in[0,1)
$$

where $\left\{\varepsilon_{t}\right\}$ is mean-zero white noise. Apply the method of undetermined coefficients to find the equilibrium responses of inflation and the output gap to an inflation shock. Comment briefly on your answer.

## SECTION B

## B1. Neoclassical Growth Model

Consider an economy with aggregate production function

$$
Y(t)=A K(t)^{\alpha} L(t)^{1-\alpha}, \alpha \in(0,1), A>0 .
$$

$Y$ is output, $K$ is capital, $L$ is the labour force, and $A$ is a total factor productivity parameter, which is constant over time. All markets are competitive, there is no population growth, the labour force is normalized to one, and capital depreciates at rate $\delta>0$. Capital evolves according to:

$$
\dot{K}(t)=I(t)-\delta K(t),
$$

where $I$ corresponds to investment.
The government imposes a proportional tax on labour income at rate $\tau^{l}$, on capital income at rate $\tau^{k}$, and on consumption at rate $\tau^{c}$. The government uses the proceeds from tax revenues to finance spending $G(t)$. The government budget constraint is balanced in every period. Assume that all tax rates are constant over time and $\lim _{t \rightarrow \infty} G(t)=G$. Let agents be infinite lived, with preferences

$$
U=\int_{0}^{\infty} e^{-\rho t} \frac{c(t)^{1-\theta}-1}{1-\theta} d t, \rho>0, \theta>0,
$$

where $c$ denotes consumption.
(a) Using the Hamiltonian, derive the first order conditions of the household's problem, as well as the first order conditions of the representative firm. Write the system of equations that describe the competitive equilibrium of this economy.
(b) Show how tax rates $\tau^{l}, \tau^{k}$ and $\tau^{c}$ affect the steady-state level of capital and output per capita. Is there any difference in the effects of these tax rates on capital per capita? Explain.
(c) If government spending $G$ in the long run increases and the government finances this spending with an increase in the consumption tax rate, what would happen with the long run level of capital per capita, output per capita, and consumption per capita? Explain.
(d) For this economy calculate the golden rule level of the capital stock per capita. Can this economy be dynamically inefficient? Explain.

## B2. Recursive Competitive Equilibrium and Asset Pricing

Consider an economy populated by a continuum of measure one of identical households, each with preferences given by $\mathbf{E}_{0}\left[\sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right)\right]$ where $c_{t}$ is consumption at period $t, u\left(c_{t}\right)$ is a function that is strictly increasing, strictly concave and twice differentiable, and $\beta \in(0,1)$ corresponds to the subjective discount factor. There is a single asset tree which yields perishable dividends $d_{t}$ (fruits). Households are initially endowed with some number of shares of the tree: $s_{0}$. Assume that $d_{t}$ is a random variable taking values from the set $\mathbf{D} \equiv\left\{d_{1}, \ldots, d_{N}\right\}$ and is distributed according to a first-order Markov chain with constant transition probability matrix. Households can trade the share of the tree. The price of the tree when the current realization of $d_{t}$ is $d$ is given by the function $p: \mathbf{D} \rightarrow R_{+}$. Households have an income flow $0<y<\infty$ at every period $t$ and $y$ is not a random variable. There are no other assets in this economy.
(a) State the problem of the representative household in recursive form. Show why the Bellman equation associated to this problem is a contraction mapping when $0<p<\infty$.
(b) Define the recursive competitive equilibrium for this economy.
(c) Assuming that the value function is differentiable, derive the stochastic Euler equation. Explain.
Now, assume that $u(c)=\frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma>0$. Let $x_{t+1}=p_{t+1}+d_{t+1}$. Using the equilibrium Euler equation, let's price the following assets in terms of the expected payoff $x_{t+1}$, the expected marginal rate of substitution between present and future consumption, and the risk-premium term.
(d) Consider a one-period discount bond. The agent pays $p_{t}$ at date $t$ for one unit of the consumption good at period $t+1$. What is the equilibrium price of this asset? Explain your result intuitively.
(e) Consider the risky asset described above. What is the equilibrium price of this asset? Write the price in terms of the expected return of the asset and the risk premium. Explain your result intuitively.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Monday 9 May 2011 2.00-4.00

M230
APPLIED MACROECONOMICS
Answer three questions only. Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet

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Metric Graph Paper
Rough work pads

[^0]1 You are given information on one quarter ahead forecasts of German GDP made by the IMF, the OECD and the World Bank. You are also given the actual outcome for GDP. The data is from the first quarter of 1980 to the last quarter of 2009.
(a) Describe a method to rank the accuracy of these forecasts.
(b) Suppose that you found that the OECD was, on average, the best forecaster. Would you discard the other two forecasts? Carefully explain your answer.
(c) You are asked to produce your own forecasts for Germany. Would you also produce nowcasts? Carefully explain your answer.

2 The relationship between fiscal debt and borrowing is described by the following relationship.

$$
\Delta b_{t}=d_{t}+\left(r_{t}-g_{t}\right) b_{t}
$$

(a) explain carefully each of the terms in the expression.
(b) Show in a diagram how debt evolves when $r_{t}>g_{t}$, for
(i) a high level of initial debt.
(ii) a low level of initial debt.
(c) Do the same as in (b) for the case in which $r_{t}<g_{t}$.
(d) Explain how you can use this relationship to interprete recent fiscal events in Europe or elsewhere around the world.

3 Why do macroeconomists generally look to aggregate shocks to explain business cycle fluctuations? Are there reasons why they need not always do so?

4 Outline the main features of 'Inflation Targeting' regimes. Have they been more successful at reducing inflation and output volatility compared to other policy regimes? How have they faired during the present financial and economic crisis?

5 What is the rationale for the use of Quantatitive Easing (QE)? Evaluate the empirical evidence on the effects of QE..

6 A model for output and inflation can be written:

$$
\begin{align*}
& y_{t}=-\lambda r r_{t}+\eta_{t}  \tag{1}\\
& \pi_{t}=\pi_{t}^{e}+\beta y_{t}+\varepsilon_{t}  \tag{2}\\
& L_{t}=\delta\left[y_{t}^{2}\right]+\left(\pi_{t}\right)^{2} \tag{3}
\end{align*}
$$

$y_{t}$ is the output gap in time period $\mathrm{t}, r r_{t}$ is defined as $r r_{t}=r_{t}-\pi_{t}-r^{*}$, with $r^{*}$ the long term (natural) real interest rate and $r$ the nominal interest rate. $\pi_{t}$ is the inflation rate.Expected inflation is assumed to be zero and the policymaker observes the current period shocks $\eta_{t}$ and $\varepsilon_{t}$ perfectly. These shocks have zero means and variances $\sigma_{\eta}^{2}$ and $\sigma_{\varepsilon}^{2}$ respectively. The optimal feedback rule for the nominal interest rate is:

$$
\begin{equation*}
r_{t}=r^{*}+\phi_{1} \eta_{t}+\phi_{2} \varepsilon_{t} \tag{4}
\end{equation*}
$$

where $\phi_{1}=[1 / \lambda]$ and $\phi_{2}=\left[\beta / \lambda\left(\delta+\beta^{2}\right)\right]$.
(a) Explain carefully what these 4 equations represent.
(b) Which variables can be considered as targets of policy and which as instruments of policy?
(c) Define controllability. Is the model above controllable? Explain your answer.
(d) What is the efficient policy frontier?
(e) What does a rise in $\sigma_{\eta}^{2}$ and $\sigma_{\varepsilon}^{2}$ do to the efficient policy frontier?

7 'The Solow Growth Model does not provide a convincing explanation for the variation in per capita incomes around the world'. Discuss in the context of the empirical studies by Mankiw, Romer and Weil (1992) and Hall and Jones (1999).

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY.
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Monday 16 May $2011 \quad 2.00-4.00$
Module 240
INTERNATIONAL FINANCE
Candidates are required to answer four our of five questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS

20 Page booklet x 2 Approved calculators allowed
Rough work pads
Graph paper
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## Question 1

## Complete markets

Consider two symmetric countries, each inhabited by many identical individuals, with utility functions $U\left(C_{t}\right)$ and $U\left(C_{t+1}^{*}\right)$, in the Home and the Foreign country, respectively. Consumption in both countries include both domestically produced goods, and foreign goods. There is home bias in consumption.

1. Assume that markets are complete across borders: write and interpret the equilibrium condition pricing the real exchange rate.
2. Do complete markets imply that the real interest rate is equalized across countries at each point in time? Explain.
3. In the above economy, suppose that at time $t$ there are news about the future productivity in the Home country: Home output will be much higher from period $t+1$ on. Do you agree with the following analysis "By the permanent income hypothesis, higher domestic wealth will translate into higher domestic consumption; the consumption demand boom in turn will appreciate the exchange rate"? Why or why not? Explain.

Question 2
Covered interest parity and risk
Let $i^{\$}, i, \mathcal{E}$ and $\mathcal{F}$ denote the dollar interest rate; the peso interest rate; the spot and forward exchange rates (dollars per peso), respectively.

1. Write the 'covered interest parity' (CIP) condition; explain when the CIP condition can be expected to hold, by which mechanism, and what can cause deviations from it.
2. Suppose that during period $t$ the country issuing pesos runs into fiscal problems, and there are rumors that the government may be forced to default partially on its liabilities in the future. International investors are fully informed about this possibility, and attach a probability $\omega$ that the interest income from government bonds will be cut by $\tau$ cents per peso in period $t+1$.
(a) Write the cash flow $x^{\S}$ in dollar accruing to international investors who borrow in dollars to take a covered position in one-period pesodenominated bonds, depending on whether the default does and does not occur. Will the CIP conditions still hold?
(b) Define a generic stochastic discount factor in real terms $\mathcal{D}$. Derive an expression relating deviations from the covered interest parity to risk. In doing so, be careful in distinguishing between variables in nominal and real terms.

## Question 3

## Uncovered interest parity and the peso problem

1. Define the Uncovered Interest Parity (UIP) condition and describe the conditions under which it can be derived from the equilibrium price of a nominal bond (use a generic stochastic discount rate $\mathcal{D}$ ).
2. What is the 'peso problem'? After defining it, explain why the peso problem may explain large deviations from the UIP condition. Provide a numerical example, focusing for simplicity on the case of a small open economy which take the foreign interest rate as given, and pursues a fixed exchange rate against the dollar.
3. What is the role of risk, if any, in peso-problem based explanations of deviations from the UIP?

## Question 4

## 1. Financial Fragility and Currency Crises

Consider the following Aghion, Bacchetta and Banerjee (EER 2000) twoperiod model of a small open economy. Suppose that asset market equilibrium is described by

$$
\begin{equation*}
S_{1}=\frac{1+i^{*}}{1+i_{1}} \frac{M_{2}}{L\left(Y_{2}, i_{2}\right)} \tag{IPLM}
\end{equation*}
$$

where $S_{t}$ is the nominal exchange rate, defined as the domestic price of foreign currency, $i_{t}$ and $i_{t}^{*}$ the domestic and foreign nominal interest rate, respectively, $M_{t}$ the nominal money supply, and $Y_{t}$ aggregate real output. Assume that real money demand is described by $L(Y, i)=Y e^{\alpha-\beta i}$, where $\alpha$ and $\beta$ are positive parameters.
The technology and financing constraints imply

$$
\begin{equation*}
Y_{2}=\max \left\{0,\left(1+e^{-\gamma i_{1}}\right)(1-\alpha)\left[Y_{1}-\left(1+r_{0}\right) \bar{B}-\left(1+i^{*}\right) \frac{S_{1}}{P_{1}}\left(B_{1}-\bar{B}\right)\right]\right\} \tag{FC}
\end{equation*}
$$

where $r_{t}$ is the domestic real interest rate, $B_{t}$ the domestic firm's borrowing, $\bar{B}$ the size of the domestic credit market, $P_{t}$ the aggregate price level, which is preset at the end of period $t-1$, and $\alpha$ and $\gamma$ are positive parameters, with $0<\alpha<1$. Assume that $B_{1}>\bar{B}$.
(a) Explain intuitively the relationship between $S_{1}$ and $Y_{2}$ according to (IPLM) and according to (FC). What kind(s) of financial fragility does this economy suffer from?
(b) Show and explain carefully how many equilibria there are (if any). Does this depend on the level of $B_{1}-\bar{B}$ ?
(c) Suppose the central bank raises the interest rate $i_{1}$. Show graphically and explain intuitively how this affects the exchange rate $S_{1}$ and future output $Y_{2}$. Could this cause a currency crisis?

## Question 5

## Stabilization of global recession

Consider the two-country monetary model by Corsetti and Pesenti (2007) with nominal rigidities: firms preset prices one period in advance in domestic currency, and stand ready to satisfy domestic and foreign demand at that price.
Suppose that the world economy is in a global recession. In both the Home and the Foreign country, (for given productivity, exchange rate and prices), employment and output initially are below their natural rate level.

1. Holding the Foreign monetary stance fixed:
(a) Discuss what Home policymakers can do to stabilize the economy, and analyze the impact of the policy measure you identify, on the exchange rate and the terms of trade, as well as on output, consumption, and prices in Home country.
(b) Analyze the impact of Home monetary policy in the Foreign country. Are foreign residents better- or worse-offs? Explain.
2. Suppose that the Foreign central bank pursues an exchange rate peg (hold $\mathcal{E}$ fixed).
(a) What would be the consequence for the world equilibrium?
(b) Could the Foreign central bank do better?

Feel free to answer using graphs or algebra, or both. For your memory, this is the analytical synthesis of the model.

To use in the graph :
$\mathcal{E}_{t}=\mu_{t} / \mu_{t}^{*}$
$P_{t} C_{t}=\mu_{t}$
$P_{t}^{*} C_{t}^{*}=\mu_{t}^{*}$
$C_{t}=Z_{t} \ell_{t} \tau_{t}$
$C_{t}^{*}=Z_{t}^{*} \ell_{t}^{*} \tau_{t}^{*}$
$\frac{1}{\tau_{t}} \equiv \frac{P_{t}}{2}\left(\frac{1}{P_{H, t}}+\frac{1}{\mathcal{E} P_{H, t}^{*}}\right)$
$\frac{1}{\tau_{t}^{*}} \equiv \frac{P_{t}^{*}}{2}\left(\frac{1}{P_{F, t}^{*}}+\frac{\mathcal{E}_{t}}{P_{F, t}}\right)$
Plus
$P_{t}=2 P_{H, t}^{1 / 2} P_{F, t}^{1 / 2}$
$P_{t}^{*}=2 P_{H, t}^{* 1 / 2} P_{F, t}^{* 1 / 2}$
$P_{H, t} C_{H, t}=\frac{1}{2} P_{t} C_{t}$
$P_{F, t} C_{F, t}=\frac{1}{2} P_{t} C_{t}$
$P_{H, t}^{*} C_{H, t}^{*}=\frac{1}{2} P_{t}^{*} C_{t}^{*}$
$P_{F, t}^{*} C_{F, t}^{*}=\frac{1}{2} P_{t}^{*} C_{t}^{*}$
Auxiliary expressions:
$\ell_{t}=E_{t-1}\left(\ell_{t}\right)=\frac{\theta-1}{\theta \kappa}=\bar{\ell}$
$\ell_{t}^{*}=E_{t-1}\left(\ell_{t}^{*}\right)=\frac{\theta-1}{\theta \kappa}=\bar{\ell}$

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY
Wednesday 18 May $2011 \quad 10.00-12.00$

Module 250
INTERNATIONAL TRADE
Candidates are required to answer Two compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Approved calculators allowed
Rough work pads
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## Question 1 Increasing returns

The economy is populated by two identical countries trading goods in a free trade regime (no trade costs). The individual preference structure is

$$
\begin{equation*}
U=\sum_{i=1}^{n} u\left(d_{i}\right)+\sum_{i=n+1}^{n+n^{*}} u\left(d_{i}\right) \tag{1}
\end{equation*}
$$

where $n$ denotes the goods produced at home and $n^{*}$ the goods produced in the foreign country. A quantity $q_{i}$ of good $i$ is produced with the following amount of labour

$$
l_{i}=f+a q_{i}
$$

in monopolistic competition. Countries are endowed with $L$ and $L^{*}$ units of labour and the wage rates are $w$ and $w^{*}$. Assume that the generic preference in (1) yields the indirect demand function $p_{i}=p\left(q_{i}\right)$ with the following properties,

$$
\varepsilon\left(q_{i}\right)=-\frac{\partial q_{i}}{\partial p_{i}} \frac{p_{i}}{q_{i}}>0
$$

where $\varepsilon\left(q_{i}\right)$ is the price elasticity of demand, and $q_{i}=L d+L^{*} d^{*}$, implying that total quantity produced of good $i$ is equal to global demand. Note that since countries are symmetric and each good $i$ enters the utility and the production function symmetrically, domestic and foreign per capita demand for each good is the same across goods and countries, $d_{i}=d_{i}^{*}=d$, and similarly, $p_{i}=p_{i}^{*}=p$ and $w=w^{*}$ ( factor price equalization holds) This implies that price elasticity of global production is equal to the elasticity of the individual consumer demand: $\varepsilon\left(q_{i}\right)=-\frac{\partial\left(L+L^{*}\right) d}{\partial p_{i}} \frac{p_{i}}{\left(L+L^{*}\right) d}=\varepsilon(d)=-\frac{\partial d}{\partial p_{i}} \frac{p_{i}}{d}>1$. The elasticity is greater than one because monopolists operates on the elastic segment of its demand function. Assume that the elasticity of demand $\varepsilon(d)$ declines with individual consumption $d$.
i. Determine the equilibrium production in this economy. Denoting $d^{A}$ and $q^{A}$ as the per-capita demand and the total quantity produced in autarky and $d^{T}$ and $q^{T}$ the demand and quantity produced under free trade, show that comparing autarky with free trade individual demand for each variety is lower and the total quantity of each variety produced is higher under trade $\left(d^{A}>d^{T}\right.$ and $\left.q^{A}<q^{T}\right)$.
ii. Show that the number of goods produced by a single country under free trade is less than that produced in autarky $\left(n^{T}<n^{A}\right)$, but the total number of varieties available to consumers under free trade is larger than in autarky $\left(n^{T}+n^{* T}<n^{A}\right)$. What are the channels of gains from trade in this economy? Although the whole economy gains from trade, are there any losers within the economy?
iii. Remove the assumption that the elasticity is decreasing in individual consumption, and assume constant price elasticity of demand $(\varepsilon(d)=\varepsilon)$ repeat the exercise in point i) and ii) and discuss the differences.

Question 2 Oligopoly, endogenous markups and heterogeneous firms Consider the following simplified version of Impullitti and Licandro (2010). Agents have utility

$$
\begin{equation*}
U=\ln X+\beta \ln Y \tag{2}
\end{equation*}
$$

where $Y$, is a homogeneous good, and $X$ a composite good

$$
\begin{equation*}
X=\left(\int_{0}^{M} x_{j}^{\alpha} d j\right)^{\frac{1}{\alpha}} \tag{3}
\end{equation*}
$$

$M$ is the mass of active varieties. The economy is populated with a continuum of agents of measure 1, endowed with one unit of labour. The homogeneous good is assumed to be the numeraire. The production technology is as follows: $Y$ is produced with 1 unit of labour in perfect competition. Each variety is produced by $n$ identical oligopolistic firms ( $n$ exogenous), while productivity differs across varieties. A firm producing a variety with productivity $z$ operates technology

$$
q=z(y-\lambda)
$$

where $\lambda$ is a fixed cost. The $n$ firm within a variety compete Cournot. With probability $\delta$ firm are hit by a bad shock and exit the market.
i. Solve the household problem: determine the optimal choice of $X$ and $Y$, and the optimal choice of each varieties in the differentiated good, $x_{j}$.
ii. Solve the firm problem in closed economy and derive the price equation for a firm with productivity $z$.
iii. Consider the following assumption

Assumption 1. The productivity distribution $\digamma(z)$ is such that $z^{*} / \bar{z}\left(z^{*}\right)$ is increasing in $z^{*}$, and the following parameter restrictions hold

$$
\frac{\lambda \bar{z}_{e}}{z_{\min }}>\frac{1-\theta}{\beta+\theta}\left(\frac{1}{n}-\lambda\right) .
$$

Using assumption 1 and the following result

$$
z^{-1} q=\theta e \frac{z}{\bar{z}}
$$

where $\bar{z}$ is avg. productivity $\bar{z}=\frac{1}{M} \int_{0}^{M} z_{j} d j$ and $e \equiv E / n M$ expenditure per firm, derive the equilibrium conditions of this economy and show graphically that a unique equilibrium cutoff exists $z^{*}$. (Hint: derive the equilibrium cutoff condition (EC), the market clearing condition (MC), and the stationarity condition for varieties). Show graphically the effects of an exogenous decrease in the markup on firm selection and on average productivity. Explain how the firm selection mechanism work.
iv. Assume now that this economy trades with another identical economy and that goods incur in an iceberg transportation cost $\tau>1$ when shipped abroad. Can a decrease in the markup provide a first glance of the effects of trade liberalization on firm selection in this model? Explain. How does the selection channel in this model differ from that in Melitz (2003)? Compare this model with Melitz and Ottaviano (2008), what is the main difference?

# EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY <br> EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY <br> EXAMINATION IN FINANCE FOR THE DEGREE OF MASTER OF PHILOSOPHY 

Module M300

ECONOMETRIC METHODS

Answer all questions in Section $A$ and two questions from Section B. Part A is $60 \%$ of the total marks.

You are permitted to use your own calculator where it has been stamped as approved by the University.
New Cambridge Elementary Statistical Tables, standard graph paper, Durbin-Watson and Dickey-Fuller Tables are provided.
Credit will be given for clear presentation of relevant statistics.

STATIONERY REQUIREMENTS
20 Page Booklet
Metric Graph Paper
Rough Work Pads

SPECIAL REQUIREMENTS
Durbin-Watson and Dickey-Fuller
Tables
New Cambridge Elementary
Statistical Tables
Approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

A. 1 The mean of the exponential distribution

$$
f\left(y ; \theta_{0}\right)=\frac{1}{\theta_{0}} \exp \left(-y / \theta_{0}\right), 0 \leq y<\infty
$$

is $\theta_{0}$. Given a random sample $y_{i},(i=1, \ldots, n)$, of size $n$ from the exponential distribution
(a) Write down the log-likelihood function.
(b) Show that the Cramér-Rao lower bound is $\theta_{0}^{2} / n$.
A. 2 Let $\varepsilon_{t}$ denote a white noise process. Determine the mean and autocorrelation function of the time series

$$
y_{t}=\mu_{0}+\varepsilon_{t}+\theta_{0} \varepsilon_{t-4},(t=1,2, \ldots)
$$

A. 3 Let $z$ denote a scalar variable. Suppose that the current value $y_{t}$ of $y$ is related to the expected value $z_{t+1}^{*}$ of $z$ in the next period by the relation

$$
y_{t}=\eta_{0} z_{t+1}^{*}+u_{t}
$$

where the error term $u_{t}$ is independent of $z_{s}$ for all $s$ and $t$. Also suppose that the expected value $z_{t+1}^{*}$ evolves according to the following mechanism defined in terms of the expectation in the previous period $z_{t}^{*}$ and the current value $z_{t}$ of $z$

$$
z_{t+1}^{*}=z_{t}^{*}+\alpha_{0}\left(z_{t}-z_{t}^{*}\right), 0<\alpha_{0}<1 .
$$

(a) Show that $y_{t}$ may be expressed as a distributed lag in $z_{t}$, i.e.

$$
y_{t}=\frac{1}{1-\phi_{0} L} \delta_{0} z_{t}+u_{t}
$$

where $L$ is the lag operator. What are $\phi_{0}$ and $\delta_{0}$ in terms of $\alpha_{0}$ and $\eta_{0}$ ?
(b) Are the least squares estimators of $\phi_{0}$ and $\delta_{0}$ from a regression of $y_{t}$ on $y_{t-1}$ and $z_{t},(t=1, \ldots, T)$, consistent? Briefly justify your answer.
A. 4 Suppose that $y \mid x \sim N\left(\beta_{0}+\log \left(1+\exp \left(-\gamma_{0} x\right)\right), \sigma_{0}^{2}\right)$, i.e., the conditional probability density function of $y$ given $x$ is

$$
\frac{1}{\sqrt{2 \pi \sigma_{0}^{2}}} \exp \left(\frac{1}{2 \sigma_{0}^{2}}\left(y-\beta_{0}-\log \left(1+\exp \left(-\gamma_{0} x\right)\right)\right)^{2}\right)
$$

Consider the random sample $\left(y_{i}, x_{i}\right),(i=1, \ldots, n)$, taken from the joint distribution of $(y, x)$. Show that the $n \mathcal{R}^{2}$ form of the Lagrange multiplier (LM) test statistic for the null hypothesis $H_{0}: \gamma=0$ may be expressed in terms of a regression of $\hat{u}_{i}$ on a constant and $x_{i},(i=1, \ldots, n)$, where $\hat{u}_{i},(i=1, \ldots, n)$, are the least squares residuals obtained from regressing $y_{i}$ on a constant, $(i=1, \ldots, n)$.
A. 5 Consider the stationary $A R M A(1,1)$ model

$$
y_{t}-\phi y_{t-1}=\varepsilon_{t}+\theta \varepsilon_{t-1},(t=1,2, \ldots),
$$

where $\varepsilon_{t}$ are independent $N\left(0, \sigma_{0}^{2}\right)$ random variables. You are given the realisation $y_{t},(t=1, \ldots, T)$, together with the initial observation $y_{0}$. Suppose that $\phi$ and $\theta$ are known.
(a) What is the optimal predictor for $y_{T+l},(l=1,2)$ ?
(b) Obtain the mean squared error of the optimal predictor for $y_{T+2}$ in (a).
A. 6 The conditional probability that a binary variable $y$ takes value 1 given the $p$ vector of continuous explanatory variables $\mathbf{x}$ is $F\left(\mathbf{x}^{\prime} \boldsymbol{\beta}_{0}\right)$, where the function $F(\cdot)$ is known. Suppose that the $l$ th element of $\mathbf{x}$ increases by one unit.
(a) What is the marginal effect on the conditional probability $F\left(\mathbf{x}^{\prime} \boldsymbol{\beta}_{0}\right)$ ?
(b) Briefly describe how this effect might be estimated given the random sample $\left(y_{i}, \mathbf{x}_{i}\right),(i=1, \ldots, n)$, from the distribution of $(y, \mathbf{x})$.

## SECTION B

B. 1 Consider the random walk plus drift model

$$
y_{t}=\beta_{0}+y_{t-1}+u_{t},(t=1,2, \ldots)
$$

where $u_{t},(t=1,2, \ldots)$, is independently and identically distributed with mean 0 and variance $\sigma_{0}^{2}$. The realisation $y_{t},(t=1, \ldots, T)$, together with the initial observation $y_{0}$, is available.
(a) Is the estimator

$$
\hat{\beta}=\left(y_{T}-y_{0}\right) / T
$$

of $\beta_{0}$
i. unbiased;
ii. consistent?
(b) Show that

$$
T^{1 / 2}\left(\hat{\beta}-\beta_{0}\right) \xrightarrow{d} N\left(0, \sigma_{0}^{2}\right)
$$

(c) Suppose that $\hat{\sigma}^{2}$ is a consistent estimator of $\sigma_{0}^{2}$. Explain how the result in (b) may be used to provide an approximate $100(1-\alpha) \%$ confidence interval for $\beta_{0}$.
B. 2 Consider the linear regression model

$$
E[y \mid \mathbf{x}, z]=\mathbf{x}^{\prime} \boldsymbol{\beta}_{0}+\gamma_{0} z, \operatorname{var}[y \mid \mathbf{x}, z]=\sigma_{0}^{2}(\mathbf{x}, z)
$$

where $\mathbf{x}$ and $z$ are a $p$-vector and a scalar explanatory variable respectively. A random sample $\left(y_{i}, \mathbf{x}_{i}, z_{i}\right),(i=1, \ldots, n)$, is available.
(a) Show that in general the least squares (LS) estimator of $\boldsymbol{\beta}_{0}$ is biased if $z$ is omitted.
(b) Obtain the conditional variance of the LS estimator of $\boldsymbol{\beta}_{0}$ given $\left(\mathbf{x}_{i}, z_{i}\right)$, $(i=1, \ldots, n)$, when the restriction $\gamma_{0}=0$ is incorrectly imposed.
(c) Describe how a test for the exclusion of $z$ might be conducted.
B. 3 In a given period of time the number of arrivals $y$ at a check-in desk at an airport terminal is conditionally distributed, given covariates $\mathbf{x}$, as a Poisson random variate with probability mass function

$$
f_{y \mid x}\left(y \mid \mathbf{x} ; \boldsymbol{\beta}_{0}\right)=\frac{\left(\mathbf{x}^{\prime} \boldsymbol{\beta}_{0}\right)^{y} \exp \left(-\mathbf{x}^{\prime} \boldsymbol{\beta}_{0}\right)}{y!}, y=0,1, \ldots
$$

A random sample of observations $\left(y_{i}, \mathbf{x}_{i}\right),(i=1, \ldots, n)$, is available.
(a) Write down the log-likelihood.
(b) Obtain the score vector.
(c) Derive the Hessian.
(d) Hence, or otherwise, find the asymptotic variance matrix of the maximum likelihood estimator of $\boldsymbol{\beta}_{0}$. [Note: $E[y \mid \mathbf{x}]=\mathbf{x}^{\prime} \boldsymbol{\beta}_{0}$.] How might this matrix be estimated?
B. 4 Write short notes on TWO of the following.
(a) Likelihood ratio tests.
(b) Convergence in probability.
(c) Unit root tests.

END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

| Tuesday 17 May 2011 | $2.00-4.00$ |
| :--- | :--- |

M310
TIME SERIES \& FINANCIAL ECONOMETRICS
Candidates are required to answer SIX questions from Section $A$ and $\boldsymbol{O N E}$ question from Section B.

Section A is weighted as $60 \%$ and Section B as $40 \%$.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
None
Rough work pads
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

A. 1 The local level unobserved components model is

$$
\begin{gathered}
y_{t}=\mu_{t}+\varepsilon_{t}, \quad \varepsilon_{t} \sim N I D\left(0, \sigma_{\varepsilon}^{2}\right), \quad t=1, \ldots, T, \\
\mu_{t}=\mu_{t-1}+\eta_{t}, \quad \eta_{t} \sim N I D\left(0, \sigma_{\eta}^{2}\right),
\end{gathered}
$$

where $\operatorname{NID}\left(0, \sigma^{2}\right)$ denotes normally and independently distributed with mean zero and constant variance $\sigma^{2}$ and $E\left(\varepsilon_{t} \eta_{s}\right)=0, t, s=1, . ., T$. Obtain the autocovariance function of the first difference of $y_{t}$. Hence identify $p$ and $q$ in the reduced form $A R M A(p, q)$ model and derive expressions for the reduced form parameters in terms of $\sigma_{\varepsilon}^{2}$ and the signal-noise ratio, $Q=\sigma_{\eta}^{2} / \sigma_{\varepsilon}^{2}$.
A. 2 A linear unobserved components (UC) model for a univariate time series can be written in matrix form as

$$
\mathbf{y}=\boldsymbol{\mu}+\boldsymbol{v}
$$

where $\mathbf{y}, \boldsymbol{\mu}$ and $\boldsymbol{v}$ are $T \times 1$ vectors with zero means and fixed covariance matrices $\mathbf{V}_{y}, \mathbf{V}_{\mu}$ and $\mathbf{V}_{v}$ respectively. If $\boldsymbol{\mu}$ and $\boldsymbol{v}$, and hence $\mathbf{y}$, have multivariate normal distributions, and $\mathbf{V}_{y}$ is p.d., the minimum mean square error estimator (MMSE) of $\boldsymbol{\mu}$ is $\widetilde{\boldsymbol{\mu}}=\mathbf{V}_{\mu y} \mathbf{V}_{y}^{-1} \mathbf{y}$, where $\mathbf{V}_{\mu y}=E\left(\boldsymbol{\mu} \mathbf{y}^{\prime}\right)$. Its mean square error matrix is $\mathbf{V}_{\mu}-\mathbf{V}_{\mu y} \mathbf{V}_{y}^{-1} \mathbf{V}_{y \mu}$. If the normality assumption is dropped, show that $\widetilde{\boldsymbol{\mu}}$ is still the minimum mean square error linear estimator (MMSLE) of $\boldsymbol{\mu}$.
A. 3 Consider the model

$$
y_{t}=\varepsilon_{t}+\beta \varepsilon_{t-1}^{2} \varepsilon_{t-2}, \quad \varepsilon_{t} \sim N I D\left(0, \sigma^{2}\right), \quad t=1, \ldots, T
$$

where $N I D\left(0, \sigma^{2}\right)$ denotes normally and independently distributed with mean zero and constant variance $\sigma^{2}$, and $\varepsilon_{0}$ and $\varepsilon_{-1}$ are fixed and known. Is $y_{t}$ (i) a martingale difference, (ii) white noise?
A. 4 The sample spectrum is

$$
I^{\#}\left(\lambda_{j}\right)=\frac{1}{2 \pi}\left[c(0)+2 \sum_{\tau=1}^{T-1} c(\tau) \cos \lambda_{j} \tau\right], \quad \lambda_{j}=2 \pi j / T, \quad j=0, \ldots,[T / 2]
$$

Explain briefly why the sample spectrum is not a good estimator of the spectrum of an indeterministic stationary process. How might you weight the sample autocovariances, $c(\tau)$, in order to produce a better estimator?
A. 5 Consider an integrated random walk plus noise model, that is

$$
\begin{aligned}
y_{t} & =\mu_{t}+\varepsilon_{t}, \quad \varepsilon_{t} \sim \operatorname{NID}\left(0, \sigma_{\varepsilon}^{2}\right), \quad t=1, \ldots, T, \\
\Delta^{2} \mu_{t} & =\zeta_{t-1}, \quad \zeta_{t} \sim \operatorname{NID}\left(0, \sigma_{\zeta}^{2}\right),
\end{aligned}
$$

where $N I D\left(0, \sigma^{2}\right)$ denotes normally and independently distributed with mean zero and constant variance, $\sigma^{2}$, and $E\left(\varepsilon_{t} \zeta_{s}\right)=0, t, s=1, . ., T$. Show how this model can be put in state space form. Could you use such a model to compute a detrended series equivalent to that obtained from the HodrickPrescott filter?
A. 6 Show that the probability integral transform (PIT) of a random variable with probability density function $f(y)$ is uniformly distributed. Find the PIT of a random variable with probability density function

$$
f(y)= \begin{cases}\psi-\frac{\psi^{2}}{2} y, & 0 \leq y \leq \frac{2}{\psi} \\ 0, & \text { elsewhere }\end{cases}
$$

where $\psi$ is a positive parameter.
A. 7 Derive the autocorrelation function of the squared observations, from an $A R C H(1)$ model

$$
\begin{gathered}
y_{t}=\sigma_{t} \varepsilon_{t}, \quad \varepsilon_{t} \sim N I D(0,1), \quad t=1, . ., T, \\
\sigma_{t}^{2}=\gamma+\alpha y_{t-1}^{2}, \quad \gamma>0, \quad 0<\alpha<\sqrt{1 / 3},
\end{gathered}
$$

where $\alpha$ and $\gamma$ are parameters, and $\operatorname{NID}(0,1)$ denotes normally and independently distributed with mean zero and unit variance.
A. 8 The copula is a joint distribution function of standard uniform random variables, that is

$$
C\left(u_{1}, u_{2}\right)=\operatorname{Pr}\left(U_{1} \leq u_{1}, U_{2} \leq u_{2}\right), \quad 0 \leq u_{1}, u_{2} \leq 1
$$

How is the copula linked to the quantiles, $\xi\left(\tau_{1}\right)$ and $\xi\left(\tau_{2}\right)$, of two variables $y_{1}$ and $y_{2}$ ? Define the survival function and explain what is meant by positive quadrant dependency.

## SECTION B

B. 1 (a) (i) Determine which, if any, of the parameters $(\alpha, \beta, \gamma)$ in the following supply and demand model are identified:

$$
\begin{gathered}
q_{t}^{S}=\alpha p_{t}+\varepsilon_{1 t} \\
q_{t}^{D}=\beta+\gamma p_{t}+\varepsilon_{2 t} \\
q_{t}^{S}=q_{t}^{D}
\end{gathered}
$$

The disturbances $\varepsilon_{1 t}$ and $\varepsilon_{2 t}$ are normally distributed with zero mean and constant variances and they may be correlated.
(ii) If the price in the supply equation is lagged, that is

$$
q_{t}^{S}=\alpha p_{t-1}+\varepsilon_{1 t}
$$

does your answer change? Are any restrictions needed on the parameters?
(iii) Write down the full log-likelihood function for the model in (ii). NB If a vector is multivariate normal, that is $\mathbf{y} \sim N(\boldsymbol{\mu}, \Sigma)$, the logarithm of its pdf is

$$
\log p(\mathbf{y})=-\frac{N}{2} \log 2 \pi-\frac{1}{2} \log |\Sigma|-\frac{1}{2}(\mathbf{y}-\boldsymbol{\mu})^{\prime} \Sigma^{-1}(\mathbf{y}-\boldsymbol{\mu})
$$

(b) (i) When expectations of inflation, $\pi_{t}$, are based on the New Keynesian Phillips curve,

$$
\pi_{t}=\gamma E_{t}\left(\pi_{t+1}\right)+\beta^{*} x_{t}+\varepsilon_{t}, \quad 0 \leq \gamma \leq 1, \quad \varepsilon_{t} \sim N I D\left(0, \sigma^{2}\right)
$$

where $\beta^{*}$ and $\gamma$ are unknown parameters, show that inflation depends on the discounted sequence of future output gaps, $x_{t}$.
(ii) If $x_{t}$ is assumed to be a first-order autoregressive process,

$$
x_{t}=\phi x_{t-1}+\xi_{t}, \quad \xi_{t} \sim N I D\left(0, \sigma_{\xi}^{2}\right),
$$

with $\xi_{t}$ independent of $\varepsilon_{t}$, and $|\phi \gamma|<1$, show that there is an identifiability problem.
B. 2 Discuss TWO of the following topics. (a) The potential distortions in the properties of economic time series induced by filters. (b) Markov chains. (c) The relevance of copulas for financial time series.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Thursday 19 May 2011
2.00-4.00

Module 320

## CROSS SECTION AND PANEL DATA ECONOMETRICS

Candidates are required to answer three questions from Section $A$ and one question from Section B.

Section A is weighted as $60 \%$ and Section B as $40 \%$.
Write your number not your name on the cover sheet of each booklet

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
| :--- | :--- |
| 20 Page booklet $x 2$ | Durbin-Watson and Dickey-Fuller Tables |
| Rough work pads | New Cambridge Elementary Statistical Tables |
| Tags | Approved calculators allowed |

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

A1 In the dynamic fixed effects panel model

$$
y_{i t}=\alpha_{i}+\lambda y_{i t-1}+\varepsilon_{i t},
$$

where $i=1, \ldots, N$ and $t=1, \ldots, T$, explain carefully how the GMM estimation of the parameter $\lambda$ may be implemented.

A2 Labour market histories for a random sample of men aged between 25 and 50 in 2007 contains information on labour market status (employed or not employed), education (in five categories, indicating increasing levels of education) and age (in years).
$L=1$ if someone is employed in January 2007 and 0 if not. Education is denoted by $E$, and age by $A$.
Using a probit model, the probability that an individual was employed in 2007, is estimated using the following specification:

$$
\begin{aligned}
\operatorname{Pr}[L=1 \mid A, E] & =\Phi\left(\mathbf{x}^{\prime} \boldsymbol{\theta}\right) \\
& =\Phi\left[\theta_{0}+\theta_{1} \cdot E+\theta_{2} \cdot(A-35)+\theta_{3} \cdot(A-35)^{2}\right]
\end{aligned}
$$

where $\Phi(\cdot)$ is the standard normal distribution function.
i) A GMM estimator incorporates knowledge of the conditional employment probabilities using the following moment conditions

$$
\begin{equation*}
\psi_{1 j}(L, \mathbf{x}, \boldsymbol{\theta})=\frac{L-\Phi\left(\mathbf{x}^{\prime} \boldsymbol{\theta}\right)}{\Phi\left(\mathbf{x}^{\prime} \theta\right)\left(1-\Phi\left(\mathbf{x}^{\prime} \boldsymbol{\theta}\right)\right)} \phi\left(\mathbf{x}^{\prime} \boldsymbol{\theta}\right) \cdot x_{j} \quad j=1, \ldots, 4 \tag{1.1}
\end{equation*}
$$

where $\mathbf{x}=\left(1, E, A-35,(A-35)^{2}\right)$ and $\phi($.$) denotes the probability$ density function of the standard normal distribution.

Explain the rationale for these moment conditions
ii) In Table 1 we present information on the actual size of the male population in various age categories, with $p_{i}$ denoting the age related employment probabilities. In what sense does the information in Table 1 provide additional moment conditions relative to $\psi_{1 j}(L, \mathbf{x}, \boldsymbol{\theta}), \quad j=1, \ldots, 4$. Briefly justify your answer.

Table 1

| Estimate of age category and conditional |  |  |  |
| :---: | :---: | :---: | :---: |
| employment probabilities |  |  |  |
| Age category | Working | Total | $p_{i}$ |
| $25-29$ | 555 | 609.0 | 0.911 |
| $30-34$ | 499 | 534.8 | 0.933 |
| $35-39$ | 407 | 436.7 | 0.932 |
| $40-44$ | 370 | 396.8 | 0.932 |
| $45-49$ | 337 | 378.0 | 0.891 |
| Total | 2168 | 2355.3 | 0.921 |

A3 The sampling-theoretic approaches to random effects models typically average the model likelihood over the random effects distribution and provide no natural way to make individual-level inferences. However, a distinguishing characteristic of random-effects applications in marketing is the interest not only in the hyperparameters of the random-effects distribution but in making inferences about individual-level parameters.
Evaluate this statement, commenting on the fact that Bayesian hierarchical models provide a natural setting in which inferences can be made concerning both the parameters of the random effects distribution and individual-level parameters.

A4 Consider a model of the form

$$
y_{i t}=\gamma y_{i, t-1}+\boldsymbol{\rho}^{\prime} \mathbf{z}_{i}+\boldsymbol{\beta}^{\prime} \mathbf{x}_{i t}+v_{i t}, \quad i=1, \ldots, N, \quad t=1, \ldots, T,
$$

where $\mathbf{z}_{i}$ is a $K_{2} \times 1$ vector of time invariant exogenous variables, $\mathbf{x}_{i t}$ is a $K_{1} \times 1$ vector of time varying exogenous variables, $\gamma$ is a scalar, and $\rho$ and $\beta$ are $K_{2} \times 1$ and $K_{1} \times 1$ vectors of parameters, respectively. $|\gamma|<1$ and $v_{i t}=\alpha_{i}+u_{i t} . E\left(\alpha_{i}\right)=E\left(u_{i t}\right)=0, \quad E\left(\alpha_{i} \mathbf{z}_{i}^{\prime}\right)=\mathbf{0}^{\prime}$, $E\left(\alpha_{i} \mathbf{x}_{i t}^{\prime}\right)=\mathbf{0}^{\prime}$, and $E\left(\alpha_{i} u_{j t}\right)=0 \quad \forall t$.

$$
\begin{aligned}
& E\left(\alpha_{i} \alpha_{j}\right)= \begin{cases}\sigma_{\alpha}^{2} & \text { if } i=j \\
0 & \text { otherwise }\end{cases} \\
& E\left(u_{i t} u_{j s}\right)= \begin{cases}\sigma_{u}^{2} & \text { if } i=j, t=s, \\
0 & \text { otherwise }\end{cases}
\end{aligned}
$$

i) Explain the rationale for the following IV estimator

$$
\begin{aligned}
& \binom{\widehat{\gamma}_{i v}}{\widehat{\boldsymbol{\beta}}_{i v}}=\left[\sum_{i=1}^{N} \sum_{t=3}^{T}\left(\begin{array}{cc}
\left(y_{i t-1}-y_{i, t-2}\right)\left(y_{i, t-2}-y_{i, t-3}\right) & \left(y_{i t-2}-y_{i, t-3}\right)\left(\mathbf{x}_{i t}-\mathbf{x}_{i, t-1}\right)^{\prime} \\
\left(\mathbf{x}_{i t}-\mathbf{x}_{i, t-1}\right)\left(y_{i, t-1}-y_{i, t-2}\right) & \left(\mathbf{x}_{i t}-\mathbf{x}_{i, t-1}\right)\left(\mathbf{x}_{i t}-\mathbf{x}_{i, t-1}\right)^{\prime}
\end{array}\right)\right]^{-1} \\
& \times\left[\sum_{i=1}^{N} \sum_{t=3}^{T}\binom{y_{i, t-2}-y_{i, t-3}}{\mathbf{x}_{i t}-\mathbf{x}_{i, t-1}}\left(y_{i t}-y_{i, t-1}\right)\right] .
\end{aligned}
$$

Suggest an alternative IV estimator and compare their relative merits.
A5 A random variable $Y$ is distributed Poisson if the pdf is given by

$$
\begin{equation*}
\operatorname{Pr}[Y=y]=\frac{e^{-e\left(\mathbf{x}^{\prime} \boldsymbol{\beta}\right)} e\left(\mathbf{x}^{\prime} \boldsymbol{\beta}\right)^{y}}{y!}, \quad y=0,1,2, \ldots \tag{1.2}
\end{equation*}
$$

where $\mathbf{x}$ is a $K \times 1$ vector of exogenous variables and $\beta$ is a $K \times 1$ vector of unknown parameters.
i) Demonstrate that the consistency of the quasi-maximum likelihood estimator $\widehat{\boldsymbol{\beta}}$ for $\beta$ depends only on the correct specification of the conditional mean $\left(\mu=\exp \left(\mathbf{x}^{\prime} \boldsymbol{\beta}\right)\right)$ and not any additional assumptions regarding the distribution of $y \mid \mathbf{x}$.
ii) Show that the Poisson MLE is a method of moments estimator
iii) The conditional mean function with multiplicative errors is given by

$$
E\left(y_{i} \mid \mathbf{x}_{i}, \tau_{i}\right)=\exp \left(\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}+\tau_{i}\right)=\mu_{i} v_{i} .
$$

where $v_{i}=\exp \left(\tau_{i}\right)$.
Assuming that one or more of the $\mathbf{x}$ are endogenous and that instruments are available write down the moment conditions and the GMM criterion function.

## SECTION B

B. 1 Consider the following cross-sectional regression model

$$
\begin{equation*}
y_{i}=\alpha+x_{i}^{*} \beta+v_{i} . \tag{1.3}
\end{equation*}
$$

Suppose we actually observe $y_{i}$ and $x_{i} . x_{i}$ is a noisy measure of $x_{i}^{*}$ subject to an additive measurement error $\varepsilon_{i}$

$$
x_{i}=x_{i}^{*}+\varepsilon_{i} .
$$

A second noisy measure of $x_{i}^{*}$ is available such that

$$
z_{i}=x_{i}^{*}+\zeta_{i} .
$$

Measurement error $\zeta_{i}$ is independent of $\varepsilon_{i}$ and the other unobservables.
i) Show that $\beta$ in (1.3) is identified with two noisey measures $x$ and $z$. How many overidentifying restrictions are there?
ii) Another cross-sectional model combines measurement error and unobserved heterogeneity

$$
\begin{align*}
y_{i} & =x_{i}^{*} \beta+\eta_{i}+v_{i}  \tag{1.4}\\
x_{i} & =x_{i}^{*}+\varepsilon_{i},
\end{align*}
$$

where all unobservables are independent, except $x_{i}^{*}$ and $\eta_{i}$.
Demonstrate and interpret the two components of the bias of the OLS estimator of $\beta$.
iii) We now observe both $y$ and $x$ for multiple time periods. Write down a set of valid orthogonality conditions assuming the measurement error is white noise and
a) there is no time invariant unobserved heterogeneity;
b) time invariant unobserved heterogeneity is present. Write down the orthogonality conditions for $T=3$.
iv) If measured persistence in $x_{i t}^{*}$ is exclusively due to unobserved heterogeneity, such that

$$
\begin{equation*}
x_{i t}^{*}=\mu_{i}+\xi_{i t}, \tag{1.5}
\end{equation*}
$$

where $\xi_{i t}$ is iid over $i$ and $t$, and independent of $\mu_{i}$, comment on the identifiability of $\beta$.
B.2. The utility to person $i$ from choosing alternative $j$ on purchase occasion $t$ is given by

$$
\begin{equation*}
U_{i j t}=\mathbf{x}_{i j t}^{\prime} \boldsymbol{\beta}+\varepsilon_{i j t} / \sigma \tag{1.6}
\end{equation*}
$$

where $\mathbf{x}_{i j t}$ is a $K \times 1$ vector of observed attributes of alternative $j, \beta$ is a conformable vector of taste weights and $\varepsilon_{i j t} \sim$ i.i.d. extreme value and $\sigma$ is the scale of the error term.
$\beta$ and $\sigma$ are not separately identified and so we initially normalise $\sigma$ to 1 .
i) Given the model in (1.6) demonstrate the Independence of Irrelevant Alternatives property.
Comment on the consequences of this property for making inference on choice behaviour.
ii) We now accommodate individual-specific deviations from the vector of mean attribute weights, $\boldsymbol{\beta}_{i}=\boldsymbol{\beta}+\boldsymbol{\eta}_{i}$, where $\boldsymbol{\eta}_{i}$ is $\operatorname{MVN}\left(0, \boldsymbol{\Sigma}_{\eta}\right)$.
(a) Write down an expression for the probability that individual $i$ with $\mathbf{x}_{i j t}$ chooses $j$. Use this to derive an expression for the log-likelihood function.
(b) In many applications $\Sigma_{\eta}$ is assumed to be diagonal.

Comment on the fact that this rules out correlation in tastes across attributes and show that it does not rule out correlation across alternatives.

In what sense is correlation across alternatives important?
(c) For a model given by (1.6) and $\beta_{i}=\beta+\eta_{i}$, with $\boldsymbol{\eta}_{i}$ is $\operatorname{MVN}\left(0, \boldsymbol{\Sigma}_{\eta}\right)$, demonstrate how choice probabilities can be simulated by taking R draws from $\operatorname{MVN}\left(0, \Sigma_{\eta}\right)$.
iii) In considering (1.6), now suppose that $\sigma$ is heterogenous in the population with $\sigma_{i}$ denoting the scale parameter for individual $i$.
Show that heterogeneity in scale is observationally equivalent to a particular type of heterogeneity in the taste weights.
(iv) For a vector of hyperparameters $\boldsymbol{\theta}$, let $g(\boldsymbol{\beta} \mid \boldsymbol{\theta})$ describe the distribution of tastes in the population. Assuming (1.6), carefully outline a method by which that the analyst can determine, insofar as possible, where each individual lies in this distribution.
Critically assess the problems in operationalising such a method.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY.
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Tuesday 10 May $2011 \quad 2.00-4.00$
M330
APPLIED ECONOMETRICS
Candidates are required to answer four compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Durbin-Watson and Dickey-Fuller Tables
Rough work pads
Tags
New Cambridge Elementary Statistical Tables
Engle-Yoo Cointegration Tables
Approved calculators allowed

## ANSWER ALL QUESTIONS

## Question 1

## EITHER

(a) Write brief notes about two of the following:
(i) Robust estimation
(ii) Granger Causality
(iii) Residual autocorrelation in regressions

OR
(b) Discuss the problems that occur and how they may be solved in a regression of dependent variable $y$ on a set of explanatory variables $x$ when the sample consists of data observed only if $y$ is less than some threshold.

## Question 2

(a) Difference-in-Difference (DD) estimators are the best method of obtaining unbiased estimates of treatment effects. Discuss reasons why DD estimates might not be persuasive and hence suggest checks of DD strategies that might be implemented in order to persuade the sceptic that they do offer an unbiased estimate of the treatment effect.
(b) Explain the differences between
(i) average treatment effect (ATE)
(ii) average treatment effect on the treated (ATT)
(iii) local average treatment effect (LATE).

Discuss the implications of the ATE > ATT, using a suitable example to illustrate your answer.

## Question 3

The government of Ruritania has proudly announced the results of the nutritional impact analysis of its recent pilot health programme, that involved the distribution of free nutrition supplements for children to all women visiting its health clinics in forty eight out of ninety districts in the country. The results were based on the 2005 round of the regular nationally representative household panel survey of socio-economic and health conditions, conducted three years after the nutrition programme had started. The results showed that the "height for age" for children receiving the supplements was about 11 percent higher than for children not receiving the supplements. (Height-forage in children measures child growth, controlling for age and is a measure of health for children from birth to 5 years of age).

A local research team has argued strongly that this is not the appropriate way of assessing the programme impact. Instead, it ran a regression across the sample of individuals of the relevant age group, with $H$, the height-forage, $X$, a set of individual (child) characteristics (age and sex) and $Z$, a set of household and community characteristics (including wealth, whether the household head is male or female, whether the village has a clinic) and $D$ measured whether the child lives in a household receiving the nutritional supplement:

$$
H_{i}=\alpha+\beta X_{i}+\gamma Z_{i}+\delta D_{i}+\epsilon_{i}
$$

This team found that a significant negative coefficient of $\delta$, suggesting that receiving supplements actually worsened nutritional status by 4 percent.
(a) Do you believe either result? Discuss, using the potential outcomes framework.
(b) Two teams are asked to design better evaluations. One team argues that the only way to get an unbiased estimate of programme impact is by randomly selecting health facilities that would be allowed to distribute the nutritional supplements so that the entire evaluation work has to be done again. The other team proposes to use a previous round of the national survey from 2000, and redo the analysis. Discuss the merits and potential problems of each approach. Which approach would you favour, and why?

## Question 4

To investigate the relationship between house prices (hprice) and earnings (earn) data were collected for 327 local authorities in England and Wales on median earnings and average house prices (in Ls)

The following relationship is estimated by OLS (standard errors are in parentheses throughout)

$$
\widehat{\text { hprice }}=-\underset{(23233)}{282776}+\underset{(1.0491)}{23.7364} \text { earn }
$$

Diagnostic tests reject the null of normality and of no heteroscedasticity in the residuals.
(i) Explain briefly what problems in estimation and inference may arise from non-normality and heteroscedasticity of the errors in an OLS regression.

The estimation is repeated using White's Heteroscedasticity Corrected Standard errors (HCSEs) to obtain

$$
\widehat{\text { hprice }}=-\underset{(56880)}{282776}+\underset{(2.6890)}{23.7364} \text { earn }
$$

(ii) What are the merits of using HCSEs rather than transforming the data to eliminate heteroscedasticity?

The actual and fitted relationship is graphed below


Two points are labelled in the graph. Obs 1 is Kensington and Chelsea (a wealthy suburb of London) and Obs 2 is Copeland (a rural area in NW England). These have the largest positive and negative residuals respectively.
(iii) Discuss what problems Obs 1 and Obs 2 may present for estimation and inference.
(iv) How might the estimation procedure be modified to solve these problems?

Examination of the data and the residuals suggests there may be a break in the relationship at earn $=21000$ rather than pure heteroscedasticity. The following Table reports OLS estimates of the original model (Col 1) and the model reestimated allowing for a break in the relationship (in Cols 2 and 3). Col 4 reports an alternative specification adding (earn) ${ }^{2}$ as an explanatory variable using the entire sample.

|  | Dependent |  |  | variable |
| :---: | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | hprice |  |
| constant | -282776 <br> $(23232.5)$ | 119008 <br> $(62868)$ | -489973 <br> $(46821)$ | 596147 <br> $(83064)$ |
| earn | 23.736 <br> $(1.05)$ | 3.103 <br> $(3.29)$ | 31.719 <br> $(1.87)$ | -51.5 <br> $(6.96)$ |
| (earn) $^{2}$ | - | - | - | 0.00156 <br> $(0.00014)$ |
|  |  |  |  |  |
| Sample | all | earn<21000 | earn>21000 | all |
| No. of Obs | 327 | 169 | 158 | 327 |
| $\mathrm{R}^{2}$ | 0.61 | 0.01 | 0.65 | 0.71 |
| Sum Sq Residuals | $1.68 \times 10^{12}$ | $4.75 \times 10^{11}$ | $9.53 \times 10^{11}$ | $1.23 \times 10^{12}$ |

The actual and fitted for both the Columns 3 and 4 are also graphed below.
(v) Using the Table above and these graphs discuss which specification you would prefer as a model for the relationship between house prices and earnings in the UK? Explain the reasons for your choice.


END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY.
EXAMINATION IN FINANCE FOR THE DEGREE OF MASTER OF PHILOSOPHY

Wednesday 18 May $2011 \quad 10.00-12.00$
Module 400
ASSET PRICING
Candidates are required to answer four compulsory questions.
Section A: two compulsory questions; Section B: two compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet

STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Approved calculators allowed
Rough work pads
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

1. Suppose that the price of a dividend-paying asset follows the Geometric Wiener Process

$$
d S=\mu S d t+\sigma S d Z,
$$

and that the cumulative dividends from the asset follow the process

$$
d D=\delta S d t
$$

Suppose finally that the return on the safe asset is $r$. Assuming that $\mu, \sigma$ and $\delta$ are all constant, and that $\sigma, \delta>0$ :
(a) Give a careful derivation of the partial differential equation satisfied by the price of a put option with strike price $K>0$ and maturity $T$.
(b) Find an explicit solution of this equation, being careful to explain your working.
2. (a) What is meant by the law of one price?
(b) Under what conditions does the law of one price hold?
(c) Prove you claim in part (b) above. (You may state without proof any mathematical results that you need.)

## SECTION B

3. Consider the standard consumption Euler Equation derived using the Constant Relative Risk Aversion (CRRA) utility function for household $h$ :

$$
\left(\frac{C_{h, t+1}}{C_{h, t}}\right)^{-\gamma_{h}}(1+r) \beta_{h}=\varepsilon_{h, t+1}
$$

where $\gamma_{h}$ is household specific coefficient of relative risk aversion, $\beta_{h}$ is the household specific subjective discount factor and $C_{h, t}$ is nondurable consumption in period $t$. Expectations error $\varepsilon_{h, t+1}$ for a given household has the properties of $E\left(\varepsilon_{h, t+1}\right)=1$.
(a) Briefly explain why one might be reluctant to accept the notion of "homogeneity" of preference parameters ( $\gamma$ and $\beta$ ) across households.
(b) Explain how one can estimate the preference parameters ( $\gamma$ and $\beta$ ) without imposing homogeneity.
4. Using the life cycle framework
(a) Explain how equity premium puzzle manifests itself in the microdata on wealth and portfolio allocation.
(b) Discuss the role of stochastic labour income and risky housing investment in reconciling observed household portfolios with the standard life cycle portfolio choice theory.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Friday 20 May $2011 \quad 2.00-4.00$
M500
DEVELOPMENT ECONOMICS
Answer two questions.Each question is of equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Rough work pads

1. "The risk-sharing hypothesis is the cross-sectional equivalent of the permanent income hypothesis."
(a) Discuss the theoretical foundations of this statement.
(b) Does the statement imply an equivalence in the way either can be tested? Derive an empirical specification from a theoretical specification and explain.
(c) Do models of informal insurance arrangements imply that social sanctions in mutual support arrangements are irrelevant? Explain.
2. Suppose that you are a policy-maker faced with the choice of implementing either a welfare program (in which unconditional cash transfers are to be made to recipients) or a workfare program (in which participants self-select to perform labour in return for a payment). Because you do not have information about the earnings (or the earning abilities) of the relevant population, you will be forced to pay everyone the welfare benefit. However, by choosing an appropriate combination of work requirements and the payments for that work, you can sort out the truly poor from the non-poor.

Assume there are workers of two 'abilities', characterized by the wages they can earn in the private market. High-ability workers can earn 6 shillings an hour, while low-ability workers can earn 4 shillings an hour. Each worker has a maximum of $T$ hours available for leisure or labour, as they choose. Income, $y$, comes solely from the number of labour hours worked, $l=T-L$, where $L$ is the amount of leisure (in hours) that the worker chooses to consume. Assume that all workers have preferences defined by:
$u=y-v(l)$, where $u$ is utility, and $v(T-L)$ is the (sub)utility function associated with the disutility of labor. Specifically, let $u=y-\frac{1}{3} l^{2}$

The poverty line is set at 32 shillings per day. The total population consists of 30 workers, of whom a proportion $\alpha$ are of low ability.

Answer the questions below, explaining how you arrive at your answer.
(a) Left to themselves, (i.e., if there were no government program at all), how many hours would each type of worker work? What would be their respective incomes? Would each type of worker be above or below the poverty line?
(b) What is the total cost to eradicate poverty if you use the welfare program?
(c) Assume that the workfare program creates no useful output (i.e., its value is zero). If you decide to use a workfare program, what is the least-cost workfare program to use - i.e., how many hours of work will you require, and how much will you pay? (It is not necessary to provide a complete mathematical derivation, if you can offer an intuitive explanation of how you arrive at your answer).
(d) What is the total cost to eradicate poverty if you use the workfare program? (Your answer should be a function of $\alpha$ ).
(e) Derive a condition on $\alpha$ to determine which program (workfare or welfare) is the least-cost method of ensuring that everyone in the population has an income of at least 32 shillings. In other words, find $\alpha^{*}$ such that the workfare program is less (or more) costly than the welfare program for all values of $\alpha$ lower (or higher) than $\alpha^{*}$.
(f) Suppose that the workfare program actually created output that was of value to the government (or the agency running the workfare program). Suppose that every hour of workfare creates value 'worth' 3 shillings to the government. (In other words, the 'shadow value' of the labour in the workfare program is 3 shillings). How would this factor into your calculations? What is the cost of the workfare program now?
3. (a) Do models of credit constraints and precautionary savings illuminate our understanding of savings behaviour in developing countries? Explain.
(b) It is often argued that the main reason for political opposition to the liberalization of international labour migration is that it has 'non-neutral' effects on the incomes of workers in the recipient country. In other words, the argument is that what matters is not just the magnitude of the gains and losses that accrue to the recipient country, but also the distributional consequences, i.e., the way that these gains and losses are distributed across the population in the recipient country. Why might this matter? In particular, if a society gains, on the whole, from more liberal immigration, why is this not sufficient to ensure political support for liberalization? To take it one step further, if a majority of the population in the recipient society stand to benefit from a liberalized immigration policy, why might that policy still fail to win enough political support? Explain.
(c) (i) "If there were no asymmetric information, (i.e., if labour could be monitored perfectly, and if contracts could be written contingent on the amount of labour to be provided), then one would not expect to see sharecropping being used as a form of land tenure". Discuss.
(ii) "If there were no uncertainty in output (e.g., due to fluctuations in weather, natural disasters, etc.), then one would not expect to see sharecropping being used as a form of land tenure". Discuss.

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Tuesday 17 May $2011 \quad 2.00-4.00$
Module 510
POVERTY, ENVIRONMENT, AND SUSTAINABLE DEVELOPMENT
Candidates are required to answer two questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Approved calculators allowed
Rough work pads
Tags

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Attempt any two questions. Part (i) of each of the two 2-part questions carries $25 \%$ of the marks of the question.

1. Explain how exploitation can masquerade as cooperation in long-term relationships.
2. Explain why movements in gross domestic product and the United Nations' Human Development Index are inadequate for assessing whether an economy has been enjoying sustainable development.
3. (i) Are the concepts of "optimum development" and "sustainable development" the same? If not, how do they differ?
(ii) "Cost-benefit analysis and sustainability analysis amount to the same thing: both involve the use of shadow prices of capital assets to value changes in wealth." Discuss.
4. (i) Distinguish "hysteresis" from "irreversibility" in dynamic processes.
(ii) Construct a model of an open access fishery to show that mere openness does not ensure that the fishery is doomed. Under what additional conditions would the fishery be doomed?

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY
Wednesday 11 May $2011 \quad 2.00-4.00$

Module 600
HISTORICAL PERSPECTIVES TO FINANCIAL CRISES
Candidates are required to answer two questions.

Each question carries equal weight.
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1. Is there a relationship between long swings in economic growth and financial crises? In your answer refer in detail to at least one perspective of this relationship.
2. Why have financial crises become more frequent since the late 1970s in both developed and developing countries?
3. Evaluate the effects of financial crises drawing on the historical evidence for the periods of the classical gold standard (1870-1914) and the inter-war period.
4. Is there any evidence that the inflow controls applied by Chile and Malaysia during the 1990s were effective in reducing the likelihood of a financial crisis?
5. (a) What were the underlying causes and mechanisms that led intelligent and selfish private agents to 'over-borrow' in Latin America and East Asia during the 1990s?
(b) Why was the 'over-borrowing' in East Asia directed mainly to investment?
(c) Why was the 'over-borrowing' in Latin America directed instead to consumption?
6. "Somehow, we just missed [...] that home prices don't go up forever" (J Dimon, CEO, JP Morgan, 2008). Discuss in relation to Minsky's 'financial fragility' hypothesis, Kindleberger's cycles of 'manias, panics and crashes', and the basic logic of 'third generation' models of financial crises.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Friday 20 May $2011 \quad 10.00-12.00$
Module 220
MACROECONOMICS III
Candidates are required to answer four compulsory questions.
Section A: three compulsory questions and Section B: one multi-part question.

Each section carries equal weight.
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You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

1. Consider the following small open endowment economy in which the representative consumer maximizes the expected value of life-time utility

$$
U_{t}=\sum_{s=t}^{\infty} \beta^{s-t}\left[u\left(C_{s}\right)+v\left(\frac{M_{s}}{P_{s}}\right)\right]
$$

where $C_{s}$ denotes consumption, $M_{s}$ nominal money balances and $P_{s}$ the aggregate price level in period $s$. The subutility functions satisfy $u^{\prime}()>$.0 , $u^{\prime \prime}()<0,. v^{\prime}()>$.0 and $v^{\prime \prime}()<$.0 , and the intertemporal discount factor $\beta$ satisfies $0<\beta<1$. The budget constraint is

$$
B_{t}+\frac{\tilde{B}_{t}}{P_{t}}+\frac{M_{t}}{P_{t}}+C_{t}+T_{t}=\left(1+r_{t-1}\right) B_{t-1}+\left(1+i_{t-1}\right) \frac{\tilde{B}_{t-1}}{P_{t}}+\frac{M_{t-1}}{P_{t}}+Y_{t}
$$

where $B_{t}$ and $\tilde{B}_{t}$ denote the holdings of real and nominal bonds at the end of period $t$, respectively; $r_{t}$ and $i_{t}$ the real and nominal interest rate on real and nominal bonds acquired in period $t$, respectively; $T_{t}$ (net) lump-sum taxes; and $Y_{t}$ the perishable output endowment, which is stochastic and only observed at the beginning of period $t$.
Use the first-order conditions to show that

$$
1+r_{t}=\left(1+i_{t}\right) \mathrm{E}_{t}\left[\frac{P_{t}}{P_{t+1}}\right]+\xi_{t}
$$

and carefully derive $\xi_{t}$. Give an economic interpretation of the result.
2. In the standard one-good two-country model of the international business cycle, where cross-border trade in assets is restricted to one bond, a positive but temporary (one-period) rise in output in the Home country always prompts Home residents to lend abroad to smooth consumption. The current account is therefore pro-cyclical. Explain whether this result necessarily holds in a world economy in which each country is specialized in the production of a domestic good which is an imperfect substitute for the good produced abroad.
3. Consider a one-good world economy with two symmetric countries, Home and Foreign (indicated by an asterisk). In each country, a fraction $\alpha$ of output $Y$ is paid as wages $W$ and the rest as profits $\Pi$ :

$$
\begin{aligned}
Y & =\Pi+W \\
Y^{*} & =\Pi^{*}+W^{*}
\end{aligned}
$$

where $W=\alpha Y$ and $W^{*}=\alpha Y^{*}$. The global resource constraint is $Y+Y^{*}=$ $C+C^{*}$, where $C$ denotes consumption. Let $s \cdot \Pi$ denote the amount of Home profits accruing to the Home residents, and $(1-s) \cdot \Pi^{*}$ the amount of Foreign profits accruing to the Home residents. So, the Home and Foreign budget constraints are

$$
\begin{aligned}
C & =W+s \Pi+(1-s) \Pi^{*} \\
C^{*} & =W^{*}+s \Pi^{*}+(1-s) \Pi
\end{aligned}
$$

Assume there is perfect risk sharing such that

$$
C=C^{*}
$$

Show that there is a time-invariant $s$ such that aggregate output risk is perfectly shared across countries. Is $s$ positive? Explain.

## SECTION B

4. In a world with $N$ countries, assume that in each of them the utility of the national representative consumer is of the CRRA type and separable both inter- and intra-temporally. If markets are complete, perfect risk diversification implies that consumption growth and prices in country $j$ and $n$ obey the following relationship:

$$
\begin{equation*}
\beta^{n}\left(\frac{c_{t+1}^{n}}{c_{t}^{n}}\right)^{-\sigma_{n}} \frac{P_{t}^{n}}{P_{t+1}^{n}}=\psi_{j n} \beta^{j}\left(\frac{c_{t+1}^{j}}{c_{t}^{j}}\right)^{-\sigma_{j}} \frac{\mathcal{E}_{t}^{n, j} P_{t}^{j}}{\mathcal{E}_{t+1}^{n, j} P_{t+1}^{j}} \tag{1}
\end{equation*}
$$

where for country $n, c^{n}$ denotes consumption, $P^{n}$ the consumption price index, and $\beta^{n}$ the intertemporal discount factor (which is constant). Similar definitions apply to country $j$. In addition, $\mathcal{E}_{t}^{n, j}$ is the nominal exchange rate (units of $n$-country currency per unit of $j$ country currency), and $\psi_{j n}$ is a parameter indexing the relative economic size of the two countries.
(a) Assuming that the parameter $\sigma$ is identical across countries $\left(\sigma_{j}=\sigma_{n}=\right.$ $\sigma$ ), show that the growth rate of consumption in country $n$ is an exact log linear function of the growth rate of world consumption, and the rate of depreciation of the multilateral exchange rate in real terms.
(b) Following the logic of Obstfeld (1994), derive an empirical test for perfect risk sharing in the form of an OLS regression. Explain carefully the derivation, and define and discuss the null hypothesis.
(c) What is the relation between the version of the Obstfeld test in part (b) and the Backus and Smith (1993) correlation puzzle?
(d) Suppose that the test in part (b) is performed for a period in which countries have capital controls and a period without capital controls (say, 1950-1980 and 1982-2010), and that the null is rejected in both periods. But, testing for structural breaks in the coefficients of the regression model suggest that these coefficients are different across the two subsamples. What can you conclude about the degree of crossborder risk sharing across the two periods? Explain.
(e) Consider an economy where markets are not complete. For the assets traded across countries, define their returns in nominal terms $R_{t}^{n}$. Write down and explain the condition for efficient diversification of traded risk (i.e. through existing assets), which is the analogue to condition (1) above.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE
OF MASTER OF PHILOSOPHY
EXAMINATION IN FINANCE FOR THE DEGREE OF MASTER OF PHILOSOPHY

Monday 7 May $2012 \quad 10.00$ to 12.00
Module M100
MICROECONOMICS I
Candidates are required to answer three compulsory questions. All questions carry equal weight.

Write your number not your name on the cover sheet of each booklet

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20 Page booklet x 2
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Rough work pads

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. (i) An individual with wealth of 100 pounds strictly prefers to turn down the opportunity of winning an additional 100 pounds with probability 0.6 and losing all her wealth with probability 0.4 . On the other hand, she strictly prefers a gamble in which she wins an additional 100 pounds with probability 0.8 and loses all her wealth with probability 0.2 to one in which she gains 100 pounds with probability 0.5 and otherwise keeps her original 100 pounds. Are these choices consistent with the von-Neumann-Morgenstern axioms? Explain.
(ii) (a) Consider two probability distributions on the interval $[0,1]$ with cumulative distribution functions $F(x)$ and $G(x)$. What does it mean to say that $F$ first-order stochastically dominates $G$ ? Show that if $F$ first-order stochastically dominates $G$ then the mean of $x$ under $G$ is less than or equal to the mean of $x$ under $F$.
(b) Show that if $F$ first-order stochastically dominates $G$ then $F(x) \leq G(x)$ for all $x \in[0,1]$.
(c) Give an example of a pair $(F, G)$ such that the mean of $x$ under $F$ is greater than that under $G$ but $F$ does not first-order stochastically dominate $G$. (Hint: look for a discontinuous $F$ ).
2. Two players, $A$ and $B$, play the following game. First $A$ chooses either $L, M$ or $R$. If she chooses $L$ then the game ends and both players get a payoff of 2 . If she chooses either $M$ or $R, B$ does not observe $A$ 's choice; $B$ then chooses either $l$ or $r$, at which point the game ends. The payoffs are then (with $A$ 's payoff first): $(0,1)$ after $M$ followed by $l,(0,3)$ after $M$ followed by $r$ or after $R$ followed by $l$; and $(3,0)$ after $R$ followed by $r$.
(a) Draw the extensive form and write the normal form of the game; find all the Nash equilibria of the game (pure and mixed).
(b) Define the term Weak Perfect Bayesian Equilibrium as it applies to this game.
(c) Find all the Weak Perfect Bayesian equilibria in pure and mixed strategies.
3. A risk-neutral principal designs an incentive contract for a risk-averse agent. The agent can choose one of three possible effort levels, $\left\{e_{1}, e_{2}, e_{3}\right\}$. The agent's output, which is stochastic, can take either the value 10 or the value zero. The probability of high output (i.e., 10) is $\frac{2}{3}$ if effort is $e_{1}, \frac{1}{2}$ if effort is $e_{2}$, and $\frac{1}{3}$ if effort is $e_{3}$. The principal's utility is the expected value of output minus the expected wage paid to the agent. The agent's utility is the expected value of $\sqrt{w}-c(e)$ where $w$ is the wage and $c(e)$ is the cost of the effort, given by $c\left(e_{1}\right)=\frac{5}{3}, c\left(e_{2}\right)=\frac{8}{5}$ and $c\left(e_{3}\right)=\frac{4}{3}$. The agent's outside option (reservation utility) is zero and he can reject the principal's offered contract. The principal is not allowed to pay a strictly negative wage.
(i) Suppose that the principal can observe the agent's effort and so pay a wage only if a given effort is exerted. For each effort level, find the optimal contract for the principal. Which contract is best for the principal?
(ii) Now suppose that effort is unobservable.
(a) Write down the constraints which would have to be satisfied by a contract which induces the agent to choose effort level $e_{2}$.
(b) Show that no contract exists which induces the agent to exert effort $e_{2}$, i.e., $e_{2}$ is not feasible.
(c) What is the highest value of $c\left(e_{2}\right)$ for which $e_{2}$ would be feasible? For this value, what is the principal's optimal contract if he wishes to induce effort $e_{2}$ ?

EXAMINATION IN ECONOMICS
FOR THE DEGREE OF MASTER OF PHILOSOPHY.
EXAMINATION IN ECONOMIC RESEARCH
FOR THE DEGREE OF MASTER OF PHILOSOPHY.

25 May 2012 10.00-12.00

M110
MICROECONOMICS II

Candidates are required to answer all four questions.
Each question carries equal weight.

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## Question 1.

(a) Let $x_{i}$ stand for the amount of good $i$ consumed by an agent with Cobb-Douglas utility function of the following form

$$
U\left(x_{1}, x_{2}, \ldots, x_{n}\right)=x_{1}^{a_{1}} x_{2}^{a_{2}} \cdots x_{n}^{a_{n}},
$$

where $a_{i}>0$. Given prices $p_{i}$ for good $i$, and a budget $M$, show that utility maximising demand $x^{*}$ results in the agent demanding

$$
x_{i}^{*}=\frac{a_{i}}{a_{1}+a_{2}+\cdots+a_{n}} \cdot \frac{M}{p_{i}} .
$$

(b) Consider a pure exchange economy with two agents $A, B$, and two goods $i=$ 1,2 . The agents have the utility functions $u^{A}\left(x_{1}, x_{2}\right)=\ln x_{1}+3 \ln x_{2}$ and $u^{B}\left(x_{1}, x_{2}\right)=2 \ln x_{1}+\ln x_{2}$. The total endowment of the goods is $\omega=(1,1)$. Describe the set of Pareto optimal allocations ( $\hat{x}_{1}^{A}, \hat{x}_{2}^{A}, \hat{x}_{1}^{B}, \hat{x}_{2}^{B}$ ) by expressing $\hat{x}_{2}^{A}, \hat{x}_{1}^{B}$, and $\hat{x}_{2}^{B}$ as a function of $\hat{x}_{1}^{A}$.
(c) If the initial endowments are $\omega^{A}=(1,0)$ and $\omega^{B}=(0,1)$, evaluate the Walrasian equilibrium price and the associated equilibrium allocations. (For notational brevity, normalise the price of good two as 1, and set the price of the first good to $p$.)
(d) Suppose we can transfer good 2 between the agents before trade takes place. What transfer should we make so that the allocation

$$
\left(\frac{1}{2}, \frac{3}{4}\right) \text { to } A, \quad \text { and } \quad\left(\frac{1}{2}, \frac{1}{4}\right) \text { to } B
$$

emerges as an equilibrium outcome after the transfer?

Question 2. Consider a financial economy with a single consumption good and two dates, today and tomorrow, where trade occurs today. There are two possible states of the world tomorrow, 1 and 2. So this economy has three (contingent) commodities. The economy has two agents, $A$ and $B$. Agent $A$ is endowed with 2 units of consumption today. Tomorrow, this agent has 1 units of consumption good if state 1 occurs, and nothing if state 2 occurs. More succinctly, agent $A$ 's endowment is $\omega^{A}=(2,1,0)$. Agent $B$ 's endowment is $\omega^{B}=(0,2,4)$. The utility functions of the agents are given by
$U^{A}\left(x_{0}, x_{1}, x_{2}\right)=\ln x_{0}+2 \ln x_{1}+\ln x_{2} \quad$ and $\quad U^{B}\left(x_{0}, x_{1}, x_{2}\right)=2 \ln x_{0}+\ln x_{1}+\ln x_{2}$,
where $x_{0}$ is consumption today, $x_{1}$ is consumption tomorrow if state 1 occurs, and $x_{2}$ is consumption tomorrow if state 2 occurs.
We fix the price of good 0 at 1 .
(a) Suppose the economy has two assets $\alpha$ and $\beta$ with payoffs tomorrow. Asset $\alpha$ has a payoff of 1 if state 1 occurs and 0 otherwise; asset $\beta$ has a payoff of 1 if state 2 occurs and 0 otherwise. Find the equilibrium prices of the two assets and the quantity traded of each asset.
(b) Suppose now that the economy has two assets: $\alpha$ as specified in (a), and $\gamma$ which has a payoff of 1 tomorrow irrespective of the state. Compare the asset spans of the two sets of assets: $\{\alpha, \beta\}$ and $\{\alpha, \gamma\}$. Find the equilibrium prices of $\alpha$ and $\gamma$, the quantity traded of each asset, and the equilibrium allocation.
(c) Suppose that the economy only has the asset $\gamma$. Find the equilibrium price of $\gamma$ and the quantity of $\gamma$ traded.
(d) Verify that the allocations in parts (a) and (b) are Pareto optimal but the one in (c) is not. Elaborate on why the equilibrium allocation fails to be Pareto optimal in part (c).

Question 3. Let $X$ be a set of decisions, $N=\{1, \ldots, n\}$ be a set of agents, and for each agent $i$, let $\Theta_{i}$ be the set of possible types. Thus $\Theta=\Theta_{1} \times \cdots \times \Theta_{n}$ is the set of type profiles. If decision $x$ is taken, and every agent $i$ pays $t_{i}$, the utility of agent $i$ with type $\theta_{i}$ is given by

$$
U_{i}\left(x, t, \theta_{i}\right)=u_{i}\left(x, \theta_{i}\right)+t_{i} .
$$

(a) Given an ex-post efficient decision rule $f: \Theta \rightarrow X$, define a transfer scheme $t: \Theta \rightarrow \mathbb{R}^{n}$ such that the direct mechanism

$$
(f, t): \Theta \rightarrow\left(X, \mathbb{R}^{n}\right)
$$

is dominant strategy incentive compatible.
(b) Assume there are two agents, each with private value drawn from $[0,1]$. A project costs $3 / 2$, and if it is built the cost will be shared equally between the agents. Show that there is no budget-balanced Groves scheme which implements the efficient decision rule.
(c) Show that if we only require Bayesian Incentive Compatibility, there is a budgetbalanced mechanism which implements the efficient decision rule.

Question 4. An auctioneer of a single object faces $n$ risk-neutral bidders with private valuations for the object, independently drawn from a distribution $F$ with density $f$.
(a) State, and sketch a proof of the Revenue Equivalence Theorem (RET) for this situation.
(b) Write down how players bid in a Japanese auction (i.e., a price clock rises, bidders drop out of the auction whenever they like (but cannot re-enter), the moment when there is one bidder left, the price clock stops, the remaining bidder wins, and pays the amount shown on the price clock), and hence derive their expected payments conditional on winning, when their values are independently drawn from a uniform distribution on $[0, M]$.
(c) Consider an "All Pay" auction (i.e., a simultaneous sealed-bid auction in which the high bidder wins the object, but every bidder pays her bid). Use the RET, together with your solution to Part (b) to solve for the bidding functions for the "All Pay" auction when bidders' values are independently drawn from a uniform distribution on $[0, M]$.
(d) Write down the differential equation for a player's bid as a function of her value in the symmetric equilibrium of an "All Pay" auction when bidders' values are independently drawn from the distribution $F$. Solve the differential equation for the special case in which $F$ is uniform on $[0, M]$. (Your solution must be consistent with (c).)

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF
MASTER OF
PHILOSOPHY
Monday 4 June $2012 \quad 10.00$ to 12.00
M120
TOPICS IN ECONOMIC THEORY
Candidates are required to answer two out of three questions. All questions carry equal weight.
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20 Page booklet x 2
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| instructed that you may do so by the Invigilator |

1. Consider, in the dynamic risk sharing game (where, recall, $P$ stands for "provide a favor", $N P$ for "not provide", $\Omega=\{1,2\}$ is the set of states of nature and $\delta \in(0,1)$ is the common discount factor), the following grim-trigger strategy:
(I) The state space is $S=\{0,1\}$,
(II) The initial state is $\bar{s}=1$,
(III) The behavior function of each player $i \in\{1,2\}$ is $B_{i}(1)=P$ and $B_{i}(0)=N P$,
(IV) The transition function is $T(\omega, 1, P)=1$ for all $\omega \in \Omega$ and $T(\omega, s, a)=0$ otherwise.
(a) Show that the grim-trigger strategy is a subgame perfect equilibrium for all $\delta$ sufficiently large.

Consider next the following forgiving strategy:
(I) The state space is $S=\{0,1,2\}$,
(II) The initial state is $\bar{s}=1$,
(III) The behavior function of player 1 is $B_{1}(s)=P$ for all $s \in\{0,1\}$ and $B_{1}(2)=$ $N P$,
(IV) The behavior function of player 2 is $B_{2}(s)=P$ if $s \in\{1,2\}$ and $B_{2}(0)=N P$,
(V) The transition function on the equilibrium path is $T(1, s, P)=s+1$ if $s \in$ $\{0,1\}, T(1,2, N P)=1, T(2, s, P)=s-1$ if $s \in\{1,2\}$ and $T(2,0, N P)=1$ and
(VI) The transition function outside the equilibrium path is $T(1, s, N P)=s$ for all $0 \leq s<2, T(1,2, P)=1, T(2, s, N P)=s$ for all $s \in\{1,2\}$ and $T(2,0, P)=1$.
(b) Show that there exists $\delta^{*} \in(0,1)$ such that the forgiving strategy is not a subgame perfect equilibrium for all $\delta>\delta^{*}$.
2. Consider a multi-person bargaining game with $n$ players. Recall that a strategy profile $f=\left(f_{1}, \ldots, f_{n}\right)$ constitutes a Nash equilibrium with complexity costs (NEC, for short) if, for all $i \in N$, (1) $\pi_{i}(f) \geq \pi_{i}\left(f_{i}^{\prime}, f_{-i}\right)$ for all strategies $f_{i}^{\prime}$, and (2) there does not exist a strategy $f_{i}^{\prime}$ such that $\pi_{i}(f)=\pi_{i}\left(f_{i}^{\prime}, f_{-i}\right)$ and $f_{i}$ is more complex than $f_{i}^{\prime}$. Furthermore, recall that $S_{i}$ denotes the the sets of partial histories for player $i$ in any stage, that $H^{\infty}$ denotes the set of all possible finite histories of stages and that a strategy $f_{i}$ is more complex than another $f_{i}^{\prime}$ if there exists a $s^{\prime} \in S_{i}$ such that

$$
\begin{aligned}
& f_{i}(h, s)=f_{i}^{\prime}(h, s) \text { for all } s \neq s^{\prime} \text { and } h \in H^{\infty}, \\
& f_{i}^{\prime}\left(h, s^{\prime}\right)=f_{i}^{\prime}\left(h^{\prime}, s^{\prime}\right) \text { for all } h, h^{\prime \infty}, \text { and } \\
& f_{i}\left(h, s^{\prime}\right) \neq f_{i}\left(h^{\prime}, s^{\prime}\right) \text { for some } h, h^{\prime \infty} .
\end{aligned}
$$

(a) Show that if $f$ is a stationary Nash equilibrium, then $f$ is a NEC.
(b) Show that for every $z=\left(z_{1}, \ldots, z_{n}\right) \in \Delta^{n}$ (i.e., $z_{i} \geq 0$ for all $1 \leq i \leq n$ and $\sum_{i=1}^{n} z_{i}=1$ ) and $t \in\{1, \ldots, n\}$, there exists a NEC that induces $(z, t)$.
(c) Show that there exists a NEC that induces no agreement.
3. Consider the infinite repetition $G^{\infty}(\delta)$ of a given stage game $G=\left(A_{i}, u_{i}\right)_{i \in N}$, where $\delta$ is the (common) discount factor. Let $N=\{1, \ldots, n\}, A=\prod_{i \in N} A_{i}$ and $A^{*} \subseteq A$ be the set of Nash equilibria of $G$. Furthermore, consider the following result: For every $\hat{A} \subseteq A, \delta \in(0,1)$ and $w \in \operatorname{co}(u(\hat{A}))$, there exists $\sigma=\left\{\sigma^{k}\right\}_{k=1}^{\infty}$ such that $w=(1-\delta) \sum_{k=1}^{\infty} \delta^{k-1} u\left(\sigma^{k}\right)$ and $\sigma^{k} \in \hat{A}$ for all $k \in \mathbb{N}$.
(a) Show that for all $\delta \in(0,1)$ and $w \in \operatorname{co}\left(u\left(A^{*}\right)\right)$ there exists a subgame perfect equilibrium $f$ of $G^{\infty}(\delta)$ such that $\pi(f)=w$.
(b) Assume that $n=2$ and that the payoff of the mutual minmax profile is zero. Show that, for all $w \in\left\{w^{\prime} \in u(A): w_{i}^{\prime}>v_{i}\right.$ for all $\left.i=1,2\right\}$, there exists $\delta^{*} \in(0,1)$ such that, for all $\delta>\delta^{*}$, there exists a subgame perfect equilibrium $f$ of $G^{\infty}(\delta)$ such that $\pi(f)=w$.

Suppose next that $G$ is the prisoners' dilemma described in the following table.

| $1 \backslash 2$ | $c$ | $d$ |
| :---: | :---: | :---: |
| $c$ | 3,3 | $-1,4$ |
| $d$ | $4,-1$ | 0,0 |

Let $\sigma=\left\{\sigma^{k}\right\}_{k=1}^{\infty}$ defined by $\sigma^{k}=(c, d)$ if $k$ is odd and $\sigma^{k}=(d, c)$ if $k$ is even. Given $h \in H$ (where $H$ is the set of histories, i.e., $H=\cup_{t=0}^{\infty} A^{t}$ ), let $\ell(h)$ denote the length of $h$. Furthermore, for all $M \in \mathbb{N}$ and $h=\left(a^{1}, \ldots, a^{\ell(h)}\right) \in H$ such that $\ell(h) \geq M$, let $T^{M}(h)=\left(a^{\ell(h)-M+1}, \ldots, a^{\ell(h)}\right)$ be the $M$-tail of $h$. Define, for all $M \in \mathbb{N}$, a strategy $f^{M}$ by setting, for all $h \in H$,

$$
f^{M}(h)= \begin{cases}\sigma^{\ell(h)+1} & \text { if } \ell(h)<M \text { and } h=\left(\sigma^{1}, \ldots, \sigma^{\ell(h)}\right) \\ \sigma^{\ell(h)+1} & \text { if } \ell(h) \geq M \text { and } T^{M}(h)=\left(\sigma^{\ell(h)-M+1}, \ldots, \sigma^{\ell(h)}\right), \\ (d, d) & \text { otherwise. }\end{cases}
$$

(c) Assume that $\delta>1 / 4$. Show that, for every $M \in \mathbb{N}, f^{M}$ is not a subgame perfect equilibrium of the infinitely repeated prisoners' dilemma.

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Thursday 24 May 2012 2.00 to 4.00

M130
APPLIED MICROECONOMICS
This paper is made up of two sections. Each section carries equal weight. You should answer four questions from Section A. Within Section A, each question carries equal weight. You must answer the compulsory question in Section B.

Write your candidate number not your name on the cover sheet of each booklet.

20 Page booklet x 2 Approved calculators allowed
Rough work pads

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

A1 You have longitudinal data on a sample of men (ages 18 to 65) that follows them over 10 years, and includes information on employment, wages, and years of schooling. You notice that there are many cases where you have data on wages for men who took a break from school to work and then completed more schooling. Can you use these data to estimate an effect of years of schooling on wages? Explain. How would you expect your estimates to compare to estimates of the return to schooling in the literature?

A2 Suppose the marginal tax rate for individuals in the UK earning above $£ 150,000$ was cut. You have data on earnings and hours of work for individuals both before and after the tax cut. While $10 \%$ of your sample earns more than $£ 150,000$, only a few of these observations are women. Can you use this policy change to learn about the labour supply elasticity of married men and women? Explain. (Note: Assume that the tax for married couples is the same as if each partner were taxed individually.)

A3 You have read Rhode and Strumpf (2003), but remain convinced that there is Tiebout sorting. Briefly explain Rhode and Strumpf's strategy and findings. Explain why you are not convinced by their findings. How would you modify Rhode and Strumpf's empirical tests to look for additional evidence of Tiebout sorting? Give specific examples.

A4 You want to estimate the effect of child protective services on children's outcomes. You have the same data as Doyle (2007), but find that it also has information on outcomes for sibling pairs. These data include observations where one sibling was placed in foster care while the other remained with their biological parents. Explain how you might use this information to estimate the effect of foster care on juvenile delinquency and any shortcomings of your proposed approach. How might your estimates compare to findings in Doyle (2007)?

A5 Suppose you want to use Altonji and Pierret (2001)'s model to estimate whether there is discrimination in wages on the basis of gender. First, describe their evidence on racial discrimination. Then, explain how you might go about estimating whether there is gender discrimination using their approach (and the same data) and any potential challenges you would face.

A6 Because of Katz, Kling and Liebman (2001)'s findings regarding the Moving to Opportunity experiment in Boston, the federal government in the United States has decided to eliminate tenant-based housing for people below the poverty-line and instead move to a system that offers vouchers for housing for these individuals. Explain how/if Katz, Kling and Liebman (2001) provides evidence for this new policy.

## SECTION B

B1 You have a nationally-representative random sample of a cohort of kindergarteners in the United States in 1998 which follows them from school entry (kindergarten, age 5 or 6 ) to their fourth year in school (grade 3). You have data on performance on a mathematics test at entry into kindergarten prior to receiving any school resources (Fall) and then at the end of each year (Spring). The test scores are standardised to have mean 0 and standard deviation 1 based on the whole sample, i.e., $Y_{i t}=\frac{y_{i t}-\bar{y}}{S D(y)}$, where $y_{i t}$ denotes raw test scores, $\bar{y}$ the average for the sample, $S D(y)$ the standard deviation for the sample, so that $Y_{i t}$ is the standardised test score. You restrict the sample to black and white students, which leaves you with a total sample of 7999 whites and 1806 blacks with test performance for all 4 years of school. Table 1 provides summary statistics for the sample. Standard deviations for some variables are presented in parentheses. SES is a socioeconomic status index based on parental education and income, with smaller values denoting lower income/less-educated families.
a. Let Black $_{i}$ indicate that a student is black and $X_{i t}$ denote family characteristics listed in Table 1. You estimate the following regressions using test scores at entry to kindergarten (Fall test scores):

$$
\begin{gathered}
Y_{i t}=\alpha_{0}+\alpha_{1} \text { Black }_{i}+\epsilon_{i t} \\
Y_{i t}=\beta_{0}+\beta_{1} \text { Black }_{i}+X_{i t} \beta_{2}+\epsilon_{i t}
\end{gathered}
$$

$\alpha_{1}=-.638$ with a standard error of $0.022 . \beta_{1}=-.102$ with a standard error of 0.026 . Interpret these estimates.
b. You run the same set of regressions now using third grade test scores and find that $\beta_{1}=-.382$ with a standard error of 0.028 . Interpret these new results.
c. You estimate the effect of peer racial composition on the achievement gap at grade 3 using the following OLS regression:

$$
Y_{i t}=\beta_{0}+\beta_{1} \text { Black }_{i}+X_{i t} \beta_{2}+\beta_{3} \% \text { Black }_{i t}+\epsilon_{i t},
$$

where $\%$ Black $_{i t}$ captures the percentage black for the student $i$ 's school-year (grade). First, explain why racial composition might matter. Then, given what you know of the sample and the existing literature, explain potential shortcomings of your estimates of $\beta_{3}$ and the expected sign/magnitude of any biases. Can you improve on these estimates? Explain.
d. Suppose you want to estimate the effect of teacher quality on the achievement gap at grade 3. Explain how you would go about this, writing down the specific model(s) you would estimate. Given what you know of the sample and the existing literature, explain potential shortcomings of your approach and the expected sign/magnitude of any biases.

Table 1: Summary Statistics

| Variable | White | Black |
| :--- | :---: | :---: |
| Fall kindergarten math | $.274(1.073)$ | $-.364(.765)$ |
| Spring kindergarten math | $.304(.975)$ | $-.421(.854)$ |
| Spring third grade math | $.275(.908)$ | $-.607(.958)$ |
| Female | $.481(.594)$ | $.495(.536)$ |
| Family Background Characteristics: |  |  |
| SES index | $.202(.792)$ | $-.359(.780)$ |
| Child's birth weight (in ounces) | $120.256(22.942)$ | $110.315(25.373)$ |
| Number of children's books in home | $93.121(64.792)$ | $39.014(41.986)$ |
| School/Class Characteristics: |  |  |
| Average class size | 20.673 | 21.264 |
| Teacher has master's degree | 0.280 | 0.317 |
| Average teacher experience | 6.869 | 5.362 |
| Computer to student ratio | 1.257 | 1.260 |
| Percentage of students in school who are black | 29.83 | 49.15 |

e. You control for the remaining observable characteristics in Table 1, including in $Z_{i t}$ characteristics of the schools, teachers and peers,

$$
Y_{i t}=\beta_{0}+\beta_{1} \text { Black }_{i}+X_{i t} \beta_{2}+\beta_{3} Z_{i t}+\beta_{4} Z_{i t} * \text { Black }_{i}+\epsilon_{i t}
$$

focusing on third grade test scores. Would parameter estimates from this model provide evidence of discrimination? Explain why or why not. Could you modify the estimator to provide an alternative test for racial discrimination?

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY
Tuesday 29 May $2012 \quad 2.00$ to 4.00

Module140
BEHAVIOURAL ECONOMICS
Candidates are required to answer four compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2 Approved calculators allowed
Rough work pads
Tags

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. How robust is the calibration of inequity aversion derived in Fehr \& Schmidt (1999)? Justify your answer in detail.
2. (a) Suppose that a decision maker has the discount function $D$. What is the associated subjective rate of time preference? What is the interpretation of this rate?
(b) Suppose now that

$$
D(t)=\exp (-\mu t)(1+\nu t)^{-\frac{1-\mu}{\nu}} .
$$

Find the subjective rate of time preference. Comment carefully on your answer.
3. (a) Give a careful description of Bernheim and Rangel's (2004) model of addiction.
(b) What policy implications emerge when addiction is understood as a cuetriggered process?
(c) How robust are these policy implications?
4. In Anderson and Holt's (1997) experimental study of information cascades, a series of players use Bayes's rule to incorporate information revealed by previous players' decisions. In the experiments two urns are used: Urn A and Urn B. Urn A contains one black ball and two white balls. Urn B contains two black balls and one white ball. One of the urns is chosen randomly and its contents are turned into an unmarked urn. The players do not know whether the unmarked urn contains the balls from Urn A or Urn B, and each player's task is to decide whether Urn A or Urn B has been used. In turn each player draws a ball, replaces it and, without revealing the colour of the ball to the other players, announces to the other players their prediction of event A (Urn A used) versus event B (Urn B used).
(a) Assume Player 3 has observed the previous two players both choosing Urn A. Player 3 then draws a black ball. What is Player 3's posterior probability of A? Justify your answer.
(b) Do Anderson and Holt find support for Bayesian models of social learning? What other approaches might explain their results? Justify your answers.

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RERSEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Wednesday 30 May $2012 \quad 2.00-4.00$

## Module 150 <br> ECONOMICS OF NETWORKS

Candidates are required to answer three out of four questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2 Approved calculators allowed
Rough work pads
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. Consider network formation with $n$ individuals. Suppose that a player $i$ can create a link with player $j$ unilaterally by paying a cost $c>0$. The set of player $i$ links is denoted by $g_{i}$; the profile of linking decisions of all players is given by $g=\left(g_{1}, \ldots, g_{n}\right)$. The payoffs to player $i$ under a strategy profile $g=\left(g_{1}, \ldots, g_{n}\right)$ are

$$
\begin{equation*}
\Pi_{i}(g)=1+\sum_{j \in N_{i}(g)} \delta^{d(i, j ; g)}-\eta_{i}^{d}(g) c, \tag{1}
\end{equation*}
$$

where $d(i, j ; g)$ is the distance between $i$ and $j$ in (the undirected version of) network $g$ and $\delta \in(0,1)$ is the decay factor and $\eta_{i}^{d}(g)$ is the number of links which player $i$ forms. The distance between two nodes that have no path between them is set equal to infinity.
(a) Derive the parameter ranges - on $c, \delta$ and $n$ - for which the complete network, the empty network and different types of star networks can arise in a Nash equilibrium.
(b) A network is efficient if it maximizes aggregate welfare across all networks. Show that complete network is efficient if $c<2\left(\delta-\delta^{2}\right)$, the star network is efficient if $2\left(\delta-\delta^{2}\right)<$ $c<2 \delta+(n-2) \delta^{2}$, and the empty network is efficient if $c>2 \delta+(n-2) \delta^{2}$.
(c) Are efficient networks always sustainable in equilibrium? Discuss the economic reasons for the tension between equilibrium and efficiency.
2. Consider a game in which $n$ players are located on nodes of an undirected network $g$. Players simultaneously choose an action $a_{i} \in\{0,1\}$. Action 1 costs $c$ where $c \in(0,1)$ while action 0 costs 0 . In network $g$, let $N_{i}(g)$ be the set of neighbors of player $i$. The benefit to player $i$ under strategy profile $a=\left(a_{1}, \ldots, a_{n}\right)$ is given by $\Pi_{i}(a)=f\left(\left\{a_{j}\right\}_{i \in N_{i} \cup\{i\}}\right)-a_{i} . c$.
(a) Let $y_{i}=a_{i}+\sum_{j \in N_{i}(g)} a_{j}$ denote the sum of efforts of player $i$ and his neighbors in network $g$. Suppose that $f\left(\left\{a_{j}\right\}_{j \in N_{i} \cup\{i\}}\right)=1$ if $y_{i} \geq 1$ and 0 otherwise. Show that in any network there exists a pure strategy equilibrium and that in equilibrium the gross utility of every player is 1 . In the star network, give one example of an efficient equilibrium and one example of an inefficient equilibrium.
(b) Now suppose that player utility is given by $f\left(\left\{a_{j}\right\}_{i \in N_{i} \cup\{i\}}\right)=\prod_{j \in N_{i} \cup\{i\}} a_{j}$. Show that there exists a pure strategy equilibrium in every network. Show that in every equilibrium players earn the same payoffs, given either by 0 or $1-c$.
(c) Finally, suppose that in addition to action $\{0,1\}$, players can unilaterally form links at a small cost $d$ where $d \in(0, c)$, with each of the other players. Let $l_{i}$ be the number of links formed by player $i$, and $(a, l)$ the strategy profile in this richer game. The payoff to a player from a strategy profile $(a, l)$ is $\Pi_{i}(a, l)=f\left(\left\{a_{j}\right\}_{j \in N_{i} \cup\{i\}}\right)-a_{i} . c-l_{i} . d$. Describe an equilibrium in actions and networks under payoff functions $f($.$) given in parts (a) and$ (b), respectively.
3. Suppose that there are $n$ countries with 1 firm in each country. Initially tariffs are prohibitive, so no foreign trade is possible. Countries can form bilateral free trade agreements (FTA) which lower tariffs to zero. Let $N_{i}(g)$ be the countries with whom country $i$ signs a FTA and $\eta_{i}(g)=\left|N_{i}(g)\right|$. Let the output of firm $j$ in country $i$ be denoted by $Q_{i}^{j}$. The total output in country $i$ is given by $Q_{i}=\sum_{j \in N_{i}(g)} Q_{i}^{j}+Q_{i}^{i}$. Every country has inverse linear demand $P_{i}=1-Q_{i}$. All firms have a constant and identical marginal cost of production, $\gamma$ where $\gamma \in(0,1)$. If firm $i$ is active in market $j$, then its output is given by $Q_{j}^{i}=(1-\gamma) /\left(n_{j}(g)+2\right)$. The welfare of country $i$ in network $g$ is:

$$
\begin{equation*}
\Pi_{i}(g)=\frac{1}{2}\left[\frac{(1-\gamma)\left(\eta_{i}(g)+1\right)}{\eta_{i}(g)+2}\right]^{2}+\sum_{j \in N_{i}(g)}\left[\frac{1-\gamma}{\eta_{j}(g)+2}\right]^{2} \tag{2}
\end{equation*}
$$

(a) Show that a pairwise stable network is either a complete network or consists of two components, one completely connected group of $n-1$ countries and a single isolated single country.
(b) World welfare is the sum of welfare of all countries. Show that the complete network maximizes world welfare.
(c) Now suppose that countries lie in two regions: A and B. The transport cost between two countries within a region are 0 , but the transport costs across regions are given by $T$. Discuss how transport costs shape the FTA network.
4. Consider trade between 2 buyers and 1 seller. There are two stages. In stage 1, players choose to form links. The links determine potential trade patterns. In stage 2, buyers simultaneously make bids to the seller. If there is one bidder he gets the object at price 0 , in case of two bidders, the winner is determined using a second price auction. Assume that the valuations of the buyers are uniformly distributed on the unit interval and that bidder learns his valuation at the start of stage 2 .
(a) Solve for expected payoffs to bidders and seller in empty, single link and two link network.
(b) Suppose now that bidders can unilaterally decide to form links at cost $c$. Show that if $c>1 / 2$ then the empty network is an equilibrium, if $1 / 6<c<1 / 2$ then the single link network is an equilibrium, while if $c<1 / 6$ then the two link network is an equilibrium. Show that equilibrium and efficient networks coincide.
(c) Next suppose that bidder and seller form a link only if both agree to pay $c$. Show that in this case efficient and pairwise stable networks do not coincide.

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Wednesday 30 May $2012 \quad 2.00$ to 4.00
Module 160
POLITICAL ECONOMY
Candidates are required to answer two compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

1. Consider two countries 1 and 2 with total wealth $w_{1}$ and $w_{2}$ respectively. Let $p_{1}\left(w_{1}, w_{2}\right)$ be the probability that country 1 wins the war against country 2 , and let $p_{2}\left(w_{1}, w_{2}\right)=1-p_{1}\left(w_{1}, w_{2}\right)$ be the probability that country 2 wins the war. Assume that $p_{1}\left(w_{1}, w_{2}\right)$ is nondecreasing in $w_{1}$ and nonincreasing in $w_{2}$. A war costs a country a fraction $C$ of its wealth. If a country wins the war then it gains a fraction $G$ of the other country's wealth. Let $C+G \leq 1$. In each country the decision whether to go to war is made by a pivotal decision-maker. In country 1 the pivotal decision-maker owns a fraction $a_{1}$ of the total wealth $w_{1}$. If country 1 wins the war against 2 the pivotal decision-maker gains a fraction $a_{1}^{\prime}$ of the gains from war $\left(=G w_{2}\right)$. Similarly, in country 2 the pivotal decision-maker owns a fraction $a_{2}$ of the total wealth $w_{2}$ and if country 2 wins the war against 1 the pivotal decision-maker gains a fraction $a_{2}^{\prime}$ of the gains from war.
(a) Just for this part of the question, let $p_{1}=\frac{w_{1}}{w_{1}+w_{2}}$. Find the critical value $\bar{C}$ such that if $C<\bar{C}$ then country 1 goes to war, while if $C>\bar{C}$ then country 1 does not go to war.
(b) Now consider the general form of $p_{1}\left(w_{1}, w_{2}\right)$ and $p_{2}\left(w_{1}, w_{2}\right)$ as defined above. Show that if the ratio $a_{2}^{\prime} / a_{2}$ is large enough then country 2 wishes to go to war.
(c) Just for this part of the question, assume that:

$$
\frac{a_{1}^{\prime}}{a_{1}}=\frac{a_{2}^{\prime}}{a_{2}}=1
$$

Prove that at most one country wishes to go to war, independently of the other parameters of the model.
(d) What is the "democratic peace"? Give an interpretation of the meaning of the ratio $a_{i}^{\prime} / a_{i}$ (where $i=1,2$ ) and discuss the relevance of the result in (c) in explaining the "democratic peace."

Consider an extension of the model in which country $i$ can make a transfer of wealth $t_{i j}$ to country $j$ in order to avoid a war (where $i, j=\{1,2\}$ and $i \neq j$ ). Assume that the pivotal decision-maker in country $i$ contributes a fraction $a_{i}$ of the transfer $t_{i j}$ and receives a fraction $a_{i}^{\prime}$ of the transfer $t_{j i}$.
Assume the following:
I. Countries can commit not to go to war conditional on a transfer
II. The following condition holds:

$$
\frac{a_{2}^{\prime}}{a_{2}} p_{2}\left(w_{1}, w_{2}\right) G w_{1}>\left[C+\left(1-p_{2}\left(w_{1}, w_{2}\right)\right) G\right] w_{2}
$$

which implies that if $t_{12}=0$ then country 2 will go to war
III. The following condition holds:

$$
\frac{a_{1}^{\prime}}{a_{1}} p_{1}\left(w_{1}, w_{2}\right) G w_{2}<\left[C+\left(1-p_{1}\left(w_{1}, w_{2}\right)\right) G\right] w_{1}
$$

which implies that country 1 does not want to go to war
(e) Find the range of $\frac{C}{G}$ such that a transfer can be made that avoids a war.
(f) Assume that:

$$
\frac{a_{1}^{\prime}}{a_{1}}=\frac{a_{2}^{\prime}}{a_{2}}=1
$$

Prove that the two countries will never go to war if they can make transfers to each other
2. "Institutions are the main determinant of a country future economic growth. Geography does not matter." Discuss.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Wednesday 30 May 2012 2.00-4.00

Module 170
CORPORATE FINANCE
Candidates are required to answer two out of the three questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Approved calculators allowed
Rough work pads
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## Question 1:

The firm Acquiring Giant (AG) has earnings per share of $\$ 2$, has 10 million shares outstanding and is trading at $\$ 20$ per share. Acquiring Giant is considering buying Soft Target (ST), which has earnings per share of $\$ 1.25$, has 4 million shares outstanding and a share price of $\$ 15$. AG will pay for ST by issuing new shares. There are no expected synergies from the transaction.
(a) Assume that AG pays no premium to buy ST. What are AG's earnings per share after the takeover? Explain each step of your calculation carefully.

Assume in the following sub-questions that AG offers an exchange ratio such that at current pre-announcement share prices, the offer represents a $20 \%$ premium to buy ST.
(b) What are AG's earnings per share after the takeover?
(c) What is AG's share price immediately after the announcement?
(d) What is ST's share price immediately after the announcement?
(e) What is the actual premium that AG will pay for ST ?

## Question 2:

Monsters Incorporated (MI) is ready to launch a new product. Depending on the success of this product, MI will have a value of either $\$ 100$ million, $\$ 150$ million, or $\$ 191$ million, with each outcome being equally likely. The cash flows are unrelated to the state of the economy (i.e. risk from the project is diversifiable) so that the cost of capital is equal to the risk-free rate, which is currently $5 \%$. Assume that the capital markets are perfect. Throughout, assume that in the event of default, $20 \%$ of the value of MI's assets will be lost in bankruptcy costs.
(a) What is the initial value of MI's equity without leverage?
(b) Suppose that MI has debt with a $\$ 125$ million face value due next year. (i) What is the initial value of MI's debt? (ii) What is the initial value of MI's equity? (iii) What is the initial total value of MI? (iv) What is the initial value of MI's financial distress costs?
(c) Suppose that at the start of the year, MI has no debt outstanding, but has 5.6 million shares of stock outstanding. If MI does not issue debt, what is MI's initial share price?
(d) Suppose that at the start of the year, MI has no debt outstanding, but has 5.6 million shares of stock outstanding. If MI issues debt of $\$ 125$ million due next year and uses the proceeds to repurchase shares, what would MI's initial share price be following the announcement of the repurchase?
(e) Suppose that MI has debt with a $\$ 140$ million face value due next year. (i) What is the initial value of MI's levered equity? (ii) What is the initial value of MI's debt? (iii) What is the initial total value of MI with leverage? How does your answer to this sub-question compare to that of question (b)-iii given above? Briefly comment.

## Question 3:

Omicron Technologies has $\$ 50$ million in excess cash and no debt. The firm expects to generate additional free cash flows of $\$ 40$ million per year in subsequent years and will pay out these future free cash flows as regular dividends. Omicron's unlevered cost of capital is $10 \%$ and there are 10 million shares outstanding. Omicron's board is meeting to decide whether to pay out its $\$ 50$ million in excess cash as a special dividend or to use it to repurchase shares of the firm's stock.
(a) What is the current value of Omicron's free cash flow?
(b) What is Omicron's total market value?
(c) Assume that Omicron uses the entire $\$ 50$ million in excess cash to pay a special dividend. (i) What is the amount of the special dividend? (ii) What is the amount of the regular yearly dividends?
(d) (i) With the entire $\$ 50$ million in excess cash paid a special dividend, what is Omicron's cum-dividend share price? (ii) With the entire $\$ 50$ million in excess cash paid a special dividend, what is Omicron's ex-dividend share price?
(e) (i) Assume that Omicron uses the entire $\$ 50$ million to repurchase shares. What is the number of shares that Omicron will repurchase? (ii) What is the number of shares that Omicron will have outstanding following the repurchase? (iii) What is the amount of the regular yearly dividends?

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Tuesday 5 June $2012 \quad 10.00$ to 12.00
Module 180
INDUSTRIAL ORGANISATION
Candidates are required to answer two out of the three questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
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Tags

## Question 1:

Consider the following Hotelling model. A mass of consumers of measure one are uniformly distributed on the unit interval $[0,1]$. Each consumer has unit demand and has utility from consumption $r>0$. The market is served by two firms, labelled 1 and 2. Competition occurs in two stages. At the first stage, firms simultaneously (and non-cooperatively) choose locations $l_{1}$ and $l_{2}$ respectively, with $l_{1}, l_{2} \in[0,1]$ and $l_{1} \leq l_{2}$. At the second stage, firms simultaneously (and noncooperatively) choose prices $p_{1}$ and $p_{2}$ respectively. Firms have identical constant marginal costs of production $c>0$.
(a) Assume that the consumers have linear transport costs so that the utility of a consumer located at $x$ who buys from firm $i=1,2$ is given by

$$
u=r-p_{i}-t\left|x-l_{i}\right|
$$

with $t>0$.
(i) For given locations and prices, determine the demand functions $Q_{i}\left(p_{i}, p_{j} ; l_{i}, l_{j}\right)$ for the two firms.
(ii) For given locations and prices, determine the profit functions $\pi_{i}\left(p_{i}, p_{j} ; l_{i}, l_{j}\right)$ for the two firms.
(iii) For given locations, assume that there is a unique interior equilibrium in prices. Find expressions $p_{i}\left(l_{i}, l_{j}\right)$ for the equilibrium prices of the two firms. Briefly comment on the role of the transport cost $t$ in determining market power.
(iv) Briefly discuss the potential problems with finding a pure strategy Nash equilibrium at the location stage of this game.
(b) Assume that the consumers have quadratic transport costs so that the utility of a consumer located at $x$ who buys from firm $i=1,2$ is given by

$$
u=r-p_{i}-t\left|x-l_{i}\right|^{2}
$$

with $t>0$.
(i) For given locations and prices, determine the demand functions $Q_{i}\left(p_{i}, p_{j} ; l_{i}, l_{j}\right)$ for the two firms.
(ii) For given locations and prices, determine the profit functions $\pi_{i}\left(p_{i}, p_{j} ; l_{i}, l_{j}\right)$ for the two firms.
(iii) For given locations, find expressions $p_{i}\left(l_{i}, l_{j}\right)$ for the equilibrium prices of the two firms.
(iv) Find the Nash equilibrium location choices of the two firms and briefly comment on these.

## Question 2:

Consider a Salop city model with an industry that consists of four firms on a circle with unit circumference located $1 / 4$ from each other. The firms are labeled $A, B, C$ and $D$ respectively. For simplicity, let the firms be located so that $A$ 's neighbour to the right is firm $B$, $B$ 's neighbour on the right is firm $C, C$ 's neighbour on the right is firm $D$ and $D$ 's neighbour on the right is firm $A$. The firms produce at identical constant marginal costs $c>0$ and set prices $p_{A}, p_{B}, p_{C}$ and $p_{D}$ respectively. There is a mass one of consumers with unit demand, who are distributed uniformly on the circle. The consumers have linear transport costs so that the utility of a consumer located at $x$ who buys from a firm at location $y$ at price $p$ is given by

$$
u=r-p-t|x-y|
$$

with $t>0$. A consumer who is indifferent between buying from firms $i$ and $j$ when they set prices $p_{i}$ and $p_{j}$ respectively is denoted by $I_{i j}$. Thus $I_{A B}$ is exactly indifferent between buying from $A$ and from $B$.
(a) Assume that the four firm behave as independent firms and compete in prices.
(i) Find the demand functions for the firms.
(ii) Carefully write down a typical firm's maximisation problem.
(iii) Find the equilibrium prices and profits of the firms.
(b) Assume that firms $A$ and $B$ merge into a new firm $A B$ and that firms $C$ and $D$ remain independent.
(i) Carefully write down the maximisation problem of firms $A$ and $B$ and of firms $C$ and $D$.
(ii) Confirm that the equilibrium prices are

$$
\begin{aligned}
& p_{A}=p_{B}=c+\frac{2 t}{5} \\
& p_{C}=p_{D}=c+\frac{3 t}{10}
\end{aligned}
$$

and find the equilibrium quantities and profits of the firms.
(iii) Compare the equilibrium outcomes with the merger to those when all firms remain independent. Do insiders to the merger $(A$ and $B$ ) benefit? Do the outsiders to the merger ( $C$ and $D$ ) benefit? Briefly discuss.

## Question 3:

An incumbent firm $I$ faces potential entry by a firm $E$. Formally, the following two-stage game is played: First, the incumbent $I$ chooses some investment level $K$. Upon observing $K$, the entrant decides whether to enter or stay out. Depending on the entry decision, the following takes place:

1. In case $E$ stays out, he earns zero profits while the incumbent $I$ remains a monopolist and earns $\pi_{1}^{m}\left(K, \sigma_{1}^{M}(K)\right)$ where $\sigma_{1}^{M}(K)$ is firm I's monopoly choice as a function of its investment $K$.
2. In case $E$ enters, the two firms simultaneously make decisions $\sigma_{1}$ and $\sigma_{2}$, earning $\pi_{1}\left(K, \sigma_{1}, \sigma_{2}\right)$ and $\pi_{2}\left(K, \sigma_{1}, \sigma_{2}\right)$ respectively. The latter expression is net of any entry costs.

Denote equilibrium choices by $\sigma_{1}^{*}(K)$ and $\sigma_{2}^{*}(K)$ respectively and assume that these are differentiable. Last, assume that the functions $\pi_{1}^{m}\left(K, \sigma_{1}^{M}(K)\right)$ and $\pi_{1}\left(K, \sigma_{1}^{*}(K), \sigma_{2}^{*}(K)\right)$ are strictly concave in $K$.
(a) Assume that the incumbent finds it optimal to deter entry.
(i) Write down the entry deterrence condition.
(ii) Carefully write down an expression of the effects that investment choice $K$ has on the entry deterrence condition, distinguishing between direct and strategic effects.
(iii) Under which conditions will the incumbent under-invest/over-invest relative to the monopoly investment level, in order to deter entry? Interpret these conditions and give brief examples where these may be met. In this sub-question, you may assume that the direct effect is zero.
(b) Assume that the incumbent finds it optimal to accommodate entry.
(i) Write down the incumbent's maximisation problem.
(ii) Carefully write down an expression of the effects that investment choice $K$ has on the incumbent's maximisation condition, distinguishing between direct and strategic effects.
(iii) Under which conditions will the incumbent under-invest/over-invest relative to the monopoly investment level, when entry is accommodated? Interpret these conditions and give brief examples where these may be met.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

| Monday 28 May 2012 | 2.00 to 4.00 |
| :--- | :--- |

Module 190
BEHAVIOURAL FINANCE
Candidates are required to answer two out of the three questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

1. Many, if not all, papers in behavioural finance relax the assumption of perfectly rational agents in the neoclassical sense.
a) Why would we not expect all agents on the financial markets to be perfectly rational? Explain.
b) Many models assume that there are heterogenous agents of different degrees of rationality, with one typically being perfectly rational. Explain why this makes sense, at least in theory.
c) Is it enough to assume that not all agents are perfectly rational for market outcomes to differ from the predictions of models that assume rationality? Begin by outlining the basic argument in either Alchian (1950), Becker (1960) or Blume \& Easley (1992). Relate your discussion to this paper. What do you think? What does the evidence suggest?
2. Let us turn our attention to the so called "equity premium puzzle".
a) What is the "equity premium puzzle"? Explain. Relate your answer to standard theory and the empirical evidence.
b) What is prospect theory? Focus on the key characteristics.
c) How has prospect theory been used to explain the "equity premium puzzle"? Do you think the approaches that have been suggested make sense? Focus your analysis on Barberis, Huang \& Santos (2001) and Mehra \& Prescott (2003).
3. Arbitrage and discounting are two important concepts in finance.
a) What do we mean with "limits of arbitrage? What are "noise" traders? Give an example where limited arbitrage could have contributed to a pricing anomaly.
b) Laibson (1997) suggests a simplified model of hyperbolic discounting. What is hyperbolic discounting? Describe the quasi-hyperbolic utility function that Laibson suggests.
c) Discuss the relationship between self-control and hyperbolic discounting as well its potential implications for outcomes on the financial markets.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Wednesday 9 May $2012 \quad 10.00$ to 12.00

Module 200
MACROECONOMICS I

Answer four compulsory questions from Section $A$ and one question from Section B. Each section carries equal weight.
Write legibly.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS

20 Page booklet x 2
Approved calculators allowed
Rough work pads
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

A. 1 Consider a simple Real Business Cycle model in which the logarithm of technology in period $t, \tilde{A}_{t}$, evolves according to

$$
\tilde{A}_{t}=\rho \tilde{A}_{t-1}+\varepsilon_{t}
$$

and the percent deviation of output from its steady state in period $t, \hat{y}_{t}$, is given by

$$
\hat{y}_{t}=(\alpha+\rho) \hat{y}_{t-1}-\rho \alpha \hat{y}_{t-2}+\varepsilon_{t}
$$

where $\varepsilon_{t}$ is a zero-mean i.i.d. process, $\rho \in[0,1)$, and $\alpha \in(0,1)$ is the capital share of output. Suppose the economy is initially at the steady state, such that $\hat{y}_{t-1}=\hat{y}_{t-2}=$ 0 . In period $t$ there is a one-time positive shock to technology, $\varepsilon_{t}=1$.
Compute the values of $\hat{y}_{t}, \hat{y}_{t+1}$ and $\hat{y}_{t+2}$. Can $\hat{y}_{t+1}$ be greater than $\hat{y}_{t}$ ? Explain intuitively why or why not.
A. 2 Consider Hayashi's marginal $q$ model described by the following two equations:

$$
\begin{aligned}
\dot{q} & =r q-\pi(K) \\
q & =1+C^{\prime}(\dot{K})
\end{aligned}
$$

where $q$ denotes the marginal value of capital, $K$ the aggregate capital stock, and $r$ the real interest rate. In addition, $\pi(\cdot)$ denotes the marginal product of capital, where $\pi^{\prime}(\cdot)<0$, and $C(\cdot)$ is the adjustment cost of investment, where $C(0)=0, C^{\prime}(0)=0$ and $C^{\prime \prime}(\cdot)>0$.
Carefully draw the phase diagram, clearly indicating the steady state and dynamics for $K$ and $q$. Using the phase diagram, analyse the effect of a temporary increase in the marginal product of capital, $\pi(K)$.
A. 3 Suppose a representative firm facing perfect competition in the goods market maximizes real profits

$$
\Pi=Y-w L
$$

where $Y$ is output, $L$ employment, and $w$ the real wage. The firm's production technology is given by

$$
Y=(e L)^{\alpha} \quad(0<\alpha<1)
$$

where $e$ is employee effort, which is described by

$$
e=(w-\bar{w})^{\beta} \quad(0<\beta<1)
$$

for $w \geq \bar{w}$ and $e=0$ otherwise, where $\bar{w}$ is the minimum wage.
Derive the optimal wage $w^{*}$ for the firm. Explain intuitively what determines its level.
A. 4 Consider the Barro tax-smoothing model. Assume that government purchases $G$ and the real interest rate $r$ are constant, and that government debt is initially zero: $B_{0}=0$. Suppose a financial crisis suddenly leads to a drop in output $Y$ for $\omega$ periods. Explain how this affects the optimal time path of the tax rate $\tau$ and government debt $B$.

## SECTION B

## B. 1 Consumption and fiscal policy

Consider the following consumption-savings problem of the representative household:

$$
\begin{gathered}
\max _{\left\{c_{t}, a_{t+1}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right) \\
\text { subject to } \quad a_{t}(1+r)+y_{t}-T_{t}=c_{t}+a_{t+1} \quad t=0,1,2, \ldots
\end{gathered}
$$

where $c_{t}, a_{t}, T_{t}$ and $y_{t}$ denotes consumption, assets, lump sum taxes, and income in period $t$, respectively. Initial wealth, $a_{0}$, and the entire sequence of income and taxes $\left\{y_{t}, T_{t}\right\}_{t=0}^{\infty}$ is taken as given. The utility function satisfies $u^{\prime}(\cdot)>0$ and $u^{\prime \prime}(\cdot)<0$, and $\beta \in(0,1)$. The real interest rate, $r$, is exogenous and such that $\beta(1+r)=1$.
(a) Derive the intertemporal budget constraint associated with the above problem. Make sure to apply the no-Ponzi game condition.
(b) Derive the Euler equation, and solve the optimization problem for $c_{0}$. Is there consumption smoothing? Explain intuitively why.
(c) Suppose the consumption sequence $\left\{c_{t}\right\}_{t=0}^{\infty}$ solves the above optimization problem. Does the sequence $\left\{\gamma c_{t}\right\}_{t=0}^{\infty}$ for $\gamma \in(0,1)$ satisfy the Euler equation? Is this sequence also a solution to the above optimization problem? Why or why not?
(d) Now, suppose that the government borrows $G_{0}>0$ for government purchases in period zero, but $G_{1}=G_{2}=\ldots=0$. The government's period-by-period budget constraint is given by

$$
B_{t+1}=B_{t}(1+r)+G_{t}-T_{t} \quad t=0,1, \ldots
$$

where $B_{t}$ denotes the amount of real government bonds at the start of period $t$, with $B_{0}=0$.
i. Derive the government's intertemporal budget constraint. Assuming that $T_{0}=0$ and $T_{1}=T_{2}=\ldots=T$, compute the value of $T$ that satisfies this budget constraint given $G_{0}$.
ii. Let aggregate output $Y_{t}$ be determined according to $Y_{t}=c_{t}+G_{t}$. Given your solution for $T$, and given the optimal consumption choice in part (b), derive the fiscal multiplier $\partial Y_{0} / \partial G_{0}$.

## B. 2 Monetary policy and transparency

Suppose the central bank maximizes the objective function

$$
W=-\frac{1}{2} \alpha(\pi-\tau)^{2}-\frac{1}{2}(1-\alpha)(y-\kappa)^{2}
$$

where $\pi$ denotes inflation, $y$ the output gap, $\tau$ the central bank's inflation target, $\kappa$ its output gap target, and $\alpha$ the relative weight on inflation stabilization, with $0<\alpha<1$. The economy is described by an expectations-augmented Phillips equation:

$$
\pi=\pi^{e}+y+s
$$

where $\pi^{e}$ denotes private sector inflation expectations, and $s$ is an i.i.d. supply shock with $\mathrm{E}[s]=0$ and $\operatorname{Var}[s]=\sigma_{s}^{2}>0$. The central bank's inflation and output gap targets are also stochastic and independently distributed with $\mathrm{E}[\tau]=\mathrm{E}[\kappa]=0$, $\operatorname{Var}[\tau]=\sigma_{\tau}^{2}>0$ and $\operatorname{Var}[\kappa]=\sigma_{\kappa}^{2}>0$. The timing is as follows. First, the central bank's targets $\tau$ and $\kappa$ are realized. Then, the private sector forms $\pi^{e}$ using rational expectations. Subsequently, the supply shock $s$ is observed. Finally, the central bank optimally sets the output gap $y$. Assume initially that the central bank targets $\tau$ and $\kappa$ are observed by both the central bank and the private sector, so there is 'perfect transparency' about the central bank's targets.
(a) Derive $y, \pi$ and $W$ for a given level of $\pi^{e}$. What level of $\pi^{e}$ would maximize $W$ ?
(b) Compute $\pi^{e}, y, \pi$ and $\mathrm{E}[W]$ when both the central bank and the private sector observe $\tau$ and $\kappa$ (i.e. 'perfect transparency'). Explain intuitively how $y$ and $\pi$ depend on $\tau, \kappa$ and $s$.
(c) Now assume that there is 'imperfect transparency' such that
i. the central bank's inflation target $\tau$ is no longer observed by the private sector;
ii. the central bank's output gap target $\kappa$ is no longer observed by the private sector.

Compute $\pi^{e}$ and $\mathrm{E}[W]$ for each case. Explain whether transparency about (i) the inflation target $\tau$, and (ii) the output gap target $\kappa$ is beneficial (ex ante) to the central bank.

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Wednesday 23 May $2012 \quad 10.00$ to 12.00
Module 210
MACROECONOMICS II
Answer four compulsory questions from Section A and one question from Section B. Each section carries equal weight.

Write legibly.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS

20 Page booklet x 2 Approved calculators allowed
Rough work pads
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

## A. 1 Search Model

Each period an unemployed worker draws one offer to work forever at wage $w$ from a cumulative distribution function $F(w)$, where $w \in[0, B]$, so $F(0)=0$ and $F(B)=1$. The worker seeks to maximize $\mathbf{E}\left[\sum_{t=0}^{\infty} \beta^{t} y_{t}\right]$, where $y_{t}$ is the income of the agent in period $t$, such that $y_{t}=w$ if the worker is employed, and $\beta \in(0,1)$. The worker is entitled to unemployment benefits in the amount $b$ only during the first period that she is unemployed. After one period on unemployment compensation, the worker receives no income. Formulate the problem of this agent in recursive form. Intuitively argue whether or not the reservation wage is constant over time.

## A. 2 Asset Pricing

Consider an economy populated by a continuum of measure one of identical households, each with preferences given by $\mathbf{E}_{0}\left[\sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right)\right]$ where $c_{t}$ is consumption at period $t, u(c)=$ $\frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma>0$, and $\beta \in(0,1)$ corresponds to the subjective discount factor. There is a single asset tree which yields perishable dividends $d_{t}$ (fruits). Assume that $d_{t}$ is a random variable taking values from the set $\mathbf{D} \equiv\left\{d_{1}, \ldots, d_{N}\right\}$ and is distributed according to a firstorder Markov chain with constant transition probability matrix. Households can trade the share of the tree. In equilibrium, the price of this tree is

$$
p_{t}=\beta E_{t}\left[\frac{u^{\prime}\left(d_{t+1}\right)\left(p_{t+1}+d_{t+1}\right)}{u^{\prime}\left(d_{t}\right)}\right] .
$$

Write this price in terms of the expected return of the asset and the risk premium. Explain your result intuitively.

## A. 3 Optimising IS Equation

Suppose the representative consumer maximizes the expected value of lifetime utility

$$
U_{t}=\sum_{s=t}^{\infty} e^{-\rho(s-t)} \frac{C_{s}^{1-\sigma}-1}{1-\sigma}
$$

where $C_{t}$ denotes consumption in period $t, \rho$ the subjective intertemporal discount rate $(\rho>0)$, and $\sigma$ the coefficient of relative risk aversion $(\sigma>0)$. The budget constraint of the representative consumer is given by

$$
C_{t}+Q_{t} B_{t}=B_{t-1}+Y_{t}
$$

where $B_{t}$ denotes one-period riskless real discount bonds purchased in period $t$ at price $Q_{t}$, and $Y_{t}$ the output endowment in period $t$, which is stochastic. The goods market equilibrium condition is

$$
C_{t}=Y_{t}
$$

Derive the dynamic, optimising IS equation relating $y_{t}$ to $r_{t}$, where $y_{t} \equiv \ln Y_{t}$ and $r_{t} \equiv$ $-\ln Q_{t}$. Briefly comment on its properties.

## A. 4 Model Determinacy

Consider the following monetary model:

$$
\begin{aligned}
\pi_{t} & =\beta \mathrm{E}_{t}\left[\pi_{t+1}\right]+\gamma m_{t} \\
m_{t+1} & =\mu m_{t}+\varepsilon_{t+1}
\end{aligned}
$$

where $\pi_{t}$ denotes inflation in period $t, m_{t}$ money growth, $\varepsilon_{t}$ an i.i.d. money shock with $\mathrm{E}_{t}\left[\varepsilon_{t+1}\right]=0$ and $\mathrm{E}_{t}[$.$] the expectation conditional on the information avail-$ able in period $t$. The parameters satisfy $0<\beta<1, \gamma>0$ and $0<\mu<1$.
Use the Blanchard-Kahn method to show whether the conditions for a unique equilibrium are satisfied for this model. Derive a solution for $\pi_{t}$ and briefly comment on its properties.

## SECTION B

## B. 1 Dynamic Programming

Consider the following social planner's problem:

$$
\max _{\left\{c_{t}, k_{t+1}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^{t}\left(\log c_{t}+\gamma \log c_{t-1}\right)
$$

subject to $c_{t}+k_{t+1} \leq A k_{t}^{\alpha}, A>0, \alpha \in(0,1), \beta \in(0,1)$ and given $k_{0}$ and $c_{-1}$.
(a) Interpret the utility function.
(b) Define the optimal value function for this problem.
(c) Is this value function a Contraction? If it is, what can we conclude from that?
(d) Use the method of guess and verify to find the optimal policy and value functions.
B. 2 Neoclassical Growth Model

Consider an economy with aggregate production function

$$
Y(t)=K(t)^{\alpha}(A(t) L(t))^{1-\alpha}, \alpha \in(0,1) .
$$

$Y$ is output, $K$ is capital, $L$ is labour, and $A$ is labour productivity, which grows at rate $g$ over time. All markets are competitive, there is no population growth, and capital depreciates at rate $\delta>0$. Capital evolves according to:

$$
\dot{K}(t)=I(t)-\delta K(t),
$$

where $I$ corresponds to investment and $K(0)$ is given. Agents are infinitely lived with preferences

$$
U=\int_{0}^{\infty} e^{-\rho t}[\log (c(t))+\log (1-l(t))] d t, \quad \rho>0
$$

where $c(t)$ denotes consumption, and $l(t)$ is labour supply.
(a) Set up the Hamiltonian that the representative household solves, taking prices as given, and determine the necessary and sufficient conditions for the allocation of consumption over time and the leisure-labour trade-off.
(b) Derive the first order conditions of the representative firm, and specify the market clearing conditions.
(c) Derive the growth rates of capital per capita, consumption per capita and labour supply along the balanced growth path equilibrium. Give an intuitive explanation.
(d) Set up the social planner's problem and derive the necessary and sufficient conditions for this problem. Explain whether the allocations of consumption, capital and labour supply are the same for these two problems.

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE
OF MASTER OF PHILOSOPHY
Tuesday 22 May $2012 \quad 10.00-12.00$
Module 230
APPLIED MACROECONOMICS
Answer three questions only. Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet

STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Approved calculators allowed
Metric Graph Paper
Rough work pads

[^1]1. $\pi_{t}^{d}$ is the desired (target) rate of inflation and $\pi_{t}$ is the actual rate of inflation at time period $t$. Define $e_{t}=\left(\pi_{t}-\pi_{t}^{d}\right)$. The policymaker has a single instrument of policy, the short term interest rate, $r_{t}$.
(a) Write down a PID controller and explain carefully what it means.
(b) Is inflation controllable in this case?
(c) Suppose now the policymaker was also concerned about output. How does this affect your answer to part (b)?
(d) Under what circumstances might both inflation and output be controllable?
(e) If they are not how might the policymaker characterise the problem she faced?
2. "In order to explain business cycle fluctuations we need to appeal to the presence of aggregate shocks". Is this always true?
3. In January of 2012 the Federal Reserve Board adopted for the first time an explicit target for the rate of inflation of $2 \%$. Was this the right thing to do?
4. The growth of national debt over time as a proportion of national income depends crucially on the interaction of the initial stock of debt, the primary balance, the real interest rate and the growth rate of the economy.
(a) Carefully explain this interaction.
(b) Under what circumstances will the debt-income ratio be stable?
(c) What changes might disturb this stable position?
(d) Suppose a country has an unsustainable level of indebtedness and successfully negotiates with its creditors a reduction in the debt to half what it was. Will this necessarily improve the position of the country?
5. Many central banks publish both point and density forecasts.
(a) Explain what these are.
(b) Why is there so much uncertainty associated with macroeconomic forecasting?
(c) What role does nowcasting play?
(d) How might a central bank improve its forecasts by using the forecasts of others?
6. In order to explain the variation in per capita incomes across countries, Mankiw, Romer and Weil estimate the model using cross country data of the form:

$$
\begin{equation*}
\log (Y / L)=a+\frac{\alpha}{1-\alpha} \log (s)-\frac{\alpha}{1-\alpha} \log (n+g+\delta)+\varepsilon \tag{1}
\end{equation*}
$$

where $Y(t)$ is output, $L(t))$ is the labour input, $\alpha$ is the share of capital in national income, $s$ is the savings rate, $n$ is the rate of growth of the population, $g$ is the rate of growth of technology, $\delta$ is the depreciation rate and $\varepsilon$ is a random error.
(a) Explain how they obtain this regression equation.
(b) How well do they find the model performs empirically?
(c) What improvements to this model do they propose?
(d) How well does the new model perform empirically?
7. (a) Explain carefully how the Taylor Principle is undermined when policymakers come up against the zero interest rate floor.
(b) How might policymakers reduce the likelihood of the zero interest floor?
(c) Faced with the zero interest rate floor how might the policymaker react?
(d) How effective is this action likely to be?

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY.
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Monday 4 June $2012 \quad 10.00$ to 12.00
Module 240
INTERNATIONAL FINANCE
Candidates are required to answer four out of five questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS

20 Page booklet x 2 Approved calculators allowed
Rough work pads
Graph paper
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## Question 1 <br> Uncovered Interest Parity

a) The UIP condition states that differences between nominal interest rates in different currencies ( $i$ and $i^{*}$ ) should be entirely explained by the expected change in the exchange rate ( $\mathcal{E}$, defined in units of domestic currency per unit of foreign currency) over the relevant time horizon. For one period bonds:

$$
\frac{1+i}{1+i^{*}}=\frac{E_{t}\left(\mathcal{E}_{t+1}\right)}{\mathcal{E}_{t}}
$$

Assume a representative domestic investor with (generic) stochastic discount rate $\mathcal{D}$. Using the expression for the equilibrium price of nominal bonds denoted in domestic and foreign currency, explain which other determinants should be added to the above expression.
b) Under what condition we may have $\frac{1+i}{1+i^{*}}>1$ in equilibrium, when $\frac{E_{t}\left(\mathcal{E}_{t+1}\right)}{\mathcal{E}_{t}}<$ 1 ?
c) Suppose that a government imposes a tax $\tau$ on trading foreign-currency denominated bonds. What are the effects of this tax on the interest differential in equilibrium? Can capital controls of this sort provide an explanation for the empirical observation that positive nominal interest differentials are systematically associated with exchange rate appreciation?

## Question 2 <br> The Peso Problem

a) As documented by a vast body of literature, the uncovered interest parity condition

$$
\frac{1+i}{1+i^{*}}=\frac{E_{t}\left(\mathcal{E}_{t+1}\right)}{\mathcal{E}_{t}}
$$

fails to hold when tested based on regression models under rational expectations. One of the candidate explanations for this empirical failure is the 'peso problem.' Define the peso problem and illustrate it, using the equilibrium pricing equation for an uncovered position in domestic currency denominated bonds, financed by borrowing in foreign currency denominated bonds.
b) Consider a country pursuing a fixed exchange rate. Suppose that investors expect a 50 percent devaluation of the exchange rate to occur in the next period with probability 1 percent. For simplicity, assume $1+i=1.05$ and $1+i^{*}=1 ; P=1$ and only 2 states of the world. Can these expectations of devaluation explain an interest differential of 500 basis points if agents are risk neutral? Under what conditions could this occur? Explain.

## Question 3 <br> Risk and Global Imbalances

Consider two countries, Home and Foreign, existing for two periods only. In either country, there are many identical household with discount factor

$$
D=\beta \frac{U\left(C_{i, t+1}\right)}{U\left(C_{i, t}\right)}, \quad i=H, F
$$

They produce an identical good. Output in the first period is identical across countries $Y_{H, t}=Y_{F, t}=Y$. In the second period, it varies randomly:

$$
Y_{H, t+1}=Y+\eta \quad \text { and } \quad Y_{F, t+1}=Y_{H, t+1}+\varepsilon
$$

where $\varepsilon$ is independent of $Y_{H, t+1}$. The two countries are in financial autarky, so that

$$
C_{i}^{A}=Y_{i} \quad i=H \text { and } F
$$

in both periods.
Answer the following:
a) Suppose the capital account is liberalized, and agents in the two countries can trade a non-contingent bond in the first period. The bond is traded at the price $q_{b o n d}$, and delivers a cash flow $x=1$. Which country would borrow from the other, running a current account deficit? Explain carefully which economic 'law' you are using in your answer, and list precisely your assumptions.
b) What insights would you derive from your answer to the previous question on the debate on 'global imbalances', reflecting persistent current account deficits of the US against persistent current account surpluses of many fastgrowing emerging markets?

## Question 4 <br> Debt and Confidence Crisis (Calvo 1988)

In this question, you are asked to write a variant of Calvo 1988 model on debt and confidence crisis. Consider a small open economy with a two period $(0,1)$ horizon. In period 0 the asset market opens and households allocate their wealth investing in an international asset $K$ paying $R$, and in domestic public debt $b$, promising a market rate $\widetilde{R}$. In period 1 , households consume their income net of taxes and the returns on their investment; given taxes, the government chooses optimally how much to spend $(G)$, and whether and to what extent to default on its debt. To keep analysis simple, assume:

- Debt $b$ is entirely public and is entirely owned by domestic investors
- Domestic residents are risk neutral.

The period 1 budget constraint of the government is

$$
T-G=\overbrace{(1-\theta) b \widetilde{R}}^{\text {interest payment }}+\overbrace{\alpha \theta b \widetilde{R}}^{\text {budget cost of default }}
$$

where $T$ denotes taxes; $G$ is spending; $\theta$ denotes the extent of default, comprised between 0 , no default, and 1 , complete default; and by setting $0<\alpha<1$ we assume that the government suffers a budget cost proportional to the extent of default. Different from the model by Calvo 1988, taxes $T$ are given, but the government can vary $G$. Assume that spending on public goods $G$ gives agents utility, measured by $U(G)$. Producing $G$ however uses resources. Namely, it requires $Z(G)$ units of output, with $\partial Z / \partial G>0$ and $\partial^{2} Z /(\partial G)^{2}<0$.
Private agents utility in period 1 is given by the sum of private consumption and the utility from public goods $C+U(G)$. Consumption is equal to net output (total output minus the resources used to produce public goods) minus taxes, plus the net revenue from investment in the safe asset and government debt:

$$
C=[Y-Z(G)]-T+K \cdot R+(1-\theta) b \cdot \widetilde{R}
$$

Answer the following:
a) Assuming rational expectations, write the equilibrium relation between $R$ and $\widetilde{R}$.
b) Posit that the goal of the government is to maximize private agents utility, i.e. $C+U(G)$. For a given $\widetilde{R}$, write the problem of the government (recall that taxes are given). Derive and interpret the condition that characterizes the optimal choice of spending $\widehat{G}$ in period 1 .

Question continued on next page
c) How does the optimality condition differ depending on whether the government defaults or does not default? In which case is optimal spending larger? Why?
d) Now, assume that the given tax rate satisfies

$$
T=\widehat{G}^{N D}+R b
$$

where $\widehat{G}^{N D}$ denotes optimal spending when there is no default. The answers to the previous questions give you two equilibrium conditions, one for $\widetilde{R}$ and one for either $\widehat{G}$ or $\widehat{G}^{N D}$. Using these conditions together with the budget constraint of the government, verify that, given taxes at the above level, the equilibrium may not be unique.
e) What lessons would you derive from this model about the possibility and costs of confidence crises?

## Question 5

## Fundamentals and Currency Crises

Suppose the government runs a structural budget deficit and forces the central bank to monetize government debt by increasing the domestic assets $B$ held by the central bank at a constant rate so that $\dot{B} / B=\gamma>0$. The central bank balance sheet is given by

$$
M=B+R
$$

where $M$ denotes the money supply and $R$ foreign reserves. The central bank aims to fix the nominal exchange rate through (sterilized) foreign exchange market interventions so that $s=\bar{s}$, where $s$ is the log domestic price of foreign currency. If the central bank has depleted its foreign reserves, it will adopt a free float. Money market equilibrium is given by

$$
\begin{equation*}
m-p=y-\theta i \tag{LM}
\end{equation*}
$$

where $m$ is the $\log$ money supply, $p$ the log aggregate price level, $i$ the nominal interest rate, $y \log$ aggregate output, which is exogenous, and $\theta$ a positive parameter. The aggregate price level $p$ is flexible and purchasing power parity holds:

$$
\begin{equation*}
p=s+p^{*} \tag{PPP}
\end{equation*}
$$

where the asterisk $*$ indicates foreign variables, which are exogenous. Economic agents have rational expectations and there is no uncertainty. International capital mobility leads to uncovered interest parity:

$$
\begin{equation*}
i=i^{*}+\dot{s} \tag{UIP}
\end{equation*}
$$

For simplicity, normalize $p^{*}=0$.
a) Derive the level of the money supply $m$ that is required to keep the exchange rate fixed at $\bar{s}$. Explain intuitively why the exchange rate peg is unsustainable.
b) Derive the shadow exchange rate $\tilde{s}$ and compute the time $T$ of the speculative attack. Explain intuitively how it depends on the growth rate $\gamma$ of monetized government debt and on the initial level of foreign reserves $R_{0}$.
c) Suppose that after fixing the exchange rate at $\bar{s}$, there is a sudden economic downturn at time $\tau$ leading to a decline in output from $y_{0}$ to $y_{\tau}$ (with $0<\tau<$ $T$ and $y_{\tau}<y_{0}$ ). Explain how this would affect the timing of the speculative attack.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Monday 4 June $2012 \quad 10.00$ - 12.00
Module 250
INTERNATIONAL TRADE
Candidates are required to answer Two compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Approved calculators allowed
Rough work pads
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## Question 1 Increasing returns

Consider the economic environment of Krugman (1980): the economy is populated by $L$ individuals choosing their consumption plans according to the following CES preference structure over a continuum of varieties $\omega \in(0, n)$

$$
\begin{equation*}
U=\left(\int_{0}^{n} q(\omega)^{\frac{\sigma-1}{\sigma}} d \omega\right)^{\frac{\sigma}{\sigma-1}} \tag{Utility}
\end{equation*}
$$

where $\sigma>1$ is the elasticity of substitution between varieties. Each variety is produced by monopolistically competitive firms using labor with the following technology

$$
\begin{equation*}
l=f+\frac{q}{\varphi} \tag{technology}
\end{equation*}
$$

where $l$ is labor needed to produce $q$ units of each differentiated good, $\varphi$ is a labor productivity parameter common to all firms and $f$ is a fixed operating cost.
i. Set up the household problem and derive the demand function for each variety $\omega$. Set up the firm problem and derive the profit maximizing price for each firm. Using the free entry and the labor market clearing conditions derive the optimal number of firms/variety in the economy.
ii. Now assume that this economy can trade with another country paying an iceberg transportation cost $\tau>1$, implying that in order to send one unit of product abroad firms must ship $1 / \tau>1$ units. Assume that the two countries are symmetric in every feature but size, with the domestic country being smaller than the foreign country: $L<L^{*}$. Derive demand and prices for goods sold domestically and exported. Derive the optimal number of firms/varieties in both countries. Explain why countries produce different varieties.
iii. Using the inverse of the price index as an indicator of welfare, show that when going from autarky to costly trade countries experience an increase in welfare. What are the sources of the welfare gains from trade? Discuss.
iv. Similarly, show the welfare effects of trade liberalization operated through a reduction in the variable trade cost $\tau$. Are the sources of the welfare gains from trade different from the previous question? Discuss.

## Question 2 Increasing returns and heterogeneous firms

Following Melitz (2003) consider an economy populated by $L$ individuals with CES preferences over a continuum of varieties $\omega \in(0, M)$

$$
\begin{equation*}
U=\left(\int_{0}^{M} q(\omega)^{\rho} d \omega\right)^{\frac{1}{\rho}} \tag{Utility}
\end{equation*}
$$

with $0<\rho<1$ and $\sigma=1 /(1-\rho)>1$ being the elasticity of substitution between varieties. Monopolistically competitive firms use labor to produce each variety according to the following technology

$$
\begin{equation*}
l=f+\frac{q}{\varphi} \tag{technology}
\end{equation*}
$$

where the productivity parameter $\varphi$ varies across firms. At entry firms sunk an entry cost $f_{e}$ and draw their productivity $\varphi$ from an initial productivity distribution with c.d.f. $G(\varphi)$. Firms stay on the market if their productivity is high enough to operate profitably. At any point in time firms can be hit by a bad shock which with probability $\delta>0$ forces them out of business.
i. Using the wage as the numeraire (i.e. setting $w=1$ ), set up the household problem and derive the demand function for each variety $\omega$. Set up the firm problem and derive the profit maximizing price for each firm. Notice that if you think there are similarities with the answers to the previous questions, you can explain them and go directly to the results skipping the derivation.
ii. Write down the profit function and define the equilibrium cutoff condition (you can help yourself with a graph). Using the definition of aggregate productivity

$$
\widetilde{\varphi}\left(\varphi^{*}\right)=\left(\int_{\varphi^{*}}^{\infty} \varphi^{\sigma-1} \mu(\varphi) d \varphi\right)^{\frac{1}{\sigma-1}}
$$

where $\mu(\varphi)=\frac{g(\varphi)}{\left(1-G\left(\varphi^{*}\right)\right)}$ is the equilibrium productivity distribution and $\varphi^{*}$ is the productivity cutoff above which firm can profitably survive. Derive the Zero Cutoff Profit (ZCP) condition as in Melitz (2003), an equilibrium condition in two unknowns, the average profit $\bar{\pi}$ and the productivity cutoff $\varphi^{*}$. Write down the free entry condition and show graphically the determination of the equilibrium average profit and cutoff $\left(\bar{\pi}, \varphi^{*}\right)$.
iii. Assume now that this country can trade with $n$ symmetric countries. Firms incur in a fixed entry cost into the export market whose per-period portion is $f_{X}$ and an iceberg trade cost $\tau>1$. Recall that the free entry condition will be exactly as in the closed economy, hence you only need to focus on
the ZCP. The ZCP in open economy is

$$
\begin{aligned}
\bar{\pi} & =\pi_{d}(\widetilde{\varphi})+p_{x} n \pi_{x}\left(\widetilde{\varphi}_{x}\right) \\
& =\left[\left(\frac{\widetilde{\varphi}\left(\varphi^{*}\right)}{\varphi^{*}}\right)^{\sigma-1}-1\right] f+p_{x} n\left[\left(\frac{\widetilde{\varphi}_{x}\left(\varphi^{*}\right)}{\varphi_{x}^{*}\left(\varphi^{*}\right)}\right)^{\sigma-1}-1\right] f_{x}
\end{aligned}
$$

where

$$
\begin{aligned}
\widetilde{\varphi} & =\left\{\frac{1}{1-G\left(\varphi^{*}\right)} \int_{\varphi^{*}}^{\infty} \varphi^{\sigma-1} g(\varphi) d \varphi\right\}^{\frac{1}{\sigma-1}} \text { and } \\
\widetilde{\varphi}_{x} & =\left\{\frac{1}{1-G\left(\varphi_{x}^{*}\right)} \int_{\varphi_{x}^{*}}^{\infty} \varphi^{\sigma-1} g(\varphi) d \varphi\right\}^{\frac{1}{\sigma-1}}
\end{aligned}
$$

are the average productivities of all firms and of exporters respectively, and where we can show that the exporters cutoff is a function of the domestic cutoff, that is $\varphi_{x}^{*}=\varphi^{*} \tau\left(\frac{f_{x}}{f}\right)^{\frac{1}{\sigma-1}}$. The probability of exporting is $p_{x}=$ $\left(1-G\left(\varphi_{x}^{*}\right)\right) / 1-G\left(\varphi^{*}\right)$. Using the free entry condition, show the equilibrium graphically and compare the cutoff under $\varphi^{*}$ autarky with that under trade and discuss the differences.
iv. Provide a brief discussion of the effect on the productivity cutoffs for domestic producers $\varphi^{*}$ and for exporters $\varphi_{X}^{*}$ of a reduction in the iceberg trade $\operatorname{cost} \tau$. Discuss the welfare effects associated with such trade liberalization scenario and compare them with the welfare gains in the Krugman model discussed in point (iv) of the previous question. Are there any losers from trade liberalization? Focus only on the intuition, you do not need to derive the effects formally.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN FINANCE FOR THE DEGREE OF MASTER OF PHILOSOPHY

Friday 11 May $2012 \quad 10.00$ to 12.00
Module 300
ECONOMETRIC METHODS
Answer all questions in Section A and two questions from Section B. Part A is $60 \%$ of the total marks.

You are permitted to use your own calculator where it has been stamped as approved by the University.
New Cambridge Elementary Statistical Tables, standard graph paper, Durbin-Watson and Dickey-Fuller Tables are provided.
Credit will be given for clear presentation of relevant statistics.
Write your number not your name on the cover sheet of each booklet
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Rough work pads

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## Section A

A. 1 Let $y_{1}, y_{2}, \ldots, y_{n}$ be a random sample from the normal distribution with mean zero, variance $\theta$, and density

$$
p(y)=\frac{1}{\sqrt{2 \pi \theta}} \exp \left\{-\frac{1}{2} \frac{y^{2}}{\theta}\right\} .
$$

Find the maximum likelihood estimator of $\theta$ and its asymptotic variance.
A. 2 Consider the following AR model

$$
y_{t}=\rho y_{t-1}+\varepsilon_{t},
$$

where the white noise $\varepsilon_{t}$ has mean 0 and variance 1 . Suppose that the process described by this model has the following $\mathrm{MA}(\infty)$ representation:

$$
y_{t}=\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^{k} \varepsilon_{t-k} .
$$

a) What is the value of $\rho$ ?
b) Compute the mean squared error of the optimal forecast of $y_{T+3}$ given $y_{T}$.
A. 3 Consider the following regression model

$$
\begin{equation*}
y_{i}=\beta x_{i}+u_{i}, i=1, \ldots, n, \tag{}
\end{equation*}
$$

where $x_{i}$ is a dummy variable taking on value 1 with probability $p>0$ and value 0 with probability $1-p, E\left(u_{i} \mid x_{i}\right)=0$, and $E\left(u_{i}^{2} \mid x_{i}\right)=1+3 x_{i}$. Suppose that a random sample $\left(y_{i}, x_{i}\right), i=1, \ldots, n$ is available.
a) Transform model ( ${ }^{*}$ ) to a homoskedastic model.
b) Find the asymptotic variance of the generalized least squares (GLS) estimator of $\beta$.
A. 4 Consider the following regression for earnings of 428 randomly sampled married women participating in the labour force

$$
\ln \text { earnings }=\beta_{1}+\beta_{2} \text { age }+\beta_{3} \text { age }^{2}+\beta_{4} \text { education }+\beta_{5} \text { kids }+\varepsilon
$$

where "kids" is a binary variable which equals one if there are children under 18 in the household. Assume that the conditional distribution of $\varepsilon$ given the explanatory variables is normal with mean zero and variance $\sigma^{2}$. Explain how would you run a Lagrange Multiplier test of the null hypothesis that $\beta_{3}=0$.
A. 5 The random process $\left\{y_{t}, t=1,2, \ldots\right\}$ is generated by the following equation

$$
y_{t}=\rho y_{t-1}+\varepsilon_{t},
$$

where $y_{0}=0$ and $\varepsilon_{t}, t=1,2, \ldots$ are i.i.d. normal random variables with mean zero and constant positive variance. The predicted values $\hat{y}_{t}$ from the OLS regression of $y_{t}, t=1, \ldots, T$, on $y_{t-1}$ are

$$
\hat{y}_{t}=\underset{(0.03)}{0.9} y_{t-1}
$$

where the standard error is given in the parentheses, and the sample size $T=250$.
a) Test the null hypothesis that $\rho=1$ against the alternative of stationarity at $5 \%$ significance level.
b) How would you test the null hypothesis $\rho_{1}+\rho_{2}=1$ against the alternative of stationarity if the process $\left\{y_{t}, t=1,2, \ldots\right\}$ were generated by equation $y_{t}=\rho_{1} y_{t-1}+\rho_{2} y_{t-2}+\varepsilon_{t}$ ?
A. 6 You are given the following autoregressive distributed lag model

$$
\begin{equation*}
y_{t}=2+\frac{1}{2} y_{t-1}+x_{t}-\frac{1}{3} x_{t-1}+\varepsilon_{t}, t=\ldots,-1,0,1, \ldots \tag{**}
\end{equation*}
$$

where $E\left(\varepsilon_{t} \mid \mathcal{F}_{t-1}\right)=0$ and $\mathcal{F}_{t-1}=\left\{\ldots,\left(y_{t-2}, x_{t-2}\right),\left(y_{t-1}, x_{t-1}\right)\right\}$.
a) What is the value of the long-run effect on $y$ of a permanent increase in $x$ by one unit?
b) What is the value of the mean lag in the distributed lag representation, $y_{t}=$ $\alpha+\sum_{i=0}^{\infty} \beta_{i} x_{t-i}$, of model ( $\left.{ }^{* *}\right)$ ?

## Section B

B. 1 Consider the linear regression model

$$
y_{i}=\alpha+\beta x_{i}+u_{i}, i=1, \ldots, n,
$$

where $\left(y_{i}, x_{i}\right)$ with $i=1, \ldots, n$ are i.i.d., $\mathrm{E}\left(x_{i}\right)=1, \mathrm{E}\left(x_{i}^{2}\right)=2, \mathrm{E}\left(u_{i} \mid x_{i}\right)=0$, and $\mathrm{E}\left(u_{i}^{2} \mid x_{i}\right)=1$. Assume that the fourth moments of $y_{i}$ and $x_{i}$ exist and are finite. Let $\bar{y}$ be the sample average of $y_{i}$, that is $\bar{y}=\frac{1}{n} \sum_{i=1}^{n} y_{i}$.
a) Define the concept of convergence in probability.
b) Derive the probability limit of $\frac{1}{n} \sum_{i=1}^{n}\left(y_{i}-\bar{y}\right)^{2}$ as $n \rightarrow \infty$.
c) Prove that, as $n$ goes to infinity, the coefficient of determination $R^{2}$ converges in probability to $\frac{\beta^{2}}{1+\beta^{2}}$.
d) Suppose that the OLS estimate $\hat{\alpha}$ of the intercept, $\alpha$, equals 2 . Construct the asymptotic $95 \%$ confidence interval for $\alpha$.
B. 2 Let $y$ be a random variable with Bernoulli distribution. That is,

$$
y= \begin{cases}1 & \text { with probability } \theta \\ 0 & \text { with probability } 1-\theta\end{cases}
$$

Suppose that you observe a random sample $y_{1}, \ldots, y_{n}$ from the above distribution. Let $n_{1}$ and $n_{0}$ be the number of the observations equal to 1 and 0 respectively.
a) Write down the log likelihood function and find the maximum likelihood estimator of $\theta$.
b) Find the Cramér-Rao lower bound as a function of $n$ and $\theta$.
c) Find expressions (in terms of $n, n_{1}$ and $n_{0}$ ) for the Likelihood Ratio statistic, Wald statistic, and Lagrange Multiplier statistic for testing the null hypothesis that $\theta=\frac{1}{2}$.
d) For $n=23, n_{1}=7$, and $n_{0}=16$, the values of the Likelihood Ratio statistic, Wald statistic, and Lagrange Multiplier statistic from c) are 3.62, 4.16, and 3.52 , respectively. Do the corresponding tests reject the null hypothesis $\theta=\frac{1}{2}$ at $5 \%$ significance level? Comment on the tests' disagreement.
B. 3 Consider an $\mathrm{AR}(1)$ process

$$
x_{t}=\rho x_{t-1}+\varepsilon_{t}, t=\ldots,-1,0,1, \ldots
$$

where $|\rho|<1$, and $\varepsilon_{t}$ is strict white noise with mean zero and variance one. Let

$$
y_{t}=x_{t}+\varepsilon_{t}, t=\ldots,-1,0,1, \ldots
$$

a) Define the concept of strict stationarity.
b) Find the first-order and the second-order autocorrelations of $y_{t}$. That is, find $\operatorname{Corr}\left(y_{t}, y_{t-1}\right)$ and $\operatorname{Corr}\left(y_{t}, y_{t-2}\right)$.
c) A researcher suggests that there might exist white noise $\eta_{t}$ with zero mean and positive variance, and a scalar $\delta$ with $|\delta|<1$ such that $y_{t}$ has an $\operatorname{AR}(1)$ representation $y_{t}=\delta y_{t-1}+\eta_{t}$. Using your results from b ) or otherwise show that $y_{t}$ cannot have such a representation, unless $\rho=0$.
d) Assuming that you only observe $y_{t}, t=1, \ldots, T$, how would you test the null hypothesis that $y_{t}$ is an i.i.d. time series?
B. 4 Discuss TWO of the following topics
a) Estimation of MA(1) models
b) Measurement error
c) Hetroskedasticity-consistent inference

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Monday 28 May $2012 \quad 2.00$ to 4.00
Module 310
TIME SERIES \& FINANCIAL ECONOMETRICS
Candidates are required to answer six questions from Section $A$ and one question from Section B.

Section A is weighted as $60 \%$ and Section B as $40 \%$.
Write your number not your name on the cover sheet of each booklet
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Rough work pads
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

A. 1 Consider an unobserved components model in which the observation at time $t$ is the sum of an $\operatorname{AR}(1)$ process, an MA(1) process and white noise. Put the model in state space form. How would you inititialize the Kalman filter? What are the values of $p$ and $q$ in the $\operatorname{ARMA}(p, q)$ reduced form?
A. 2 Describe the portmanteau test for serial correlation. What is the problem with using it for testing that the returns in a typical financial time series are serially uncorrelated? How would you overcome the difficulty?
A. 3 Show that the seasonally differenced series $\Delta_{s} y_{t}$ is stationary, but not invertible, when $y_{t}$ is a random walk. Derive an expression for its spectrum when $s=2$, sketch the spectrum and comment on its form.
A. 4 Write down a model for a stochastic cycle. How does a cyclical trend model differ from a trend plus cycle model? Show how a cyclical trend model can be put in state space form.
A. 5 Write down the exact log-likelihood function of a stationary first order autoregressive process that is observed at times $t=1,2$ and 4 .
A. 6 The logarithm of the range, $R_{t}$, of a stock price or index over a day is sometimes assumed to be normal. If this is the case, explain how you would use the range to estimate changing volatility, $h_{t}$, when the equation linking range and volatility is

$$
R_{t}=\varepsilon_{t} \exp \left(h_{t}\right), \quad t=1, \ldots, T
$$

What is the distribution of $\varepsilon_{t}$ ?
A. 7 What is a band-pass filter? Are such filters useful in economics?
A. 8 Consider the VARMA(1,1) model

$$
\mathbf{y}_{t}=\phi \mathbf{y}_{t-1}+\boldsymbol{\xi}_{t}+\Theta \boldsymbol{\xi}_{t-1}, \quad t=1, \ldots, T
$$

where $\mathbf{y}_{t}$ is an $N \times 1$ vector of observations and $\boldsymbol{\xi}_{t}$ is an $N \times 1$ multivariate white noise process with zero mean vector and covariance matrix $\boldsymbol{\Sigma}$. What restrictions, if any, are needed on the $N \times N$ parameter matrices, $\phi$ and $\Theta$, in order for $\mathbf{y}_{t}$ to be covariance stationary? Derive an expression for the lag one autocovariance matrix, $\boldsymbol{\Gamma}(1)$, in terms of $\boldsymbol{\phi}, \Theta, \boldsymbol{\Sigma}$ and $\boldsymbol{\Gamma}(0)$, the covariance matrix of $\mathbf{y}_{t}$. Given $\boldsymbol{\Gamma}(1)$, show how the remaining autocovariance matrices can be computed recursively.

## SECTION B

B. 1 (a). Two random variables are connected by a Clayton copula

$$
C\left(u_{1}, u_{2}\right)=\left(u_{1}^{-\theta}+u_{2}^{-\theta}-1\right)^{-1 / \theta}, \quad \theta \in[-1, \infty)
$$

(i). Write down the probability that an observation in series one is less than the $\tau_{1}$-quantile given that the observation in the second series is known to be below the $\tau_{1}$-quantile. Hence derive an expression for lower tail dependency, $\lambda_{L}$, and evaluate it when $\theta=1$.
(ii). Explain how Blomqvist's beta measures association between two variables. What is the value of Blomqvist's beta for a Clayton copula with $\theta=5$ ?
(iii). Write down the survival function for the Clayton copula. What is its connection with upper tail dependency?
(iv). Suppose that the marginal distributions of the two random variables, $Y_{1}$ and $Y_{2}$, are both exponential, that is the $p d f$ is $\alpha_{i}^{-1} \exp \left(-y / \alpha_{i}\right)$, $i=1,2$, and that they are connected by a Clayton copula. Write down the joint distribution function of $Y_{1}$ and $Y_{2}$.
(b). Gumbel's bivariate exponential distribution is
$F\left(y_{1}, y_{2}\right)=1-\exp \left(-y_{1}\right)-\exp \left(-y_{2}\right)+\exp \left(-y_{1}-y_{2}-\theta y_{1} y_{2}\right), \quad 0 \leq \theta \leq 1$.
The marginals are exponential. Use the formula for $F^{-1}(u)$ for an exponential to write down an expression for the Gumbel copula, $C\left(u_{1}, u_{2}\right)$.
B. 2 Consider the unobserved component model

$$
\begin{aligned}
y_{t} & =\mu_{t}+\lambda w_{t}+\varepsilon_{t}, \quad t=1, \ldots, T, \\
\mu_{t} & =\mu_{t-1}+\eta_{t},
\end{aligned}
$$

where $\left\{\varepsilon_{t}\right\}$ and $\left\{\eta_{t}\right\}$ are mutually independent normally distributed white noise disturbances with variances $\sigma_{\varepsilon}^{2}$ and $\sigma_{\eta}^{2}$ respectively, and $w_{t}$ is an intervention variable

$$
w_{t}= \begin{cases}0 & \text { if } t<\tau \\ 1 & \text { if } t \geq \tau .\end{cases}
$$

(i). Briefly explain the role of the intervention variable.
(ii). Suppose that $\lambda$ is estimated by regressing $\Delta y_{t}$ on $\Delta w_{t}$, where $\Delta=1-L$ is the first difference operator. Explaining the reasoning behind your answers, determine whether this estimator, denoted $\hat{\lambda}$, is (a) unbiased, and (b) consistent. How would you obtain an efficient estimator of $\lambda$ ?
(iii). Explain how data on a control group could be used to obtain a more efficient estimator of $\lambda$. What is the significance of co-integration in this context? Does it have any connection with consistency?
(iv). Briefly describe an application where a control group is used.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Tuesday 5 June $2012 \quad 2.00-4.00$
Module 320
CROSS SECTION AND PANEL DATA ECONOMETRICS
Candidates are required to answer three questions from Section $A$ and one question from Section B.

Section A is weighted as $60 \%$ and Section B as $40 \%$.
Write your number not your name on the cover sheet of each booklet

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
| :--- | :--- |
| 20 Page booklet $x 2$ | Durbin-Watson and Dickey-Fuller Tables |
| Rough work pads | New Cambridge Elementary Statistical Tables |
| Tags | Approved calculators allowed |

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## SECTION A

A1 Briefly explain what you understand by the following.
a) identification versus weak identification tests in linear regression models
b) strict exogeneity assumption in linear unobserved effects panel data models
c) an error components versus a random parameters specification for the mixed logit model

A2 Consider the following linear unobserved effects dynamic panel data model

$$
\begin{equation*}
y_{i t}=\alpha y_{i t-1}+\mathbf{x}_{i t}^{\prime} \boldsymbol{\beta}+u_{i t} \quad i=1, \ldots, N \quad t=1, \ldots, T, \tag{1}
\end{equation*}
$$

a) Discuss the properties of the OLS estimator under the following assumptions:
i) $u_{i t}=\eta_{i}+v_{i t} \quad$ where $v_{i t}$ is $i . i . d . \quad N\left(0, \sigma^{2}\right)$ across $i$ and $t . \eta_{i}$, $i=1, \ldots, N$, are fixed parameters.
ii) $u_{i t}=\eta+v_{i t} \quad$ where $v_{i t}$ is i.i.d. $N\left(0, \sigma^{2}\right)$ across $i$ and $t . \eta$ is a fixed parameter
Under (i) discuss how unbiased estimates of the parameters may be obtained.
b) Given $u_{i t}=\eta_{i}+v_{i t}$ and the assumption

$$
\begin{equation*}
E\left(v_{i t} \mid \mathbf{x}_{i 1}, \ldots, \mathbf{x}_{i T}, \eta_{i}\right)=0 \quad(t=1, \ldots, T) \tag{2}
\end{equation*}
$$

write down a set of moment conditions that, subject to a rank condition, will ensure identification given the endogeneity of lagged $y$ in (1) and serial correlation in $v_{i t}$ of unspecified form.

A3 Consider the following linear regression

$$
\begin{aligned}
y_{i} & =\rho S_{i}+\eta_{i} \\
S_{i} & =\pi z_{i}+\varepsilon_{i}
\end{aligned}
$$

where $y_{i}, S_{i}$, denote, respectively, wages and a measure of schooling for the $i^{t h}$ individual. $\eta_{i}$ and $\varepsilon_{i}$ are stochastic errors, and $\operatorname{Cov}\left(S_{i}, \eta_{i}\right) \neq 0$. $z_{i}$ is a scalar instrument where $\operatorname{Cov}\left(z_{i}, \eta_{i}\right)=0 . \rho$ and $\pi$ are unknown parameters.
a) In the case where $z_{i}$ is a binary instrument, show that the IV estimator for $\rho$ is

$$
\rho=\frac{E\left(y_{i} \mid z_{i}=1\right)-E\left(y_{i} \mid z_{i}=0\right)}{E\left(S_{i} \mid z_{i}=1\right)-E\left(S_{i} \mid z_{i}=0\right)}
$$

b) Table 1 presents estimates of the effects of education on the logarithm of men's weekly earnings. Standard errors are given in parenthesis.
Column 1 presents results for the OLS estimator using different age controls. Given the presumed endogeneity of education, columns 2-4 present parameter estimates for a number of specifications based upon the use of instrumental variables (IV), with differing degrees of identification.
The F statistic $\left(F_{E}\right)$ for the test of the joint statistical significance of the excluded instruments is reported.
Provide a succinct summary of the main results.
A4 Consider the following representation of utility for choice $j$ and individual $i$

$$
\begin{equation*}
U_{i j}=\omega^{\prime} \mathbf{v}_{j}+\mu_{i}^{\prime} \mathbf{z}_{j}+\varepsilon_{i j}, \quad i=1, \ldots, n, \quad j=1, \ldots, J \tag{3}
\end{equation*}
$$

where $\boldsymbol{\omega}$ denotes a vector of unknown parameters, $\mathbf{v}_{j}$ and $\mathbf{z}_{j}$ are vectors of observed data for alternative $j, \mu_{i}$ is a vector of stochastic terms with zero mean, and $\varepsilon_{i j}$ is a stochastic term which is distributed iid Type 1 extreme value.
a) Derive an expression for the correlation of utility over any two alternatives, say $j$ and $l$. What restriction imposed on (3) delivers the standard logit model?
b) The model in (3) is often referred to as an error components discrete choice model. Show that the error component and random coefficients specifications are formally equivalent.

A5 Denote by $U_{i j}$ the unobserved utility perceived by individual $i$ who chooses alternative $j$. This utility may be modelled as follows:

$$
\begin{equation*}
U_{i j}=\mathbf{x}_{i j}^{\prime} \boldsymbol{\beta}_{i}+\epsilon_{i j} \tag{4}
\end{equation*}
$$

where $i=1, \ldots, n$, and $j=1, \ldots, J$, index, respectively, individuals and alternatives. $\mathbf{x}_{i j}$ is a k-dimensional vector of explanatory variables, $\boldsymbol{\beta}_{i}$ is a k -dimensional parameter vector and $\epsilon_{i j}$ is a random shock known to individual $i$.

Table 1: Estimated Effect of Completed Years of Education on Men's Log Weekly Earnings


Calculated from the 5\% Public-Use Sample of the 1980 US Census for men born 1930-1938. Sample size is 329,509 . All specifications include Race ( $1=$ black), SMSA ( $1=$ central city), Married ( $1=$ married, living with spouse), and 8 Regional dummies as control variables. Column 4 also includes 50 state dummies as controls.

SMSA denotes Standard Metropolitan Statistical Area.
a) Assume that the $\epsilon_{i j}$ are distributed type 1 extreme value, that $\boldsymbol{\beta}_{i}=\boldsymbol{\beta} \quad \forall i$, and that the choice set comprises three kinds of vehicles: large gas cars (lgc), small gas cars (sgc), and small electric cars (sec). The probability that a given household will choose each of these is, respectively, $P_{\mathrm{lgc}}=0.66, P_{\mathrm{sgc}}=0.33$, and $P_{\text {sec }}=0.01$.
Suppose a subsidy for electric cars is introduced and that this subsidy raises $P_{\text {sec }}$ from 0.01 to 0.10 . Given the error assumptions, what are the probabilities for $\operatorname{lgc}$ and sgc after the subsidy has been introduced. To what extent are these probabilities unexpected?
b) We now rewrite (4) with $\boldsymbol{\beta}_{i}$ decomposed into a mean and deviation from its mean

$$
\begin{equation*}
U_{i j}=\mathbf{x}_{i j}^{\prime} \overline{\boldsymbol{\beta}}+\mathbf{x}_{i j}^{\prime} \tilde{\boldsymbol{\beta}}_{i}+\epsilon_{i j}, \tag{5}
\end{equation*}
$$

where $\boldsymbol{\beta}_{i} \sim N\left(\overline{\boldsymbol{\beta}}, \mathbf{W}_{\beta}\right)$. We let $\eta_{i j}=\mathbf{x}_{i j}^{\prime} \tilde{\boldsymbol{\beta}}_{i}+\epsilon_{i j}$.
Assume that there are two choices, that that there is a single covariate $x_{i j}$ and that the errors, $\epsilon_{i 1}, \epsilon_{i 2}$ are distributed independent and identically normal, with variance $\sigma_{\epsilon}$. Show that the covariance matrix of the errors, $\operatorname{Cov}\left(\eta_{i 1}, \eta_{i 2}\right)$, is given by

$$
\Omega_{\eta}=\sigma_{\beta}\left(\begin{array}{ll}
x_{i 1}^{2} & x_{i 1} x_{i 2} \\
x_{i 1} x_{i 2} & x_{i 2}^{2}
\end{array}\right)+\sigma_{\epsilon}\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right) .
$$

where $\sigma_{\beta}$ denotes the variance of $\beta_{i}$.
Are all parameters in $\Omega_{\eta}$ identified? Explain your answer.

## SECTION B

B. 1 The relationship between female labour force participation $(y)$ and fertility $\left(d_{1}\right)$ is given by

$$
y_{i t}=\mathbf{1}\left(\alpha+\delta_{1} d_{1, i t}+\delta_{2} d_{2, i t}+\mathbf{x}_{i}^{\prime} \boldsymbol{\beta}+\xi_{i t}>0\right)
$$

where $1($.$) denotes the indicator function. y_{i t}$ is a binary indicator of participation in year $t$ (defined as positive annual hours of work), fertility $\left(d_{1, i t}\right)$ is a dummy variable which takes the value one if the age of the youngest child in $t+1$ is $1 . d_{2, i t}$ is a dummy indicator equal to one if the woman has a child between 2 and $6 . \mathbf{x}_{i}$ is vector of additional controls. $\xi_{i t}=\alpha_{i}+u_{i t}$ is a standard composite panel data error term: $\alpha_{i}$ represents an individual-specific effect and $u_{i t}$ denote time varying errors including idiosyncratic labour market shocks.
a) Briefly discuss why fertility is most likely an endogenous variable. Provide justification for the use of an indicator of whether a woman has two children of the same sex as an external instrument for fertility?
b) In Table 2 we report results based on the application of a number of linear estimators. Columns 2, 3, and 4, report, respectively, results based upon the use of an OLS, 2SLS and within-group (WG) estimator. Columns 5 and 6 present results for two firstdifference GMM estimators, that exploit, respectively, strict and weak exogeneity assumptions.
i) For each estimator outline the identification assumptions and in particular discuss how each estimator accounts for the endogeneity of fertility.
ii) In what sense does the use of the WG estimator represent a tradeoff between heterogeneity and bias.
c) Write down the moment equations which underly the results for the GMM-2 estimator.
d) Using the results in columns 4 and 5 test for strict exogeneity. What do you conclude?

Table 2: Female Labour Force Participation
Linear Probability Models ( $\mathrm{N}=1442$, 1986-1989)

| Variable | OLS | 2 SLS $^{1}$ | WG | GMM-1 ${ }^{2}$ | GMM-2 ${ }^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Fertility | $\begin{gathered} -0.15 \\ (8.2) \end{gathered}$ | $\begin{gathered} -1.01 \\ (2.1) \end{gathered}$ | $\begin{gathered} -0.06 \\ (3.8) \end{gathered}$ | $\begin{gathered} -0.08 \\ (2.8) \end{gathered}$ | $\begin{gathered} -0.13 \\ (2.2) \end{gathered}$ |
| Kids 2-6 | $\begin{gathered} -0.08 \\ (5.2) \end{gathered}$ | $\begin{gathered} -0.24 \\ (2.6) \end{gathered}$ | $\begin{aligned} & 0.001 \\ & (0.04) \end{aligned}$ | $\begin{gathered} -0.005 \\ (0.4) \end{gathered}$ | $\begin{gathered} -0.09 \\ (2.7) \end{gathered}$ |
| Sargan (d.f.) |  |  |  | 48. <br> (22) | $\begin{gathered} 18 . \\ (10) \end{gathered}$ |

## Notes

i) Heteroskedasticity robust $t$-ratios shown in parentheses
ii) ${ }^{1}$ External instrument: whether a woman has two children of the same sex.
iii) The variable Kids2-6 is equal to one if a woman has a child between 2 and 6 . iv) ${ }^{2}$ Treats fertility as endogenous and uses all lags and leads of Kids 2-6 and the external instrument same sex.
v) ${ }^{3}$ Treats fertility as endogenous and uses lags of Kids 2-6 and same sex up to t-1.
vi) Estimators which do not account for individual effects also include a constant, age, race, and education dummies (not reported).
vii) Sargan denotes the Sargan test statistic for overidentification.

The sample consists of 1442 women aged between 18-55 in 1986.
B.2. a) Consider the following utility model for individual $i$, choice $j$

$$
\begin{equation*}
U_{i j}=\mathbf{v}_{i j}^{\prime} \boldsymbol{\omega}_{i}+\varepsilon_{i j}, \tag{6}
\end{equation*}
$$

where $\mathbf{v}_{i j}$ is an $L \times 1$ parameter vector, $\boldsymbol{\omega}_{i}$ is $L \times 1$ parameter, and $\varepsilon_{i j}$ is distributed type 1 extreme value. We write the distribution of $\boldsymbol{\omega}_{i}$ as

$$
\begin{align*}
\boldsymbol{\omega}_{i} & \sim g(\boldsymbol{\omega} \mid \boldsymbol{\eta})  \tag{7}\\
\eta & =\left(\overline{\boldsymbol{\omega}}, \boldsymbol{\Sigma}_{\boldsymbol{\omega}}\right),
\end{align*}
$$

where $\bar{\omega}$ denotes an $L \times 1$ vector of mean parameters, and $\Sigma_{\omega}$ is a $L \times L$ matrix of covariance parameters.
i. In what ways does the specification in (6) and (7) represent an improvement over standard logit.
ii. Write down the likelihood component for individual $i$ alternative $j$

1) conditional upon $\omega_{i}$
2) accounting for that fact that for each individual $\boldsymbol{\omega}_{i}$ is not known.
iii. Evaluate the statement that relative to the multinomial probit model, the dimensionality problem for mixed logit is invariant to the size of the choice set.
b) i. Write down a specific form of $\boldsymbol{\Sigma}_{\omega}$ that delivers the standard logit model.
ii. The Log-Likelihood statistics based upon two models are:
standard logit -4959
mixed logit -3619
In the case where $\widehat{\boldsymbol{\Sigma}}_{\widehat{\omega}}$ is a $5 \times 5$ diagonal matrix, evaluate whether the data supports the standard logit model.
c) Briefly contrast the use of data augmentation when evaluating the joint posterior distribution for the multinomial probit and the mixed logit models.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Monday 21 May $2012 \quad 10.00$ to 12.00
Module 330
APPLIED ECONOMETRICS
Candidates are required to answer four compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
| :--- | :--- |
| 20 Page booklet x 2 | Durbin-Watson and Dickey-Fuller Tables |
| Rough work pads | New Cambridge Elementary Statistical Tables |
| Tags | Engle-Yoo Cointegration Tables |
|  | Approved calculators allowed |

> | You may not start to read the questions printed on |
| :--- |
| the subsequent pages of this question paper until |
| instructed that you may do so by the Invigilator |

## Question 1

To investigate whether the slope of the yield curve may have predictive power for future inflation an econometrician obtains monthly data on the US price level, one year and five year nominal interest rates. The following variables are defined

$$
\begin{aligned}
\text { inflation }_{t} & =100 \ln \left(\frac{\text { pcex }_{t+12}}{\text { pcex }_{t}}\right) \\
\text { inflation }_{t} & =\text { inflation }_{t}-\text { inflation }_{t-12} \\
\text { slope }_{t} & =R 5_{t}-R 1_{t}
\end{aligned}
$$

where inflation is the future 12 month change in the US personal consumption expenditure excluding food and energy price index (pcex), and slope is the difference between the 5 year ( $R 5$ ) and one year ( $R 1$ ) US Treasury constant maturity nominal interest rates. inflation $_{t}$ is then the difference between last 12 months and next 12 months inflation rate (note that at any time $t$ both last 12 months inflation rate and $R 5_{t}$ and $R 1_{t}$ are all known).

The following regression results are obtained over the sample period $1964 \mathrm{M} 2-$ 2010M2. Standard errors are in parentheses throughout.

$$
\begin{gathered}
\Delta \text { inflation }_{t}=\underset{(0.056)}{0.2320}-\underset{(0.053)}{0.3666 \mathrm{SLOPE}_{t}} \\
T=553 \quad \vec{R}^{2}=0.079
\end{gathered}
$$

She concludes that there is strong evidence that the slope of the yield curve is able to predict how the future 12 month inflation rate will differ from the last 12 months inflation rate.
(a) To what extent do these results support this conclusion?

Worried about structural stability she obtains the CUSUM and CUSUMsquares graphs for this regression.

(b) How would you interpret these graphs? What do they tell you about the underlying regression?
(c) Having examined these graphs she decides there is evidence of structural instability and calculates a Chow test for structural stability with break point 1985M2. What problems are there with this procedure?

She then reestimates the regression from 1985M2-2010M2 by OLS and obtains

$$
\begin{gathered}
\text { inflation }_{t}=\underset{(0.043)}{-0.0246}-\underset{(0.036)}{0.0768} \mathrm{SLOPE}_{t} \\
T=301 \quad \bar{R}^{2}=0.012 \quad D W=0.26
\end{gathered}
$$

where DW is the Durbin-Watson statistic. Worried about serial correlation she then uses the Cochrane-Orcutt technique and obtains

$$
\begin{aligned}
\Delta \text { inflation }_{t} & =-\underset{(0.124)}{0.1366}+\underset{(0.068)}{0.0436} \mathrm{SLOPE}_{t} \\
T=300 \quad \bar{R}^{2} & =0.76 \quad D W=2.03 \quad \hat{\rho}=0.88
\end{aligned}
$$

(Statistics based on $\rho$-differenced data)
where $\hat{\rho}$ is the estimated first order autocorrelation of the OLS residuals
(d) Explain how you would interpret these OLS and Cochrane-Orcutt results.
(e) What do you conclude about the ability of the term structure to forecast inflation over this period?

## Question 2

(a) Show that for a set of $n$ observations $y_{1}, \ldots, y_{n}$ the solution to the minimisation problem

$$
\min _{\xi} \sum_{i=1}^{n} \rho_{\tau}\left(y_{i}-\xi\right)
$$

where $\rho_{\tau}($.$) is the function$

$$
\rho_{\tau}(z)= \begin{cases}z \tau & \text { if } z \geq 0 \\ z(\tau-1) & \text { if } z<0\end{cases}
$$

finds the $\tau^{\text {th }}$ quantile of the data.
(b) Explain how this can be used to model the relationship between a set of independent variables $x$ and a dependent variable $y$ at different quantiles of the dependent variable $y$. What are the advantages or disadvantages of quantile regression relative to ordinary least squares?
(c) An alternative strategy is to segment the response variable $y$ into quantiles according to its unconditional distribution and then doing an ordinary least squares regression within each of these subsets. Explain why this would not give unbiased estimates of the relationship between $y$ and $x$.

## Question 3

(a) Explain the importance of the assumptions of conditional independence and common support in estimating treatment effects.
(b) What is a matching estimator? Discuss carefully, the differences between the OLS and matching estimator.
(c) Discuss the role of the variables used to construct the matching estimator, distinguishing between variables that affect selection into the programme and the outcome of interest. Would it be best to have as many variables as possible? Explain.

## Question 4

(a) We are interested in estimating the effect of government-sponsored training on earnings. We have a sample of trainees who started their training in the first 3 months of 1964. Their earnings were tracked both prior, during and after training. A random sample of the working population was used as a comparison group. The average earnings for males for the two groups in the years 1959-69 are shown in the following Figure. Explain why a simple estimate of the difference in earnings between the treatment group (trainees) and comparison group in 1965 might not not yield a reliable estimate of the returns to training. Would a difference-in-difference estimator, using data pre- and post-training (based on the years 1963 and 1965 do better? Explain.

(b) Most children in India are in school but surveys suggest that $44 \%$ of children, $7-12$ years old, cannot read a basic paragraph. A remedial education programme hired young women to teach students lagging behind in basic literacy and numeracy skills. A sample of 100 schools was selected and remedial education was provided to grade 3 students in half the schools in the first year
and to grade 4 students in the remainder of the schools in the second year, so that all schools received the treatment at the end of the experiment. The experimental design is described as follows:

Treatment: 2 hours per day extra class for children in a grade who do not master basic skills, by assigning a helper (teacher) to that grade.

Year 1: Half schools given remedial education in grade 3, other half in grade 4.

Year 2: First half get follow up remedial help, second half schools get remedial education in grade 3

Year 3: No treatment, follow up

Given this design, in each year, children in grade three in schools that received the program for grade four form the comparison group for children that receive the program for grade three, and vice versa. The main outcome of interest is whether the interventions resulted in any improvement in learning levels. Learning was measured using annual pre-tests, given during the first few weeks of the school year, and post-tests, given at the end of the term. Table 1 presents the results of the pre- and post-tests for Treatment (Treat) and Comparison (Comp) groups over the two years, with results for Grades 3 and 4 pooled together since they do not differ very much. All scores are normalised relative to the distribution of the pretest score in the comparison group in each grade, and year.

TABLE 1

| Test Score Summary |  | Statistics (Grades 3 \& 4) <br> Pre-Test (standard error) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Post-Test (standard error) |
| Year 1 | Maths |  |  |  | $\begin{aligned} & \text { Treat } \\ & -0.01 \end{aligned}$ | $\begin{aligned} & \text { Comp } \\ & 0.00 \end{aligned}$ | Difference | $\begin{aligned} & \text { Treat } \\ & 0.35 \end{aligned}$ | $\begin{aligned} & \text { Comp } \\ & 0.17 \end{aligned}$ | Difference |
|  |  | -0.01 | 0.18 |  |  |  |  |
|  |  | (0.05) | (0.07) |  |  |  |  |
|  | Language | 0.02 | 0.00 | 0.02 | 0.79 | 0.67 | 0.13 |  |  |
|  |  |  |  | (0.06) |  |  | (0.07) |  |  |
| Year 2 |  |  |  |  |  |  |  |  |  |
|  | Maths | 0.05 | 0.00 | 0.05 | 1.45 | 1.04 | 0.40 |  |  |
|  |  |  |  | (0.05) |  |  | (0.08) |  |  |
|  | Language | 0.06 | 0.00 | 0.06 | 1.08 | 0.79 | 0.29 |  |  |
|  |  |  |  | (0.06) |  |  | (0.06) |  |  |

(i) Discuss briefly the results in Table 1. What does it aim to demonstrate?

Table 2 presents the results, for various years and grades from a specification which regresses the change in a student's test score (post-test score minus pretest score) on the treatment status of the child's school-grade, controlling for the pre-test score of child $i$ in grade $g$ and school $j$, where $D_{j g}$ is a dummy taking the value 1 if the school received a helper in the child's grade $g$, and 0 otherwise:

$$
Y_{i g j, P O S T}-Y_{i g j, P R E}=\lambda+\delta D_{j g}+\theta Y_{i g j, P R E}+\varepsilon_{i g j, P O S T}
$$

TABLE 2

|  |  | No. of Obs | Math | Language | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Grade 3 | Year 1 | 4230 | 0.179 | 0.102 | 0.152 |
|  |  | $(0.086)$ | $(0.085)$ | $0.005)$ |  |
|  | Year 2 | 5819 | 0.418 | 0.233 | 0.354 |
|  |  | $(0.107)$ | $(0.089)$ | $(0.100)$ |  |
| Grade 4 | Year 1 | 4196 | 0.190 | 0.114 | 0.166 |
|  |  |  | $(0.072)$ | $(0.076)$ | $(0.073)$ |
|  | Year 2 | 6131 | 0.307 | 0.240 | 0.289 |
|  |  |  | $(0.078)$ | $(0.068)$ | $(0.074))$ |

(ii) What does the specification above capture? Explain using the results in Table 2 above whether the programme was successful?
(iii) What are the potential sources of bias in this study? Explain, with suggestions for the kind of information that would be required, how these could be addressed.
(iv) You are asked whether such a programme has demonstrated internal validity and has external validity. What is the evidence you would need in order to answer these questions?

## END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN FINANCE FOR THE DEGREE OF MASTER OF PHILOSOPHY
Monday 4 June $2012 \quad 2.00$ to 4.00

Module 400
ASSET PRICING
Candidates are required to answer four compulsory questions.
Section A: two compulsory questions; Section B: two compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Approved calculators allowed
Rough work pads
Tags

## Section A

1. (a) What does it mean to say that a utility function $u$ has constant relative risk aversion?
(b) What functional forms can $u$ take?
(c) Explain carefully how to derive your answer to part (b) from your answer to part (a).
2. (a) Suppose that an investor lives for two periods. She has utility function $u$ and initial wealth $w_{0}>0$. In the first period, she can either consume her wealth or invest it in a risky asset, the gross return on which is given by the random variable $\widetilde{R}_{1}>0$. In the second period, she consumes the proceeds of her first-period investment. Her discount factor is $\delta$. Write out the investor's optimization problem carefully, and find the first-order condition for her optimal investment.
(b) Now suppose that the original risky asset is replaced by a new risky asset, the gross return on which is given by the random variable $\widetilde{R}_{2}=\widetilde{R}_{1}+\varepsilon$, where $E\left[\varepsilon \mid \widetilde{R}_{1}\right]=0$. Suppose further that $u$ has constant relative risk aversion. Under what conditions on $u$ does the investor's investment in the risky asset increase? Justify your answer carefully.

## Section B

3. There are two assets, a safe asset and a risky asset. The price of the safe asset evolves according to the sde

$$
d B=r B d t
$$

and the price of the risky asset evolves according to the sde

$$
d S=\mu S d t+\sigma S d Z
$$

where $Z$ is a standard Wiener process.
(a) What is the final payoff of a European put option with maturity $T$ written on the risky asset?
(b) Derive a formula for the price of such an option as a function of the current time and the current price of the risky asset.
4. (a) Explain carefully what is meant by the term structure of bond rates and the term structure of forward rates.
(b) Suppose that, at time 0 , the term structure of bond prices and the term structure of forward rates observed on the market are $p_{0}^{*}$ and $f_{0}^{*}$ respectively. What relationship must hold between $p_{0}^{*}$ and $f_{0}^{*}$ ? Why?
(c) Suppose now that the risk-neutral dynamics of the spot rate are given by

$$
d r=(\Theta(t)-a r) d t+\sigma^{2} d Z,
$$

where: $Z$ is a standard Wiener process; $a$ and $\sigma^{2}$ are scalars; and $\Theta$ is a function. Derive a formula for the term structure $p_{t, r}$ of bond prices (at time $t$ when the spot rate is $r$ ) in terms of $p_{0}^{*}$ and $f_{0}^{*}$ (but not $\Theta$ or $\left.\left(f_{0}^{*}\right)^{\prime}\right)$.

END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Monday 4 June $2012 \quad 2.00$ to 4.00
Module 500
DEVELOPMENT ECONOMICS
Answer two questions. Each question is of equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Rough work pads

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. (a) Risk looms large for poor households in developing countries. Describe (using a model) how decisions to invest or produce are affected when households cannot insure against production risk.
(b) It has been observed that, in rural areas in developing countries, the terms of the contract between landlords and tenant farmers are often determined at the same time as the terms of a credit contract between those same parties. Discuss why this might be more likely to happen in the agricultural sector in developing countries than in, say, urban markets in rich countries. What are the implications of asymmetries in bargaining power between tenants and landlords on the contracts that are likely to be observed? It is not necessary to develop a formal model in answer to these questions, if you can convey the arguments intuitively.
2. (a) It is argued that rainfall insurance solves the supply problems affecting agricultural insurance. Is there any justification for this claim in theory?
(b) How would you test the role of knowledge and comprehension in the demand for rainfall insurance? What does the existing evidence say as to whether farmers in developing countries understand rainfall insurance products? Is the evidence consistent with farmers viewing rainfall insurance as a solution against the weather risk they face?
3. (a) Why might governments reward special interests in inefficient ways, even when there exist more efficient ways of transferring resources to those special interests? Explain, being careful to define terms where necessary.
(b) Suppose that the payoffs from two alternative policies being considered in an economy with 100 workers can be described as follows. There are three 'types' of workers in the economy, denoted as type A, B, and C. Of the 100 workers, 30 are of 'type' A, another 30 are of type B, and the remaining 40 workers are of type C. If Policy 1 is implemented, then workers of type A and C will lose $\$ 1$ each, while B-type workers will gain $\$ 2$ each. If Policy 2 is implemented, then workers of type B and type C will lose $\$ 1$ each, while workers of type A will gain $\$ 2$ each. All this information is known to all the workers.

For each question below, please provide a brief explanation, and an intuitive discussion, of your answer.
(i) If Policy 1 is enacted, what will be the net change in social welfare? If voting is costless, and each of the workers knows his or her own type, will Policy 1 win support from a majority of worker-voters? Same questions for Policy 2: if enacted, what will be the net change in social welfare? Will it win majority support, if voting is costless and workers know their own type?
(ii) If each of the 100 workers knows his own type, but the opportunity cost of voting is $\$ 1.50$ for each voter, will Policy 1 win majority support among those who cast votes? Discuss the implications for the political economy of trade policy formation.
(iii) Now suppose that the type of A workers is already known, both to themselves and to all other workers. However, each of the other workers faces individual-specific uncertainty, in the sense that his or her type will not be known until after the policy is implemented. If voting is costless, and each voter maximizes his or her expected payoff, will Policy 1 win majority support? Does this policy suffer from a 'status quo bias'? Be careful to define what you mean by 'status quo bias'.
(iv) Now suppose again that each voter knows their own type. Suppose that the proponents of Policy 1 and Policy 2 join forces and propose a policy bundle which combines Policy 1 and Policy 2. If this policy combination is enacted, what will be the net change in social welfare? If voting is costless, will this policy combination win majority support? Compare your answer with your answer to part (i).

END OF PAPER

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY
Wednesday 30 May $2012 \quad 2.00$ to 4.00

Module 510
POVERTY, ENVIRONMENT, AND SUSTAINABLE DEVELOPMENT
Candidates are required to answer two questions. Each question carries equal weight. Part (i) of each of the two 2-part questions carries $25 \%$ of the marks of the question.
Write your number not your name on the cover sheet of each booklet.

STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. "The idea of establishing markets for externalities so as to eliminate the externalities is a non-starter." Discuss.
2. Assess the thesis that the large difference in fertility rates between subSaharan Africa and Western Europe is attributable to differences between the two continents in the demand for children.
3. (i) What does the shadow price of investment measure?
(ii) What data would you seek if you were asked to estimate the shadow price of investment in health in a poor country?
4. (i) Consumption rates of discount are usually taken to be positive. Why? Under what circumstances would they be negative?
(ii) "The pure rate of time preference is an irrelevance in discussions on whether we should try to ensure that mean global temperature does not increase by more than 2 degrees Celsius from what it is today." Discuss.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Tuesday 5 June $2012 \quad 10.00$ to 12.00
Module M600
HISTORICAL PERSPECTIVES TO FINANCIAL CRISES
Candidates are required to answer two questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2 Approved calculators allowed
Rough work pads
Tags

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1.     - Evaluate the relationship between the main technology-cycles and financial crises using historical evidence.
2.     - Discuss the view that although excess capital inflows are at the root of both the Latin American and the East Asian crises during the 1990s, the main difference in their dynamics is that in the former there was an 'exogenous push' of foreign capital, while in the latter there was an 'endogenous pull' of capital inflows.
3.     - "The main lesson emerging from the genesis of the current world-wide financial crisis is that 'over-liquid' and 'under-regulated' financial markets are unable to self-correct." Discuss.
4.     - 'Given that international financial markets are now characterized by both huge volatilities across all asset and currency markets, and high asset return correlations, developing countries should think again about the need for capital controls'. Discuss.
5.     - Evaluate the economic effects of national and global financial crises drawing on historical evidence from the period of the classical gold standard (18701914) and the interwar period.
6.     - Either:
"Globalization opened up opportunities to find new people to exploit their ignorance. And we found them." (Joseph Stiglitz). Discuss the possible effects of financial globalisation on the quality of financial assets from the point of view of the 'greater-fool' theory.

Or:
"The problem is that [...] the theories embedded in general equilibrium dynamics [...] don't let us think about [issues such as ...] financial crises and their real consequences in Asian and Latin America [...]." (Robert Lucas). Discuss.

EXAMINATION IN ECONOMICS FOR THE DEGREE OF MASTER OF PHILOSOPHY
EXAMINATION IN ECONOMIC RESEARCH FOR THE DEGREE OF MASTER OF PHILOSOPHY

Monday 4 June $2012 \quad 10.00$ to 12.00
Module 620
INSTITUTIONS AND LONG-DISTANCE TRADE
Candidates are required to answer two compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. In the context of long-distance trade in medieval and early modern Europe:
(a) Explain briefly the theoretical reasons for thinking that markets may not have provided the efficient quantity of information to traders.
(b) Present one theoretical model showing how a 'private-order' institution might have been better than markets at ensuring availability of information in longdistance trade.
(c) How well does the model you have presented stand up to confrontation with the empirical findings?
2. Compare and contrast, in the light of the empirical evidence, the theoretical models that have been advanced to characterize the contract-enforcement institutions used by any two of the following groups of merchants:
(a) the medieval Italian merchants
(b) the German Hanseatic merchants
(c) the 'Maghribi traders'
(d) foreign merchants trading in Bruges
(e) merchants trading at the Champagne Fairs

ECM9/ECM10/ECM11/ MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil in Finance
Monday 5 May 2014 2:00pm to 4:00pm

## M100

## Microeconomics I

Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. A profit-maximizing monopolist produces a single good with constant marginal cost $c>0$ and zero fixed cost. Consumers have utility function $\theta \sqrt{q}-t$ where $q$ is quantity consumed, $t$ is total payment and $\theta>0$ is a parameter. The reservation utility of consumers is zero.
(a) Sketch the indifference curves in $q-t$ space of the monopolist and a typical consumer. Derive the optimal contract which the monopolist will offer to the consumer.

Now suppose that there are two types of consumer. A proportion $p$ have $\theta=\theta_{h}$ and the rest have $\theta=\theta_{l}<\theta_{h}$. The value of $\theta$ is private information to the consumer.
(b) Consider a pair of contracts which satisfies the incentive compatibility and participation constraints but both constraints are slack. Sketch the relevant indifference curves for the two contracts. Show by means of diagrams that two of the constraints must bind for the monopolist's optimal pair of contracts.
(c) Hence find the optimal pair of contracts. You may assume that the monopolist sells a strictly positive amount to each type of consumer.
(d) Show that the optimal contract is not Pareto-efficient and briefly account for the form of the inefficiency.
2. Two players play the following game. There are three periods. In each period, supposing the game has not yet ended, the moves are as follows. Player 1 chooses either to exit, in which case the game ends, or not. If player 1 does not exit player 2 then decides whether to exit, in which case the game ends, or not. If a player exits in period $t(t=1,2,3)$ that player's final payoff is $t^{2}$ and the other player's payoff is zero. If neither player ever exits both players get a payoff of 8 .
(a) Draw the extensive form of the game. How many subgames are there in the game? What is the set of strategies for each player?
(b) Which strategies, if any, are strictly dominated? Explain your reasoning.
(c) Describe all subgame-perfect Nash equilibria and explain your reasoning.
(d) Now suppose that at the third stage the first player to move is chosen randomly, each with probability 0.5 . What is the strategy set for each player? Describe all the subgameperfect equilibria of this game and explain your reasoning.
3. Two people bargain over a surplus of size 1 in the following manner. Each simultaneously makes a claim (a number in the unit interval $[0,1]$ ). Let player $i$ 's claim be $x_{i}$ $(i=1,2)$. If $x_{1}+x_{2}>1$ then both players get a payoff of zero. If $x_{1}+x_{2} \leq 1$ then player $i$ 's payoff is $x_{i}(i=1,2)$.
(a) Find all the pure-strategy Nash equilibria of this game.

Now suppose that, with probability $\theta$, player 2 is only able to make a claim of 0.5 ; with probability $1-\theta$ player 2 can choose any $x_{2} \in[0,1]$. Otherwise the game is as described above. Player 1 does not know which type of player 2 she is facing.
(b) Define the term pure-strategy Bayes-Nash Equilibrium in the context of this game.
(c) Suppose that $\theta=0.25$. Let $y$ be the value of $x_{2}$ in the case in which 2 is free to choose $x_{2}$. Show that if $y>0.5$ then 1's best response is $x_{1}=0.5$ for $y \in\left[\frac{7}{8}, 1\right]$ and otherwise $x_{1}=1-y$.
(d) Describe all the pure-strategy Bayes-Nash equilibria for the case $\theta=0.25$.
4. (a) Describe the Independence Axiom in the von Neumann-Morgenstern theory of Choice under Uncertainty.
(b) An individual strictly prefers a certain gain of 200 pounds to a $50 \%$ chance of winning 500 pounds. She also strictly prefers a $50 \%$ chance of winning 500 pounds and a $20 \%$ chance of winning 200 pounds to a $20 \%$ chance of winning 500 pounds and an $80 \%$ chance of winning 200 pounds. Does she satisfy the independence axiom?
(c) A second individual satisfies the independence axiom. He is indifferent between winning 500 pounds with probability $\frac{1}{2}$ and winning 200 pounds with probability $\frac{2}{3}$. Sketch her indifference curves in the space of lotteries with the three possible prizes $(500,200,0)$. Using the independence axiom (not the expected utility representation), show that he must be indifferent between winning 500 with probability $\frac{1}{4}$ and winning 200 with probability $\frac{1}{3}$.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Tuesday 20 May 2014 2:00pm to 4:00pm

M110

## Microeconomics II

Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. (a) Consider an economy with three types of goods. Suppose that an agent's utility function is given by

$$
U(x, y, z)=x^{a} y^{b} z^{c}
$$

where $a, b, c$ are constants. Express the necessary and sufficient conditions on $a, b, c$ for $U$ to be strictly increasing. Then for such a utility function, given an initial endowment of $\left(\omega_{a}, \omega_{b}, \omega_{c}\right)$, and given prices $p_{a}, p_{b}, p_{c}$, compute the agent's utility maximising demand. (You are asked to solve the optimisation problem explicitly.)
(b) Consider a pure exchange economy with two agents $A, B$, and two goods $i=1,2$. The agents have the utility functions

$$
u^{A}\left(x_{1}, x_{2}\right)=2 \ln x_{1}+3 \ln x_{2} \quad \text { and } \quad u^{B}\left(x_{1}, x_{2}\right)=\ln x_{1}+2 \ln x_{2} .
$$

The total endowment of the goods is $\omega=(1,1)$.
Describe the set of Pareto optimal allocations $\left(\hat{x}_{1}^{A}, \hat{x}_{2}^{A}, \hat{x}_{1}^{B}, \hat{x}_{2}^{B}\right)$ by expressing $\hat{x}_{2}^{A}, \hat{x}_{1}^{B}$, and $\hat{x}_{2}^{B}$ as a function of $\hat{x}_{1}^{A}$.
(c) If the initial endowments are $\omega^{A}=\omega^{B}=\left(\frac{1}{2}, \frac{1}{2}\right)$, evaluate the Walrasian equilibrium price and the associated equilibrium allocations. (For notational brevity, normalise the price of good two as 1 , and set the price of the first good to $p$.)
(d) Suppose we can transfer good 2 between the agents before trade takes place. What transfer should we make so that the allocation

$$
\left(\frac{1}{4}, \frac{1}{5}\right) \text { to } \mathrm{A} \text {, and }\left(\frac{3}{4}, \frac{4}{5}\right) \text { to } \mathrm{B}
$$

emerges as an equilibrium outcome after the transfer?
2. (a) Describe a financial economy with a single consumption (numeraire) good, two dates ( 0 and 1 ), $k$ states in date 1 . What is an asset? What is an equilibrium price vector? Explain carefully.
(b) State and prove the first welfare theorem for financial economies.
(c) State the fundamental theorem (about the relationship between equilibrium prices of assets and state multipliers), and comment on the interpretation of state multipliers.
3. An seller of a single object faces $n$ risk-neutral bidders with private valuations for the object, independently drawn from a distribution $F$ with density $f$.
(a) State the Revenue Equivalence Theorem (RET) for this situation.
(b) Consider an "All Pay" auction (i.e., a simultaneous sealed-bid auction in which the highest bidder wins the object, but every bidder pays her bid). Write down the differential equation for a player's bid as a function of her value in the symmetric equilibrium of this.
(c) Solve the differential equation for the special case in which $F$ is uniform on $[0, M]$.
(d) What would be the bidding behaviour if the auctioneer was instead running a Japanese auction, where a price clock rises, bidders drop out of the auction whenever they like (but cannot re-enter), the moment when there is one bidder left, the price clock stops, the remaining bidder wins, and pays the amount shown on the price clock? If bidders' values are independently drawn from a uniform distribution on $[0, M]$, what is the expected payment by a bidder with valuation $x$ ?
(e) Compare your answers to parts (c) and (d), and comment briefly.
4. Consider the first price auction for a single indivisible object between two bidders $i=1,2$. Each bidder $i$ draws an independent signal $t_{i}$ which is uniformly distributed on the unit interval. Given signals $t_{i}$ and $t_{j}$, the valuations of the bidders are

$$
\begin{aligned}
& v_{1}=2 t_{1}+t_{2} \\
& v_{2}=2 t_{2}+t_{1} .
\end{aligned}
$$

If bidder $i$ receives the object and pays $p$, her utility is $2 t_{i}+t_{j}-p$. Each bidder knows her own signal, but not the other's.
(a) What is expected value of the item to a bidder with type $t_{i}$ ?
(b) Assume a symmetric equilibrium of the sort $b\left(t_{i}\right)=k t_{i}+m$, and solve for $k$ and $m$.
(c) Alternatively, the seller considers selling the object with a fixed posted price. That is, he posts a price $q$, and if any of the bidders wants to buy the object at that price, they do so. If both want to buy it at that price, the seller decides randomly whom he will trade with, again at that price $q$. If the seller wants to maximise his expected revenue from this form of sale, what is his optimal choice of $q$ ? What is the associated revenue?
(d) Now, suppose that the bidders' valuation are given by $v_{i}=2 t_{i}+\frac{1}{2}$. Compare the expected revenue of the first price auction in this case to the expected revenue of the first price auction in part (b).

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Wednesday 4 June 2014 10:00am to 12:00pm

## M120

## Topics in Economic Theory

Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Consider the marriage problem with finitely many agents and strict preferences.
(a) Give a formal definition of a stable matching.
(b) Is there always a stable matching? If yes, prove it. If not, give an example to the contrary.
(c) State the singles theorem and prove it.
(d) Give a formal definition of strategy-proofness in the context of matching mechanisms. Is there a strategy-proof mechanism whose outcome is stable whenever a stable matching exists? If yes, describe the mechanism and prove that it is strategy-proof. If not, give an example and explain clearly why no strategy-proof mechanism can always return a stable matching.
(e) Let $f$ be a direct mechanism whose outcome is stable with respect to the revealed preferences. Consider the preference revelation game induced by this mechanism. Prove or disprove: every individually rational matching is a Nash equilibrium outcome of this game.
2. A subordinate $(S)$ has to make a report to her boss $(B)$ who will then take one of three decisions, denoted $x, y$ and $z$. There are two possible reports that she can make, $M_{1}$ and $M_{2}$. The payoffs for each party depend on the decision but also on the state of the world, which could be either $a$ or $b$. These two states are equally likely. $S$ knows the true state of the world but $B$ does not. The payoffs are given in the following table, with $S$ 's payoff first.

|  | $x$ | $y$ | $z$ |
| :---: | :---: | :---: | :---: |
| $a$ | 0,4 | 4,6 | $-2,0$ |
| $b$ | 4,4 | 2,0 | 0,6 |

(a) Describe informally what is meant by a Weak Perfect Bayesian Equilibrium (WPBE) for this game.
(b) Describe a WPBE in which only one of the two possible reports could be received by $B$.
(c) Suppose that there is a WPBE in which both messages have a strictly positive probability of being received, and $B$ chooses $z$ with zero probability after receiving $M_{1}$ and chooses $y$ with zero probability after receiving $M_{2}$. Find $B$ 's belief after receiving $M_{2}$. Show that there cannot be a WPBE of this kind, unless both reports are followed by action $x$ with probability 1.
(d) Now suppose that $M_{2}$ is not available, but $S$ can choose between sending no report or sending $M_{1}$, in which case it has only a 0.5 chance of arriving. Describe a WPBE which Pareto-dominates the WPBE you found in part (b). Show that it is an equilibrium and find the equilibrium payoffs.
(e) Briefly discuss your results.
3. Consider the infinite horizon alternating bargaining game with 2 players (Rubinstein model). Here the rules are in odd periods player 1 makes an offer of a split $x \in[0,1]$ to player 2 , which player 2 may accept or reject and in even periods player 2 makes an offer of a split to player 1 , which player 1 may accept or reject. If at any period a player accepts an offer, the proposed split is immediately implemented and the game ends. If she rejects, nothing happens until the next period. Assume that each player $i$ discounts future payoffs by a common factor $\delta<1$.
(a) Show that there exists a subgame perfect equilibrium with the following outcome: there is agreement at the first date and the players receive a unique payoff described by the following split: player 1 receives $\frac{1}{1+\delta}$ and player 2 receives $\frac{\delta}{1+\delta}$.
(b) Explain why the equilibrium outcome described in part (i) is the unique subgame perfect equilibrium outcome (A sketch of the arguments will suffice).
4. Consider a repeated random state oligopoly market with $n$ firms who produce a homogenous commodity at zero cost at each date. In each period there is a random demand shock (state) $s>0$ that is first drawn independently with the probability $q(s)$ from the finite set $S$ and revealed to the firms. After observing the states, the firms simultaneously choose non-negative prices for that period. The consumers buy from the cheapest firm (ties are split equally) and the aggregate demand of the consumers at price $p$ at each period is given by $s-p$.
(a) Compute the Nash equilibrium outcome, the mutual minmax outcome and the monopoly price in any stage game if the state is $s$.
(b) Suppose that along an equilibrium path of the repeated game, at every date the firms set a price $p(s)$ whenever the state is $s$. Characterise the expected equilibrium payoff for each firm from such a symmetric stationary equilibrium strategy.
(c) Show that for sufficiently large $\delta$, setting the monopoly price at every date and for every state $s$ can be sustained as an equilibrium.
(d) Fix any $\delta$, and consider any strongly (most collusive) symmetric stationary equilibrium.
i. Show that on the equilibrium path the firms set prices countercyclically in high states. Provide an intuition for this result.
ii. What happens to the equilibrium prices in low states?
(e) Does the countercyclical result in part (d.ii) persist if the random demand shocks were persistent not independent as above. Explain your answer.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Tuesday 27 May 2014 2:00pm to 4:00pm

## M130

## Applied Microeconomics

This paper is in two sections. Candidates are required to answer four out of six questions in Section A; and the one compulsory question in Section $B$.

Questions within Section A carry equal weight. Each section carries equal weighting.

Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Section A

A1 Consider the following regression of the budget share for clothing on prices, demographics and expenditure.

| Source | SS | $d f$ | MS |  | Number of obs | 14994 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | $F(9,14984)$ |  | 273.29 |
| Model | 9.80415378 | 91.0 | 935042 |  | Prob > F | $=$ | 0.0000 |
| Residual | 59.7266979 | 14984.00 | 986032 |  | R-squared | $=$ | 0.1410 |
|  |  |  |  |  | Adj R-squared |  | 0.1405 |
| Total | 69.5308516 | 14993.00 | 637554 |  | Root MSE | $=$ | . 06314 |
| bsclth | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | In | nterval] |
| lhhsize | -. 029766 | . 0010464 | -28.45 | 0.000 | -. 0318171 |  | . 0277149 |
| lpfath | . 0639738 | . 0178202 | 3.59 | 0.000 | . 029044 |  | . 0989036 |
| lprest | -. 066385 | . 0151091 | -4.39 | 0.000 | -. 0960006 |  | . 0367694 |
| lptranall | . 0050483 | . 0065405 | 0.77 | 0.440 | -. 0077719 |  | . 0178684 |
| lpserv | -. 0165291 | . 017966 | -0.92 | 0.358 | -. 0517446 |  | . 0186864 |
| lprecr | -. 0047786 | . 0212658 | -0.22 | 0.822 | -. 0464622 |  | .036905 |
| lpvice | . 0149331 | . 0070191 | 2.13 | 0.033 | . 0011748 |  | . 0286915 |
| lpclth | -. 0462183 | . 0160131 | -2.89 | 0.004 | -. 0776059 |  | . 0148308 |
| lrxtot | . 0419853 | . 0011517 | 36.46 | 0.000 | . 0397278 |  | .0442427 |
| _cons | -. 2887095 | . 0107603 | -26.83 | 0.000 | -. 309801 |  | . 2676181 |

bsclth is the share of clothing at home in total expenditure. The mean clothing share in the sample is $10.5 \%$. Total expenditure is deflated using a Stone price index. "lrxtot" is the $\log$ of deflated expenditure. "ad" indicates the number of adults in the household and "kids" indicates the number of children. "lpfath" through "lpclth" are the log of the prices that make up total expenditure.
(a) Calculate the elasticity of expenditure on clothing at home with respect to total expenditure and with respect to own price.
(b) How do your answers to (a) depend on the assumption of the household as a unitary decision unit?

A2 The Almost Ideal Demand System can be represented by the cost function:

$$
\begin{gathered}
\log c(p, u)=a(p)+b(p) u \\
a(p)=a_{0}+\sum_{k} \alpha_{k} \log p_{k}+\frac{1}{2} \sum_{k} \sum_{l} \gamma_{k l} \log p_{k} \log p_{l} \\
b(p)=\beta_{0} \Pi p_{k}^{\beta_{k}}
\end{gathered}
$$

and corresponding budget shares:

$$
w_{k}=\alpha_{k}+\sum_{i} \gamma_{k i} \log p_{i}+\beta_{k}[\log x-\log P]
$$

where

$$
\log P=a(p)
$$

What restrictions does demand theory place on $\alpha_{k}, \beta_{k}$ and $\gamma_{k i}$ ?

A3 What assumptions are necessary for the use of information on other family members education and wages to be useful for estimating the returns to education? How would such information best be used?

A4 The government is considering cutting the rate of tax on income above $£ 50,000$ by $10 \%$.
(a) Derive the Slutsky equation showing the likely effects on labour supply in a static model.
(b) How would your answer on the likely effects change in a dynamic context if the tax cut was to last only one year.

A5 An individual has a utility function

$$
u\left(q_{1}, q_{2}\right)=q_{1}^{\beta} q_{2}^{1-\beta}
$$

(a) By how much does the cost of living increase when the price of good 1 increases by $\lambda \%$ ?
(b) When will the cost of living be independent of the level of utility?

A6 (a) What are the likely theoretical costs and benefits of increasing the generosity of the disability insurance system?
(b) What do we learn from alternative approaches to measuring these costs and benefits?

## Section B

B1 A second-order approximation to the Euler equation yields the following relationship:

$$
E_{t}\left[\Delta \ln c_{t+1}\right]=\frac{1}{\gamma}\left(E_{t}\left[r_{t+1}\right]-\delta\right)+\frac{1}{2}(\gamma+1) \sigma_{\Delta \ln c_{t+1}}^{2}
$$

where $E_{t}$ is expectations at time $t ; c_{t}$ is consumption in period $t ; r_{t+1}$ is the interest rate between period $t$ and $t+1 ; \delta$ is the discount rate; and $\sigma_{\Delta \ln c_{t+1}}^{2}$ is the variance of consumption growth.
(a) How shoud the coefficients on $E_{t}\left[r_{t+1}\right]$ and $\sigma_{\Delta \ln c_{t+1}}^{2}$ be interpreted?
(b) How might the coefficient be estimated and, in particular, what are the difficulties introduced by uncertainty over the interest rate?
(c) What can be learnt from this specificaion about the amount of consumption in any given period?
(d) If the government is considering stimulating consumption through an income tax rebate or a cut in VAT, how useful are the estimates of this equation to predicting the effects?

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Thursday 29 May 2014 2pm to 4pm

## M140

## Behavioural Economics

Candidates are required to answer all four questions.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. (a) Describe the public-goods experiments conducted by Fehr and Gächter (2000).
(b) To what extent did those experiments establish that cooperators punish the free-riders?
(c) How would you improve on their experimental design?
2. (a) Name and describe the two most important findings of neuroscience for economics.
(b) Why do you regard these two findings as more important than other findings of neuroscience for economics?
3. (a) Define the endowment effect.
(b) Describe an experiment that shows its existence.
(c) Does the effect disappear in real markets with experienced traders? Justify your answer.
4. Consider an urn filled with red and white balls. Suppose that $\frac{2}{3}$ of the balls are of one color and $\frac{1}{3}$ of the other color. Suppose that individual 1 has drawn 5 balls (with replacement) and found 4 red and 1 white, and that individual 2 has drawn 20 balls (with replacement) and found 12 red and 8 white.
(a) Assuming equal prior probabilities, which individual should feel more confident that the urn contains $\frac{2}{3}$ red balls?
(b) Experimental evidence shows that the majority of people give the wrong answer to (a). Define the bias that leads to this counterintuitive experimental finding and give one example of a context where this bias may affect economic decisions.
(c) How does the bias in (b) relate to mean-regression bias? Define meanregression bias and describe an adverse consequence of this bias.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Thursday 22 May 2014 2:00pm to 4:00pm

M150

## Economics of Networks

Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Consider network formation with $N=\{1,2, . . n\}, n \geq 3$ individuals. Suppose that a player $i$ can create a link with player $j$ unilaterally by paying a cost $c>0$. The set of player $i$ links is denoted by $g_{i}$; the profile of linking decisions of all players is given by $g=\left(g_{1}, \ldots, g_{n}\right)$. The payoffs to player $i$ under a strategy profile $g=\left(g_{1}, \ldots, g_{n}\right)$ are

$$
\begin{equation*}
\Pi_{i}(g)=1+\sum_{j \in N \backslash\{i\}} \delta^{d(i, j ; g)}-\eta_{i}^{d}(g) c, \tag{1}
\end{equation*}
$$

where $d(i, j ; g)$ is the distance between $i$ and $j$ in (the undirected version of) network $g$ and $\delta \in(0,1)$ is the decay factor and $\eta_{i}^{d}(g)$ is the number of links which player $i$ forms. The distance between two nodes that have no path between them is set equal to infinity.
(a) Derive the parameter ranges - on $c, \delta$ and $n$ - for which the complete network, the empty network and different types of star networks can arise in a Nash equilibrium.
(b) A network is efficient if it maximizes aggregate welfare across all networks. Show that complete network is efficient if $c<2\left(\delta-\delta^{2}\right)$, the star network is efficient if $2\left(\delta-\delta^{2}\right)<c<2 \delta+(n-2) \delta^{2}$, and the empty network is efficient if $c>2 \delta+(n-2) \delta^{2}$.
(c) Show that if $c>1, n \geq 4, \delta$ is close to 1 , then the periphery sponsored star network is the unique non-empty strict Nash equilibrium network.

2 Suppose that there are two price setting firms selling a homogenous product to a unit mass of consumers. The consumers do not know the prices set by firms. They receive a free sample price from one firm at random. In addition, they can search for prices (at cost $c>0$ for the other price) and can share information with $k$ other consumers. Assume that consumers value the product at 1 and that $c<1$. The questions below ask you to study the nature of search and pricing in equilibrium.
(a) Suppose that $k=0$ : show that there exists an equilibrium in which firms set monopoly price 1 and no consumer searches.
(b) Now suppose that $k \geq 1$. Show that if $c$ is small, then there is a unique equilibrium in which consumers must mix in search intensity and firms must mix in prices.
(c) What are the effects of greater communication among consumers, represented by an increase in the value of $k$, on market prices and consumer welfare?

3 There is a source and a destination and a collection of $n$ intermediary traders located on nodes of an undirected connected network in between the source and destination. The value of exchange is 1 . The network of traders and the valuations are common knowledge. All traders post prices; they have zero costs. If the Buyer and Seller have a (direct) link they exchange the good and split the surplus. If they don't have a direct link then they compute the cost of using every path between them. They pick the lowest cost path if it is less than 1 (randomizing across paths if there are multiple lowest cost paths). Source and destination divide the residual surplus, after paying the cost of the path.
(a) Formalize this economic exchange as a game of simultaneous moves.
(b) Prove that for any network there exists a pure strategy Nash equilibrium.
(c) Define an outcome as efficient if trade occurs with probability 1. Show that there exists an efficient equilibrium in every network.
(d) Now suppose that the network is common knowledge but the location of source and destination is unknown. So traders set a uniform price for all trade that goes through them, irrespective of the distance that it travels.
i. Fix the network to be a ring with 10 traders. Show that there exists a symmetric Nsh equilibrium with price equal to $0,1,1 / 2$ or $1 / 3$.

4 Consider trade between 2 buyers and 1 seller (with zero costs). There are two stages. In stage 1, players choose to form links. The links determine potential trade patterns. In stage 2, buyers simultaneously make bids to the seller. If there is one bidder he gets the object at price 0 ; in case of two bidders, the winner is determined using a second price auction. Assume that the valuations of the buyers are uniformly distributed on the unit interval and that bidder learns his valuation at the start of stage 2.
(a) Solve for expected payoffs to bidders and seller in empty, single link and two link network.
(b) Suppose now that bidders can unilaterally decide to form links at cost $c$. Show that if $c>1 / 2$ then the empty network is an equilibrium, if $1 / 6<c<1 / 2$ then the single link network is an equilibrium, while if $c<1 / 6$ then the two link network is an equilibrium. Show that equilibrium and efficient networks coincide.
(c) Next suppose that bidder and seller form a link only if both agree to pay $c$. Show that in this case efficient and pairwise stable networks do not coincide.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research
XXXXXX X00 to X. 00
Module 160
POLITICAL ECONOMY
Candidates are required to answer three compulsory questions.

Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

1. Consider $N$ (where $N$ is odd) voters that have single-peaked preferences over policies $p \in P$, where $p \in \mathbb{R}$ is a unidimensional policy variable. Denote by $\pi_{i}$ the unique favourite policy of individual $i$. Assume that individual 1 is the one with the smallest favourite policy and individual $N$ is the one with the largest favourite policy, and individuals are ordered such that $\pi_{1}<\pi_{2}<\ldots<\pi_{N}$. Assume sincere voting throughout the question.
(a) Give formal definitions of the following:

- Single-peaked preferences
- Median voter
(b) State the Median Voter Theorem
(c) Show by using a counterexample that the Median Voter Theorem does not hold if the policy space is two-dimensional, i.e. $p \in \mathbb{R}^{2}$
(d) Show by using a counterexample that the Median Voter Theorem does not hold if individuals have non-single-peaked preferences. Assume a voting procedure of direct democracy with an open agenda.
(e) State and give a proof of the Downsian Policy Convergence Theorem. Make sure you present the formal set-up of the model and you explicitly state the assumptions that you are making.

2. "No great man lives in vain. The history of the world is but the biography of great men." Comment.
3. How did the introduction of Fox News affect the US presidential election in 2000? Is this consistent with the theoretical predictions in Mullainathan and Shleifer (2005)?

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 19 May 2014 2:00pm to 4:00pm

M170

## Industrial Organisation

Candidates are required to answer all three questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION <br> Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Consider a Hotelling duopoly in which consumers who wish to buy one unit of the good are uniformly located on a line $[0,1]$. Consumers pay a linear transport cost $t x$ to buy at a distance $x$ from their location on the line. Firm $A$ is located at 0 with marginal cost $c \in(0, t)$, and firm $B$ is located at 1 with a marginal cost of zero.
(a) Assume that firms simultaneously set prices $p_{A}$ and $p_{B}$ respectively. Compute firms' equilibrium prices, market shares, and profits.
(b) How much does a small increase in firm $A$ 's marginal cost $d c>0$ reduce its profits? Is this more than, less than, or equal to the change in $A$ 's marginal cost times its market share? Why?
(c) Assume that, instead of each firm offering the same price to all consumers, firms price separately to each individual consumer on the line, i.e., firms can perfectly price discriminate. Compute firms' equilibrium prices, market shares, and profits.
(d) Comment on whether the equilibrium with price discrimination in (c) is better or worse than uniform pricing in (a) from the viewpoints of firms and consumers respectively.
2. Consider a market for a homogeneous product, characterised by the demand function $q=a-p$. An upstream manufacturer $M$ produces the good at constant marginal cost $c>0$. Assume that $a>c$. A downstream distributor $D$ buys the goods from $M$ and sells them to final consumers. Assume that the distributor has no marginal costs over and above the input price charged by the manufacturer. Assume that the manufacturer sets a linear wholesale price $w$ for each unit of the good.
(a) For a given wholesale price $w$, derive the quantity, price and profits of the distributor.
(b) Derive the wholesale price $w$ that maximises the manufacturer's profits, the resulting quantity, price and profits of the distributor and the profits of the manufacturer.
(c) Compare total industry profits under linear wholesale prices with the profits $\pi^{V I}$ of a vertically integrated monopolist.
(d) Consider an entrant firm $E$ that can produce the homogeneous product at constant marginal cost $c$. Assume that $E$ competes in prices against the distributor $D$. Calculate the equilibrium profits of the manufacturer $M$, the distributor $D$ and the entrant $E$.
3. Consider two firms $i=1,2$ producing goods 1 and 2 at zero cost, that consumers consider to be perfect complements. That is, consumers have demand for bundles consisting of one of each good. Denote the price of good $i=1,2$ by $p_{i}$. Assume that aggregate demand for bundles is given by

$$
q=a-\left(p_{1}+p_{2}\right)
$$

with $a>0$. Denote the the amount demanded of good $i=1,2$ by $q_{i}$, and note that $q_{1}=q_{2}=q$.
(a) Suppose that the two firms set prices non-cooperatively. Write down the firms' maximisation problems and derive the reaction functions.
(b) Find the equilibrium prices, quantities and profits of the two firms.
(c) Suppose the two firms merge and set prices in order to maximise joint profits. Write down the merged firm's maximisation problem and derive the equilibrium price, quantity and profits.
(d) Carefully compare the prices, quantities and profits under non-cooperative price setting with those of the merged firm. Briefly provide an intuition for the differences. Assess whether the merger increases social welfare, distinguishing between changes in producer surplus and consumer surplus.

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Wednesday 7 May 2014 2:00pm to 4:00pm

M200

## Macroeconomics I

The paper is in two sections:
Section A - candidates must answer all four questions.
Section B - candidates must answer one out of two questions.
Each section carries equal weight.
Write your candidate number (not your name) on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

A. 1 Suppose the dynamics of a continuous-time Solow growth model with physical and human capital can be represented by the following two equations:

$$
\begin{gathered}
\dot{k}=s_{k} k^{\alpha}(u h)^{1-\alpha}-\delta k \\
\dot{h}=(1-u) h^{\beta}-\delta h
\end{gathered}
$$

where $k$ is the physical capital stock per capita, $h$ is human capital per capita, $s_{k} \in(0,1)$ is the saving rate, and $\alpha \in(0,1), \beta \in(0,1)$ and $\delta>0$ are constant parameters. In addition, $u \in(0,1)$ is the fraction of time allocated to production and $(1-u)$ is the fraction of time allocated to human capital acquisition.
Draw and explain the phase diagram describing the dynamics of human and physical capital per capita ( $h$ and $k$ ). Compute the steady-state levels of human and physical capital per capita ( $h^{*}$ and $k^{*}$ ), and explain how they are affected by $u$.
A. 2 Suppose that a firm facing a gross real interest rate $1+r>0$ has a production function given by $Y_{t}=A F\left(K_{t}\right)$, where $A>0, F^{\prime}(\cdot)>0$, and $F^{\prime \prime}(\cdot)<0$. The firm's objective function is to maximise the present discounted value of profits over an infinite horizon. In addition to the direct real cost of investment $p_{I, t} I_{t}$, where $p_{I, t}$ is the price of investment $I_{t}$ in terms of output $Y_{t}$, the firm also faces real adjustment costs $\frac{\chi I_{t}^{2}}{2 K_{t}}$ with $\chi>0$ in any period in which it invests (or disinvests) $I_{t}$. Capital $K_{t}$ accumulates according to the following equation:

$$
K_{t+1}=K_{t}+I_{t}
$$

where $K_{0}>0$ is given.
Set up the firm's optimization problem and find the first-order conditions characterising optimal investment together with the boundary conditions. Interpret these conditions.
A. 3 Consider the following insider-outsider model. The representative firm maximizes real profits $\Pi_{t}=Y_{t}-w_{t} L_{t}$, subject to the production technology $Y_{t}=A_{t} \sqrt{L_{t}}$, where $Y_{t}$ is output, $L_{t}$ labour, and $A_{t}=\bar{A} \xi_{t}$, where $\xi_{t}$ is an i.i.d. shock with $\mathrm{E}\left[\ln \xi_{t}\right]=0$. There is perfect competition in the goods market. The real wage $w_{t}$ is set at the end of period $t-1$ by the workers who are employed in that period ('insiders') such that they expect to maintain their $\log$ employment: $\mathrm{E}\left[\ln L_{t}\right]=$ $\ln L_{t-1}$. After the shock $\xi_{t}$ has been realized, the firm decides whether to hire additional workers ( $L_{t}>L_{t-1}$ ) or fire workers $\left(L_{t}<L_{t-1}\right)$ in period $t$.
Suppose there is a negative shock $\xi_{t}<0$ such that $A_{t}$ temporarily drops $2 \%$ below $\bar{A}$ in period $t$. Explain the effect of this on employment $L_{t}$ from period $t$ onwards.
A. 4 Consider the following flexible price monetary model based on purchasing power parity, money market equilibrium and uncovered interest parity, respectively:

$$
\begin{aligned}
p_{t} & =s_{t}+p_{t}^{*} \\
m_{t}-p_{t} & =y_{t}-i_{t} \\
i_{t} & =i_{t}^{*}+\mathrm{E}_{t}\left[s_{t+1}\right]-s_{t}
\end{aligned}
$$

where $p$ denotes the log aggregate price level, $s$ the $\log$ nominal exchange rate, defined as the domestic price of foreign currency, $m$ the log money supply, $y$ log aggregate output, and $i$ the nominal interest rate. Foreign variables are indicated by an asterisk and subscripts indicate the time period.
Suppose that in period $t$ it is suddenly expected that Home money supply in period $t+1$ will be $2 \%$ lower than previously anticipated. Explain the effect of this on the nominal exchange rate in period $t$.

## SECTION B

## B. 1 Real Business Cycles

Consider the following two-period economy. There is a continuum of measure one of identical individuals $i \in[0,1]$. A representative individual maximises the following utility function:

$$
U=\sum_{t=0}^{1} \beta^{t}\left[\ln \left(c_{t}\right)+\gamma \ln \left(1-l_{t}\right)\right]
$$

where $c_{t}$ denotes consumption, $l_{t}$ labour supply, the subscript $t=0,1$ the time period, and $\beta \in(0,1)$ and $\gamma>0$ are constant parameters. The time endowment is equal to one in each period. Assume that individuals start with no initial assets $\left(b_{0}=0\right)$ and cannot die with a debt. Individuals supply labour to firms at a real wage rate $w_{t}$ per unit of labour. The consumption good is the numéraire.
There is also a continuum of measure one of identical, profit maximising firms. The production technology of the representative firm is represented by the following function:

$$
Y_{t}=A_{t} L_{t}
$$

where $Y_{t}$ is output, $L_{t}$ is labour used in production, and $A_{t}>0$ is an exogenous productivity factor. There is perfect competition in the product and labour market. There is no government and the real interest rate is $r$.
(a) Formulate the problem of the representative firm and take the first-order condition. Give an economic interpretation of the result.
(b) Formulate the problem of a representative individual and derive the first-order conditions associated with this problem. Provide some economic intuition for the conditions which characterise the trade-offs of this problem.
(c) Derive the optimal allocations for consumption $c_{0}$ and $c_{1}$, labour supply $l_{0}$ and $l_{1}$ and assets $b_{1}$, for given $w_{0}, w_{1}$ and $r$. Explain intuitively how they depend on the real interest rate $r$.
(d) Suppose that the economy is closed. Derive the equilibrium real interest rate $r$. Explain the effect of an increase in productivity $A_{1}$ on the real wage $w_{0}$ and $w_{1}$, the real interest rate $r$, consumption $c_{0}$ and $c_{1}$, and labour supply $l_{0}$ and $l_{1}$.

## B. 2 Effect of Price Rigidities

Consider an economy with monopolistic competition in the goods market and perfect competition in the labour market. The representative consumer $i \in[0,1]$ maximizes utility

$$
U_{i}=C_{i}-\frac{1}{\gamma} L_{i}^{\gamma}
$$

subject to the budget constraint

$$
C_{i}=\Pi_{i}+w L_{i}
$$

where $C_{i}, L_{i}$ and $\Pi_{i}$ denote consumption, labour and real profits for agent $i, w$ the real wage, and $\gamma>1$ a preference parameter. The representative firm $i \in[0,1]$ maximizes real profits

$$
\Pi_{i}=\frac{P_{i}}{P} Q_{i}-w L_{i}
$$

where $P_{i}$ denotes the price of good $i, P \equiv \int_{0}^{1} P_{i} d i$ the aggregate price level, and $Q_{i}$ the production of good $i$, which is described by the production technology

$$
Q_{i}=A L_{i}
$$

where $A$ denotes productivity. The demand for good $i$ is given by

$$
Q_{i}=\left(\frac{P_{i}}{P}\right)^{-\eta} Y
$$

where $Y \equiv \int_{0}^{1} Q_{i} d i$ denotes aggregate output, and $\eta>1$ is a demand parameter. The aggregate demand relation is described by

$$
Y=\frac{M}{P}
$$

where $M$ denotes aggregate demand.
(a) Use the first-order conditions to derive the optimal price $P_{i} / P$ for the firm and the optimal labour supply $L_{i}$ for the consumer. Give an economic interpretation of the results.
(b) Derive the real wage $w$, employment $L \equiv \int_{0}^{1} L_{i} d i$ and output $Y$ with flexible prices. How are they affected by a change in aggregate demand $M$ ? Give an economic interpretation of the results.
(c) Suppose now that all firms keep their price $P_{i}$ fixed and that there is a drop in aggregate demand $M$. Derive and explain the effect on the real wage $w$, employment $L$ and output $Y$.
(d) Starting from the flexible price equilibrium and assuming $A=1, \gamma=2$ and $\eta=5$, compute the real wage $w$, output $Y$ and profit $\Pi_{i}$ when all firms keep their price $P_{i}$ fixed after a $10 \%$ drop in aggregate demand $M$. Compute the profit $\Pi_{j}$ if firm $j$ optimally sets its price $P_{j}$ while all other firms $i \neq j$ keep their price $P_{i}$ fixed, and explain under what circumstances it would be optimal for firms to keep their price fixed.

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 2 June $2014 \quad$ 2:00pm to 4:00pm

M210

## Macroeconomics II

This paper is in two sections. Section A: Candidates are required to answer three out of four questions. Section B: Candidates are required to answer one compulsory question.

Each section carries equal weighting.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 2
Rough Work Pad
Tags

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

## A. 1 New Keynesian Model

Consider the following New Keynesian model:

$$
\begin{aligned}
\pi_{t} & =\beta E_{t}\left[\pi_{t+1}\right]+x_{t} \\
x_{t} & =E_{t}\left[x_{t+1}\right]-\left(i_{t}-E_{t}\left[\pi_{t+1}\right]-r_{t}^{n}\right)
\end{aligned}
$$

where $\pi_{t}$ denotes inflation, $x_{t}$ the output gap, $i_{t}$ the nominal interest rate, $r_{t}^{n}$ the natural real interest rate, the subscript $t$ indicates the time period, and $\beta$ is a parameter with $0<\beta<1$. The central bank sets the nominal interest rate subject to the lower-bound constraint that $i_{t} \geq 0$. Assume that in the initial steady state in period $t=0$, the central bank optimally sets $i_{t}=r_{t}^{n}=\bar{r}>0$.
Suppose that there is a sudden shock in period $t=1$ so that $r_{1}^{n}=\underline{r}<0$ and $i_{1}=0$. In each subsequent period $s$ there is a probability $p$, with $0<p<1 / 3$, that the shock persists so $r_{s}^{n}=\underline{r}<0$ and $i_{s}=0$; and a probability $1-p$ that the natural real interest rate permanently reverts back with $i_{s}=r_{s}^{n}=\bar{r}>0$ from period $s$ onwards.
Compute the level of inflation $\bar{\pi}$ and the output gap $\bar{x}$ for the initial steady state. In addition, derive the level of inflation $\underline{\pi}$ and the output gap $\underline{x}$ that prevail while the central bank sets $i=0$. Give an economic interpretation of the result.

## A. 2 Searching for ideas

Consider an entrepreneur who begins at $t=0$ with the following technology $a_{0}=0$ and has two alternative actions. The entrepreneur can engage in production using her current technology or she can search for a new technology. When engaged in production the income of the entrepreneur is $y_{t}=a_{t}$ where $a_{t}$ is her current technology. At each period she engages in such a search, she gets an independent draw of a new technology from a time-invariant distribution function $H(a)$ defined over a bounded interval $[0, \bar{a}]$ with $\bar{a}<\infty$. The income of the entrepreneur when searching is $y_{t}=0$. The entrepreneur devises a strategy to maximize $E\left[\sum_{t=0}^{\infty} \beta^{t} y_{t}\right]$, where $\beta \in(0,1)$ is a discount factor. Write the Bellman's functional equation for this problem and explain whether or not this equation is a Contraction Mapping.

## A. 3 OLG Model

Consider a discrete time economy, $t=0,1,2, \ldots$ Agents in this economy live for two periods only. In the first period of life (young period), agents have one unit of productive time, but in the second period (old period) of life agents do not work and they consume their savings from the first period of their lives. Agents have preferences over consumption when young, $c_{1}(t)$, and when old, $c_{2}(t+1)$ and they are represented by the following utility function

$$
U=\frac{c_{1}(t)^{1-\theta}-1}{1-\theta}+\beta \frac{c_{2}(t+1)^{1-\theta}-1}{1-\theta},
$$

$\theta>0, \beta \in(0,1)$. Firms in this economy have access to a technology to produce the consumption good $Y(t)$ and this technology is represented by a Cobb-Douglas production function:

$$
Y(t)=A K(t)^{\alpha} L(t)^{1-\alpha}, \alpha \in(0,1), A>0 .
$$

$K(t)$ and $L(t)$ denote capital and labour used in production, respectively. Capital fully depreciates after being used, $\delta=1$. At period $t=0$ there exists a unit measure of old agents with capital stock $K(0)$. What is the equilibrium equation of motion of the aggregate capital stock? Explain.

## A. 4 Asset Pricing

Consider an economy populated by a continuum of measure one of identical households, each with preferences given by $\mathbf{E}_{0}\left[\sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right)\right]$ where $c_{t}$ is consumption at period $t, u\left(c_{t}\right)$ is a function that is strictly increasing, strictly concave and twice differentiable, and $\beta \in(0,1)$ corresponds to the subjective discount factor. There is a single asset tree which yields perishable dividends $d_{t}$ (fruits). Households are initially endowed with some number of shares of the tree: $s_{0}$. Assume that $d_{t}$ is a random variable taking values from the set $\mathbf{D} \equiv\left\{d_{1}, \ldots, d_{N}\right\}$ and is distributed according to a first-order Markov chain with constant transition probability matrix. Households can trade the share of the tree. The price of the tree when the current realization of $d_{t}$ is $d$ is given by the function $p: \mathbf{D} \rightarrow R_{+}$. Households have an income flow $0<y<\infty$ at every period $t$ and $y$ is not a random variable. There are no other assets in this economy. State the problem of the representative household in recursive form and assuming that the value function is differentiable, derive the stochastic Euler equation. Interpret this equation.

## SECTION B

## B. 1 Neoclassical Growth Model

Consider an economy with continuum of firms of measure one and each firm has access to the following technology:

$$
Y(t)=K(t)^{\alpha}(A(t) N(t))^{1-\alpha}, \alpha \in(0,1) .
$$

$Y$ is output, $K$ is capital, $N$ is labour, and $A$ is labour productivity. Technological progress grows at rate $g$ and the initial technology is given. All markets are competitive.
There is a continuum of measure one of households who are infinite lived, with preferences

$$
U=\int_{0}^{\infty} e^{-\rho t} u\left(c^{i}(t)\right) d t, \quad \rho>0
$$

where $c^{i}(t)$ denotes consumption of household $i$ at $t$, and

$$
u(c)=\frac{\left(c^{i}\right)^{1-\theta}-1}{1-\theta}, \theta>0 .
$$

Each household has an instantaneous unit of productive time. Household $i$ has $k^{i}(0)$ units of capital at $t=0$ and $k^{i}(0)$ can be different across households. Capital evolves according to:

$$
\dot{k}^{i}(t)=x^{i}(t)-\delta k^{i}(t),
$$

where $x^{i}$ corresponds to investment in capital of household $i$ and $\delta>0$ is the depreciation rate.
(a) Set up the Hamiltonian that household $i$ solves, taking prices as given and determine the necessary and sufficient conditions for the allocation of consumption for each household over time.
(b) For a given path of prices, will consumption of each household $i$ be the same across all households? (Hint: Write down the present value budget constraint of each household.)
(c) Derive also the first-order conditions of the representative firm, and define the market clearing conditions.
(d) Derive the growth rates of the aggregate capital stock and consumption of each household along a balanced growth path equilibrium. Is the growth rate of consumption of each household $i$ the same across all households along the balanced growth path equilibrium? Explain.
(e) Derive a parameter restriction ensuring that the transversality condition is satisfied for a balanced growth path with a positive growth rate of the aggregate capital stock and consumption of each household $i$.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 26 May 2014 10:00am to 12:00pm

M220

## Macroeconomics III

This paper is in two sections:
Section A - candidates are required to answer three out of four questions.
Section $B$ - candidates are required to answer one compulsory question.
Each section carries equal weighting.
Write your candidate number (not your name) on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.
A. 1 Consider the following two-period $N$-country model. The representative consumer of a representative country $n \in\{1,2, \ldots, N\}$ maximises expected lifetime utility

$$
U^{n}=u\left(C_{1}^{n}\right)+\beta \sum_{s=1}^{\mathcal{S}} \pi(s) u\left[C_{2}^{n}(s)\right]
$$

where $C_{1}^{n}$ denotes consumption in period $1, C_{2}^{n}(s)$ consumption in period 2 in state $s \in\{1,2, \ldots, \mathcal{S}\}, \pi(s) \in(0,1)$ the probability of state $s$, the utility function satisfies $u^{\prime}(\cdot)>0$ and $u^{\prime \prime}(\cdot)<0$, and $\beta \in(0,1)$ is the intertemporal discount factor. In period 1 there is international asset trade in competitive world markets for a risk-free bond, which pays a real interest rate $r$, and $N$ GDP securities, which are a claim to uncertain period-two output $Y_{2}^{m}$ of country $m \in\{1,2, \ldots, N\}$. The budget constraints of the representative consumer of country $n$ are given by

$$
\begin{aligned}
C_{1}^{n}+B_{2}^{n}+\sum_{m=1}^{N} x_{m}^{n} V_{1}^{m} & =Y_{1}^{n}+V_{1}^{n} \\
C_{2}^{n}(s) & =(1+r) B_{2}^{n}+\sum_{m=1}^{N} x_{m}^{n} Y_{2}^{m}(s) \quad \text { for } s \in\{1,2, \ldots, \mathcal{S}\}
\end{aligned}
$$

where $B_{2}^{n}$ is the stock of risk-free bonds in country $n$ at the end of period $1, x_{m}^{n}$ the share that country $n$ holds in the GDP security of country $m$, and $V_{1}^{m}$ the market value in period 1 of the GDP security of country $m$.
Use the first-order conditions to derive an expression for the market value $V_{1}^{m}$ of the GDP security of country $m$ in terms of $Y_{2}^{m}, C_{1}^{n}$ and $C_{2}^{n}$. Give an intuitive explanation of the result.
A. 2 Write import prices at the Home border as

$$
\bar{P}_{F}=m k p \cdot \mathcal{E} \cdot m c^{*}
$$

where $m k p$ is the Foreign firms' markup in the Home market, $m c^{*}$ the Foreign marginal costs in foreign currency, and $\mathcal{E}$ the nominal exchange rate, defined as the Home currency price of Foreign currency. Consider the results from regressing import prices at the consumer level $P_{F}$ on the nominal exchange rate and (a good proxy for) Foreign marginal costs:

$$
\ln P_{F}=\alpha+\gamma \ln \mathcal{E}+\beta m c^{*}+v
$$

Explain whether we can consider $\gamma$ a reliable estimate of the "structural exchange rate pass through" for import prices.
A. 3 Assume that a social planner maximises the following social welfare function:

$$
W=\frac{1}{2} \ln C+\frac{1}{2} \ln C^{*}
$$

The aggregate consumption index in Home $C$ and Foreign $C^{*}$ is given by

$$
\begin{aligned}
C & =C_{H}^{\theta} C_{F}^{1-\theta} \\
C^{*} & =C_{H}^{* \theta} C_{F}^{* 1-\theta}
\end{aligned}
$$

where $0<\theta<1$, and $C_{H / F}$ and $C_{H / F}^{*}$ denote consumption of the Home/Foreign good in the Home and Foreign country, respectively. In addition, goods market equilibrium holds:

$$
\begin{aligned}
C_{H}+C_{H}^{*} & =Y_{H} \\
C_{F}+C_{F}^{*} & =Y_{F}
\end{aligned}
$$

where $Y_{H / F}$ denotes the (tradable) output of the Home/Foreign good. Use the first order conditions describing the efficient allocation to derive
(i) an expression that can be interpreted as the terms of trade of a country, and (ii) an expression that can be interpreted as the real exchange rate. Briefly explain their properties.
A. 4 Consider a Cole and Obstfeld two-country endowment economy where the consumption aggregator has a unit elasticity of substitution (a Cobb-Douglas aggregator).
Carefully explain why the endowment risk is perfectly diversified with no need for international trade in assets. Discuss how this result can provide guidance in understanding the direction of capital flows in a bond economy in response to temporary endowment shocks, for substitution elasticities above or below unity.

## SECTION B

B. 1 Consider a Home firm producing for both the Home and the Foreign markets. The firm produces a variety $h$ using the production function $Y_{t}(h)=Z_{\mathrm{H}, t} L_{t}(h)$, where $L_{t}(h)$ denotes labour employed and $Z_{\mathrm{H}, t}$ Home productivity. The firm faces imperfect competition with the following Home and Foreign demand functions:

$$
\begin{aligned}
C_{t}(h) & =\left(\frac{p_{t}(h)}{P_{\mathrm{H}, t}}\right)^{-\theta} C_{\mathrm{H}, t} \\
C_{t}^{*}(h) & =\left(\frac{p_{t}^{*}(h)}{P_{\mathrm{H}, t}^{*}}\right)^{-\theta} C_{\mathrm{H}, t}^{*}
\end{aligned}
$$

where $p_{t}(h)$ is the consumer-level price of variety $h, P_{\mathrm{H}, t}$ the price index associated with the composite consumption index $C_{\mathrm{H}, t}, \theta>1$ and Foreign variables are indicated by an asterisk. The consumer-level price is the sum of the producerlevel price $\bar{p}_{t}(h)$ and the cost of retailers, which depends on wages $W_{t}$ in local currency:

$$
\begin{aligned}
& p_{t}(h)=\bar{p}_{t}(h)+\eta W_{t} \\
& p_{t}^{*}(h)=\bar{p}_{t}^{*}(h)+\eta W_{t}^{*}
\end{aligned}
$$

where $\eta>0$.
(a) Derive the profit maximizing prices for the firm. Give an economic interpretation of the result.
(b) Are the profit maximizing prices identical across markets of destination? Carefully explain under what conditions this is the case.
(c) Suppose now that retailers in one country have the option to buy the product $h$ directly from retailers from the other country, rather than from the producer. This possibility is a constraint on the ability of the producer to charge the optimal price. Using the optimal prices you derived in part (a), show that the range of exchange rate values for which the producer is able to charge the optimal markup can be written as a function of $\theta$ and $W / W^{*}$. Explain whether this range is increasing or decreasing in the monopoly power of the firm.
(d) Derive the optimal pricing under the no arbitrage constraint on pricing. Explain whether the law of one price will hold at either producer or consumer level.

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Wednesday 21 May 2014 2:00pm to 4:00pm

M230

## Applied Macroeconomics

Candidates are required to answer three out of seven questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 2
Metric Graph Paper
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Country A at time period $t$ has a real Gross Domestic Product of 1000 bitcoins, a Price level of 1 and a nominal national Debt of 600 bitcoins. It faces a real interest rate of $3 \%$ and a real growth rate of $2.5 \%$. The government spends 200 bitcoins and raises taxes of 200 bitcoins.
(a) Write down the government budget constraint and the debt dynamics describing the evolution of national debt.
(b) Draw the phase diagram for the evolution of debt and comment upon its features.
(c) Suppose the government wishes to stabilise the debt-income ratio. What does it have to do?
(d) Suppose the real interest rate falls to $2.5 \%$ and the real growth rate of the economy rises to $3 \%$. What effect does this have on your answer?
2. Forward Guidance is meant to improve the communication and transparency of monetary policy.
(a) How does the distinction between state and time contingent policies help to do this?
(b) How might the successful pursuit of forward guidance show up in the spead between short and long term interest rates?
(c) Is there any empirical evidence to suggest that forward guidance actually works?
3. Consider the vector autoregresive model with the four variables $\left[\begin{array}{c}p \\ y \\ s \\ x\end{array}\right]$, where $p$ is the price level, $y$ is output, $s$ is the short term interest rate and $x$ is the exchange rate. The relationship between the reduced form errors and the structural shocks is given by $A e_{t}=B u_{t}$. The matrix $B$ is an identity matrix and:

$$
\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
a_{21} & 1 & 0 & 0 \\
a_{31} & a_{32} & 1 & 0 \\
a_{41} & a_{42} & a_{43} & 1
\end{array}\right]\left[\begin{array}{c}
\varepsilon^{p} \\
\varepsilon_{t}^{y} \\
\varepsilon_{t}^{s} \\
\varepsilon_{t}^{x}
\end{array}\right]=\left[\begin{array}{c}
u_{t}^{p} \\
u_{t}^{y} \\
u_{t}^{s} \\
u_{t}^{x}
\end{array}\right]
$$

(a) Explain the relationship between the structural equation and the reduced form.
(b) Is the model identified?
(c) What is an economic interpretation of the restrictions placed on the $A$ matrix?
(d) Do these restrictions seem plausible?
(e) What other forms of identifying restriction could be used?
4. Is it true that if you want to understand why aggregate economies fluctuate at business cycle frequencies it is necessary to appeal to aggregate shocks and not to idiosyncratic firm or sector specific shocks?"
5. (a) Why do forecasters draw a distinction between point forecasts and density forecasts?
(b) If more than one point and density forecast were available, how might we combine them?
(c) Why does nowcasting play such an important role in real-time forecasting?
6. Why does the constraint of a zero nominal interest rate floor provide such a limitation to conventional monetary policy? What measures have been introduced to ameliorate this constraint?
7. Despite the many challenges to the conduct of macroeconomic policy inflation targeting has survived the events of the last 6 years. Why is this so?

END OF PAPER

ECM9/ECM10/ECM11/ MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil in Finance

Friday 9 May 2014 10:00am to 12:00pm

M300

## Econometric Methods

The paper is in two sections. Section A: Candidates are required to answer all six questions. Section B: Candidates are required to answer two out of four questions.

Weighting: Section $A$ as $60 \%$, Section $B$ as $40 \%$.

Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Table of values of the standard normal density
New Cambridge Elementary Statistical Tables
Durbin-Watson and Dickey-Fuller Tables
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Section A

A. 1 Consider the following joint probability density

$$
f_{X, Y}(x, y)=\left\{\begin{array}{l}
\frac{3}{2}\left(x^{2}+y^{2}\right) \text { if } x \in[0,1], y \in[0,1] \\
0 \text { otherwise }
\end{array}\right.
$$

(i) Find the conditional expectation function $E(Y \mid X=x)$.
(ii) Compute the minimum mean squared error prediction of $Y$ given that $X=1 / 2$.
A. 2 You are given the following estimates of a Probit and a Linear Probability Model for success of failure of mortgage applications

$$
\begin{aligned}
\widehat{\operatorname{Pr}}\left(y_{i}=1 \mid x_{i}\right) & =\Phi\left(\underset{(0.16)}{-2.19}+\underset{(0.47)}{3.00} x_{i}\right), \\
\hat{y}_{i} & =\underset{(0.032)}{\left(0.604 x^{2}\right.}+\underset{(0.098)}{0.08 x_{i}},
\end{aligned}
$$

where $y_{i}=1$ if the application is successful, $y_{i}=0$ otherwise, and $x_{i}$ is the payment-toincome ratio for applicant $i$. The median value of $x_{i}$ is 0.35 .
(i) Assuming that the Probit specification is correct and has a valid causal interpretation, compute the estimate of the marginal causal effect of the payment-to-income ratio on the probability of success at the median value of $x_{i}$.
(ii) Without providing explicit formulas, relate the estimate of the slope in the Linear Probability Model to the causal effects of $x_{i}$ on the probability of success.
A. 3 Consider the following regression

$$
y_{i}=\alpha+\beta x_{i}+u_{i}, i=1, \ldots, n,
$$

where $x_{i}$ is an endogenous variable, in the sense that $\operatorname{Cov}\left(x_{i}, u_{i}\right) \neq 0$, and $n=1000$. Suppose that you have two instruments $z_{1 i}$ and $z_{2 i}$.
(i) Formally state conditions for the validity of $z_{1 i}$ and $z_{2 i}$.
(ii) The value of the $F$-statistic from the first stage regression is 4.56 . What does this imply about the quality of the 2SLS estimator based on $z_{1 i}$ and $z_{2 i}$ ?
A. 4 A random sample $\left(y_{i}\right),(i=1, \ldots, 1000)$, is taken from the exponential distribution with density

$$
f(y)=\theta_{0} e^{-\theta_{0} y} \text { for } y>0 .
$$

In particular, $E y_{i}=\theta_{0}^{-1}$ and $\operatorname{Var}\left(y_{i}\right)=\theta_{0}^{-2}$.
(i) Find the sample's Fisher information, $I\left(\theta_{0}\right)$.
(ii) The maximum likelihood estimate of $\theta_{0}$ equals 1.04. Test a hypothesis that $\theta_{0}=1$ at $5 \%$ asymptotic significance level.
A. 5 The continuously updated GMM procedure based on a random sample $\left(y_{i}, x_{i}\right), i=1, \ldots, n$, and moment conditions

$$
E\left(\varepsilon_{i}\right)=0, E\left(\varepsilon_{i} x_{i}\right)=0, \text { and } E\left(\varepsilon_{i} x_{i}^{2}\right)=0,
$$

where $\varepsilon_{i}=y_{i}-\alpha-\frac{1}{x_{i}-\beta}$, is used to obtain the following estimates: $\hat{\beta}_{G M M}=0.5, \hat{\alpha}_{G M M}=0.63$.
(i) In the context of the above example, describe the continuously updated GMM procedure.
(ii) Suppose that the value of the $J$-statistic is 4.567 . What would you conclude?
A. 6 An investigator would like to test a hypothesis that $\Delta c_{t}$ is an i.i.d. sequence with mean zero and unknown variance $\sigma^{2}$, where $\Delta c_{t}$ is the demeaned quarterly growth of total consumption expenditure in the UK. The alternative of interest is that $\Delta c_{t}$ has a nontrivial first-order autocorrelation. Using data from 1955:Q1 $(t=1)$ to 2013:Q2 $(t=234)$, the investigator computes

$$
\hat{\rho}_{1}=\frac{\frac{1}{T} \sum_{t=2}^{T} \Delta c_{t} \Delta c_{t-1}}{\widehat{\sigma}^{2}}=-0.0346,
$$

where $T=234$, and $\widehat{\sigma}^{2}$ is a consistent estimate of $\sigma^{2}$.
(i) Derive the asymptotic distribution of $\hat{\rho}_{1}$ as $T \rightarrow \infty$, assuming that the null hypothesis is true.
(ii) Use your result in (i) to test the investigator's hypothesis.

## Section B

B. 1 Consider a random sample $\left\{\left(y_{i}, x_{i}\right), i=1, \ldots, n\right\}$, where the conditional distribution of $y_{i}$ given $x_{i}$ is normal with mean zero and variance $\sigma^{2} x_{i}^{2}$, i.e.,

$$
y_{i} \mid x_{i} \sim N\left(0, \sigma^{2} x_{i}^{2}\right) .
$$

Suppose that the marginal distribution of $x_{i}$ does not depend on $\sigma^{2}$ and is such that $\operatorname{Pr}\left(x_{i}=0\right)=0$.
(i) Find the maximum likelihood estimator of $\sigma^{2}$ as a function of $\left(y_{i}, x_{i}\right), i=1, \ldots, n$.
(ii) Show that the maximum likelihood estimator from (i) is conditionally unbiased. Find the conditional Cramer-Rao lower bound on the conditional variance of a conditionally unbiased estimator of $\sigma^{2}$.
(iii) Show that the maximum likelihood estimator obtained in (i) achieves the conditional Cramer-Rao lower bound. [Hint: the raw fourth moment of a standard normal distribution equals 3].
(iv) Find the method of moments estimator of $\sigma^{2}$ that uses the moment condition $E\left(y_{i}^{2} \mid x_{i}\right)=\sigma^{2} x_{i}^{2}$. Is it conditionally unbiased? Find its variance and compare with the variance of the maximum likelihood estimator.
B. 2 Consider a version of the consumption capital asset pricing model, which implies that

$$
E_{t}\left(\beta\left(\frac{c_{t+1}}{c_{t}}\right)^{-\gamma}\left(1+r_{t+1}\right)-1\right)=0
$$

where $E_{t}$ is the expectation conditional on the information available at time $t, c_{t}$ is consumption, and $r_{t}$ is the real rate of interest on a financial asset. To estimate $\beta$ and $\gamma$, a researcher proposes to use GMM based on the uncorrelatedness of $\beta\left(\frac{c_{t+1}}{c_{t}}\right)^{-\gamma}\left(1+r_{t+1}\right)-1$ with a constant, the current consumption growth $c_{t} / c_{t-1}$, and the current rate of interest $r_{t}$.
(i) Write down the GMM's criterion function corresponding to a weighting matrix $W$.
(ii) Suppose that there are 200 observations and that the value of the minimum of the optimal GMM criterion function equals 0.004 . Test the validity of the overidentifying restrictions at $5 \%$ significance level.
(iii) A colleague argues that the consumption growth and the rate of return may be i.i.d. sequences. Assuming that the colleague is right, what does this imply for the asymptotic limit of the GMM criterion function from (i)? What does this imply about the quality of the GMM?
B. 3 A researcher estimates the following regressions

$$
\begin{aligned}
\widehat{\Delta R_{1 t}} & =\underset{(0.084)}{0.245}-\underset{(0.0003)}{0.0008 t}-\underset{(0.0084)}{0.0318} R_{1 t-1}+\sum_{i=1}^{3} \hat{\theta}_{1 i} \Delta R_{1 t-i}, \\
\widehat{\Delta R_{2 t}} & =\underset{(0.063)}{0.206}-\underset{(0.0002)}{0.0007 t}-\underset{(0.0064)}{0.0202} R_{2 t-1}+\sum_{i=1}^{3} \hat{\theta}_{2 i} \Delta R_{2 t-i}, \\
\Delta R_{1 t}-\Delta R_{2 t} & =\underset{(0.008)}{0.022}-\underset{(0.0405)}{0.2604}\left(R_{1 t-1}-R_{2 t-1}\right),
\end{aligned}
$$

where $R_{1 t}$ is Sterling one month mean interbank lending rate, $R_{2 t}$ is the official Bank of England rate, estimates of the standard errors are given in the parentheses, and the data are monthly from Jan 1990 to Oct 2013 (286 observations in total).
(i) Test the hypotheses that $R_{1 t}$ and $R_{2 t}$ are integrated of order one series. Use $1 \%$ significance level.
(ii) Discuss possible consequences for your tests in (i) of not including the time trend in the first two regressions above.
(iii) On theoretical grounds, the researcher decides that the difference between $R_{1 t}$ and $R_{2 t}$ must be stationary. Test the hypothesis that the researcher is wrong at $1 \%$ significance level.
(iv) To forecast the interbank lending rate, the researcher formulates and estimates the following regression

$$
\widehat{\Delta R_{1 t}}=\underset{(0.0138)}{0.0246}+\underset{(0.0472)}{0.424 \Delta} R_{1 t-1}-\underset{(0.0691)}{0.6396}\left(R_{1 t-1}-R_{2 t-1}\right) .
$$

The values of $R_{1 t}$ and $R_{2 t}$ in Oct 2013 were, respectively, $0.48 \%$ and $0.5 \%$. The values of these variables in Sep 2013 were, respectively, $0.476 \%$ and $0.5 \%$. Compute the reseacher's forecast of $R_{1 t}$ for Dec 2013. Describe a method that you would use to evaluate the accuracy of this forecast.
B. 4 Discuss TWO of the following topics
a) Clustered standard errors.
b) Spurious regression.
c) Selection bias and the potental output framework for estimating causal effects.

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance \& Economics
MPhil in Finance

Wednesday 28 May 2014 2:00pm to 4:00pm

M310

## Time Series

Candidates are required to answer five questions from Section $A$ and one question from Section B.

Section A is weighted as $60 \%$ and Section $B$ as $40 \%$
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 2
Rough Work Pad
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

A1. Show that the minimum mean square error predictor of an observation $\ell$ steps ahead is the conditional expectation

$$
\tilde{y}_{T+l \mid T}=E\left(y_{T+l} \mid y_{T}, y_{T-1}, \ldots . .\right) .
$$

A2. Compare and contrast the autocovariance matrices of the differenced observations from a multivariate random walk plus noise (local level) model,

$$
\begin{gathered}
\mathbf{y}_{t}=\boldsymbol{\mu}_{t}+\boldsymbol{\varepsilon}_{t}, \quad \boldsymbol{\varepsilon}_{t} \sim N I D\left(\mathbf{0}, \Sigma_{\varepsilon}\right), \quad t=1, \ldots, T, \\
\boldsymbol{\mu}_{t}=\boldsymbol{\mu}_{t-1}+\boldsymbol{\eta}_{t}, \quad \eta_{t} \sim N I D\left(\mathbf{0}, \Sigma_{\eta}\right),
\end{gathered}
$$

with the autocovariance of a vector $\mathrm{MA}(1)$ process. Show that a common trend in a bivariate local level model implies co-integration.

A3. Express the exponentially weighted moving average (EWMA)

$$
m_{t}=\lambda \sum_{j=0}^{\infty}(1-\lambda)^{j} y_{t-j}, \quad 0<\lambda<1
$$

in terms of the lag operator, $L$. Hence show that detrending a series that is integrated of order one with an EWMA will give a stationary series. Obtain the spectrum of the series obtained when a detrending EWMA filter is appled to a random walk.

A4. When expectations of the rate of inflation, $\pi_{t}$, are based on the New Keynesian Phillips Curve (NKPC),

$$
\pi_{t}=\gamma E_{t}\left(\pi_{t+1}\right)+\beta^{*} x_{t}+\varepsilon_{t}, \quad 0 \leq \gamma \leq 1
$$

where $x_{t}$ is the output gap and $\varepsilon_{t} \sim \operatorname{NID}(0,1)$. If it is assumed that $x_{t}$ is a first-order autoregressive process with parameter $\phi$, obtain an expression for the parameter $\beta$ in the equivalent model

$$
\pi_{t}=\beta x_{t}+\varepsilon_{t}, \quad t=1, \ldots, T .
$$

State any assumptions you make.
A5. A random variable $y_{t}, t=1, \ldots, T$, is defined as

$$
\begin{aligned}
& y_{t}=\varepsilon_{t}, \quad \text { for } t \text { even } \\
& y_{t}=\left(\varepsilon_{t-1}^{2}-1\right) / \sqrt{2}, \quad \text { for } t \text { odd }
\end{aligned}
$$

where $\varepsilon_{t} \sim \operatorname{NID}(0,1)$. Show that $y_{t}$ is white noise. Given a sample in which $T$ is even, write down the minimum mean square linear estimator of $y_{T+1}$ and find its MSE. Can you construct a better predictor ? NB. The kurtosis of a standard normal variate is three.

A6. Write down the definition of a heavy-tailed distribution. The PDF of the Weibull distribution is

$$
f(y ; \alpha, v)=\frac{v}{\alpha}\left(\frac{y}{\alpha}\right)^{v-1} \exp \left(-(y / \alpha)^{v}\right), \quad 0 \leq y<\infty, \quad \alpha, v>0
$$

where $\alpha$ is the scale and $v$ is the shape parameter. Its CDF is $F(y)=1-$ $\exp \left(-(y / \alpha)^{v}\right)$. Determine whether there are any values of the shape parameter for which the distribution has a heavy tail. Sketch the score against $y$ for $v=2$ and $v=0.5$ and comment.

A7. Two variables, $x_{1 t}$ and $x_{2 t}$, have a bivariate normal distribution with zero means, unit variances and a correlation $\rho$. Why is it advantageous to make the arctanh transformation

$$
\gamma=\frac{1}{2} \ln \frac{1+\rho}{1-\rho}=\tanh ^{-1} \rho,
$$

when $\rho$ changes over time? In a dynamic conditional score (DCS) model with time-varying correlation, $\rho_{t \mid t-1}$, the conditional score with respect to $\gamma_{t \mid t-1}$ is

$$
u_{\gamma t}=\frac{1}{4}\left(x_{1 t}+x_{2 t}\right)^{2} \exp \left(-2 \gamma_{t \mid t-1}\right)-\frac{1}{4}\left(x_{1 t}-x_{2 t}\right)^{2} \exp \left(2 \gamma_{t \mid t-1}\right)+\frac{\exp \left(-2 \gamma_{t \mid t-1}\right)-1}{\exp \left(-2 \gamma_{t \mid t-1}\right)+1}
$$

or, equivalently,

$$
u_{\gamma t}=\frac{1}{4}\left(x_{1 t}+x_{2 t}\right)^{2} \frac{1-\rho_{t \mid t-1}}{1+\rho_{t \mid t-1}}-\frac{1}{4}\left(x_{1 t}-x_{2 t}\right)^{2} \frac{1+\rho_{t \mid t-1}}{1-\rho_{t \mid t-1}}+\rho_{t \mid t-1} .
$$

Show that the expected value of $u_{\gamma t}$ is zero. Evaluate $u_{\gamma t}$ when $\rho_{t \mid t-1}=0,-0.5$ and -0.99 for (i) $x_{1 t}=-2$ and $x_{2 t}=2$ and (ii) $x_{1 t}=x_{2 t}=0$ and comment.

## SECTION B

B1. A seasonal component, $\gamma_{t}$, may be added to a model consisting of a trend, $\mu_{t}$, and (white noise) irregular, $\varepsilon_{t}$, to give

$$
y_{t}=\mu_{t}+\gamma_{t}+\varepsilon_{t}, \quad t=1, \ldots, T .
$$

The seasonal pattern may be allowed to change over time by letting the coefficients evolve as random walks. If $\gamma_{j t}$ denotes the effect of season $j$ at time $t$, then

$$
\gamma_{j t}=\gamma_{j, t-1}+\omega_{j t}, \quad j=1, \ldots, s
$$

where $\omega_{j t}, j=1, \ldots, s$, are zero mean disturbance terms. Although all $s$ seasonal components are continually evolving, only one affects the observations at any particular point in time, that is $\gamma_{t}=\gamma_{j t}$ when season $j$ is prevailing at time $t$.
(a) Show that the requirement that the seasonal components evolve in such a way that they always sum to zero is enforced by setting

$$
\operatorname{Var}\left(\boldsymbol{\omega}_{t}\right)=\sigma_{\omega}^{2}\left(\mathbf{I}-s^{-1} \mathbf{i} \mathbf{i}^{\prime}\right), \quad \sigma_{\omega}^{2} \geq 0,
$$

where $\boldsymbol{\omega}_{t}=\left(\omega_{1 t}, \ldots, \omega_{s t}\right)^{\prime}$ and $\mathbf{i}$ is a vector of ones, coupled with $\sum_{j=1}^{s} \gamma_{j 0}=0$ and an initial covariance matrix proportional to $\operatorname{Var}\left(\boldsymbol{\omega}_{t}\right)$.
(b) Put the model for $y_{t}$ in state space form, assuming that the trend is a random walk. If the Kalman filter is started with a diffuse prior, how many observations are needed to obtain a proper prior? Explain briefly how you would estimate the unknown parameters by maximum likelihood.
(c) Write down the (seasonal) ARMA reduced form model for the specification in (ii). Show that this process becomes strictly noninvertible when $\sigma_{\omega}^{2}=0$. Illustrate for the case of $s=2$ and investigate the implications in the frequency domain by obtaining the power spectrum.

B2. Discuss TWO of the following topics and explain their relevance to applied work in macroeconomics and/or finance: (a) Vector error correction models, (b) EGARCH models, (c) the Hodrick-Prescott filter.

ECM9/ECM10/ECM11/ MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil in Finance

Tuesday 3 June 2014 10:00am to 12:00pm

## M320

## Cross Section and Panel Data Econometrics

Candidates are required to answer three questions from Section $A$ and one question from Section B.

Section A is weighted as $60 \%$ and Section B as 40\%
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

A1 Consider the linear static panel data model

$$
\begin{equation*}
\mathbf{y}_{i}=\mathbf{X}_{i} \boldsymbol{\beta}+\boldsymbol{\varepsilon}_{i}, \tag{1}
\end{equation*}
$$

where $\mathbf{y}_{i}$ is a $T \times 1$ vector, $\varepsilon_{i}$ is a $T \times 1$ vector and $\mathbf{X}_{i}$ is $T \times K$ matrix. $\mathbf{Z}_{i}=\left\{\mathbf{z}_{i t}\right\}$ is a $T \times R$ matrix of instruments which satisfies the moment conditions

$$
\begin{equation*}
E\left[\mathbf{Z}_{i}^{\prime} \varepsilon_{i}\right]=\mathbf{0} \tag{2}
\end{equation*}
$$

$\mathbf{z}_{i t}$ is an $R \times 1$ vector and $R \geq K$.
a) Write down the GMM estimator based upon these moment conditions.
b) Noting that the moment conditions in (2) are satisfied if

$$
\begin{equation*}
E\left[\sum_{t=1}^{T} z_{i t} \varepsilon_{i t}\right]=0 \tag{3}
\end{equation*}
$$

comment on the observation that under (3), it is just as difficult to find instruments with panel data as it is with cross-section data.
c) Consider the following alternative set of moment conditions

$$
\begin{equation*}
E\left[\mathbf{z}_{i t} \varepsilon_{i t}\right]=0, \quad t=1, \ldots, T \tag{4}
\end{equation*}
$$

What are the advantages and disadvantages of these moment conditions relative to (3)?

A2 Decision maker $i$ faces a finite choice set $\Omega_{J}$ of dimension $J$. Writing the utility of choice $j$ for individual $i$ as

$$
U_{i j}=V_{i j}+\varepsilon_{i j}
$$

alternative $j$ is chosen if $U_{i j}-U_{i j^{\prime}}>0 \quad \forall_{j} \neq j^{\prime} \in \Omega_{j}$. The probability that $j$ is chosen may then be written

$$
\begin{equation*}
P_{i j}=\int_{\varepsilon} 1\left(\varepsilon_{i j}-\varepsilon_{i j^{\prime}}>V_{i j^{\prime}}-V_{i j}\right) f\left(\varepsilon_{i}\right) d \varepsilon_{i \prime} \tag{5}
\end{equation*}
$$

where $\varepsilon_{i}=\left(\varepsilon_{i 1}, \ldots, \varepsilon_{i j}\right)$.
a) (i.) Given $V_{i j}=\mathbf{v}_{j}^{\prime} \omega$, for fixed parameters $\omega$, and $\varepsilon_{i} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon}\right)$, what is the dimension of the integral in (5). How many parameters are estimable?
(ii.) Given $V_{i j}=\mathbf{v}_{j}^{\prime} \omega_{i}$, with $\omega_{i} \sim N\left(\bar{\omega}, \Sigma_{\omega}\right)$, and $\varepsilon_{i} \sim$ Type 1 Extreme Value, how does the tractability of the probability expression compare to that of (i.) above?
b) For $V_{i j}=\mathbf{v}_{j}^{\prime} \omega$ and $\varepsilon_{i} \sim$ Type 1 Extreme Value, write down the choice probability for alternative $j$.
c) In what sense do the two models implied by a) (ii) and b) represent a trade-off between tractability and the representation of choice behaviour in discrete choice?

A3 Consider the following unobserved effects panel data model

$$
\begin{equation*}
y_{i t}=\alpha+\beta^{\prime} \mathbf{x}_{i t}+\mu_{i}+\varepsilon_{i t} \tag{6}
\end{equation*}
$$

where the $\mu_{i}$ denotes an individual-specific effect and $\varepsilon_{i t}$ is an i.i.d error term. $\mathbf{x}_{i t}$ denotes a $K \times 1$ vector of exogenous variables.
The random effects (or GLS) estimator may be written as

$$
\begin{align*}
\widehat{\boldsymbol{\beta}}_{G L S}= & {\left[\frac{1}{T} \sum_{i=1}^{N} X_{i}^{\prime} \mathbf{Q} X_{i}+\psi \sum_{i=1}^{N}\left(\overline{\mathbf{x}}_{i}-\overline{\mathbf{x}}\right)\left(\overline{\mathbf{x}}_{i}-\overline{\mathbf{x}}\right)^{\prime}\right]^{-1} }  \tag{7}\\
& \times\left[\frac{1}{T} \sum_{i=1}^{N} X_{i}^{\prime} \mathbf{Q} \mathbf{y}_{i}+\psi \sum_{i=1}^{N}\left(\overline{\mathbf{x}}_{i}-\overline{\mathbf{x}}\right)\left(\bar{y}_{i}-\bar{y}\right)\right]
\end{align*}
$$

where $\sigma_{\mu}^{2}$ and $\sigma_{\varepsilon}^{2}$ denote, respectively, the variance of $\mu_{i}$ and $\varepsilon_{i t}$. $\psi=\sigma_{\varepsilon}^{2} /\left(\sigma_{\varepsilon}^{2}+T \sigma_{\mu}^{2}\right), \mathbf{Q}=\mathbf{I}_{T}-\frac{1}{T} \mathbf{i i}, \mathrm{i}$ is a $T \times 1$ unit vector, and $\mathbf{I}_{T}$ is a $T \times T$ identity matrix.
a) Making reference to (7), evaluate the statement that treating $\mu_{i}$ as a random effect provides a solution intermediate between treating all effects as different and treating them all as equal.
b) Let $\kappa=\widehat{\boldsymbol{\beta}}_{G L S, s}-\widehat{\boldsymbol{\beta}}_{F E, s}$ denote a subvector of differences in parameter estimates for time-varying regressors for the GLS and FE estimator.
State the implied null and alternative hypothesis that follow directly from the use of the test statistic $\widehat{H}=\boldsymbol{\kappa}^{\prime}[\operatorname{Var}(\boldsymbol{\kappa})]^{-1} \boldsymbol{\kappa}$.
Explain why one might want to consider a robust form of this test statistic.
A. 4 Consider the following bivariate linear regression model

$$
Y_{i}=\beta_{1}+\beta_{2} X_{i}+\varepsilon_{i}
$$

where $X_{i}$ is a scalar regressor and $E\left(\varepsilon_{i} \mid X_{i}\right)=\tau\left(X_{i}\right)$. A single instrument $Z_{i}$ is available with $\sigma_{\varepsilon_{i} Z_{i}}=0$ and $\sigma_{X_{i}, Z_{i}} \neq 0$.
a) Find an expression for the IV estimator $\widehat{\beta}_{2}^{I V}$.
b) Show that $\widehat{\beta}_{2}^{I V}$ is consistent provided that $\sigma_{\mathrm{ZX}} \neq 0$.
c) Using the results from $b$ ) or otherwise, show that the IV estimator is preferred to OLS on asymptotic grounds when
$\left(\rho_{z, u} / \rho_{z, x}\right)<\rho_{x, u}$.
$\rho_{z, u}$ denotes the correlation coefficient between $z$ and $u$.
d) Carefully explain why in a just-identified model neither the Hansen nor Sargan test for instrument exogeneity can be computed.
A. 5 Elaborate upon the following statements.
a) Relative to the linear probability model, the advantages of the threshold nonlinear binary response model are less when the design matrix is sparse, and/or when there are endogenous binary regressors.
b) In the context of dynamic panel data models, the use of GMMstyle instruments circumvents the sample size efficiency tradeoff.
c) If in a linear fixed effects (FE) static panel data model, time series variation around regressors $\mathrm{x}_{i t}$ is limited, the evaluation of the FE and competing estimators using mean square error may be appropriate.

## SECTION B

B. 1 (a) Suppose $y_{2} \in\{0,1\}$ denotes an endogenous binary treatment and $y_{1} \in\{0,1\}$ is the outcome of interest. $y_{1}(1)$ and $y_{1}(0)$, denote, respectively, the outcome with and without the treatment. $z \in\{0,1\}$ is a scalar binary instrument, independent of $y_{1}(1)$ and $y_{1}(0)$, and correlated with $y_{2}$.
Suppose you have an i.i.d sample of $N$ individuals.
i. Find an expression for the IV estimator for the average treatment effect.
ii. We now impose the following structure on the model:

$$
\begin{aligned}
& y_{1}=\mathbf{1}\left(y_{1}^{*}>0\right) \\
& y_{1}^{*}=\kappa_{y_{1}}+\gamma y_{2}+\varepsilon_{y_{1}} \\
& y_{2}=\mathbf{1}\left(y_{2}^{*}>0\right) \\
& y_{2}^{*}=\kappa_{y_{2}}+\alpha z+\varepsilon_{y_{2}}
\end{aligned}
$$

where $y_{1}^{*}$ and $y_{2}^{*}$ denote latent variables. $\left(\varepsilon_{y_{2}}, \varepsilon_{y_{1}}\right)$ is distributed standard bivariate normal with correlation $\rho$, and independent of $z .1($.$) denotes the indicator function. \kappa_{y_{1}}$, $\kappa_{y_{2}}, \gamma$, and $\alpha$ are unknown parameters.
Find an expression for the average treatment effect for $y_{2}$.
(b) We now consider an alternative model given by

$$
\begin{align*}
& y_{1}^{*}=\mathbf{z}_{1}^{\prime} \delta_{1}+\alpha_{1} y_{2}+u_{1}  \tag{8}\\
& y_{1}=\mathbf{1}\left(y_{1}^{*}>0\right)  \tag{9}\\
& y_{2}=\mathbf{z}_{1}^{\prime} \delta_{21}+\mathbf{z}_{2}^{\prime} \delta_{22}+v_{2}=\mathbf{z}^{\prime} \delta_{2}+v_{2} \tag{10}
\end{align*}
$$

(8) and (9) denotes the structural equation and (10) is the reduced form for $y_{2}$. $\left(u_{1}, v_{2}\right) \sim N(0, \Sigma)$, and is independent of z. We impose the normalisation $\operatorname{Var}\left(u_{1}\right)=1$. $\alpha_{1}, \delta_{1}, \delta_{21}, \delta_{22}$, and $\delta_{2}$ are unknown parameters, and $1($.$) denotes the indicator$ function.
i. Given the distributional assumptions explain why in this instance $y_{2}$ cannot be a discrete random variable.
ii. Find an expression for $\operatorname{Pr}\left(y_{1}=1 \mid \mathbf{z}\right)$.

Show that this method estimates the parameters of (8) up to a common scale, say $1 / \vartheta$. Find an expression for $\vartheta^{2}$.
iii. Using this method, describe a two-step estimation procedure for the parameters of the structural equation.
iv. Contrast the use of $\operatorname{Pr}\left(y_{1}=1 \mid \mathbf{z}\right)$ versus $\operatorname{Pr}\left(y_{1}=1 \mid \mathbf{z}, v_{2}\right)$ as alternate estimation strategies.
B. $2 U_{i j t}$ denotes the utility for individual $i$ who chooses alternative $j$ at time $t$. This utility may be written as follows

$$
\begin{equation*}
U_{i j t}=\mathbf{x}_{i j t}^{\prime} \beta_{i}+\epsilon_{i j t} \tag{11}
\end{equation*}
$$

where $\mathbf{x}_{i j t}$ is a $k \times 1$ vector of explanatory variables, $\beta_{i}$ is a $k \times 1$ parameter vector and $\epsilon_{i j t}$ is a random shock distributed Type 1 extreme value. Individual $i$ chooses alternative $j$ in period $t$ if

$$
U_{i j t}>U_{i m t} \quad \forall j \neq m
$$

We observe $\mathrm{y}_{i}=\left(y_{i 1}, \ldots, y_{i T}\right)^{\prime}$, where $y_{i t}=j$ if individual $i$ chooses alternative $j$ at time $t$.
(a) For $\beta_{i} \sim g(\beta \mid \theta)$, where $\theta$ is a vector of hyperparameters and $g($.$) is the distribution of \beta$ in the population, find:
i. $\operatorname{Pr}\left(\mathbf{y}_{i} \mid \mathbf{x}_{i}, \beta_{i}\right)$ and
ii. $\operatorname{Pr}\left(\mathbf{y}_{i} \mid \mathbf{x}_{i}, \boldsymbol{\theta}\right)$.
$x_{i}$ collects, for each $i$, variables for all alternatives and choice situations.
(b) Find an expression for $h\left(\beta \mid \mathbf{y}_{i}, \mathbf{x}_{i}, \boldsymbol{\theta}\right)$, the density of $\beta$ in the subpopulation of individuals who would choose $\mathbf{y}_{i}$ when facing $\mathrm{x}_{\mathrm{i}}$.
(c) In what sense does a model with preference heterogeneity, as in (11), circumvent the independence of irrelevant alternatives property.
(d) Writing $g\left(\beta \mid \theta=\left(\bar{\beta}, \Sigma_{\beta}\right)\right)$, where $\bar{\beta}$ and $\Sigma_{\beta}$ denote, respectively mean and covariance parameters, and letting $k\left(\bar{\beta}, \Sigma_{\beta}\right)$ denote the prior distribution, find a general expression for the joint posterior distribution for $\bar{\beta}$ and $\Sigma_{\beta}$.
Comment on any difficulties encountered in taking draws from this posterior and how augmenting the posterior with $\beta_{i}$ might help.

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Friday 23 May 2014 2:00pm to 4:00pm

M330

## Applied Econometrics

Candidates are required to answer all four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Durbin-Watson and Dickey-Fuller Tables
New Cambridge Elementary Statistical Tables
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

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1. We examine a sample of $N$ individuals who are eligible for treatment. Let ( $Y_{i}, X_{i}, D_{i} ; i=1,2 . . N$ ), be a vector of observations on the scalar-valued outcome $Y$, a vector of observable covariates $X$ and a binary indicator of treatment $D$.
a) Discuss, in brief, the assumption of conditional independence for the identification of the average treatment effect (ATE) and the average treatment on the treated effect (ATT). Explain the difference in assumptions and the behavioural implications of these assumptions.
b) Discuss the assumption of support or overlap in identifying the two parameters. Again, explain how the assumption required differs between the ATE and ATT and why?
c) What is the key assumption in identifying the treatment effect in a regression discontinuity design? Is the effect thus identified an average treatment effect? Explain and contrast this with the requirements for identification using instrumental variables or matching estimators.
2. The questions below refer to a large-scale randomised evaluation of the impact of teacher performance pay in schools. The aim was to examine whether offering teachers pay based on student performance on tests improved student achievement. The incentive treatment consisted of an announcement that bonuses would be paid at the beginning of the next school year conditional on average improvements in test scores during the current school year.

The overall design of the project was as follows. The eligible sample consisted of 300 schools. 150 schools were in the control group and there was no intervention while in the remaining 150 schools, teachers received bonuses which were conditional on student test scores. Baseline tests of student learning were conducted in the 300 sampled schools two months before the interventions were announced. The baseline test covered material from the previous school year. At the end of the program a final test was conducted, with similar content.

The following specification examines the impact of the incentive program. The variable Incentives, is a binary variable representing treatment, taking the value 1 if it is a treated school and zero otherwise.

$$
T_{i j k m}^{1}=\alpha+\beta T_{i j k m}^{0}+\gamma(\text { Incentives })+\delta Z_{m}+\epsilon_{k}+\epsilon_{j k}+\epsilon_{i j k}
$$

where $T^{t}$ denotes the normalised test score ( $t=0$ denotes the baseline while $t=1$ refers to the end of the program year), $i, j, k, m$ indicate student, grade, school and region respectively, and $Z_{m}$ is a vector of region dummies or region fixed effects. A further specification also includes school and household level variables. Table 1 summarises the results. (Standard errors in brackets).

Table 1: Impact of Incentives on Student Test Scores Dependent Variable: Normalised End-of-Year Test Score

|  | $(1)$ |  |
| :---: | :---: | :---: |
| Normalised lagged test score | $0.503^{* * *}(0.01)$ | $0.498^{* * *}(0.01)$ |
| Incentive school | $0.149^{* * *}(0.04)$ | $0.152^{* * *}(0.04)$ |
| School and household controls | No | Yes |
| $\mathrm{R}^{2}$ | 0.31 | 0.34 |

( $^{* * *}$ Significant at 1 percent; ${ }^{* *}$ Significant at 5 percent; * Significant at 10 percent)
a) Discuss both the specification and results in brief. What further information would you wish to have to persuade yourself that the impact of the intervention is identified?
b) The study also collected information on teacher characteristics such as education, experience and other individual information. The specification below was estimated and the results are presented in Table 2.

$$
\begin{aligned}
& T_{i j k m}^{1}=\alpha+\beta T_{i j k m}^{0}+\gamma_{1}(\text { Incentives })+\gamma_{2}(\text { Characteristics })+ \\
& \gamma_{3} \text { (Incentives*Characteristics) }+\delta Z_{m}+\epsilon_{k}+\epsilon_{j k}+\epsilon_{i j k},
\end{aligned}
$$

Table 2: Impact of Incentives and Teacher Characteristics Dependent Variable: Normalised End-of-Year Test Score

|  | Education | Training | Years of Experience |
| :---: | :---: | :---: | :---: |
| Incentive | $0.113(.163)$ | $0.224(.176)$ | $0.258^{* * *}(.059)$ |
| Characteristic | $0.003(.032)$ | $0.051(.041)$ | $0.001(.003)$ |
| Interaction | $.086^{*}(.050)$ | $0.138^{* *}(.061)$ | $0.009^{* *}(.004)$ |
| $\mathrm{R}^{2}$ | 0.29 | 0.29 | 0.29 |
| (*** Significant at 1 percent; ${ }^{* *}$ Significant at 5 percent; * Significant at 10 |  |  |  |
| percent) |  |  |  |

Explain the rationale for the second specification and interpret the results. An additional specification that included household characteristics such as parental education and incomes and their interaction with the dummy variable, Incentives, was also estimated but none of the coefficients was found significant. Discuss the rationale for this specification as well.
c) Discuss the error structure specified. What is the implication for the computation of standard errors? What is the econometric rationale for the inclusion of the baseline test score?
d) Discuss two issues that might compromise identification in this context.
3. We are interested in analysing the historical performance of the Phillips curve, i.e. the inverse relation between the rate of unemployment $\left(U_{t}\right)$ and the rate of inflation $\left(\pi_{t}\right)$. We start by obtaining least square estimates of the following equation

$$
\pi_{t}=\alpha+\beta U_{t}+\varepsilon_{t}, \quad t=1, \ldots, T,
$$

where $\varepsilon_{t}$ is assumed to be i.i.d. normal with mean zero and variance $\sigma^{2}$. We are concerned that one of the Gauss-Markov assumptions - i.i.d. disturbances - is not satisfied and entertain the hypothesis of autocorrelated disturbances.
a) What are the consequences of residual autocorrelation for least squares estimates? Describe the Lagrange multiplier test procedure and how you could use it to dispel such concerns.
b) Describe the steps involved in obtaining Cochrane-Orcutt estimates of the Phillips curve under the assumption that $\varepsilon_{t}$ follows an autoregressive process of order one.
c) Show that the original model augmented with the assumption of $\mathrm{AR}(1)$ disturbances can be rewritten as an autoregressive distributed lag (ADL) model. Does the original model impose any parameter restrictions on the ADL model? Ignoring such restrictions, is least squares inference based on the (unrestricted) ADL model valid?
4. A researcher is interested in understanding the impact on food expenditures of an income subsidy program for poor households in a village in the Congo. The researcher collected data on household food expenditure and income for 40 randomly sampled households. The following relationship was estimated by OLS:

$$
\widehat{\log } \widehat{\_} \text {food }=\underset{(0.434)}{0.189}+\underset{(0.115)}{0.694} \log \_ \text {income },
$$

where log_food gives the logarithm of household food expenditure, log_income gives the logarithm of household income and figures in parenthesis give standard errors. The researcher stored the residuals, $\widehat{u}$, and then ran the regression:

$$
\widehat{u}^{2}=\widehat{\alpha}+\widehat{\beta} \log \_ \text {income }+\widehat{\gamma} \log \_ \text {income }^{2}
$$

to obtain the following results:

| F-statistic | 3.809082 | Prob. F(2,37) | 0.0313 |
| :--- | :--- | :--- | :--- |
| Obs*R-squared | 6.829652 | Prob. Chi-Square(2) | 0.0329 |
| Scaled explained SS | 4.115153 | Prob. Chi-Square(2) | 0.1278 |

```
Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date:03/09/14 Time:19:00
Sample: }14
Included observations:40
White heteroskedasticity-consistent standard errors & covariance
```

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| :--- | ---: | :--- | ---: | ---: |
| C | 1.775739 | 0.811223 | 2.188966 | 0.0350 |
| LOG_INCOME^2 | 0.124622 | 0.051554 | 2.417298 | 0.0207 |
| LOG_INCOME | -0.933226 | 0.410481 | -2.273493 | 0.0289 |
| R-squared | 0.170741 | Mean dependent var | 0.066338 |  |
| Adjusted R-squared | 0.125917 | S.D. dependent var | 0.077632 |  |
| S.E. of regression | 0.072580 | Akaike info criterion | -2.336208 |  |
| Sum squared resid | 0.194912 | Schwarz criterion | -2.209542 |  |
| Log likelihood | 49.72415 | Hannan-Quinn criter. | -2.290409 |  |
| F-statistic | 3.809082 | Durbin-Watson stat | 1.918104 |  |
| Prob(F-statistic) | 0.031316 |  |  |  |

Table 3: Squared residuals regression.
a) What test is the researcher performing in the squared residuals regression? Do the results in Table 3 cast doubt on the OLS estimates? If so, why?
b) The researcher then calculated White's robust standard errors. What are the merits and drawbacks of pursuing this strategy compared with reestimating the equation by Generalised Least Squares?
c) Figure 1 shows the fitted relationship between income and food expenditure (in levels) and labels the two highest income households, A and B. Do you think these are high leverage observations? Are they influential observations? Justify your answers and name one measure that would help your reasoning.


Figure 1: Scatter plot of household income vs. food expenditure. Solid line gives OLS fit.
d) Why would a robust regression estimation technique - such as Least Absolute Deviations - be of help in this case?
e) Suppose that the researcher decides to drop household $B$ from the sample and reestimates the regression using OLS. Under what condition(s) would this be equivalent to Least Trimmed Squares?

END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Friday 6 June $2014 \quad$ 2:00pm to 4:00pm

M500
Development Economics
Candidates are required to answer two out of three questions.
Each question carries equal weight.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 2
Rough Work Pad
Tags

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Each landlord in a village needs two workers for his land. One worker performs tasks that can be monitored by the landlord. The other worker performs tasks where the effort cannot be observed during each period, while he is working, but is revealed at the end of the period. Wages are paid at the beginning of each period.
(a) The worker assigned to the tasks where effort cannot be observed will evidently shirk if he is hired for a single period. One solution to this moral hazard problem is for the landlord to hire workers, who are infinitelylived, on a permanent basis. There is now a cost to shirking because the permanent worker will be fired by his employer at the end of the period when it is discovered that he failed to put in effort. Let the market clearing wage for casual workers, who perform the monitored task, be $W_{c}$. Let the wage received by the permanent worker be $W_{p}$. Both types of workers incur the same cost of effort $e$, which the permanent workers avoid when they shirk. Casual workers can be monitored, so they never shirk. The landlords in the village communicate with each other, so any permanent worker who shirks will never be hired again as a permanent worker, but will be hired in the future as a casual worker (assume that there is no unemployment). Derive $W_{p}$ as a function of $W_{c}, e$ and $\delta$ (the discount factor).
(b) Now suppose we introduce imperfect communication among the landlords. In particular, let the probability of being rehired as a permanent worker in the period after shirking be $0<P<1$. If a shirker is rehired in the subsequent period, we assume that he stays honest thereafter. If he is not immediately rehired as a permanent worker he will remain as a casual worker forever after. What happens to $W_{p}$ as $P$ increases? Derive the result mathematically and then explain the result.
(c) Suppose, instead, that we introduce an exogenous probability of separation $\tau$ in the model. This means that a permanent worker who puts in effort could still be asked to leave by his landlord. He will become a casual worker for at least one period, but could be rehired in the future as a permanent worker by another landlord. In contrast, if he shirks and is then fired, he will always remain a casual worker. What happens to $W_{p}$ as $\tau$ increases? Explain without explicitly deriving the result mathematically
(d) We effectively assumed in part (a) that no landlord would hire a previous shirker as a permanent worker, which implies that $P=0$. Assuming an exogenous probability of separation, as above, explain why no landlord would want to deviate and hire a previous shirker.

2 Each firm in an industry consists of a single entrepreneur and the firm's output is described by the function: $Y=\omega K^{1 / 2}$, where $\omega$ is the entrepreneur's ability and $K$ is the amount of capital (in dollars) that he invests in the firm. Assume that capital markets are efficient and that the unit cost of capital (the interest rate) is $r$. Normalize so that the unit price of the product is set to one.
(a) Show that (i) high ability entrepreneurs invest more and (ii) higher ability entrepreneurs have higher output.
(b) Now suppose that capital markets are imperfect and firms must turn to informal sources of credit such as their community networks. Half the entrepreneurs in the industry belong to a wealthy caste who moved recently from farming into business and the other half belong to castes that have been engaged in business for generations.
When the firms in the industry were surveyed by a research team, it was discovered that entrepreneurs from the traditional farming community invested more, but had lower output, on average, than entrepreneurs from the traditional business communities. This appears inconsistent with the result in part (a), where entrepreneurs who invested more had higher output. Reconcile this apparent discrepancy, using the framework set up in part (a) to help you. Derive the precise mathematical condition under which this empirical finding could be obtained and then provide the intuition for your answer.
(c) Apart from capital market imperfections, can you think of other explanations for the empirical finding? [Hint: modify the production function or consider imperfections in other markets].
3. a) Marriage in India is endogamous, which implies that individuals can only marry within their sub-caste. In contrast, marriage in sub-Saharan Africa is exogamous, which implies that individuals must marry outside their clan. This implies that marriage in sub-Saharan Africa is accompanied by membership in a new affine network organized around the wife's family and that men must often bring home their brides from far away. Can you provide an economic explanation for these differences in marriage rules?
b) Traditional clan-based networks in Africa have been transplanted to the city in recent decades, with urban migrants receiving support from both their birth network and their affine network (if married) in finding jobs. In turn, they send home remittances and support members of the network in the city, by providing them with accommodation, credit, and other forms of assistance. A team of researchers conducted a survey of male migrants in Kisumu, the third largest city in Kenya, in 2001. They were interested in estimating the effect of marriage on migrant incomes and so ran the following regression:

$$
y_{i}=\alpha+\beta M_{i}+\gamma X_{i}+\varepsilon_{i}
$$

where $y_{i}$ is migrant $i$ 's income, $M_{i}=1$ if he was married, 0 if he was single, $X_{i}$ is the migrant's age, and $\varepsilon_{i}$ is the error term. All men ultimately marry, so the regression was restricted to men aged 20-30. The researchers ran two regressions: OLS and instrumental variables. They found that $\beta$ was positive in each case, but larger in magnitude with the instrumental variable regression (you can assume that the instrument is valid). When they replaced the migrant's income with the fraction of his income that he sent home to his rural network, they obtained the same pattern; $\beta$ was positive, but larger in magnitude with the instrumental variable regression. Can you interpret this result by taking advantage of findings from other papers that you studied in class? [Hint:- think about selection into and out of networks.]

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Wednesday 4 June $2014 \quad$ 2:00pm to 4:00pm

M610

## British Industrialisation

Candidates are required to answer two out of four questions.
Each question carries equal weight.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Describe the key elements that de Vries identifies as characterising the Industrious Revolution. Does the evidence for Britain indicate that an Industrious Revolution occurred?
2. Do we have a satisfactory explanation of population growth in the long-eighteenth century?
3. Early accounts of the Industrial Revolution emphasised rapid growth in output brought about by the widespread use of new technology. To what extent has the work of Crafts challenged this version of events? How does the recent work of ShawTaylor and Wrigley challenge Crafts' interpretation?
4. Assess the role played by agriculture in Britain's industrialisation.

## ECM11

MPhil in Finance and Economics

Thursday 8 May 2014 2:00pm to 4:00pm

## F100

## Finance 1

Candidates are required to answer all four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Suppose Gozilla Inc and the Super-Mario Company have expected returns and volatilities (standard deviations) shown below, with a correlation of $\rho_{G S}=22 \%$.

|  | Expected return | Volatility |
| :--- | :---: | :---: |
| Gozilla Inc | $\bar{r}_{G}=7 \%$ | $\sigma_{G}=16 \%$ |
| Super-Mario | $\bar{r}_{S}=10 \%$ | $\sigma_{S}=20 \%$ |

(a) Calculate (i) the expected return and (ii) the volatility (standard deviation) of a portfolio that is equally invested in Gozilla's and Super-Mario's stock.
(b) If only the correlation between Gozilla's and Super-Mario's stock were to increase, (i) Would the expected return of the portfolio rise or fall?
(ii) Would the volatility of the portfolio rise or fall?
(c) Calculate (i) the expected return and (ii) the volatility (standard deviation) of a portfolio that consists of a long position of $\$ 10,000$ in Gozilla and a short position of $\$ 2000$ in Super-Mario's.
2. A 6 percent six-year bond yields $12 \%$ and a $10 \%$ six-year bond yields $8 \%$. Calculate the six-year spot rate. (Assume annual coupon payments.)
3. Suppose that $c_{1}, c_{2}$, and $c_{3}$ are the prices of European call options with strike prices $X_{1}, X_{2}$, and $X_{3}$, respectively, where $X_{1}<X_{2}<X_{3}$ and $X_{2}-X_{1}=X_{3}-X_{2}$. All options have the same maturity. Show that

$$
c_{2} \leq \frac{c_{1}+c_{3}}{2}
$$

What is the corresponding result for European put options?
4. Consider a firm whose activities have value, $V_{t}$, which follows a Geometric Brownian Motion:

$$
d V_{t}=\mu V_{t} d t+\sigma V_{t} d z_{t}
$$

where $z_{t}$ is a standard Brownian motion. Assume that a riskless asset exists that pays a constant rate of interest $r$. Let $\psi\left(T_{\underline{V}} \mid V_{t}\right)$ denote the probability density function of the
first passage time $T_{\underline{V}}$ to a lower boundary $\underline{V}$, conditional on the current state being $V_{t}$ (where $V_{t}>\underline{V}$ ). The Laplace transform of $\psi\left(T_{\underline{V}} \mid V_{t}\right)$ is here equal to

$$
\int_{t}^{\infty} e^{-r\left(T_{\underline{V}}-t\right)} d \psi\left(T_{\underline{V}} \mid V_{t}\right)=\left(\frac{V_{t}}{\underline{V}}\right)^{\lambda}
$$

where $\lambda=-2 r / \sigma^{2}$.
The firm has outstanding only equity and a perpetual debt contract which promises an infinitely long stream of constant coupon payments, $c$, each unit of time, as well as the residual value of the firm in case of bankruptcy. In the absence of taxes and bankruptcy costs, if the debt remains outstanding without time limit unless bankruptcy is triggered by the value of the firm's assets reaching the value $\underline{V}$, then the value of the debt and equity are

$$
\begin{aligned}
D\left(V_{t}\right) & =\frac{c}{r}+\left[\underline{V}-\frac{c}{r}\right]\left(\frac{V_{t}}{\underline{V}}\right)^{\lambda} \\
S\left(V_{t}\right) & =V_{t}-\frac{c}{r}+\left\{0-\left[\underline{V}^{*}-\frac{c}{r}\right]\right\}\left(\frac{V_{t}}{\underline{V}}\right)^{\lambda} .
\end{aligned}
$$

(a) Consider that (i) debt service payments are tax deductible with $\tau$ being the tax advantage of debt and (ii) bankruptcy entails a proportional cost, $\alpha V_{t}$.
Derive the value of the debt and equity.
(b) Consider that (i) debt service payments are tax deductible with $\tau$ being the tax advantage of debt, (ii) bankruptcy entails a proportional cost, $\alpha V_{t}$, and (iii) shareholders freely choose when to abandon servicing the debt, in a non-cooperative fashion, and that in this event, debt-holders respond triggering bankruptcy.
Derive the share-holders non-cooperative optimal closure rule.

## END OF PAPER

ECM11
MPhil in Finance and Economics

Wednesday 21 May 2014 10:00am to 12:00pm

## F200

## Finance II

Candidates are required to answer all four questions.
All questions carry equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. In a recent closely contested lawsuit, Apex sued Bpex for patent infringement. The jury came back today with its decision.

The market today responded to the announcement of this decision and the rate of return on Apex was $r_{A}=3.1 \%$. The rate of return on Bpex was only $r_{B}=2.5 \%$.

The market today also responded to very encouraging news about the unemployment rate, and $r_{M}=3 \%$. The historical relationship between returns on these stocks and the market portfolio has been estimated from index model regressions as:

Apex: $\quad r_{A}=.2 \%+1.4 r_{M}$
Bpex: $\quad r_{B}=-.1 \%+.6 r_{M}$
Based on these data, which company do you think won the lawsuit?
2. The price of a 1 -year zero-coupon bond is $\$ 934.6$. The price of a 2 -year $4 \%$ coupon bond is $\$ 949.2$. The price of a 3 -year $8 \%$ coupon bond is $\$ 1036.4$.
(a) What are the spot and forward rates embedded in these bond prices?
(b) A 3-year $4 \%$ coupon bond is selling at $\$ 950$. Is there a profit opportunity? If so, how would you take advantage of it?
3. A stock price is currently 80 . Over this coming year it is expected to go up by $50 \%$ or down by $50 \%$. Over the following year it is expected to go up by $70 \%$ or down by $30 \%$. The risk-free interest rate is $10 \%$ per annum over these two years.
(a) Using the binomial model, what is the value of a two years European option to sell with a strike price of 80 ?
(b) Discuss whether you could use the Black-Scholes model to give a more accurate estimate.
(c) Would there be a difference in value if the option was American?
4. Consider a model with two dates $(t=1,2)$, and no discounting. At $t=1$, an entrepreneur is running a firm with assets in place which generate $X \in\left\{X^{L}, X^{H}\right\}$ at $t=2$, with $\Delta_{X} \equiv X^{H}-X^{L}>0$ and $\theta \equiv \operatorname{Pr}\left[X=X^{H}\right]$.
Existing investors hold claims on $X$ with promised repayments at $t=2$ equal to $R^{L}$ if $X=X^{L}$ and $R^{H} \equiv R^{L}+\Delta_{R}$ if $X=X^{H}$. More precisely, they hold debt with face value $K$ so that $R^{L}=\min \left\{K ; X^{L}\right\}$ and $R^{H}=\min \left\{K ; X^{H}\right\}$.
At $t=1$, the entrepreneur considers undertaking a project: Investing $F$ increases $\theta$ to $\left(\theta+\Delta_{\theta}\right)$ and this investment has a positive value, i.e., $\Delta_{\theta} \Delta_{X}-F>0$.
(a) Show that with risky debt, the investment policy is distorted towards underinvestment.
(b) Show that debt forgiveness can be Pareto improving, establishing the face value of the debt, $\hat{R}$, the debtholders are willing to accept, conditionally on the investment being made.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Thursday 22 May 2014 10:00am to 12:00pm

F300
Corporate Finance
Candidates are required to answer all four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. You work for a leveraged buyout firm and are evaluating a potential buyout of Origami Inc. Origami's stock price is $\$ 15$ and it has 10 million shares outstanding. You believe that if you buy the company and replace its management, its value will increase by $50 \%$. You are planning on doing a leveraged buyout of Origami and will offer $\$ 20$ per share for control of the company.
a) Assume you get $50 \%$ control of Origami. What is the value of Origami with the LBO?
b) Assume you get $50 \%$ control of Origami. What is the price of the non-tendered shares? At this price, will shareholders choose to tender their shares?
c) Assuming you get $50 \%$ control of Origami, then what is your gain from this transaction?
2. Skinny Chip Company (SCC) has assets with a market value of $\$ 600$ million, $\$ 70$ million of which are cash. It has debt of $\$ 250$ million, and 20 million shares outstanding. Assume perfect capital markets.
a) What is SCC's current stock price?
b) If SCC distributes the $\$ 70$ million cash as a dividend, then what will the stock price be after the dividend has been paid?
c) If SCC distributes the $\$ 70$ million cash as a share repurchase, then what will the stock price be after the share repurchase?
d) If SCC distributes the $\$ 70$ million cash as a dividend, then what will its debt-toequity ratio be after the dividend has been paid?
e) If SCC distributes the $\$ 70$ million as a share repurchase, then what will its debt-toequity ratio be after the share repurchase?
3. Screwdriver Ltd is an all-equity firm with 60 million shares outstanding, which are currently trading at $\$ 20$ per share. Last month, Screwdriver announced that it will change its capital structure by issuing $\$ 200$ million in debt. The $\$ 200$ million raised by this issue, plus another $\$ 200$ million in cash that Screwdriver already has, will be used to repurchase existing shares of stock. Assume that capital markets are perfect.
a) What is the market capitalization of Screwdriver before and after this transaction takes place?
b) At the conclusion of this transaction, what will Screwdriver's share price be?
c) Suppose you are a shareholder in Screwdriver holding 300 shares, and you disagree with the decision to lever the firm. Show how you can undo the effect of this decision by changing your own portfolio.
4. A risk neutral entrepreneur has cash $A>0$ and wants to undertake a project that costs $I>A$. He seeks to raise $I-A$ from a competitive market of lenders who require zero excess return. If financing is obtained, the entrepreneur exerts effort, either high or low, influencing the probability of success. If effort is high, project succeeds with probability $p_{H}$ while if effort is low, probability of success is $p_{L}<p_{H}$. Let $\Delta p=p_{H}-p_{L}$. In case of success, the project yields $R>0$ while if there is failure, the project yields nothing. Low effort yields a private benefit $B>0$, while high effort yields no private benefit. The entrepreneur is protected by limited liability, and effort is not observed by the lenders. The parameters are such that $p_{H} R-I>0$ and $p_{L} R-I+B<0$. A sharing rule between the lenders and the entrepreneur is a split such that $R=R_{b}+R_{\ell}$. Last, all parties are protected by limited liability.
a) Write up a constraint on the borrower's share $R_{b}$ that ensures that the entrepreneur puts in high effort
b) Write up a constraint that ensures that the lenders are willing to lend to the borrower and find a condition on $A$ that ensures that the project is financed. Explain briefly why there is a tradeoff between NPV and pledgeable income.
c) What is the entrepreneur's net utility if financed and how does it vary when $B$ is changed (supposing that he remains financed after the change)?
d) Show that it is without loss of optimality that the agent always invests his entire net worth $A$ in the project. Hint: Show that if the agent invests $A^{\prime}<A$ in the project and keeps the remainder $\left(A-A^{\prime}\right)$, he can at best do as well as when he invests the entire net worth $A$ (and may be worse off).

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil in Finance

Tuesday 20 May 2014 10:00am to 12:00pm

F400

## Asset Pricing

Candidates are required to answer all four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. There are two assets, the gross returns on which follow a bivariate normal distribution. The gross return on asset $j \in\{1,2\}$ is distributed normally with mean $\mu_{j}$ and variance $\sigma_{j}^{2}$; the correlation coefficient between the two gross returns is $c ; \mu_{2}>\mu_{1}$; and $\sigma_{2}^{2}>\sigma_{1}^{2}>0$. There is also an investor. She has one unit of wealth, which she must allocate between the two assets; and she has coefficient of absolute risk aversion $\rho>0$.
(a) What is meant by the certainty-equivalent wealth of the investor?
(b) Find a formula for her certainty-equivalent wealth.
(c) What is her optimal allocation to asset 2?
(d) What form does this allocation take when $c$ takes on the values $-1,0$ and 1? Comment on your answers.
2. Consider an economy with two dates, namely 0 and 1 , and two possible states at date 1 , namely 1 and 2 . Suppose that only only one asset is traded at date 0 , namely the risky asset with payoff vector

$$
y=\binom{1}{2} .
$$

Suppose further that the price of this asset is 1 .
(a) How would you model a bond in this context?
(b) Find a lower bound and an upper bound for the price of your bond.
(c) Now consider an arbitrary asset

$$
x=\binom{x_{1}}{x_{2}} .
$$

Find the best lower bound and the best upper bound for the price of this asset.
(d) What does your answer to part (c) yield as the best upper and lower bounds for the price of your bond?
3. Suppose that the price $p^{T}$ of a $T$-bond satisfies the stochastic differential equation

$$
d p^{T}(t)=p^{T}(t) m(t, x(t), T) d t+p^{T}(t) v(t, x(t), T) d Z(t)
$$

for $t \in[0, T)$ and

$$
p^{T}(T)=1,
$$

where $Z$ is a standard Wiener process and $m$ and $v$ are given functions. Suppose furthermore that the economic state $x$ evolves according to the stochastic differential equation

$$
d x(t)=\zeta(t, x(t)) d t+\eta(t, x(t)) d Z(t)
$$

where $Z$ is the same Wiener process as above and $\zeta$ and $\eta$ are given functions.
(a) What restrictions would you want to place on the characteristics $m$ and $v$ of the dynamics of bond prices?
(b) What is meant by a forward rate in this model?
(c) Find the stochastic differential equation satisfied by forward rates.
4. (a) What is meant by put-call parity?
(b) Give a careful derivation of the put-call parity relation.

Now suppose that we are given two parameters, namely $0<a<K$. Consider the derivative with final payoff

$$
\Phi(S)=\left\{\begin{array}{ll}
0 & \text { if } S \leq K-a \\
S-(K-a) & \text { if } K-a \leq S \leq K \\
(K+a)-S & \text { if } K \leq S \leq K+A \\
0 & \text { if } K+a \leq S
\end{array}\right\}
$$

where $S$ is the price of the underlying at maturity $T$.
(c) Draw a graph of $\Phi$ in the case $K=5$ and $a=1$.
(d) What is the interpretation of this derivative?
(e) What can you say about the price of this derivative at time 0 ?

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Friday 30 May 2014 2:00pm to 4:00pm

F500

## Empirical Finance

This paper is in two sections. Section A: Candidates are required to answer all four questions. Section B: Candidates are required to answer two out of three questions.

Each section carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION<br>Campbell, Lo \& Mackinlay Tables<br>Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Section A

A1. Suppose that daily stock returns follow a moving average process

$$
r_{t}=\varepsilon_{t}+\theta \varepsilon_{t-1}
$$

where $\varepsilon_{t} \sim N(0,1)$.
(a) What is the mean, variance, and autocovariance function of the daily return series?
(b) Suppose we consider weekly returns

$$
r_{W s}=r_{5 s}+r_{5 s-1}+r_{5 s-2}+r_{5 s-3}+r_{5 s-4}
$$

for $s=1,2, \ldots$ What is the mean, variance, and autocovariance function of the weekly return series? (Hint: You may find it helpful to look at $\operatorname{cov}\left(r_{W 2}, r_{W 3}\right)$ )
(c) What does this say about the autocorrelation function of weekly returns? And about monthly returns?
(d) Are these results consistent with those presented in the table (Table 2.4 from CLM)?
(e) Suppose instead that

$$
r_{t}=\varepsilon_{t}+\theta \varepsilon_{t-5}
$$

where $\varepsilon_{t} \sim N(0,1)$. How does this affect the answers to b and c ?
A2. True, False, Explain
(a) Rational Bubbles are logically impossible
(b) In the absence of uninformed traders, bid ask spreads would be narrower, and so markets would be more efficient.
(c) The equity premium puzzle is explained by long run risks
(d) The hypothesis RW1 (Random Walk) implies RW2 which implies RW2.5 (The Martingale Hypothesis) which implies RW3. Therefore, if one rejects RW3, then one rejects RW1.
(e) The presence of stock splits can be explained only by tradition.

A3. Describe what is meant by the "matching approach" to carrying out event studies. What are the strengths and weakness of this approach in comparison with the usual approach based on CAR comparisons?

A4. Suppose that (excess) returns obey a linear K-factor model

$$
Z_{i t}=\mu_{i}+\sum_{j=1}^{K} b_{i j} f_{j t}+\varepsilon_{i t}
$$

where $\varepsilon_{i t}$ is an (iid over $t$ ) idiosyncratic error term with

$$
\operatorname{cov}\left(\varepsilon_{i t}, f_{j s}\right)=0 \quad ; \quad E\left(\varepsilon_{i t} \varepsilon_{j s}^{\top}\right)=\left\{\begin{array}{cc}
\sigma_{i j} & \text { if } t=s \\
0 & \text { else }
\end{array} .\right.
$$

You observe the panel $\left\{Z_{i t}, i=1, \ldots, N, t=1, \ldots, T\right\}$.
(a) The factors are themselves not directly observed. Describe how you might estimate the factors $\left\{f_{j t}, j=1, \ldots, K\right\}$ in two cases:
i. The time series $T$ is large compared with the number of assets $N$;
ii. The time series $T$ is short compared with the number of assets $N$.
(b) Suppose now that factor returns are assumed to be observed. How would you test the Arbitrage Pricing Theory?

## Section B

B1. Explain Campbell's Approximate Model of stock prices

$$
\begin{aligned}
p_{t} & =\frac{k}{1-\rho}+E_{t} \sum_{j=0}^{\infty} \rho^{j}\left[(1-\rho) d_{t+1+j}-r_{t+1+j}\right] \\
& =\frac{k}{1-\rho}+(1-\rho) \sum_{j=0}^{\infty} \rho^{j} E_{t} d_{t+1+j}-\sum_{j=0}^{\infty} \rho^{j} E_{t} r_{t+1+j},
\end{aligned}
$$

where $p_{t}$ is $\log$ stock prices, $d_{t}$ is $\log$ dividends, and $r_{t}$ is stock returns, while $E_{t}$ denotes expectation conditional on all information at time $t$. Specifically, explain how it is derived and what is the constant $\rho$ ? Explain how this equation might be used to explain the observed variability of stock prices. Explain how one might use Vector Autoregressions to obtain estimates of $E_{t} r_{t+1+j}$.

B2. Describe the consumption capital asset pricing model (CCAPM). How is it derived and how it restricts consumption and asset returns. How could you test this hypothesis in practice? Describe the data you would need and the econometric methods you might use. Consider the table (Table 8.1 from CLM). What does this say about the plausibility of the CCAPM?

B3. Explain the calendar time hypothesis and the trading time hypothesis. How would you distinguish between these two hypotheses on the basis of stock return data. Explain the type of data you would need and how you would carry out the test. What if the results lie between the two cases, why are the possible explanations for this, and how might you go about distinguishing them?

## END OF PAPER

CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Tuesday 3 June $2014 \quad$ 2:00pm to 4:00pm

F510
International Finance
Candidates are required to answer four out of five questions.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## 1. Carry trade

Let $\mathcal{F}_{t}$ and $\mathcal{E}_{t}$ denote forward and spot exchange rates (units of domestic currency per unit of foreign currency). Suppose the foreign currency trades at a forward premium.
(a) Write up and explain a 'carry trade' strategy using forward contracts (possibly using bid and ask spread)?
(b) Based on the readings discussed during lectures, if you were to form a diversified carry-trade portfolio of many currencies, you would expect a lower variance of return and a lower excess return?

## 2. Covered Interest Parity

(a) Define the covered interest parity (CIP) condition.
(b) Carefully discuss conditions under which the CIP is violated.
(c) Provide an illustration of one of this condition making use of the asset pricing condition $q=E(D x)$, where $q$ is the price of an asset at $\mathrm{t}=0, D$ is the stochastic discount rate between $t=0$ and $\mathrm{t}=1, x$ is the stochastic cash flow at $\mathrm{t}=1, E$ denotes expectations.

## 3. International borrowing and sovereign risk

Consider a country populated by a continuum of mass 1 residents, existing for two periods. These residents are ex-ante identical, but their endowment may differ ex post, depending on the realization of individual income shocks. Specifically, in period 0 , all residents have the same endowment equal to $.5 y$. In period 1 , each one of thern may get 2 y with probability $1 / 2$ (if lucky); or get $y$ with the same probability (if unlucky). The preference and endowment of individual $i$ is

$$
\begin{aligned}
& U\left(c_{i, 0}\right)+E U\left(c_{i, 1}\right) \\
& y_{i, 0}=.5 y \quad \text { and } \quad y_{i, 1}=\left\{\begin{array}{cl}
2 y & \text { with prob. } 1 / 2 \\
y & \text { with prob. } 1 / 2
\end{array}\right.
\end{aligned}
$$

Residents can trade assets (financial contracts) with international investors who are risk neutral and do not discount the future.

The domestic government is interested in the welfare of its citizens. Output is verifiable, but there is no international institution that enforces international contracts.

## Sovereign risk

(a) Describe the contract that would allow individuals to smooth consumption completely across time and contingencies (hence achieve efficient risk sharing). Describe (a) cash flow $x$ from and to each individual $i$ conditional on having high or low income at time 1 and (b) its price $q$ at time 0 .
(b) Explain carefully whether, with no international authority able to enforce cross-border contracts, residents in the country will be able to access international markets and enter these contracts.

## The role of penalties

Suppose that the international community can impose a penalty equal to $.4 y$ if the country residents do not pay their debt.
(c) To what extent will the residents be able to borrow? (hint: calculate the price of the expected cash flow).
(d) Calculate the consumption of the 'lucky' and 'unlucky' residents in period 1. Is there scope for domestic residents to insure each others' income?

## 4. Currency Crises

Suppose the central bank of an emerging market economy aims to peg its currency, the peso, to the US dollar so that $S=\bar{S}$, where $S$ is the peso price of US dollars. The central bank balance sheet is given by

$$
M=B+R
$$

where $M$ denotes the money supply, $B$ domestic government bonds held by the central bank and $R$ foreign reserves. If the central bank has depleted its foreign reserves, it adopts a free float. Assume that the government runs a structural budget deficit and forces the central bank to buy government bonds so that $\dot{B} / B=\gamma>0$, where $\dot{X} \equiv \mathrm{~d} X / \mathrm{d} t$ denotes the time derivative of $X$.
Money market equilibrium is given by

$$
\frac{M}{P}=Y e^{-i}
$$

where $P$ is the aggregate price level, which is flexible, $i$ is the nominal interest rate, and $Y$ is aggregate output, which is exogenous and normalized to $Y=1$. Purchasing power parity holds so that

$$
P=S P^{*}
$$

where $P^{*}$ is the US aggregate price level, which is exogenous and normalized to $P^{*}=1$. In addition, assume that

$$
i=i^{*}+\dot{S} / S
$$

where $i^{*}$ is the US nominal interest rate.
(a) Derive the level of the money supply $M$ that is required to keep the exchange rate fixed at $S=\bar{S}$. Explain intuitively why the exchange rate peg is unsustainable.
(b) Derive the time $T$ of the speculative attack (assuming no speculative bubbles). Explain how it depends on the initial level of foreign reserves $R_{0}$.
(c) Suppose that after fixing the exchange rate at $S=\bar{S}$ at $t=0$, there is suddenly an increase in the US interest rate $i^{*}$ at time $\tau$, where $0<\tau<T$. Explain whether this could trigger a currency crisis.

## 5. Risk and international borrowing

Suppose you have two individuals (or countries) with identical preferences over two periods

$$
U\left(C_{t}\right)+\beta E_{t} U\left(C_{t+1}\right)
$$

so that their fundamental stochastic discount will be

$$
D=\frac{\beta U^{\prime}\left(C_{t+1}\right)}{U^{\prime}\left(C_{t}\right)}
$$

Their current endowment is identical $Y_{t}=Y_{t}^{*}$. Their future endowment has the same expected value $E_{t} Y_{t+1}=E_{t} Y_{t+1}^{*}$ but different variance: the domestic endowment is more variable than the foreign one, i.e. $\operatorname{Var}\left(Y_{t+1}^{*}\right)>\operatorname{Var}\left(Y_{t+1}\right)$. Suppose the two can trade in a sure bond $B$, that is exchanged at the price $q_{t}$ today, and promise one unit of goods at $t+1$.
(a) Which individual/country will tend to borrow? Under what condition on the utility function? (Hint: compare asset prices before trade in assets). Explain intuitively.
(b) What lessons can you draw about the implications of integrating countries/regions with different level of output variability for global imbalances and the real interest rate?

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
Monday 26 May 2014 10:00am to 12:00pm

F520

## Behavioural Finance

Candidates are required to answer two out of three questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Behavioural corporate finance is a fairly recent area of research.
(a) What is behavioural corporate finance? Broadly outline the major directions of research and how they relate to standard economic theory and corporate finance.
(b) In behavioural models, what are the implications of manager overconfidence or optimism for firms' investment behaviour and capital structure?
(c) Using Dr Siming's lecture as a starting point, discuss the pros and cons of national honours systems from the point of view of firm owners.
2. Risk preferences are important on financial markets.
(a) Prospect theory is about decision making under risk. Outline the key characteristics of the theory and contrast them to neoclassical theory.
(b) What is the equity premium puzzle and how has prospect theory been used to explain it?
(c) Empirically, how do risk taking and risk preferences compare to the assumptions in neoclassical theory? How can we explain the deviations?
3. Momentum, under and overreaction are three concepts that we have encountered in the course.
(a) Outline what these are and how they are related to each other.
(b) What behavioural explanations have been given for these phenomena?
(c) Although stock markets seemingly overreact so called contrarian investment strategies are not really profitable. Outline the basic idea behind such strategies and discuss why they may not be profitable. In your discussion, draw on concepts that we have encountered in the course.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Thursday $5^{\text {th }}$ June $2014 \quad$ 2:00pm to $4: 00 \mathrm{pm}$

F540

## Topics in Applied Asset Management

This paper is in two sections:
Section A: candidates are required to answer one compulsory question.
Section B: Candidates are required to answer two out of four questions.
Section A is weighted at $40 \%$ and section B is weighted at $60 \%$.
Provide as much mathematical detail as you can to justify your answers.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad
New Cambridge Elementary Statistical Tables

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION <br> New Cambridge Elementary Statistical Tables <br> Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

A1 Provide as much mathematical detail as you can when answering these questions.
(a) Diversification;
i. Derive a formula that shows how the portfolio variance in a portfolio with $\frac{1}{N}$ weights declines to some limit as the number of assets to the portfolio is increased. Interpret the result and discuss any limitations given the assumptions you have used to derive the formula.
ii. Derive the expression given by Booth and Fama which shows the return contribution due to diversification of an individual asset to a portfolio. Interpret the result.
(b) Describe what is meant by a Risk Parity Strategy and comment on their relative advantages/disadvantages.
(c) Show the steps in the derivation of the Parametric Portfolios of Brandt, Santa Clara and Valkanov and explain carefully why this approach might be preferred to direct computation of the portfolio weights through the use of "plug" in estimates of the moments as in classical Mean Variance.
(d) Explain the results obtained by Kan and Zhou on the conditions under which we might expect mean variance to dominate $\frac{1}{N}$
(e) DeMiguel, Garlappi and Uppal (DGU) (2007) argue that Mean Variance allocations are generally dominated by $\frac{1}{N}$ allocations;
i. Describe two approaches that have been put forward to "rescue" Mean Variance.
ii. Explain what is meant by Volatility Timing Strategies and explain why they might dominate $\frac{1}{N}$ and the tangency portfolio.
iii. What critical elements of the practical portfolio allocation problem became apparent in the results of DGU and other papers you have studied which would suggest their strong conclusions should be qualified.

## SECTION B

B1 Predictability;
(a) Criticise the methodology of Goyal and Welch and describe the modifications to their analysis made by Campbell and Thompson when considering the issue of whether asset returns are predicable with conditioning information.
(b) Show, following Mark Britten-Jones, how the optimal weights for the tangency portfolio can be computed through a linear regression. Hence explain the technical difficulties you can see that are common between determining robust predictability and robust portfolio weights.
(c) Derive from first principles the formula Campbell and Thompson develop that demonstrates the relative importance of the $R^{2}$ in the predictive conditional regression in comparison of the Sharpe Ratio of the asset in question to determine if there is likely to be profitable conditioning information.
(d) In the light of the formula in (c) above, do you believe Campbell and Thompson have provided sufficient evidence of profitable conditional predictability and if so provide some detail?

B2 Case Studies: Trend Following and Carry;
(a) Define formally what is meant by "Carry" and how it is measured in the case of Equities, Foreign Exchange and Commodities
(b) Explain the distinction between cross sectional momentum and the form of trend following followed by CTA's
(c) Why are trend following CTA's often considered to be cheap forms of lookback straddle option strategies?
(d) Provide two rationalisations for the degree of predictability observed historically in managed futures returns?
(e) Show how the returns from a cross section momentum strategy can be decomposed into three components; terms reflecting time series autocorrelation, cross covariances and a mean term, following Lo and MacKinlay as discussed by Moskowitz, Oi and Pedersen. What results did Moskowitz, Oi and Pedersen find as to the relative values of these three components with futures data with what implications?

B3 Volatility Measurement;
(a) Define what is meant by a GARCH, EGARCH and an ARCH-M model. What are their relative advantages?
(b) Clark introduced an Information flow model; explain how this explains why asset returns are non- Gaussian and why the ARCH family will usually be unable to correctly capture the true underlying volatility process.
(c) Explain how the parameters of a general $\operatorname{GARCH}(\mathrm{p}, \mathrm{q})$ model may be estimated.
(d) In a multivariate context Risk Management often requires the estimation of large covariance matrices for a portfolio. Explain how the number of parameters to be estimated may be reduced through the use of
i. Constant Correlation GARCH
ii. Orthogonal GARCH

B4 Risk Management;
(a) What is the relationship between Value at Risk and Expected Shortfall?
(b) Explain what is meant by the Sortino Ratio, Calmar Ratio and Maximum Drawdown.
(c) What is meant by Filtered historical simulation when used to compute VaR on a single asset
(d) Suppose that the change in the value of a portfolio over a 1 day time period is Normal with a mean of zero and a standard deviation of $\$ 2$ million
i. What is the (a) 1-day $97.5 \% \mathrm{VaR}$, (b) the 5 -day $97.5 \% \mathrm{VaR}$ and (c) the 5 -day $99 \%$ VaR
ii. What for questions (b) and (c) would be the answer be if there is first order autocorrelation with the autocorrelation parameter equal to 0.16 ?
(e) Suppose that each of two investments has a $4 \%$ chance of loss of $\$ 10$ million, a $2 \%$ chance of a loss of $\$ 1$ million and a $94 \%$ chance of a profit of $\$ 1$ million. They are independent of each other.
i. What is the VaR for one of the investments when the confidence interval is $95 \%$
ii. What is the expected shortfall when the confidence level is $95 \%$
iii. What is the VaR for a portfolio consisting of the two investments when the confidence interval is $95 \%$
iv. What is the expected shortfall for a portfolio consisting of the two investments when the confidence level is $95 \%$
v. Show that, in this example, VaR does not satisfy the sub-additivity condition whereas expected shortfall does?

## END OF PAPER

1. In a recent closely contested lawsuit, Apex sued Bpex for patent infringement. The jury came back today with its decision.

The market today responded to the announcement of this decision and the rate of return on Apex was $r_{A}=3.1 \%$. The rate of return on Bpex was only $r_{B}=2.5 \%$.

The market today also responded to very encouraging news about the unemployment rate, and $r_{M}=3 \%$. The historical relationship between returns on these stocks and the market portfolio has been estimated from index model regressions as:

Apex: $\quad r_{A}=.2 \%+1.4 r_{M}$
Bpex: $\quad r_{B}=-.1 \%+.6 r_{M}$
Based on these data, which company do you think won the lawsuit?
2. The price of a 1 -year zero-coupon bond is $\$ 934.6$. The price of a 2 -year $4 \%$ coupon bond is $\$ 949.2$. The price of a 3 -year $8 \%$ coupon bond is $\$ 1036.4$.
(a) What are the spot and forward rates embedded in these bond prices?
(b) A 3 -year $4 \%$ coupon bond is selling at $\$ 950$. Is there a profit opportunity? If so, how would you take advantage of it?
3. A stock price is currently 80 . Over this coming year it is expected to go up by $50 \%$ or down by $50 \%$. Over the following year it is expected to go up by $70 \%$ or down by $30 \%$. The risk-free interest rate is $10 \%$ per annum over these two years.
(a) Using the binomial model, what is the value of a two years European option to sell with a strike price of 80 ?
(b) Discuss whether you could use the Black-Scholes model to give a more accurate estimate.
(c) Would there be a difference in value if the option was American?
4. Consider a model with two dates $(t=1,2)$, and no discounting. At $t=1$, an entrepreneur is running a firm with assets in place which generate $X \in\left\{X^{L}, X^{H}\right\}$ at $t=2$, with $\Delta_{X} \equiv X^{H}-X^{L}>0$ and $\theta \equiv \operatorname{Pr}\left[X=X^{H}\right]$.
Existing investors hold claims on $X$ with promised repayments at $t=2$ equal to $R^{L}$ if $X=X^{L}$ and $R^{H} \equiv R^{L}+\Delta_{R}$ if $X=X^{H}$. More precisely, they hold debt with face value
$K$ so that $R^{L}=\min \left\{K ; X^{L}\right\}$ and $R^{H}=\min \left\{K ; X^{H}\right\}$.
At $t=1$, the entrepreneur considers undertaking a project: Investing $F$ increases $\theta$ to $\left(\theta+\Delta_{\theta}\right)$ and this investment has a positive value, i.e., $\Delta_{\theta} \Delta_{X}-F>0$.
(a) Show that with risky debt, the investment policy is distorted towards underinvestment.
(b) Show that debt forgiveness can be Pareto improving, establishing the face value of the debt, $\hat{R}$, the debtholders are willing to accept, conditionally on the investment being made.

## END OF PAPER

## ECM11

MPhil in Finance and Economics

Thursday 8 May 2014 2:00pm to 4:00pm

## F100

## Finance 1

Candidates are required to answer all four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Suppose Gozilla Inc and the Super-Mario Company have expected returns and volatilities (standard deviations) shown below, with a correlation of $\rho_{G S}=22 \%$.

|  | Expected return | Volatility |
| :--- | :---: | :---: |
| Gozilla Inc | $\bar{r}_{G}=7 \%$ | $\sigma_{G}=16 \%$ |
| Super-Mario | $\bar{r}_{S}=10 \%$ | $\sigma_{S}=20 \%$ |

(a) Calculate (i) the expected return and (ii) the volatility (standard deviation) of a portfolio that is equally invested in Gozilla's and Super-Mario's stock.
(b) If only the correlation between Gozilla's and Super-Mario's stock were to increase, (i) Would the expected return of the portfolio rise or fall?
(ii) Would the volatility of the portfolio rise or fall?
(c) Calculate (i) the expected return and (ii) the volatility (standard deviation) of a portfolio that consists of a long position of $\$ 10,000$ in Gozilla and a short position of $\$ 2000$ in Super-Mario's.
2. A 6 percent six-year bond yields $12 \%$ and a $10 \%$ six-year bond yields $8 \%$. Calculate the six-year spot rate. (Assume annual coupon payments.)
3. Suppose that $c_{1}, c_{2}$, and $c_{3}$ are the prices of European call options with strike prices $X_{1}, X_{2}$, and $X_{3}$, respectively, where $X_{1}<X_{2}<X_{3}$ and $X_{2}-X_{1}=X_{3}-X_{2}$. All options have the same maturity. Show that

$$
c_{2} \leq \frac{c_{1}+c_{3}}{2}
$$

What is the corresponding result for European put options?
4. Consider a firm whose activities have value, $V_{t}$, which follows a Geometric Brownian Motion:

$$
d V_{t}=\mu V_{t} d t+\sigma V_{t} d z_{t}
$$

where $z_{t}$ is a standard Brownian motion. Assume that a riskless asset exists that pays a constant rate of interest $r$. Let $\psi\left(T_{\underline{V}} \mid V_{t}\right)$ denote the probability density function of the
first passage time $T_{\underline{V}}$ to a lower boundary $\underline{V}$, conditional on the current state being $V_{t}$ (where $V_{t}>\underline{V}$ ). The Laplace transform of $\psi\left(T_{\underline{V}} \mid V_{t}\right)$ is here equal to

$$
\int_{t}^{\infty} e^{-r\left(T_{\underline{V}}-t\right)} d \psi\left(T_{\underline{V}} \mid V_{t}\right)=\left(\frac{V_{t}}{\underline{V}}\right)^{\lambda}
$$

where $\lambda=-2 r / \sigma^{2}$.
The firm has outstanding only equity and a perpetual debt contract which promises an infinitely long stream of constant coupon payments, $c$, each unit of time, as well as the residual value of the firm in case of bankruptcy. In the absence of taxes and bankruptcy costs, if the debt remains outstanding without time limit unless bankruptcy is triggered by the value of the firm's assets reaching the value $\underline{V}$, then the value of the debt and equity are

$$
\begin{aligned}
D\left(V_{t}\right) & =\frac{c}{r}+\left[\underline{V}-\frac{c}{r}\right]\left(\frac{V_{t}}{\underline{V}}\right)^{\lambda} \\
S\left(V_{t}\right) & =V_{t}-\frac{c}{r}+\left\{0-\left[\underline{V}^{*}-\frac{c}{r}\right]\right\}\left(\frac{V_{t}}{\underline{V}}\right)^{\lambda} .
\end{aligned}
$$

(a) Consider that (i) debt service payments are tax deductible with $\tau$ being the tax advantage of debt and (ii) bankruptcy entails a proportional cost, $\alpha V_{t}$.
Derive the value of the debt and equity.
(b) Consider that (i) debt service payments are tax deductible with $\tau$ being the tax advantage of debt, (ii) bankruptcy entails a proportional cost, $\alpha V_{t}$, and (iii) shareholders freely choose when to abandon servicing the debt, in a non-cooperative fashion, and that in this event, debt-holders respond triggering bankruptcy.
Derive the share-holders non-cooperative optimal closure rule.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Thursday 22 May 2014 10:00am to 12:00pm

F300

## Corporate Finance

Candidates are required to answer all four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. You work for a leveraged buyout firm and are evaluating a potential buyout of Origami Inc. Origami's stock price is $\$ 15$ and it has 10 million shares outstanding. You believe that if you buy the company and replace its management, its value will increase by $50 \%$. You are planning on doing a leveraged buyout of Origami and will offer $\$ 20$ per share for control of the company.
a) Assume you get $50 \%$ control of Origami. What is the value of Origami with the LBO?
b) Assume you get $50 \%$ control of Origami. What is the price of the non-tendered shares? At this price, will shareholders choose to tender their shares?
c) Assuming you get $50 \%$ control of Origami, then what is your gain from this transaction?
2. Skinny Chip Company (SCC) has assets with a market value of $\$ 600$ million, $\$ 70$ million of which are cash. It has debt of $\$ 250$ million, and 20 million shares outstanding. Assume perfect capital markets.
a) What is SCC's current stock price?
b) If SCC distributes the $\$ 70$ million cash as a dividend, then what will the stock price be after the dividend has been paid?
c) If SCC distributes the $\$ 70$ million cash as a share repurchase, then what will the stock price be after the share repurchase?
d) If SCC distributes the $\$ 70$ million cash as a dividend, then what will its debt-toequity ratio be after the dividend has been paid?
e) If SCC distributes the $\$ 70$ million as a share repurchase, then what will its debt-toequity ratio be after the share repurchase?
3. Screwdriver Ltd is an all-equity firm with 60 million shares outstanding, which are currently trading at $\$ 20$ per share. Last month, Screwdriver announced that it will change its capital structure by issuing $\$ 200$ million in debt. The $\$ 200$ million raised by this issue, plus another $\$ 200$ million in cash that Screwdriver already has, will be used to repurchase existing shares of stock. Assume that capital markets are perfect.
a) What is the market capitalization of Screwdriver before and after this transaction takes place?
b) At the conclusion of this transaction, what will Screwdriver's share price be?
c) Suppose you are a shareholder in Screwdriver holding 300 shares, and you disagree with the decision to lever the firm. Show how you can undo the effect of this decision by changing your own portfolio.
4. A risk neutral entrepreneur has cash $A>0$ and wants to undertake a project that costs $I>A$. He seeks to raise $I-A$ from a competitive market of lenders who require zero excess return. If financing is obtained, the entrepreneur exerts effort, either high or low, influencing the probability of success. If effort is high, project succeeds with probability $p_{H}$ while if effort is low, probability of success is $p_{L}<p_{H}$. Let $\Delta p=p_{H}-p_{L}$. In case of success, the project yields $R>0$ while if there is failure, the project yields nothing. Low effort yields a private benefit $B>0$, while high effort yields no private benefit. The entrepreneur is protected by limited liability, and effort is not observed by the lenders. The parameters are such that $p_{H} R \quad I>0$ and $p_{L} R \quad I+B<0$. A sharing rule between the lenders and the entrepreneur is a split such that $R=R_{b}+R_{\ell}$. Last, all parties are protected by limited liability.
a) Write up a constraint on the borrower's share $R_{b}$ that ensures that the entrepreneur puts in high effort
b) Write up a constraint that ensures that the lenders are willing to lend to the borrower and find a condition on $A$ that ensures that the project is financed. Explain briefly why there is a tradeoff between NPV and pledgeable income.
c) What is the entrepreneur's net utility if financed and how does it vary when $B$ is changed (supposing that he remains financed after the change)?
d) Show that it is without loss of optimality that the agent always invests his entire net worth $A$ in the project. Hint: Show that if the agent invests $A^{\prime}<A$ in the project and keeps the remainder $\left(A-A^{\prime}\right)$, he can at best do as well as when he invests the entire net worth $A$ (and may be worse off).

## END OF PAPER

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil in Finance

Tuesday 20 May 2014 10:00am to 12:00pm

F400

## Asset Pricing

Candidates are required to answer all four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. There are two assets, the gross returns on which follow a bivariate normal distribution. The gross return on asset $j \in\{1,2\}$ is distributed normally with mean $\mu_{j}$ and variance $\sigma_{j}^{2}$; the correlation coefficient between the two gross returns is $c ; \mu_{2}>\mu_{1}$; and $\sigma_{2}^{2}>\sigma_{1}^{2}>0$. There is also an investor. She has one unit of wealth, which she must allocate between the two assets; and she has coefficient of absolute risk aversion $\rho>0$.
(a) What is meant by the certainty-equivalent wealth of the investor?
(b) Find a formula for her certainty-equivalent wealth.
(c) What is her optimal allocation to asset 2?
(d) What form does this allocation take when $c$ takes on the values $-1,0$ and 1? Comment on your answers.
2. Consider an economy with two dates, namely 0 and 1 , and two possible states at date 1 , namely 1 and 2 . Suppose that only only one asset is traded at date 0 , namely the risky asset with payoff vector

$$
y=\binom{1}{2} .
$$

Suppose further that the price of this asset is 1 .
(a) How would you model a bond in this context?
(b) Find a lower bound and an upper bound for the price of your bond.
(c) Now consider an arbitrary asset

$$
x=\binom{x_{1}}{x_{2}} .
$$

Find the best lower bound and the best upper bound for the price of this asset.
(d) What does your answer to part (c) yield as the best upper and lower bounds for the price of your bond?
3. Suppose that the price $p^{T}$ of a $T$-bond satisfies the stochastic differential equation

$$
d p^{T}(t)=p^{T}(t) m(t, x(t), T) d t+p^{T}(t) v(t, x(t), T) d Z(t)
$$

for $t \in[0, T)$ and

$$
p^{T}(T)=1,
$$

where $Z$ is a standard Wiener process and $m$ and $v$ are given functions. Suppose furthermore that the economic state $x$ evolves according to the stochastic differential equation

$$
d x(t)=\zeta(t, x(t)) d t+\eta(t, x(t)) d Z(t)
$$

where $Z$ is the same Wiener process as above and $\zeta$ and $\eta$ are given functions.
(a) What restrictions would you want to place on the characteristics $m$ and $v$ of the dynamics of bond prices?
(b) What is meant by a forward rate in this model?
(c) Find the stochastic differential equation satisfied by forward rates.
4. (a) What is meant by put-call parity?
(b) Give a careful derivation of the put-call parity relation.

Now suppose that we are given two parameters, namely $0<a<K$. Consider the derivative with final payoff

$$
\Phi(S)=\left\{\begin{array}{ll}
0 & \text { if } S \leq K-a \\
S-(K-a) & \text { if } K-a \leq S \leq K \\
(K+a)-S & \text { if } K \leq S \leq K+A \\
0 & \text { if } K+a \leq S
\end{array}\right\}
$$

where $S$ is the price of the underlying at maturity $T$.
(c) Draw a graph of $\Phi$ in the case $K=5$ and $a=1$.
(d) What is the interpretation of this derivative?
(e) What can you say about the price of this derivative at time 0 ?

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Friday 30 May 2014 2:00pm to 4:00pm

F500

## Empirical Finance

This paper is in two sections. Section A: Candidates are required to answer all four questions. Section B: Candidates are required to answer two out of three questions.

Each section carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION<br>Campbell, Lo \& Mackinlay Tables<br>Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Section A

A1. Suppose that daily stock returns follow a moving average process

$$
r_{t}=\varepsilon_{t}+\theta \varepsilon_{t-1}
$$

where $\varepsilon_{t} \sim N(0,1)$.
(a) What is the mean, variance, and autocovariance function of the daily return series?
(b) Suppose we consider weekly returns

$$
r_{W s}=r_{5 s}+r_{5 s-1}+r_{5 s-2}+r_{5 s-3}+r_{5 s-4}
$$

for $s=1,2, \ldots$ What is the mean, variance, and autocovariance function of the weekly return series? (Hint: You may find it helpful to look at $\operatorname{cov}\left(r_{W 2}, r_{W 3}\right)$ )
(c) What does this say about the autocorrelation function of weekly returns? And about monthly returns?
(d) Are these results consistent with those presented in the table (Table 2.4 from CLM)?
(e) Suppose instead that

$$
r_{t}=\varepsilon_{t}+\theta \varepsilon_{t-5}
$$

where $\varepsilon_{t} \sim N(0,1)$. How does this affect the answers to b and c ?
A2. True, False, Explain
(a) Rational Bubbles are logically impossible
(b) In the absence of uninformed traders, bid ask spreads would be narrower, and so markets would be more efficient.
(c) The equity premium puzzle is explained by long run risks
(d) The hypothesis RW1 (Random Walk) implies RW2 which implies RW2.5 (The Martingale Hypothesis) which implies RW3. Therefore, if one rejects RW3, then one rejects RW1.
(e) The presence of stock splits can be explained only by tradition.

A3. Describe what is meant by the "matching approach" to carrying out event studies. What are the strengths and weakness of this approach in comparison with the usual approach based on CAR comparisons?

A4. Suppose that (excess) returns obey a linear K-factor model

$$
Z_{i t}=\mu_{i}+\sum_{j=1}^{K} b_{i j} f_{j t}+\varepsilon_{i t}
$$

where $\varepsilon_{i t}$ is an (iid over $t$ ) idiosyncratic error term with

$$
\operatorname{cov}\left(\varepsilon_{i t}, f_{j s}\right)=0 \quad ; \quad E\left(\varepsilon_{i t} \varepsilon_{j s}^{\top}\right)=\left\{\begin{array}{cc}
\sigma_{i j} & \text { if } t=s \\
0 & \text { else }
\end{array} .\right.
$$

You observe the panel $\left\{Z_{i t}, i=1, \ldots, N, t=1, \ldots, T\right\}$.
(a) The factors are themselves not directly observed. Describe how you might estimate the factors $\left\{f_{j t}, j=1, \ldots, K\right\}$ in two cases:
i. The time series $T$ is large compared with the number of assets $N$;
ii. The time series $T$ is short compared with the number of assets $N$.
(b) Suppose now that factor returns are assumed to be observed. How would you test the Arbitrage Pricing Theory?

## Section B

B1. Explain Campbell's Approximate Model of stock prices

$$
\begin{aligned}
p_{t} & =\frac{k}{1-\rho}+E_{t} \sum_{j=0}^{\infty} \rho^{j}\left[(1-\rho) d_{t+1+j}-r_{t+1+j}\right] \\
& =\frac{k}{1-\rho}+(1-\rho) \sum_{j=0}^{\infty} \rho^{j} E_{t} d_{t+1+j}-\sum_{j=0}^{\infty} \rho^{j} E_{t} r_{t+1+j},
\end{aligned}
$$

where $p_{t}$ is $\log$ stock prices, $d_{t}$ is $\log$ dividends, and $r_{t}$ is stock returns, while $E_{t}$ denotes expectation conditional on all information at time $t$. Specifically, explain how it is derived and what is the constant $\rho$ ? Explain how this equation might be used to explain the observed variability of stock prices. Explain how one might use Vector Autoregressions to obtain estimates of $E_{t} r_{t+1+j}$.

B2. Describe the consumption capital asset pricing model (CCAPM). How is it derived and how it restricts consumption and asset returns. How could you test this hypothesis in practice? Describe the data you would need and the econometric methods you might use. Consider the table (Table 8.1 from CLM). What does this say about the plausibility of the CCAPM?

B3. Explain the calendar time hypothesis and the trading time hypothesis. How would you distinguish between these two hypotheses on the basis of stock return data. Explain the type of data you would need and how you would carry out the test. What if the results lie between the two cases, why are the possible explanations for this, and how might you go about distinguishing them?

## END OF PAPER

CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Tuesday 3 June $2014 \quad$ 2:00pm to 4:00pm

F510
International Finance
Candidates are required to answer four out of five questions.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## 1. Carry trade

Let $\mathcal{F}_{t}$ and $\mathcal{E}_{t}$ denote forward and spot exchange rates (units of domestic currency per unit of foreign currency). Suppose the foreign currency trades at a forward premium.
(a) Write up and explain a 'carry trade' strategy using forward contracts (possibly using bid and ask spread)?
(b) Based on the readings discussed during lectures, if you were to form a diversified carry-trade portfolio of many currencies, you would expect a lower variance of return and a lower excess return?

## 2. Covered Interest Parity

(a) Define the covered interest parity (CIP) condition.
(b) Carefully discuss conditions under which the CIP is violated.
(c) Provide an illustration of one of this condition making use of the asset pricing condition $q=E(D x)$, where $q$ is the price of an asset at $\mathrm{t}=0, D$ is the stochastic discount rate between $t=0$ and $\mathrm{t}=1, x$ is the stochastic cash flow at $\mathrm{t}=1, E$ denotes expectations.

## 3. International borrowing and sovereign risk

Consider a country populated by a continuum of mass 1 residents, existing for two periods. These residents are ex-ante identical, but their endowment may differ ex post, depending on the realization of individual income shocks. Specifically, in period 0 , all residents have the same endowment equal to $.5 y$. In period 1 , each one of thern may get 2 y with probability $1 / 2$ (if lucky); or get $y$ with the same probability (if unlucky). The preference and endowment of individual $i$ is

$$
\begin{aligned}
& U\left(c_{i, 0}\right)+E U\left(c_{i, 1}\right) \\
& y_{i, 0}=.5 y \quad \text { and } \quad y_{i, 1}=\left\{\begin{array}{cl}
2 y & \text { with prob. } 1 / 2 \\
y & \text { with prob. } 1 / 2
\end{array}\right.
\end{aligned}
$$

Residents can trade assets (financial contracts) with international investors who are risk neutral and do not discount the future.

The domestic government is interested in the welfare of its citizens. Output is verifiable, but there is no international institution that enforces international contracts.

## Sovereign risk

(a) Describe the contract that would allow individuals to smooth consumption completely across time and contingencies (hence achieve efficient risk sharing). Describe (a) cash flow $x$ from and to each individual $i$ conditional on having high or low income at time 1 and (b) its price $q$ at time 0 .
(b) Explain carefully whether, with no international authority able to enforce cross-border contracts, residents in the country will be able to access international markets and enter these contracts.

## The role of penalties

Suppose that the international community can impose a penalty equal to $.4 y$ if the country residents do not pay their debt.
(c) To what extent will the residents be able to borrow? (hint: calculate the price of the expected cash flow).
(d) Calculate the consumption of the 'lucky' and 'unlucky' residents in period 1. Is there scope for domestic residents to insure each others' income?

## 4. Currency Crises

Suppose the central bank of an emerging market economy aims to peg its currency, the peso, to the US dollar so that $S=\bar{S}$, where $S$ is the peso price of US dollars. The central bank balance sheet is given by

$$
M=B+R
$$

where $M$ denotes the money supply, $B$ domestic government bonds held by the central bank and $R$ foreign reserves. If the central bank has depleted its foreign reserves, it adopts a free float. Assume that the government runs a structural budget deficit and forces the central bank to buy government bonds so that $\dot{B} / B=\gamma>0$, where $\dot{X} \equiv \mathrm{~d} X / \mathrm{d} t$ denotes the time derivative of $X$.
Money market equilibrium is given by

$$
\frac{M}{P}=Y e^{-i}
$$

where $P$ is the aggregate price level, which is flexible, $i$ is the nominal interest rate, and $Y$ is aggregate output, which is exogenous and normalized to $Y=1$. Purchasing power parity holds so that

$$
P=S P^{*}
$$

where $P^{*}$ is the US aggregate price level, which is exogenous and normalized to $P^{*}=1$. In addition, assume that

$$
i=i^{*}+\dot{S} / S
$$

where $i^{*}$ is the US nominal interest rate.
(a) Derive the level of the money supply $M$ that is required to keep the exchange rate fixed at $S=\bar{S}$. Explain intuitively why the exchange rate peg is unsustainable.
(b) Derive the time $T$ of the speculative attack (assuming no speculative bubbles). Explain how it depends on the initial level of foreign reserves $R_{0}$.
(c) Suppose that after fixing the exchange rate at $S=\bar{S}$ at $t=0$, there is suddenly an increase in the US interest rate $i^{*}$ at time $\tau$, where $0<\tau<T$. Explain whether this could trigger a currency crisis.

## 5. Risk and international borrowing

Suppose you have two individuals (or countries) with identical preferences over two periods

$$
U\left(C_{t}\right)+\beta E_{t} U\left(C_{t+1}\right)
$$

so that their fundamental stochastic discount will be

$$
D=\frac{\beta U^{\prime}\left(C_{t+1}\right)}{U^{\prime}\left(C_{t}\right)}
$$

Their current endowment is identical $Y_{t}=Y_{t}^{*}$. Their future endowment has the same expected value $E_{t} Y_{t+1}=E_{t} Y_{t+1}^{*}$ but different variance: the domestic endowment is more variable than the foreign one, i.e. $\operatorname{Var}\left(Y_{t+1}^{*}\right)>\operatorname{Var}\left(Y_{t+1}\right)$. Suppose the two can trade in a sure bond $B$, that is exchanged at the price $q_{t}$ today, and promise one unit of goods at $t+1$.
(a) Which individual/country will tend to borrow? Under what condition on the utility function? (Hint: compare asset prices before trade in assets). Explain intuitively.
(b) What lessons can you draw about the implications of integrating countries/regions with different level of output variability for global imbalances and the real interest rate?

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
Monday 26 May 2014 10:00am to 12:00pm

F520

## Behavioural Finance

Candidates are required to answer two out of three questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Behavioural corporate finance is a fairly recent area of research.
(a) What is behavioural corporate finance? Broadly outline the major directions of research and how they relate to standard economic theory and corporate finance.
(b) In behavioural models, what are the implications of manager overconfidence or optimism for firms' investment behaviour and capital structure?
(c) Using Dr Siming's lecture as a starting point, discuss the pros and cons of national honours systems from the point of view of firm owners.
2. Risk preferences are important on financial markets.
(a) Prospect theory is about decision making under risk. Outline the key characteristics of the theory and contrast them to neoclassical theory.
(b) What is the equity premium puzzle and how has prospect theory been used to explain it?
(c) Empirically, how do risk taking and risk preferences compare to the assumptions in neoclassical theory? How can we explain the deviations?
3. Momentum, under and overreaction are three concepts that we have encountered in the course.
(a) Outline what these are and how they are related to each other.
(b) What behavioural explanations have been given for these phenomena?
(c) Although stock markets seemingly overreact so called contrarian investment strategies are not really profitable. Outline the basic idea behind such strategies and discuss why they may not be profitable. In your discussion, draw on concepts that we have encountered in the course.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Thursday $5^{\text {th }}$ June $2014 \quad$ 2:00pm to $4: 00 \mathrm{pm}$

F540

## Topics in Applied Asset Management

This paper is in two sections:
Section A: candidates are required to answer one compulsory question.
Section B: Candidates are required to answer two out of four questions.
Section A is weighted at $40 \%$ and section B is weighted at $60 \%$.
Provide as much mathematical detail as you can to justify your answers.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad
New Cambridge Elementary Statistical Tables

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION <br> New Cambridge Elementary Statistical Tables <br> Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

A1 Provide as much mathematical detail as you can when answering these questions.
(a) Diversification;
i. Derive a formula that shows how the portfolio variance in a portfolio with $\frac{1}{N}$ weights declines to some limit as the number of assets to the portfolio is increased. Interpret the result and discuss any limitations given the assumptions you have used to derive the formula.
ii. Derive the expression given by Booth and Fama which shows the return contribution due to diversification of an individual asset to a portfolio. Interpret the result.
(b) Describe what is meant by a Risk Parity Strategy and comment on their relative advantages/disadvantages.
(c) Show the steps in the derivation of the Parametric Portfolios of Brandt, Santa Clara and Valkanov and explain carefully why this approach might be preferred to direct computation of the portfolio weights through the use of "plug" in estimates of the moments as in classical Mean Variance.
(d) Explain the results obtained by Kan and Zhou on the conditions under which we might expect mean variance to dominate $\frac{1}{N}$
(e) DeMiguel, Garlappi and Uppal (DGU) (2007) argue that Mean Variance allocations are generally dominated by $\frac{1}{N}$ allocations;
i. Describe two approaches that have been put forward to "rescue" Mean Variance.
ii. Explain what is meant by Volatility Timing Strategies and explain why they might dominate $\frac{1}{N}$ and the tangency portfolio.
iii. What critical elements of the practical portfolio allocation problem became apparent in the results of DGU and other papers you have studied which would suggest their strong conclusions should be qualified.

## SECTION B

B1 Predictability;
(a) Criticise the methodology of Goyal and Welch and describe the modifications to their analysis made by Campbell and Thompson when considering the issue of whether asset returns are predicable with conditioning information.
(b) Show, following Mark Britten-Jones, how the optimal weights for the tangency portfolio can be computed through a linear regression. Hence explain the technical difficulties you can see that are common between determining robust predictability and robust portfolio weights.
(c) Derive from first principles the formula Campbell and Thompson develop that demonstrates the relative importance of the $R^{2}$ in the predictive conditional regression in comparison of the Sharpe Ratio of the asset in question to determine if there is likely to be profitable conditioning information.
(d) In the light of the formula in (c) above, do you believe Campbell and Thompson have provided sufficient evidence of profitable conditional predictability and if so provide some detail?

B2 Case Studies: Trend Following and Carry;
(a) Define formally what is meant by "Carry" and how it is measured in the case of Equities, Foreign Exchange and Commodities
(b) Explain the distinction between cross sectional momentum and the form of trend following followed by CTA's
(c) Why are trend following CTA's often considered to be cheap forms of lookback straddle option strategies?
(d) Provide two rationalisations for the degree of predictability observed historically in managed futures returns?
(e) Show how the returns from a cross section momentum strategy can be decomposed into three components; terms reflecting time series autocorrelation, cross covariances and a mean term, following Lo and MacKinlay as discussed by Moskowitz, Oi and Pedersen. What results did Moskowitz, Oi and Pedersen find as to the relative values of these three components with futures data with what implications?

B3 Volatility Measurement;
(a) Define what is meant by a GARCH, EGARCH and an ARCH-M model. What are their relative advantages?
(b) Clark introduced an Information flow model; explain how this explains why asset returns are non- Gaussian and why the ARCH family will usually be unable to correctly capture the true underlying volatility process.
(c) Explain how the parameters of a general $\operatorname{GARCH}(\mathrm{p}, \mathrm{q})$ model may be estimated.
(d) In a multivariate context Risk Management often requires the estimation of large covariance matrices for a portfolio. Explain how the number of parameters to be estimated may be reduced through the use of
i. Constant Correlation GARCH
ii. Orthogonal GARCH

B4 Risk Management;
(a) What is the relationship between Value at Risk and Expected Shortfall?
(b) Explain what is meant by the Sortino Ratio, Calmar Ratio and Maximum Drawdown.
(c) What is meant by Filtered historical simulation when used to compute VaR on a single asset
(d) Suppose that the change in the value of a portfolio over a 1 day time period is Normal with a mean of zero and a standard deviation of $\$ 2$ million
i. What is the (a) 1-day $97.5 \% \mathrm{VaR}$, (b) the 5 -day $97.5 \% \mathrm{VaR}$ and (c) the 5 -day $99 \%$ VaR
ii. What for questions (b) and (c) would be the answer be if there is first order autocorrelation with the autocorrelation parameter equal to 0.16 ?
(e) Suppose that each of two investments has a $4 \%$ chance of loss of $\$ 10$ million, a $2 \%$ chance of a loss of $\$ 1$ million and a $94 \%$ chance of a profit of $\$ 1$ million. They are independent of each other.
i. What is the VaR for one of the investments when the confidence interval is $95 \%$
ii. What is the expected shortfall when the confidence level is $95 \%$
iii. What is the VaR for a portfolio consisting of the two investments when the confidence interval is $95 \%$
iv. What is the expected shortfall for a portfolio consisting of the two investments when the confidence level is $95 \%$
v. Show that, in this example, VaR does not satisfy the sub-additivity condition whereas expected shortfall does?

## END OF PAPER

ECM9/ECM10/ECM11/ MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil in Finance
Monday 5 May 2014 2:00pm to 4:00pm

## M100

## Microeconomics I

Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. A profit-maximizing monopolist produces a single good with constant marginal cost $c>0$ and zero fixed cost. Consumers have utility function $\theta \sqrt{q}-t$ where $q$ is quantity consumed, $t$ is total payment and $\theta>0$ is a parameter. The reservation utility of consumers is zero.
(a) Sketch the indifference curves in $q-t$ space of the monopolist and a typical consumer. Derive the optimal contract which the monopolist will offer to the consumer.

Now suppose that there are two types of consumer. A proportion $p$ have $\theta=\theta_{h}$ and the rest have $\theta=\theta_{l}<\theta_{h}$. The value of $\theta$ is private information to the consumer.
(b) Consider a pair of contracts which satisfies the incentive compatibility and participation constraints but both constraints are slack. Sketch the relevant indifference curves for the two contracts. Show by means of diagrams that two of the constraints must bind for the monopolist's optimal pair of contracts.
(c) Hence find the optimal pair of contracts. You may assume that the monopolist sells a strictly positive amount to each type of consumer.
(d) Show that the optimal contract is not Pareto-efficient and briefly account for the form of the inefficiency.
2. Two players play the following game. There are three periods. In each period, supposing the game has not yet ended, the moves are as follows. Player 1 chooses either to exit, in which case the game ends, or not. If player 1 does not exit player 2 then decides whether to exit, in which case the game ends, or not. If a player exits in period $t(t=1,2,3)$ that player's final payoff is $t^{2}$ and the other player's payoff is zero. If neither player ever exits both players get a payoff of 8 .
(a) Draw the extensive form of the game. How many subgames are there in the game? What is the set of strategies for each player?
(b) Which strategies, if any, are strictly dominated? Explain your reasoning.
(c) Describe all subgame-perfect Nash equilibria and explain your reasoning.
(d) Now suppose that at the third stage the first player to move is chosen randomly, each with probability 0.5 . What is the strategy set for each player? Describe all the subgameperfect equilibria of this game and explain your reasoning.
3. Two people bargain over a surplus of size 1 in the following manner. Each simultaneously makes a claim (a number in the unit interval $[0,1]$ ). Let player $i$ 's claim be $x_{i}$ $(i=1,2)$. If $x_{1}+x_{2}>1$ then both players get a payoff of zero. If $x_{1}+x_{2} \leq 1$ then player $i$ 's payoff is $x_{i}(i=1,2)$.
(a) Find all the pure-strategy Nash equilibria of this game.

Now suppose that, with probability $\theta$, player 2 is only able to make a claim of 0.5 ; with probability $1-\theta$ player 2 can choose any $x_{2} \in[0,1]$. Otherwise the game is as described above. Player 1 does not know which type of player 2 she is facing.
(b) Define the term pure-strategy Bayes-Nash Equilibrium in the context of this game.
(c) Suppose that $\theta=0.25$. Let $y$ be the value of $x_{2}$ in the case in which 2 is free to choose $x_{2}$. Show that if $y>0.5$ then 1's best response is $x_{1}=0.5$ for $y \in\left[\frac{7}{8}, 1\right]$ and otherwise $x_{1}=1-y$.
(d) Describe all the pure-strategy Bayes-Nash equilibria for the case $\theta=0.25$.
4. (a) Describe the Independence Axiom in the von Neumann-Morgenstern theory of Choice under Uncertainty.
(b) An individual strictly prefers a certain gain of 200 pounds to a $50 \%$ chance of winning 500 pounds. She also strictly prefers a $50 \%$ chance of winning 500 pounds and a $20 \%$ chance of winning 200 pounds to a $20 \%$ chance of winning 500 pounds and an $80 \%$ chance of winning 200 pounds. Does she satisfy the independence axiom?
(c) A second individual satisfies the independence axiom. He is indifferent between winning 500 pounds with probability $\frac{1}{2}$ and winning 200 pounds with probability $\frac{2}{3}$. Sketch her indifference curves in the space of lotteries with the three possible prizes $(500,200,0)$. Using the independence axiom (not the expected utility representation), show that he must be indifferent between winning 500 with probability $\frac{1}{4}$ and winning 200 with probability $\frac{1}{3}$.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Tuesday 20 May 2014 2:00pm to 4:00pm

M110

## Microeconomics II

Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. (a) Consider an economy with three types of goods. Suppose that an agent's utility function is given by

$$
U(x, y, z)=x^{a} y^{b} z^{c}
$$

where $a, b, c$ are constants. Express the necessary and sufficient conditions on $a, b, c$ for $U$ to be strictly increasing. Then for such a utility function, given an initial endowment of $\left(\omega_{a}, \omega_{b}, \omega_{c}\right)$, and given prices $p_{a}, p_{b}, p_{c}$, compute the agent's utility maximising demand. (You are asked to solve the optimisation problem explicitly.)
(b) Consider a pure exchange economy with two agents $A, B$, and two goods $i=1,2$. The agents have the utility functions

$$
u^{A}\left(x_{1}, x_{2}\right)=2 \ln x_{1}+3 \ln x_{2} \quad \text { and } \quad u^{B}\left(x_{1}, x_{2}\right)=\ln x_{1}+2 \ln x_{2} .
$$

The total endowment of the goods is $\omega=(1,1)$.
Describe the set of Pareto optimal allocations $\left(\hat{x}_{1}^{A}, \hat{x}_{2}^{A}, \hat{x}_{1}^{B}, \hat{x}_{2}^{B}\right)$ by expressing $\hat{x}_{2}^{A}, \hat{x}_{1}^{B}$, and $\hat{x}_{2}^{B}$ as a function of $\hat{x}_{1}^{A}$.
(c) If the initial endowments are $\omega^{A}=\omega^{B}=\left(\frac{1}{2}, \frac{1}{2}\right)$, evaluate the Walrasian equilibrium price and the associated equilibrium allocations. (For notational brevity, normalise the price of good two as 1 , and set the price of the first good to $p$.)
(d) Suppose we can transfer good 2 between the agents before trade takes place. What transfer should we make so that the allocation

$$
\left(\frac{1}{4}, \frac{1}{5}\right) \text { to } \mathrm{A} \text {, and }\left(\frac{3}{4}, \frac{4}{5}\right) \text { to } \mathrm{B}
$$

emerges as an equilibrium outcome after the transfer?
2. (a) Describe a financial economy with a single consumption (numeraire) good, two dates ( 0 and 1 ), $k$ states in date 1 . What is an asset? What is an equilibrium price vector? Explain carefully.
(b) State and prove the first welfare theorem for financial economies.
(c) State the fundamental theorem (about the relationship between equilibrium prices of assets and state multipliers), and comment on the interpretation of state multipliers.
3. An seller of a single object faces $n$ risk-neutral bidders with private valuations for the object, independently drawn from a distribution $F$ with density $f$.
(a) State the Revenue Equivalence Theorem (RET) for this situation.
(b) Consider an "All Pay" auction (i.e., a simultaneous sealed-bid auction in which the highest bidder wins the object, but every bidder pays her bid). Write down the differential equation for a player's bid as a function of her value in the symmetric equilibrium of this.
(c) Solve the differential equation for the special case in which $F$ is uniform on $[0, M]$.
(d) What would be the bidding behaviour if the auctioneer was instead running a Japanese auction, where a price clock rises, bidders drop out of the auction whenever they like (but cannot re-enter), the moment when there is one bidder left, the price clock stops, the remaining bidder wins, and pays the amount shown on the price clock? If bidders' values are independently drawn from a uniform distribution on $[0, M]$, what is the expected payment by a bidder with valuation $x$ ?
(e) Compare your answers to parts (c) and (d), and comment briefly.
4. Consider the first price auction for a single indivisible object between two bidders $i=1,2$. Each bidder $i$ draws an independent signal $t_{i}$ which is uniformly distributed on the unit interval. Given signals $t_{i}$ and $t_{j}$, the valuations of the bidders are

$$
\begin{aligned}
& v_{1}=2 t_{1}+t_{2} \\
& v_{2}=2 t_{2}+t_{1} .
\end{aligned}
$$

If bidder $i$ receives the object and pays $p$, her utility is $2 t_{i}+t_{j}-p$. Each bidder knows her own signal, but not the other's.
(a) What is expected value of the item to a bidder with type $t_{i}$ ?
(b) Assume a symmetric equilibrium of the sort $b\left(t_{i}\right)=k t_{i}+m$, and solve for $k$ and $m$.
(c) Alternatively, the seller considers selling the object with a fixed posted price. That is, he posts a price $q$, and if any of the bidders wants to buy the object at that price, they do so. If both want to buy it at that price, the seller decides randomly whom he will trade with, again at that price $q$. If the seller wants to maximise his expected revenue from this form of sale, what is his optimal choice of $q$ ? What is the associated revenue?
(d) Now, suppose that the bidders' valuation are given by $v_{i}=2 t_{i}+\frac{1}{2}$. Compare the expected revenue of the first price auction in this case to the expected revenue of the first price auction in part (b).

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Wednesday 4 June 2014 10:00am to 12:00pm

## M120

## Topics in Economic Theory

Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Consider the marriage problem with finitely many agents and strict preferences.
(a) Give a formal definition of a stable matching.
(b) Is there always a stable matching? If yes, prove it. If not, give an example to the contrary.
(c) State the singles theorem and prove it.
(d) Give a formal definition of strategy-proofness in the context of matching mechanisms. Is there a strategy-proof mechanism whose outcome is stable whenever a stable matching exists? If yes, describe the mechanism and prove that it is strategy-proof. If not, give an example and explain clearly why no strategy-proof mechanism can always return a stable matching.
(e) Let $f$ be a direct mechanism whose outcome is stable with respect to the revealed preferences. Consider the preference revelation game induced by this mechanism. Prove or disprove: every individually rational matching is a Nash equilibrium outcome of this game.
2. A subordinate $(S)$ has to make a report to her boss $(B)$ who will then take one of three decisions, denoted $x, y$ and $z$. There are two possible reports that she can make, $M_{1}$ and $M_{2}$. The payoffs for each party depend on the decision but also on the state of the world, which could be either $a$ or $b$. These two states are equally likely. $S$ knows the true state of the world but $B$ does not. The payoffs are given in the following table, with $S$ 's payoff first.

|  | $x$ | $y$ | $z$ |
| :---: | :---: | :---: | :---: |
| $a$ | 0,4 | 4,6 | $-2,0$ |
| $b$ | 4,4 | 2,0 | 0,6 |

(a) Describe informally what is meant by a Weak Perfect Bayesian Equilibrium (WPBE) for this game.
(b) Describe a WPBE in which only one of the two possible reports could be received by $B$.
(c) Suppose that there is a WPBE in which both messages have a strictly positive probability of being received, and $B$ chooses $z$ with zero probability after receiving $M_{1}$ and chooses $y$ with zero probability after receiving $M_{2}$. Find $B$ 's belief after receiving $M_{2}$. Show that there cannot be a WPBE of this kind, unless both reports are followed by action $x$ with probability 1.
(d) Now suppose that $M_{2}$ is not available, but $S$ can choose between sending no report or sending $M_{1}$, in which case it has only a 0.5 chance of arriving. Describe a WPBE which Pareto-dominates the WPBE you found in part (b). Show that it is an equilibrium and find the equilibrium payoffs.
(e) Briefly discuss your results.
3. Consider the infinite horizon alternating bargaining game with 2 players (Rubinstein model). Here the rules are in odd periods player 1 makes an offer of a split $x \in[0,1]$ to player 2 , which player 2 may accept or reject and in even periods player 2 makes an offer of a split to player 1 , which player 1 may accept or reject. If at any period a player accepts an offer, the proposed split is immediately implemented and the game ends. If she rejects, nothing happens until the next period. Assume that each player $i$ discounts future payoffs by a common factor $\delta<1$.
(a) Show that there exists a subgame perfect equilibrium with the following outcome: there is agreement at the first date and the players receive a unique payoff described by the following split: player 1 receives $\frac{1}{1+\delta}$ and player 2 receives $\frac{\delta}{1+\delta}$.
(b) Explain why the equilibrium outcome described in part (i) is the unique subgame perfect equilibrium outcome (A sketch of the arguments will suffice).
4. Consider a repeated random state oligopoly market with $n$ firms who produce a homogenous commodity at zero cost at each date. In each period there is a random demand shock (state) $s>0$ that is first drawn independently with the probability $q(s)$ from the finite set $S$ and revealed to the firms. After observing the states, the firms simultaneously choose non-negative prices for that period. The consumers buy from the cheapest firm (ties are split equally) and the aggregate demand of the consumers at price $p$ at each period is given by $s-p$.
(a) Compute the Nash equilibrium outcome, the mutual minmax outcome and the monopoly price in any stage game if the state is $s$.
(b) Suppose that along an equilibrium path of the repeated game, at every date the firms set a price $p(s)$ whenever the state is $s$. Characterise the expected equilibrium payoff for each firm from such a symmetric stationary equilibrium strategy.
(c) Show that for sufficiently large $\delta$, setting the monopoly price at every date and for every state $s$ can be sustained as an equilibrium.
(d) Fix any $\delta$, and consider any strongly (most collusive) symmetric stationary equilibrium.
i. Show that on the equilibrium path the firms set prices countercyclically in high states. Provide an intuition for this result.
ii. What happens to the equilibrium prices in low states?
(e) Does the countercyclical result in part (d.ii) persist if the random demand shocks were persistent not independent as above. Explain your answer.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Tuesday 27 May 2014 2:00pm to 4:00pm

## M130

## Applied Microeconomics

This paper is in two sections. Candidates are required to answer four out of six questions in Section A; and the one compulsory question in Section $B$.

Questions within Section A carry equal weight. Each section carries equal weighting.

Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Section A

A1 Consider the following regression of the budget share for clothing on prices, demographics and expenditure.

| Source | SS | $d f$ | MS |  | Number of obs | 14994 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | F ( 9, 14984) |  | 273.29 |
| Model | 9.80415378 | 91.0 | 935042 |  | Prob > F | $=$ | 0.0000 |
| Residual | 59.7266979 | 14984.00 | 986032 |  | R-squared | $=$ | 0.1410 |
|  |  |  |  |  | Adj R-squared |  | 0.1405 |
| Total | 69.5308516 | 14993.00 | 637554 |  | Root MSE | $=$ | . 06314 |
| bsclth | Coef. | Std. Err. | t | $P>\|t\|$ | [95\% Conf. | In | nterval] |
| lhhsize | -. 029766 | . 0010464 | -28.45 | 0.000 | -. 0318171 |  | . 0277149 |
| lpfath | . 0639738 | . 0178202 | 3.59 | 0.000 | . 029044 |  | . 0989036 |
| lprest | -. 066385 | . 0151091 | -4.39 | 0.000 | -. 0960006 |  | . 0367694 |
| lptranall | . 0050483 | . 0065405 | 0.77 | 0.440 | -. 0077719 |  | . 0178684 |
| lpserv | -. 0165291 | . 017966 | -0.92 | 0.358 | -. 0517446 |  | . 0186864 |
| lprecr | -. 0047786 | . 0212658 | -0.22 | 0.822 | -. 0464622 |  | .036905 |
| lpvice | . 0149331 | . 0070191 | 2.13 | 0.033 | . 0011748 |  | . 0286915 |
| lpclth | -. 0462183 | . 0160131 | -2.89 | 0.004 | -. 0776059 |  | . 0148308 |
| lrxtot | . 0419853 | . 0011517 | 36.46 | 0.000 | . 0397278 |  | .0442427 |
| _cons | -. 2887095 | . 0107603 | -26.83 | 0.000 | -. 309801 |  | . 2676181 |

bsclth is the share of clothing at home in total expenditure. The mean clothing share in the sample is $10.5 \%$. Total expenditure is deflated using a Stone price index. "lrxtot" is the $\log$ of deflated expenditure. "ad" indicates the number of adults in the household and "kids" indicates the number of children. "lpfath" through "lpclth" are the log of the prices that make up total expenditure.
(a) Calculate the elasticity of expenditure on clothing at home with respect to total expenditure and with respect to own price.
(b) How do your answers to (a) depend on the assumption of the household as a unitary decision unit?

A2 The Almost Ideal Demand System can be represented by the cost function:

$$
\begin{gathered}
\log c(p, u)=a(p)+b(p) u \\
a(p)=a_{0}+\sum_{k} \alpha_{k} \log p_{k}+\frac{1}{2} \sum_{k} \sum_{l} \gamma_{k l} \log p_{k} \log p_{l} \\
b(p)=\beta_{0} \Pi p_{k}^{\beta_{k}}
\end{gathered}
$$

and corresponding budget shares:

$$
w_{k}=\alpha_{k}+\sum_{i} \gamma_{k i} \log p_{i}+\beta_{k}[\log x-\log P]
$$

where

$$
\log P=a(p)
$$

What restrictions does demand theory place on $\alpha_{k}, \beta_{k}$ and $\gamma_{k i}$ ?

A3 What assumptions are necessary for the use of information on other family members education and wages to be useful for estimating the returns to education? How would such information best be used?

A4 The government is considering cutting the rate of tax on income above $£ 50,000$ by $10 \%$.
(a) Derive the Slutsky equation showing the likely effects on labour supply in a static model.
(b) How would your answer on the likely effects change in a dynamic context if the tax increase was to last only one year.

A5 An individual has a utility function

$$
u\left(q_{1}, q_{2}\right)=q_{1}^{\beta} q_{2}^{1-\beta}
$$

(a) By how much does the cost of living increase when the price of good 1 increases by $\lambda \%$ ?
(b) When will the cost of living be independent of the level of utility?

A6 (a) What are the likely theoretical costs and benefits of increasing the generosity of the disability insurance system?
(b) What do we learn from alternative approaches to measuring these costs and benefits?

## Section B

B1 A second-order approximation to the Euler equation yields the following relationship:

$$
E_{t}\left[\Delta \ln c_{t+1}\right]=\frac{1}{\gamma}\left(E_{t}\left[r_{t+1}\right]-\delta\right)+\frac{1}{2}(\gamma+1) \sigma_{\Delta \ln c_{t+1}}^{2}
$$

where $E_{t}$ is expectations at time $t ; c_{t}$ is consumption in period $t ; r_{t+1}$ is the interest rate between period $t$ and $t+1 ; \delta$ is the discount rate; and $\sigma_{\Delta \ln c_{t+1}}^{2}$ is the variance of consumption growth.
(a) How shoud the coefficients on $E_{t}\left[r_{t+1}\right]$ and $\sigma_{\Delta \ln c_{t+1}}^{2}$ be interpreted?
(b) How might the coefficient be estimated and, in particular, what are the difficulties introduced by uncertainty over the interest rate?
(c) What can be learnt from this specificaion about the amount of consumption in any given period?
(d) If the government is considering stimulating consumption through an income tax rebate or a cut in VAT, how useful are the estimates of this equation to predicting the effects?

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Thursday 29 May 2014 2pm to 4pm

## M140

## Behavioural Economics

Candidates are required to answer all four questions.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. (a) Describe the public-goods experiments conducted by Fehr and Gächter (2000).
(b) To what extent did those experiments establish that cooperators punish the free-riders?
(c) How would you improve on their experimental design?
2. (a) Name and describe the two most important findings of neuroscience for economics.
(b) Why do you regard these two findings as more important than other findings of neuroscience for economics?
3. (a) Define the endowment effect.
(b) Describe an experiment that shows its existence.
(c) Does the effect disappear in real markets with experienced traders? Justify your answer.
4. Consider an urn filled with red and white balls. Suppose that $\frac{2}{3}$ of the balls are of one color and $\frac{1}{3}$ of the other color. Suppose that individual 1 has drawn 5 balls (with replacement) and found 4 red and 1 white, and that individual 2 has drawn 20 balls (with replacement) and found 12 red and 8 white.
(a) Assuming equal prior probabilities, which individual should feel more confident that the urn contains $\frac{2}{3}$ red balls?
(b) Experimental evidence shows that the majority of people give the wrong answer to (a). Define the bias that leads to this counterintuitive experimental finding and give one example of a context where this bias may affect economic decisions.
(c) How does the bias in (b) relate to mean-regression bias? Define meanregression bias and describe an adverse consequence of this bias.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Thursday 22 May 2014 2:00pm to 4:00pm

M150

## Economics of Networks

Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Consider network formation with $N=\{1,2, . . n\}, n \geq 3$ individuals. Suppose that a player $i$ can create a link with player $j$ unilaterally by paying a cost $c>0$. The set of player $i$ links is denoted by $g_{i}$; the profile of linking decisions of all players is given by $g=\left(g_{1}, \ldots, g_{n}\right)$. The payoffs to player $i$ under a strategy profile $g=\left(g_{1}, \ldots, g_{n}\right)$ are

$$
\begin{equation*}
\Pi_{i}(g)=1+\sum_{j \in N \backslash\{i\}} \delta^{d(i, j ; g)}-\eta_{i}^{d}(g) c, \tag{1}
\end{equation*}
$$

where $d(i, j ; g)$ is the distance between $i$ and $j$ in (the undirected version of) network $g$ and $\delta \in(0,1)$ is the decay factor and $\eta_{i}^{d}(g)$ is the number of links which player $i$ forms. The distance between two nodes that have no path between them is set equal to infinity.
(a) Derive the parameter ranges - on $c, \delta$ and $n$ - for which the complete network, the empty network and different types of star networks can arise in a Nash equilibrium.
(b) A network is efficient if it maximizes aggregate welfare across all networks. Show that complete network is efficient if $c<2\left(\delta-\delta^{2}\right)$, the star network is efficient if $2\left(\delta-\delta^{2}\right)<c<2 \delta+(n-2) \delta^{2}$, and the empty network is efficient if $c>2 \delta+(n-2) \delta^{2}$.
(c) Show that if $c>1, n \geq 4, \delta$ is close to 1 , then the periphery sponsored star network is the unique non-empty strict Nash equilibrium network.

2 Suppose that there are two price setting firms selling a homogenous product to a unit mass of consumers. The consumers do not know the prices set by firms. They receive a free sample price from one firm at random. In addition, they can search for prices (at cost $c>0$ for the other price) and can share information with $k$ other consumers. Assume that consumers value the product at 1 and that $c<1$. The questions below ask you to study the nature of search and pricing in equilibrium.
(a) Suppose that $k=0$ : show that there exists an equilibrium in which firms set monopoly price 1 and no consumer searches.
(b) Now suppose that $k \geq 1$. Show that if $c$ is small, then there is a unique equilibrium in which consumers must mix in search intensity and firms must mix in prices.
(c) What are the effects of greater communication among consumers, represented by an increase in the value of $k$, on market prices and consumer welfare?

3 There is a source and a destination and a collection of $n$ intermediary traders located on nodes of an undirected connected network in between the source and destination. The value of exchange is 1 . The network of traders and the valuations are common knowledge. All traders post prices; they have zero costs. If the Buyer and Seller have a (direct) link they exchange the good and split the surplus. If they don't have a direct link then they compute the cost of using every path between them. They pick the lowest cost path if it is less than 1 (randomizing across paths if there are multiple lowest cost paths). Source and destination divide the residual surplus, after paying the cost of the path.
(a) Formalize this economic exchange as a game of simultaneous moves.
(b) Prove that for any network there exists a pure strategy Nash equilibrium.
(c) Define an outcome as efficient if trade occurs with probability 1. Show that there exists an efficient equilibrium in every network.
(d) Now suppose that the network is common knowledge but the location of source and destination is unknown. So traders set a uniform price for all trade that goes through them, irrespective of the distance that it travels.
i. Fix the network to be a ring with 10 traders. Show that there exists a symmetric Nsh equilibrium with price equal to $0,1,1 / 2$ or $1 / 3$.

4 Consider trade between 2 buyers and 1 seller (with zero costs). There are two stages. In stage 1, players choose to form links. The links determine potential trade patterns. In stage 2, buyers simultaneously make bids to the seller. If there is one bidder he gets the object at price 0 ; in case of two bidders, the winner is determined using a second price auction. Assume that the valuations of the buyers are uniformly distributed on the unit interval and that bidder learns his valuation at the start of stage 2.
(a) Solve for expected payoffs to bidders and seller in empty, single link and two link network.
(b) Suppose now that bidders can unilaterally decide to form links at cost $c$. Show that if $c>1 / 2$ then the empty network is an equilibrium, if $1 / 6<c<1 / 2$ then the single link network is an equilibrium, while if $c<1 / 6$ then the two link network is an equilibrium. Show that equilibrium and efficient networks coincide.
(c) Next suppose that bidder and seller form a link only if both agree to pay $c$. Show that in this case efficient and pairwise stable networks do not coincide.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 26 May $2014 \quad$ 2:00pm to $4: 00 \mathrm{pm}$

M160

## Political Economy

Candidates are required to answer all three questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 2
Rough Work Pad
Tags

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

## You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Consider $N$ (where $N$ is odd) voters that have single-peaked preferences over policies $p \in P$, where $p \in \mathbb{R}$ is a unidimensional policy variable. Denote by $\pi_{i}$ the unique favourite policy of individual $i$. Assume that individual 1 is the one with the smallest favourite policy and individual $N$ is the one with the largest favourite policy, and individuals are ordered such that $\pi_{1}<\pi_{2}<\ldots<\pi_{N}$. Assume sincere voting throughout the question.
(a) Give formal definitions of the following:

- Single-peaked preferences
- Median voter
(b) State the Median Voter Theorem
(c) Show by using a counterexample that the Median Voter Theorem does not hold if the policy space is two-dimensional, i.e. $p \in \mathbb{R}^{2}$
(d) Show by using a counterexample that the Median Voter Theorem does not hold if individuals have non-single-peaked preferences. Assume a voting procedure of direct democracy with an open agenda.
(e) State and give a proof of the Downsian Policy Convergence Theorem. Make sure you present the formal set-up of the model and you explicitly state the assumptions that you are making.

2. "No great man lives in vain. The history of the world is but the biography of great men." Comment.
3. How did the introduction of Fox News affect the US presidential election in 2000? Is this consistent with the theoretical predictions in Mullainathan and Shleifer (2005)?

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 19 May 2014 2:00pm to 4:00pm

M170

## Industrial Organisation

Candidates are required to answer all three questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION <br> Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Consider a Hotelling duopoly in which consumers who wish to buy one unit of the good are uniformly located on a line $[0,1]$. Consumers pay a linear transport cost $t x$ to buy at a distance $x$ from their location on the line. Firm $A$ is located at 0 with marginal cost $c \in(0, t)$, and firm $B$ is located at 1 with a marginal cost of zero.
(a) Assume that firms simultaneously set prices $p_{A}$ and $p_{B}$ respectively. Compute firms' equilibrium prices, market shares, and profits.
(b) How much does a small increase in firm $A$ 's marginal cost $d c>0$ reduce its profits? Is this more than, less than, or equal to the change in $A$ 's marginal cost times its market share? Why?
(c) Assume that, instead of each firm offering the same price to all consumers, firms price separately to each individual consumer on the line, i.e., firms can perfectly price discriminate. Compute firms' equilibrium prices, market shares, and profits.
(d) Comment on whether the equilibrium with price discrimination in (c) is better or worse than uniform pricing in (a) from the viewpoints of firms and consumers respectively.
2. Consider a market for a homogeneous product, characterised by the demand function $q=a-p$. An upstream manufacturer $M$ produces the good at constant marginal cost $c>0$. Assume that $a>c$. A downstream distributor $D$ buys the goods from $M$ and sells them to final consumers. Assume that the distributor has no marginal costs over and above the input price charged by the manufacturer. Assume that the manufacturer sets a linear wholesale price $w$ for each unit of the good.
(a) For a given wholesale price $w$, derive the quantity, price and profits of the distributor.
(b) Derive the wholesale price $w$ that maximises the manufacturer's profits, the resulting quantity, price and profits of the distributor and the profits of the manufacturer.
(c) Compare total industry profits under linear wholesale prices with the profits $\pi^{V I}$ of a vertically integrated monopolist.
(d) Consider an entrant firm $E$ that can produce the homogeneous product at constant marginal cost $c$. Assume that $E$ competes in prices against the distributor $D$. Calculate the equilibrium profits of the manufacturer $M$, the distributor $D$ and the entrant $E$.
3. Consider two firms $i=1,2$ producing goods 1 and 2 at zero cost, that consumers consider to be perfect complements. That is, consumers have demand for bundles consisting of one of each good. Denote the price of good $i=1,2$ by $p_{i}$. Assume that aggregate demand for bundles is given by

$$
q=a-\left(p_{1}+p_{2}\right)
$$

with $a>0$. Denote the the amount demanded of good $i=1,2$ by $q_{i}$, and note that $q_{1}=q_{2}=q$.
(a) Suppose that the two firms set prices non-cooperatively. Write down the firms' maximisation problems and derive the reaction functions.
(b) Find the equilibrium prices, quantities and profits of the two firms.
(c) Suppose the two firms merge and set prices in order to maximise joint profits. Write down the merged firm's maximisation problem and derive the equilibrium price, quantity and profits.
(d) Carefully compare the prices, quantities and profits under non-cooperative price setting with those of the merged firm. Briefly provide an intuition for the differences. Assess whether the merger increases social welfare, distinguishing between changes in producer surplus and consumer surplus.

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Wednesday 7 May 2014 2:00pm to 4:00pm

M200

## Macroeconomics I

The paper is in two sections:
Section A - candidates must answer all four questions.
Section B - candidates must answer one out of two questions.
Each section carries equal weight.
Write your candidate number (not your name) on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

A. 1 Suppose the dynamics of a continuous-time Solow growth model with physical and human capital can be represented by the following two equations:

$$
\begin{gathered}
\dot{k}=s_{k} k^{\alpha}(u h)^{1-\alpha}-\delta k \\
\dot{h}=(1-u) h^{\beta}-\delta h
\end{gathered}
$$

where $k$ is the physical capital stock per capita, $h$ is human capital per capita, $s_{k} \in(0,1)$ is the saving rate, and $\alpha \in(0,1), \beta \in(0,1)$ and $\delta>0$ are constant parameters. In addition, $u \in(0,1)$ is the fraction of time allocated to production and $(1-u)$ is the fraction of time allocated to human capital acquisition.
Draw and explain the phase diagram describing the dynamics of human and physical capital per capita ( $h$ and $k$ ). Compute the steady-state levels of human and physical capital per capita ( $h^{*}$ and $k^{*}$ ), and explain how they are affected by $u$.
A. 2 Suppose that a firm facing a gross real interest rate $1+r>0$ has a production function given by $Y_{t}=A F\left(K_{t}\right)$, where $A>0, F^{\prime}(\cdot)>0$, and $F^{\prime \prime}(\cdot)<0$. The firm's objective function is to maximise the present discounted value of profits over an infinite horizon. In addition to the direct real cost of investment $p_{I, t} I_{t}$, where $p_{I, t}$ is the price of investment $I_{t}$ in terms of output $Y_{t}$, the firm also faces real adjustment costs $\frac{\chi I_{t}^{2}}{2 K_{t}}$ with $\chi>0$ in any period in which it invests (or disinvests) $I_{t}$. Capital $K_{t}$ accumulates according to the following equation:

$$
K_{t+1}=K_{t}+I_{t}
$$

where $K_{0}>0$ is given.
Set up the firm's optimization problem and find the first-order conditions characterising optimal investment together with the boundary conditions. Interpret these conditions.
A. 3 Consider the following insider-outsider model. The representative firm maximizes real profits $\Pi_{t}=Y_{t}-w_{t} L_{t}$, subject to the production technology $Y_{t}=A_{t} \sqrt{L_{t}}$, where $Y_{t}$ is output, $L_{t}$ labour, and $A_{t}=\bar{A} \xi_{t}$, where $\xi_{t}$ is an i.i.d. shock with $\mathrm{E}\left[\ln \xi_{t}\right]=0$. There is perfect competition in the goods market. The real wage $w_{t}$ is set at the end of period $t-1$ by the workers who are employed in that period ('insiders') such that they expect to maintain their $\log$ employment: $\mathrm{E}\left[\ln L_{t}\right]=$ $\ln L_{t-1}$. After the shock $\xi_{t}$ has been realized, the firm decides whether to hire additional workers ( $L_{t}>L_{t-1}$ ) or fire workers $\left(L_{t}<L_{t-1}\right)$ in period $t$.
Suppose there is a negative shock $\xi_{t}<0$ such that $A_{t}$ temporarily drops $2 \%$ below $\bar{A}$ in period $t$. Explain the effect of this on employment $L_{t}$ from period $t$ onwards.
A. 4 Consider the following flexible price monetary model based on purchasing power parity, money market equilibrium and uncovered interest parity, respectively:

$$
\begin{aligned}
p_{t} & =s_{t}+p_{t}^{*} \\
m_{t}-p_{t} & =y_{t}-i_{t} \\
i_{t} & =i_{t}^{*}+\mathrm{E}_{t}\left[s_{t+1}\right]-s_{t}
\end{aligned}
$$

where $p$ denotes the log aggregate price level, $s$ the $\log$ nominal exchange rate, defined as the domestic price of foreign currency, $m$ the log money supply, $y$ log aggregate output, and $i$ the nominal interest rate. Foreign variables are indicated by an asterisk and subscripts indicate the time period.
Suppose that in period $t$ it is suddenly expected that Home money supply in period $t+1$ will be $2 \%$ lower than previously anticipated. Explain the effect of this on the nominal exchange rate in period $t$.

## SECTION B

## B. 1 Real Business Cycles

Consider the following two-period economy. There is a continuum of measure one of identical individuals $i \in[0,1]$. A representative individual maximises the following utility function:

$$
U=\sum_{t=0}^{1} \beta^{t}\left[\ln \left(c_{t}\right)+\gamma \ln \left(1-l_{t}\right)\right]
$$

where $c_{t}$ denotes consumption, $l_{t}$ labour supply, the subscript $t=0,1$ the time period, and $\beta \in(0,1)$ and $\gamma>0$ are constant parameters. The time endowment is equal to one in each period. Assume that individuals start with no initial assets $\left(b_{0}=0\right)$ and cannot die with a debt. Individuals supply labour to firms at a real wage rate $w_{t}$ per unit of labour. The consumption good is the numéraire.
There is also a continuum of measure one of identical, profit maximising firms. The production technology of the representative firm is represented by the following function:

$$
Y_{t}=A_{t} L_{t}
$$

where $Y_{t}$ is output, $L_{t}$ is labour used in production, and $A_{t}>0$ is an exogenous productivity factor. There is perfect competition in the product and labour market. There is no government and the real interest rate is $r$.
(a) Formulate the problem of the representative firm and take the first-order condition. Give an economic interpretation of the result.
(b) Formulate the problem of a representative individual and derive the first-order conditions associated with this problem. Provide some economic intuition for the conditions which characterise the trade-offs of this problem.
(c) Derive the optimal allocations for consumption $c_{0}$ and $c_{1}$, labour supply $l_{0}$ and $l_{1}$ and assets $b_{1}$, for given $w_{0}, w_{1}$ and $r$. Explain intuitively how they depend on the real interest rate $r$.
(d) Suppose that the economy is closed. Derive the equilibrium real interest rate $r$. Explain the effect of an increase in productivity $A_{1}$ on the real wage $w_{0}$ and $w_{1}$, the real interest rate $r$, consumption $c_{0}$ and $c_{1}$, and labour supply $l_{0}$ and $l_{1}$.

## B. 2 Effect of Price Rigidities

Consider an economy with monopolistic competition in the goods market and perfect competition in the labour market. The representative consumer $i \in[0,1]$ maximizes utility

$$
U_{i}=C_{i}-\frac{1}{\gamma} L_{i}^{\gamma}
$$

subject to the budget constraint

$$
C_{i}=\Pi_{i}+w L_{i}
$$

where $C_{i}, L_{i}$ and $\Pi_{i}$ denote consumption, labour and real profits for agent $i, w$ the real wage, and $\gamma>1$ a preference parameter. The representative firm $i \in[0,1]$ maximizes real profits

$$
\Pi_{i}=\frac{P_{i}}{P} Q_{i}-w L_{i}
$$

where $P_{i}$ denotes the price of good $i, P \equiv \int_{0}^{1} P_{i} d i$ the aggregate price level, and $Q_{i}$ the production of good $i$, which is described by the production technology

$$
Q_{i}=A L_{i}
$$

where $A$ denotes productivity. The demand for good $i$ is given by

$$
Q_{i}=\left(\frac{P_{i}}{P}\right)^{-\eta} Y
$$

where $Y \equiv \int_{0}^{1} Q_{i} d i$ denotes aggregate output, and $\eta>1$ is a demand parameter. The aggregate demand relation is described by

$$
Y=\frac{M}{P}
$$

where $M$ denotes aggregate demand.
(a) Use the first-order conditions to derive the optimal price $P_{i} / P$ for the firm and the optimal labour supply $L_{i}$ for the consumer. Give an economic interpretation of the results.
(b) Derive the real wage $w$, employment $L \equiv \int_{0}^{1} L_{i} d i$ and output $Y$ with flexible prices. How are they affected by a change in aggregate demand $M$ ? Give an economic interpretation of the results.
(c) Suppose now that all firms keep their price $P_{i}$ fixed and that there is a drop in aggregate demand $M$. Derive and explain the effect on the real wage $w$, employment $L$ and output $Y$.
(d) Starting from the flexible price equilibrium and assuming $A=1, \gamma=2$ and $\eta=5$, compute the real wage $w$, output $Y$ and profit $\Pi_{i}$ when all firms keep their price $P_{i}$ fixed after a $10 \%$ drop in aggregate demand $M$. Compute the profit $\Pi_{j}$ if firm $j$ optimally sets its price $P_{j}$ while all other firms $i \neq j$ keep their price $P_{i}$ fixed, and explain under what circumstances it would be optimal for firms to keep their price fixed.

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 2 June $2014 \quad$ 2:00pm to 4:00pm

M210

## Macroeconomics II

This paper is in two sections. Section A: Candidates are required to answer three out of four questions. Section B: Candidates are required to answer one compulsory question.

Each section carries equal weighting.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 2
Rough Work Pad
Tags

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

## A. 1 New Keynesian Model

Consider the following New Keynesian model:

$$
\begin{aligned}
\pi_{t} & =\beta E_{t}\left[\pi_{t+1}\right]+x_{t} \\
x_{t} & =E_{t}\left[x_{t+1}\right]-\left(i_{t}-E_{t}\left[\pi_{t+1}\right]-r_{t}^{n}\right)
\end{aligned}
$$

where $\pi_{t}$ denotes inflation, $x_{t}$ the output gap, $i_{t}$ the nominal interest rate, $r_{t}^{n}$ the natural real interest rate, the subscript $t$ indicates the time period, and $\beta$ is a parameter with $0<\beta<1$. The central bank sets the nominal interest rate subject to the lower-bound constraint that $i_{t} \geq 0$. Assume that in the initial steady state in period $t=0$, the central bank optimally sets $i_{t}=r_{t}^{n}=\bar{r}>0$.
Suppose that there is a sudden shock in period $t=1$ so that $r_{1}^{n}=\underline{r}<0$ and $i_{1}=0$. In each subsequent period $s$ there is a probability $p$, with $0<p<1 / 3$, that the shock persists so $r_{s}^{n}=\underline{r}<0$ and $i_{s}=0$; and a probability $1-p$ that the natural real interest rate permanently reverts back with $i_{s}=r_{s}^{n}=\bar{r}>0$ from period $s$ onwards.
Compute the level of inflation $\bar{\pi}$ and the output gap $\bar{x}$ for the initial steady state. In addition, derive the level of inflation $\underline{\pi}$ and the output gap $\underline{x}$ that prevail while the central bank sets $i=0$. Give an economic interpretation of the result.

## A. 2 Searching for ideas

Consider an entrepreneur who begins at $t=0$ with the following technology $a_{0}=0$ and has two alternative actions. The entrepreneur can engage in production using her current technology or she can search for a new technology. When engaged in production the income of the entrepreneur is $y_{t}=a_{t}$ where $a_{t}$ is her current technology. At each period she engages in such a search, she gets an independent draw of a new technology from a time-invariant distribution function $H(a)$ defined over a bounded interval $[0, \bar{a}]$ with $\bar{a}<\infty$. The income of the entrepreneur when searching is $y_{t}=0$. The entrepreneur devises a strategy to maximize $E\left[\sum_{t=0}^{\infty} \beta^{t} y_{t}\right]$, where $\beta \in(0,1)$ is a discount factor. Write the Bellman's functional equation for this problem and explain whether or not this equation is a Contraction Mapping.

## A. 3 OLG Model

Consider a discrete time economy, $t=0,1,2, \ldots$ Agents in this economy live for two periods only. In the first period of life (young period), agents have one unit of productive time, but in the second period (old period) of life agents do not work and they consume their savings from the first period of their lives. Agents have preferences over consumption when young, $c_{1}(t)$, and when old, $c_{2}(t+1)$ and they are represented by the following utility function

$$
U=\frac{c_{1}(t)^{1-\theta}-1}{1-\theta}+\beta \frac{c_{2}(t+1)^{1-\theta}-1}{1-\theta},
$$

$\theta>0, \beta \in(0,1)$. Firms in this economy have access to a technology to produce the consumption good $Y(t)$ and this technology is represented by a Cobb-Douglas production function:

$$
Y(t)=A K(t)^{\alpha} L(t)^{1-\alpha}, \alpha \in(0,1), A>0 .
$$

$K(t)$ and $L(t)$ denote capital and labour used in production, respectively. Capital fully depreciates after being used, $\delta=1$. At period $t=0$ there exists a unit measure of old agents with capital stock $K(0)$. What is the equilibrium equation of motion of the aggregate capital stock? Explain.

## A. 4 Asset Pricing

Consider an economy populated by a continuum of measure one of identical households, each with preferences given by $\mathbf{E}_{0}\left[\sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right)\right]$ where $c_{t}$ is consumption at period $t, u\left(c_{t}\right)$ is a function that is strictly increasing, strictly concave and twice differentiable, and $\beta \in(0,1)$ corresponds to the subjective discount factor. There is a single asset tree which yields perishable dividends $d_{t}$ (fruits). Households are initially endowed with some number of shares of the tree: $s_{0}$. Assume that $d_{t}$ is a random variable taking values from the set $\mathbf{D} \equiv\left\{d_{1}, \ldots, d_{N}\right\}$ and is distributed according to a first-order Markov chain with constant transition probability matrix. Households can trade the share of the tree. The price of the tree when the current realization of $d_{t}$ is $d$ is given by the function $p: \mathbf{D} \rightarrow R_{+}$. Households have an income flow $0<y<\infty$ at every period $t$ and $y$ is not a random variable. There are no other assets in this economy. State the problem of the representative household in recursive form and assuming that the value function is differentiable, derive the stochastic Euler equation. Interpret this equation.

## SECTION B

## B. 1 Neoclassical Growth Model

Consider an economy with continuum of firms of measure one and each firm has access to the following technology:

$$
Y(t)=K(t)^{\alpha}(A(t) N(t))^{1-\alpha}, \alpha \in(0,1) .
$$

$Y$ is output, $K$ is capital, $N$ is labour, and $A$ is labour productivity. Technological progress grows at rate $g$ and the initial technology is given. All markets are competitive.
There is a continuum of measure one of households who are infinite lived, with preferences

$$
U=\int_{0}^{\infty} e^{-\rho t} u\left(c^{i}(t)\right) d t, \quad \rho>0
$$

where $c^{i}(t)$ denotes consumption of household $i$ at $t$, and

$$
u(c)=\frac{\left(c^{i}\right)^{1-\theta}-1}{1-\theta}, \theta>0 .
$$

Each household has an instantaneous unit of productive time. Household $i$ has $k^{i}(0)$ units of capital at $t=0$ and $k^{i}(0)$ can be different across households. Capital evolves according to:

$$
\dot{k}^{i}(t)=x^{i}(t)-\delta k^{i}(t),
$$

where $x^{i}$ corresponds to investment in capital of household $i$ and $\delta>0$ is the depreciation rate.
(a) Set up the Hamiltonian that household $i$ solves, taking prices as given and determine the necessary and sufficient conditions for the allocation of consumption for each household over time.
(b) For a given path of prices, will consumption of each household $i$ be the same across all households? (Hint: Write down the present value budget constraint of each household.)
(c) Derive also the first-order conditions of the representative firm, and define the market clearing conditions.
(d) Derive the growth rates of the aggregate capital stock and consumption of each household along a balanced growth path equilibrium. Is the growth rate of consumption of each household $i$ the same across all households along the balanced growth path equilibrium? Explain.
(e) Derive a parameter restriction ensuring that the transversality condition is satisfied for a balanced growth path with a positive growth rate of the aggregate capital stock and consumption of each household $i$.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 26 May 2014 10:00am to 12:00pm

M220

## Macroeconomics III

This paper is in two sections:
Section A - candidates are required to answer three out of four questions.
Section $B$ - candidates are required to answer one compulsory question.
Each section carries equal weighting.
Write your candidate number (not your name) on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.
A. 1 Consider the following two-period $N$-country model. The representative consumer of a representative country $n \in\{1,2, \ldots, N\}$ maximises expected lifetime utility

$$
U^{n}=u\left(C_{1}^{n}\right)+\beta \sum_{s=1}^{\mathcal{S}} \pi(s) u\left[C_{2}^{n}(s)\right]
$$

where $C_{1}^{n}$ denotes consumption in period $1, C_{2}^{n}(s)$ consumption in period 2 in state $s \in\{1,2, \ldots, \mathcal{S}\}, \pi(s) \in(0,1)$ the probability of state $s$, the utility function satisfies $u^{\prime}(\cdot)>0$ and $u^{\prime \prime}(\cdot)<0$, and $\beta \in(0,1)$ is the intertemporal discount factor. In period 1 there is international asset trade in competitive world markets for a risk-free bond, which pays a real interest rate $r$, and $N$ GDP securities, which are a claim to uncertain period-two output $Y_{2}^{m}$ of country $m \in\{1,2, \ldots, N\}$. The budget constraints of the representative consumer of country $n$ are given by

$$
\begin{aligned}
C_{1}^{n}+B_{2}^{n}+\sum_{m=1}^{N} x_{m}^{n} V_{1}^{m} & =Y_{1}^{n}+V_{1}^{n} \\
C_{2}^{n}(s) & =(1+r) B_{2}^{n}+\sum_{m=1}^{N} x_{m}^{n} Y_{2}^{m}(s) \quad \text { for } s \in\{1,2, \ldots, \mathcal{S}\}
\end{aligned}
$$

where $B_{2}^{n}$ is the stock of risk-free bonds in country $n$ at the end of period $1, x_{m}^{n}$ the share that country $n$ holds in the GDP security of country $m$, and $V_{1}^{m}$ the market value in period 1 of the GDP security of country $m$.
Use the first-order conditions to derive an expression for the market value $V_{1}^{m}$ of the GDP security of country $m$ in terms of $Y_{2}^{m}, C_{1}^{n}$ and $C_{2}^{n}$. Give an intuitive explanation of the result.
A. 2 Write import prices at the Home border as

$$
\bar{P}_{F}=m k p \cdot \mathcal{E} \cdot m c^{*}
$$

where $m k p$ is the Foreign firms' markup in the Home market, $m c^{*}$ the Foreign marginal costs in foreign currency, and $\mathcal{E}$ the nominal exchange rate, defined as the Home currency price of Foreign currency. Consider the results from regressing import prices at the consumer level $P_{F}$ on the nominal exchange rate and (a good proxy for) Foreign marginal costs:

$$
\ln P_{F}=\alpha+\gamma \ln \mathcal{E}+\beta m c^{*}+v
$$

Explain whether we can consider $\gamma$ a reliable estimate of the "structural exchange rate pass through" for import prices.
A. 3 Assume that a social planner maximises the following social welfare function:

$$
W=\frac{1}{2} \ln C+\frac{1}{2} \ln C^{*}
$$

The aggregate consumption index in Home $C$ and Foreign $C^{*}$ is given by

$$
\begin{aligned}
C & =C_{H}^{\theta} C_{F}^{1-\theta} \\
C^{*} & =C_{H}^{* \theta} C_{F}^{* 1-\theta}
\end{aligned}
$$

where $0<\theta<1$, and $C_{H / F}$ and $C_{H / F}^{*}$ denote consumption of the Home/Foreign good in the Home and Foreign country, respectively. In addition, goods market equilibrium holds:

$$
\begin{aligned}
C_{H}+C_{H}^{*} & =Y_{H} \\
C_{F}+C_{F}^{*} & =Y_{F}
\end{aligned}
$$

where $Y_{H / F}$ denotes the (tradable) output of the Home/Foreign good. Use the first order conditions describing the efficient allocation to derive
(i) an expression that can be interpreted as the terms of trade of a country, and (ii) an expression that can be interpreted as the real exchange rate. Briefly explain their properties.
A. 4 Consider a Cole and Obstfeld two-country endowment economy where the consumption aggregator has a unit elasticity of substitution (a Cobb-Douglas aggregator).
Carefully explain why the endowment risk is perfectly diversified with no need for international trade in assets. Discuss how this result can provide guidance in understanding the direction of capital flows in a bond economy in response to temporary endowment shocks, for substitution elasticities above or below unity.

## SECTION B

B. 1 Consider a Home firm producing for both the Home and the Foreign markets. The firm produces a variety $h$ using the production function $Y_{t}(h)=Z_{\mathrm{H}, t} L_{t}(h)$, where $L_{t}(h)$ denotes labour employed and $Z_{\mathrm{H}, t}$ Home productivity. The firm faces imperfect competition with the following Home and Foreign demand functions:

$$
\begin{aligned}
C_{t}(h) & =\left(\frac{p_{t}(h)}{P_{\mathrm{H}, t}}\right)^{-\theta} C_{\mathrm{H}, t} \\
C_{t}^{*}(h) & =\left(\frac{p_{t}^{*}(h)}{P_{\mathrm{H}, t}^{*}}\right)^{-\theta} C_{\mathrm{H}, t}^{*}
\end{aligned}
$$

where $p_{t}(h)$ is the consumer-level price of variety $h, P_{\mathrm{H}, t}$ the price index associated with the composite consumption index $C_{\mathrm{H}, t}, \theta>1$ and Foreign variables are indicated by an asterisk. The consumer-level price is the sum of the producerlevel price $\bar{p}_{t}(h)$ and the cost of retailers, which depends on wages $W_{t}$ in local currency:

$$
\begin{aligned}
& p_{t}(h)=\bar{p}_{t}(h)+\eta W_{t} \\
& p_{t}^{*}(h)=\bar{p}_{t}^{*}(h)+\eta W_{t}^{*}
\end{aligned}
$$

where $\eta>0$.
(a) Derive the profit maximizing prices for the firm. Give an economic interpretation of the result.
(b) Are the profit maximizing prices identical across markets of destination? Carefully explain under what conditions this is the case.
(c) Suppose now that retailers in one country have the option to buy the product $h$ directly from retailers from the other country, rather than from the producer. This possibility is a constraint on the ability of the producer to charge the optimal price. Using the optimal prices you derived in part (a), show that the range of exchange rate values for which the producer is able to charge the optimal markup can be written as a function of $\theta$ and $W / W^{*}$. Explain whether this range is increasing or decreasing in the monopoly power of the firm.
(d) Derive the optimal pricing under the no arbitrage constraint on pricing. Explain whether the law of one price will hold at either producer or consumer level.

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Wednesday 21 May 2014 2:00pm to 4:00pm

M230

## Applied Macroeconomics

Candidates are required to answer three out of seven questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 2
Metric Graph Paper
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Country A at time period $t$ has a real Gross Domestic Product of 1000 bitcoins, a Price level of 1 and a nominal national Debt of 600 bitcoins. It faces a real interest rate of $3 \%$ and a real growth rate of $2.5 \%$. The government spends 200 bitcoins and raises taxes of 200 bitcoins.
(a) Write down the government budget constraint and the debt dynamics describing the evolution of national debt.
(b) Draw the phase diagram for the evolution of debt and comment upon its features.
(c) Suppose the government wishes to stabilise the debt-income ratio. What does it have to do?
(d) Suppose the real interest rate falls to $2.5 \%$ and the real growth rate of the economy rises to $3 \%$. What effect does this have on your answer?
2. Forward Guidance is meant to improve the communication and transparency of monetary policy.
(a) How does the distinction between state and time contingent policies help to do this?
(b) How might the successful pursuit of forward guidance show up in the spead between short and long term interest rates?
(c) Is there any empirical evidence to suggest that forward guidance actually works?
3. Consider the vector autoregresive model with the four variables $\left[\begin{array}{c}p \\ y \\ s \\ x\end{array}\right]$, where $p$ is the price level, $y$ is output, $s$ is the short term interest rate and $x$ is the exchange rate. The relationship between the reduced form errors and the structural shocks is given by $A e_{t}=B u_{t}$. The matrix $B$ is an identity matrix and:

$$
\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
a_{21} & 1 & 0 & 0 \\
a_{31} & a_{32} & 1 & 0 \\
a_{41} & a_{42} & a_{43} & 1
\end{array}\right]\left[\begin{array}{c}
\varepsilon^{p} \\
\varepsilon_{t}^{y} \\
\varepsilon_{t}^{s} \\
\varepsilon_{t}^{x}
\end{array}\right]=\left[\begin{array}{c}
u_{t}^{p} \\
u_{t}^{y} \\
u_{t}^{s} \\
u_{t}^{x}
\end{array}\right]
$$

(a) Explain the relationship between the structural equation and the reduced form.
(b) Is the model identified?
(c) What is an economic interpretation of the restrictions placed on the $A$ matrix?
(d) Do these restrictions seem plausible?
(e) What other forms of identifying restriction could be used?
4. Is it true that if you want to understand why aggregate economies fluctuate at business cycle frequencies it is necessary to appeal to aggregate shocks and not to idiosyncratic firm or sector specific shocks?"
5. (a) Why do forecasters draw a distinction between point forecasts and density forecasts?
(b) If more than one point and density forecast were available, how might we combine them?
(c) Why does nowcasting play such an important role in real-time forecasting?
6. Why does the constraint of a zero nominal interest rate floor provide such a limitation to conventional monetary policy? What measures have been introduced to ameliorate this constraint?
7. Despite the many challenges to the conduct of macroeconomic policy inflation targeting has survived the events of the last 6 years. Why is this so?

END OF PAPER

ECM9/ECM10/ECM11/ MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil in Finance

Friday 9 May 2014 10:00am to 12:00pm

M300

## Econometric Methods

The paper is in two sections. Section A: Candidates are required to answer all six questions. Section B: Candidates are required to answer two out of four questions.

Weighting: Section $A$ as $60 \%$, Section $B$ as $40 \%$.

Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Table of values of the standard normal density
New Cambridge Elementary Statistical Tables
Durbin-Watson and Dickey-Fuller Tables
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Section A

A. 1 Consider the following joint probability density

$$
f_{X, Y}(x, y)=\left\{\begin{array}{l}
\frac{3}{2}\left(x^{2}+y^{2}\right) \text { if } x \in[0,1], y \in[0,1] \\
0 \text { otherwise }
\end{array}\right.
$$

(i) Find the conditional expectation function $E(Y \mid X=x)$.
(ii) Compute the minimum mean squared error prediction of $Y$ given that $X=1 / 2$.
A. 2 You are given the following estimates of a Probit and a Linear Probability Model for success of failure of mortgage applications

$$
\begin{aligned}
\widehat{\operatorname{Pr}}\left(y_{i}=1 \mid x_{i}\right) & =\Phi\left(\underset{(0.16)}{-2.19}+\underset{(0.47)}{3.00} x_{i}\right), \\
\hat{y}_{i} & =\underset{(0.032)}{\left(0.604 x^{2}\right.}+\underset{(0.098)}{0.08 x_{i}},
\end{aligned}
$$

where $y_{i}=1$ if the application is successful, $y_{i}=0$ otherwise, and $x_{i}$ is the payment-toincome ratio for applicant $i$. The median value of $x_{i}$ is 0.35 .
(i) Assuming that the Probit specification is correct and has a valid causal interpretation, compute the estimate of the marginal causal effect of the payment-to-income ratio on the probability of success at the median value of $x_{i}$.
(ii) Without providing explicit formulas, relate the estimate of the slope in the Linear Probability Model to the causal effects of $x_{i}$ on the probability of success.
A. 3 Consider the following regression

$$
y_{i}=\alpha+\beta x_{i}+u_{i}, i=1, \ldots, n,
$$

where $x_{i}$ is an endogenous variable, in the sense that $\operatorname{Cov}\left(x_{i}, u_{i}\right) \neq 0$, and $n=1000$. Suppose that you have two instruments $z_{1 i}$ and $z_{2 i}$.
(i) Formally state conditions for the validity of $z_{1 i}$ and $z_{2 i}$.
(ii) The value of the $F$-statistic from the first stage regression is 4.56 . What does this imply about the quality of the 2SLS estimator based on $z_{1 i}$ and $z_{2 i}$ ?
A. 4 A random sample $\left(y_{i}\right),(i=1, \ldots, 1000)$, is taken from the exponential distribution with density

$$
f(y)=\theta_{0} e^{-\theta_{0} y} \text { for } y>0 .
$$

In particular, $E y_{i}=\theta_{0}^{-1}$ and $\operatorname{Var}\left(y_{i}\right)=\theta_{0}^{-2}$.
(i) Find the sample's Fisher information, $I\left(\theta_{0}\right)$.
(ii) The maximum likelihood estimate of $\theta_{0}$ equals 1.04. Test a hypothesis that $\theta_{0}=1$ at $5 \%$ asymptotic significance level.
A. 5 The continuously updated GMM procedure based on a random sample $\left(y_{i}, x_{i}\right), i=1, \ldots, n$, and moment conditions

$$
E\left(\varepsilon_{i}\right)=0, E\left(\varepsilon_{i} x_{i}\right)=0, \text { and } E\left(\varepsilon_{i} x_{i}^{2}\right)=0,
$$

where $\varepsilon_{i}=y_{i}-\alpha-\frac{1}{x_{i}-\beta}$, is used to obtain the following estimates: $\hat{\beta}_{G M M}=0.5, \hat{\alpha}_{G M M}=0.63$.
(i) In the context of the above example, describe the continuously updated GMM procedure.
(ii) Suppose that the value of the $J$-statistic is 4.567 . What would you conclude?
A. 6 An investigator would like to test a hypothesis that $\Delta c_{t}$ is an i.i.d. sequence with mean zero and unknown variance $\sigma^{2}$, where $\Delta c_{t}$ is the demeaned quarterly growth of total consumption expenditure in the UK. The alternative of interest is that $\Delta c_{t}$ has a nontrivial first-order autocorrelation. Using data from 1955:Q1 $(t=1)$ to 2013:Q2 $(t=234)$, the investigator computes

$$
\hat{\rho}_{1}=\frac{\frac{1}{T} \sum_{t=2}^{T} \Delta c_{t} \Delta c_{t-1}}{\widehat{\sigma}^{2}}=-0.0346,
$$

where $T=234$, and $\widehat{\sigma}^{2}$ is a consistent estimate of $\sigma^{2}$.
(i) Derive the asymptotic distribution of $\hat{\rho}_{1}$ as $T \rightarrow \infty$, assuming that the null hypothesis is true.
(ii) Use your result in (i) to test the investigator's hypothesis.

## Section B

B. 1 Consider a random sample $\left\{\left(y_{i}, x_{i}\right), i=1, \ldots, n\right\}$, where the conditional distribution of $y_{i}$ given $x_{i}$ is normal with mean zero and variance $\sigma^{2} x_{i}^{2}$, i.e.,

$$
y_{i} \mid x_{i} \sim N\left(0, \sigma^{2} x_{i}^{2}\right) .
$$

Suppose that the marginal distribution of $x_{i}$ does not depend on $\sigma^{2}$ and is such that $\operatorname{Pr}\left(x_{i}=0\right)=0$.
(i) Find the maximum likelihood estimator of $\sigma^{2}$ as a function of $\left(y_{i}, x_{i}\right), i=1, \ldots, n$.
(ii) Show that the maximum likelihood estimator from (i) is conditionally unbiased. Find the conditional Cramer-Rao lower bound on the conditional variance of a conditionally unbiased estimator of $\sigma^{2}$.
(iii) Show that the maximum likelihood estimator obtained in (i) achieves the conditional Cramer-Rao lower bound. [Hint: the raw fourth moment of a standard normal distribution equals 3].
(iv) Find the method of moments estimator of $\sigma^{2}$ that uses the moment condition $E\left(y_{i}^{2} \mid x_{i}\right)=\sigma^{2} x_{i}^{2}$. Is it conditionally unbiased? Find its variance and compare with the variance of the maximum likelihood estimator.
B. 2 Consider a version of the consumption capital asset pricing model, which implies that

$$
E_{t}\left(\beta\left(\frac{c_{t+1}}{c_{t}}\right)^{-\gamma}\left(1+r_{t+1}\right)-1\right)=0
$$

where $E_{t}$ is the expectation conditional on the information available at time $t, c_{t}$ is consumption, and $r_{t}$ is the real rate of interest on a financial asset. To estimate $\beta$ and $\gamma$, a researcher proposes to use GMM based on the uncorrelatedness of $\beta\left(\frac{c_{t+1}}{c_{t}}\right)^{-\gamma}\left(1+r_{t+1}\right)-1$ with a constant, the current consumption growth $c_{t} / c_{t-1}$, and the current rate of interest $r_{t}$.
(i) Write down the GMM's criterion function corresponding to a weighting matrix $W$.
(ii) Suppose that there are 200 observations and that the value of the minimum of the optimal GMM criterion function equals 0.004 . Test the validity of the overidentifying restrictions at $5 \%$ significance level.
(iii) A colleague argues that the consumption growth and the rate of return may be i.i.d. sequences. Assuming that the colleague is right, what does this imply for the asymptotic limit of the GMM criterion function from (i)? What does this imply about the quality of the GMM?
B. 3 A researcher estimates the following regressions

$$
\begin{aligned}
\widehat{\Delta R_{1 t}} & =\underset{(0.084)}{0.245}-\underset{(0.0003)}{0.0008 t}-\underset{(0.0084)}{0.0318} R_{1 t-1}+\sum_{i=1}^{3} \hat{\theta}_{1 i} \Delta R_{1 t-i}, \\
\widehat{\Delta R_{2 t}} & =\underset{(0.063)}{0.206}-\underset{(0.0002)}{0.0007 t}-\underset{(0.0064)}{0.0202} R_{2 t-1}+\sum_{i=1}^{3} \hat{\theta}_{2 i} \Delta R_{2 t-i}, \\
\Delta R_{1 t}-\Delta R_{2 t} & =\underset{(0.008)}{0.022}-\underset{(0.0405)}{0.2604}\left(R_{1 t-1}-R_{2 t-1}\right),
\end{aligned}
$$

where $R_{1 t}$ is Sterling one month mean interbank lending rate, $R_{2 t}$ is the official Bank of England rate, estimates of the standard errors are given in the parentheses, and the data are monthly from Jan 1990 to Oct 2013 (286 observations in total).
(i) Test the hypotheses that $R_{1 t}$ and $R_{2 t}$ are integrated of order one series. Use $1 \%$ significance level.
(ii) Discuss possible consequences for your tests in (i) of not including the time trend in the first two regressions above.
(iii) On theoretical grounds, the researcher decides that the difference between $R_{1 t}$ and $R_{2 t}$ must be stationary. Test the hypothesis that the researcher is wrong at $1 \%$ significance level.
(iv) To forecast the interbank lending rate, the researcher formulates and estimates the following regression

$$
\widehat{\Delta R_{1 t}}=\underset{(0.0138)}{0.0246}+\underset{(0.0472)}{0.424 \Delta} R_{1 t-1}-\underset{(0.0691)}{0.6396}\left(R_{1 t-1}-R_{2 t-1}\right) .
$$

The values of $R_{1 t}$ and $R_{2 t}$ in Oct 2013 were, respectively, $0.48 \%$ and $0.5 \%$. The values of these variables in Sep 2013 were, respectively, $0.476 \%$ and $0.5 \%$. Compute the reseacher's forecast of $R_{1 t}$ for Dec 2013. Describe a method that you would use to evaluate the accuracy of this forecast.
B. 4 Discuss TWO of the following topics
a) Clustered standard errors.
b) Spurious regression.
c) Selection bias and the potental output framework for estimating causal effects.

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance \& Economics
MPhil in Finance

Wednesday 28 May 2014 2:00pm to 4:00pm

M310

## Time Series

Candidates are required to answer five questions from Section $A$ and one question from Section B.

Section A is weighted as $60 \%$ and Section $B$ as $40 \%$
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 2
Rough Work Pad
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

A1. Show that the minimum mean square error predictor of an observation $\ell$ steps ahead is the conditional expectation

$$
\tilde{y}_{T+l \mid T}=E\left(y_{T+l} \mid y_{T}, y_{T-1}, \ldots . .\right) .
$$

A2. Compare and contrast the autocovariance matrices of the differenced observations from a multivariate random walk plus noise (local level) model,

$$
\begin{gathered}
\mathbf{y}_{t}=\boldsymbol{\mu}_{t}+\varepsilon_{t}, \quad \varepsilon_{t} \sim N I D\left(\mathbf{0}, \Sigma_{\varepsilon}\right), \quad t=1, \ldots, T, \\
\boldsymbol{\mu}_{t}=\boldsymbol{\mu}_{t-1}+\boldsymbol{\eta}_{t}, \quad \eta_{t} \sim N I D\left(\mathbf{0}, \Sigma_{\eta}\right),
\end{gathered}
$$

with the autocovariance matrices of a vector MA(1) process. Show that a common trend in a bivariate local level model implies co-integration.

A3. Express the exponentially weighted moving average (EWMA)

$$
m_{t}=\lambda \sum_{j=0}^{\infty}(1-\lambda)^{j} y_{t-j}, \quad 0<\lambda<1,
$$

in terms of the lag operator, $L$. Hence show that detrending a series that is integrated of order one with an EWMA will give a stationary series. Obtain the spectrum of the series obtained when a detrending EWMA filter is appled to a random walk.

A4. The New Keynesian Phillips Curve (NKPC) for the rate of infl ation $\pi_{t}$, is

$$
\pi_{t}=\gamma E_{t}\left(\pi_{t+1}\right)+\beta^{*} x_{t}+\varepsilon_{t}, \quad 0 \leq \gamma \leq 1
$$

where $x_{t}$ is the output gap and $\varepsilon_{t} \sim \operatorname{NID}(0,1)$. If it is assumed that $x_{t}$ is a first-order autoregressive process with parameter $\phi$, obtain an expression for the parameter $\beta$ in the equivalent model

$$
\pi_{t}=\beta x_{t}+\varepsilon_{t}, \quad t=1, \ldots, T .
$$

State any assumptions you make.

A5. A random variable $y_{t}, t=1, \ldots, T$, is defined as

$$
\begin{aligned}
& y_{t}=\varepsilon_{t}, \quad \text { for } t \text { even } \\
& y_{t}=\left(\varepsilon_{t-1}^{2}-1\right) / \sqrt{2}, \quad \text { for } t \text { odd }
\end{aligned}
$$

where $\varepsilon_{t} \sim \operatorname{NID}(0,1)$. Show that $y_{t}$ is white noise. Given a sample in which $T$ is even, write down the minimum mean square linear estimator of $y_{T+1}$ and find its MSE. Can you construct a better predictor ?

NB. The kurtosis of a standard normal variate is three.
A6. Write down the definition of a heavy-tailed distribution. The PDF of the Weibull distribution is

$$
f(y ; \alpha, v)=\frac{v}{\alpha}\left(\frac{y}{\alpha}\right)^{v-1} \exp \left(-(y / \alpha)^{v}\right), \quad 0 \leq y<\infty, \quad \alpha, v>0
$$

where $\alpha$ is the scale and $v$ is the shape parameter. Its $\operatorname{CDF}$ is $F(y)=1-$ $\exp \left(-(y / \alpha)^{v}\right)$. Determine whether there are any values of the shape parameter for which the distribution has a heavy tail. Sketch the score against $y$ for $v=2$ and $v=0.5$ and comment.

A7. Two variables, $x_{1 t}$ and $x_{2 t}$, have a bivariate normal distribution with zero means, unit variances and a correlation $\rho$. Why is it advantageous to make the arctanh transformation

$$
\gamma=\frac{1}{2} \ln \frac{1+\rho}{1-\rho}=\tanh ^{-1} \rho,
$$

when $\rho$ changes over time? In a dynamic conditional score (DCS) model with time-varying correlation, $\rho_{t \mid t-1}$, the conditional score with respect to $\gamma_{t \mid t-1}$ is

$$
u_{\gamma t}=\frac{1}{4}\left(x_{1 t}+x_{2 t}\right)^{2} \exp \left(-2 \gamma_{t \mid t-1}\right)-\frac{1}{4}\left(x_{1 t}-x_{2 t}\right)^{2} \exp \left(2 \gamma_{t \mid t-1}\right)+\frac{\exp \left(-2 \gamma_{t \mid t-1}\right)-1}{\exp \left(-2 \gamma_{t \mid t-1}\right)+1},
$$

or, equivalently,

$$
u_{\gamma t}=\frac{1}{4}\left(x_{1 t}+x_{2 t}\right)^{2} \frac{1-\rho_{t \mid t-1}}{1+\rho_{t \mid t-1}}-\frac{1}{4}\left(x_{1 t}-x_{2 t}\right)^{2} \frac{1+\rho_{t \mid t-1}}{1-\rho_{t \mid t-1}}+\rho_{t \mid t-1} .
$$

Show that the expected value of $u_{\gamma t}$ is zero. Evaluate $u_{\gamma t}$ when $\rho_{t \mid t-1}=0,-0.5$ and -0.99 for (i) $x_{1 t}=-2$ and $x_{2 t}=2$ and (ii) $x_{1 t}=x_{2 t}=0$ and comment.

## SECTION B

B1. A seasonal component, $\gamma_{t}$, may be added to a model consisting of a trend, $\mu_{t}$, and (white noise) irregular, $\varepsilon_{t}$, to give

$$
y_{t}=\mu_{t}+\gamma_{t}+\varepsilon_{t}, \quad t=1, \ldots, T .
$$

The seasonal pattern may be allowed to change over time by letting the coefficients evolve as random walks. If $\gamma_{j t}$ denotes the effect of season $j$ at time $t$, then

$$
\gamma_{j t}=\gamma_{j, t-1}+\omega_{j t}, \quad j=1, \ldots, s
$$

where $\omega_{j t}, j=1, \ldots, s$, are zero mean disturbance terms. Although all $s$ seasonal components are continually evolving, only one affects the observation at any particular point in time, that is $\gamma_{t}=\gamma_{j t}$ when season $j$ is prevailing at time $t$.
(a) Show that the requirement that the seasonal components evolve in such a way that they always sum to zero is enforced by setting

$$
\operatorname{Var}\left(\boldsymbol{\omega}_{t}\right)=\sigma_{\omega}^{2}\left(\mathbf{I}-s^{-1} \mathbf{i} \mathbf{i}^{\prime}\right), \quad \sigma_{\omega}^{2} \geq 0,
$$

where $\boldsymbol{\omega}_{t}=\left(\omega_{1 t}, \ldots, \omega_{s t}\right)^{\prime}$ and $\mathbf{i}$ is a vector of ones, coupled with $\sum_{j=1}^{s} \gamma_{j 0}=0$ and an initial covariance matrix proportional to $\operatorname{Var}\left(\boldsymbol{\omega}_{t}\right)$.
(b) Put the model for $y_{t}$ in state space form, assuming that the trend is a random walk. If the Kalman filter is started with a diffuse prior, how many observations are needed to obtain a proper prior? Explain briefly how you would estimate the unknown parameters by maximum likelihood.
(c) Write down the (seasonal) ARMA reduced form model for the specification in (b). Show that this process becomes strictly noninvertible when $\sigma_{\omega}^{2}=0$. Illustrate for the case of $s=2$ and investigate the implications in the frequency domain by obtaining the power spectrum.

B2. Discuss TWO of the following topics and explain their relevance to applied work in macroeconomics and/or finance: (a) Vector error correction models, (b) EGARCH models, (c) the Hodrick-Prescott filter.

ECM9/ECM10/ECM11/ MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil in Finance

Tuesday 3 June 2014 10:00am to 12:00pm

## M320

## Cross Section and Panel Data Econometrics

Candidates are required to answer three questions from Section $A$ and one question from Section B.

Section A is weighted as $60 \%$ and Section B as 40\%
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

A1 Consider the linear static panel data model

$$
\begin{equation*}
\mathbf{y}_{i}=\mathbf{X}_{i} \boldsymbol{\beta}+\boldsymbol{\varepsilon}_{i}, \tag{1}
\end{equation*}
$$

where $\mathbf{y}_{i}$ is a $T \times 1$ vector, $\varepsilon_{i}$ is a $T \times 1$ vector and $\mathbf{X}_{i}$ is $T \times K$ matrix. $\mathbf{Z}_{i}=\left\{\mathbf{z}_{i t}\right\}$ is a $T \times R$ matrix of instruments which satisfies the moment conditions

$$
\begin{equation*}
E\left[\mathbf{Z}_{i}^{\prime} \varepsilon_{i}\right]=\mathbf{0} \tag{2}
\end{equation*}
$$

$\mathbf{z}_{i t}$ is an $R \times 1$ vector and $R \geq K$.
a) Write down the GMM estimator based upon these moment conditions.
b) Noting that the moment conditions in (2) are satisfied if

$$
\begin{equation*}
E\left[\sum_{t=1}^{T} z_{i t} \varepsilon_{i t}\right]=0 \tag{3}
\end{equation*}
$$

comment on the observation that under (3), it is just as difficult to find instruments with panel data as it is with cross-section data.
c) Consider the following alternative set of moment conditions

$$
\begin{equation*}
E\left[\mathbf{z}_{i t} \varepsilon_{i t}\right]=0, \quad t=1, \ldots, T \tag{4}
\end{equation*}
$$

What are the advantages and disadvantages of these moment conditions relative to (3)?

A2 Decision maker $i$ faces a finite choice set $\Omega_{J}$ of dimension $J$. Writing the utility of choice $j$ for individual $i$ as

$$
U_{i j}=V_{i j}+\varepsilon_{i j}
$$

alternative $j$ is chosen if $U_{i j}-U_{i j^{\prime}}>0 \quad \forall_{j} \neq j^{\prime} \in \Omega_{j}$. The probability that $j$ is chosen may then be written

$$
\begin{equation*}
P_{i j}=\int_{\varepsilon} 1\left(\varepsilon_{i j}-\varepsilon_{i j^{\prime}}>V_{i j^{\prime}}-V_{i j}\right) f\left(\varepsilon_{i}\right) d \varepsilon_{i \prime} \tag{5}
\end{equation*}
$$

where $\varepsilon_{i}=\left(\varepsilon_{i 1}, \ldots, \varepsilon_{i j}\right)$.
a) (i.) Given $V_{i j}=\mathbf{v}_{j}^{\prime} \omega$, for fixed parameters $\omega$, and $\varepsilon_{i} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon}\right)$, what is the dimension of the integral in (5). How many parameters are estimable?
(ii.) Given $V_{i j}=\mathbf{v}_{j}^{\prime} \omega_{i}$, with $\omega_{i} \sim N\left(\bar{\omega}, \Sigma_{\omega}\right)$, and $\varepsilon_{i} \sim$ Type 1 Extreme Value, how does the tractability of the probability expression compare to that of (i.) above?
b) For $V_{i j}=\mathbf{v}_{j}^{\prime} \omega$ and $\varepsilon_{i} \sim$ Type 1 Extreme Value, write down the choice probability for alternative $j$.
c) In what sense do the two models implied by a) (ii) and b) represent a trade-off between tractability and the representation of choice behaviour in discrete choice?

A3 Consider the following unobserved effects panel data model

$$
\begin{equation*}
y_{i t}=\alpha+\beta^{\prime} \mathbf{x}_{i t}+\mu_{i}+\varepsilon_{i t} \tag{6}
\end{equation*}
$$

where the $\mu_{i}$ denotes an individual-specific effect and $\varepsilon_{i t}$ is an i.i.d error term. $\mathbf{x}_{i t}$ denotes a $K \times 1$ vector of exogenous variables.
The random effects (or GLS) estimator may be written as

$$
\begin{align*}
\widehat{\boldsymbol{\beta}}_{G L S}= & {\left[\frac{1}{T} \sum_{i=1}^{N} X_{i}^{\prime} \mathbf{Q} X_{i}+\psi \sum_{i=1}^{N}\left(\overline{\mathbf{x}}_{i}-\overline{\mathbf{x}}\right)\left(\overline{\mathbf{x}}_{i}-\overline{\mathbf{x}}\right)^{\prime}\right]^{-1} }  \tag{7}\\
& \times\left[\frac{1}{T} \sum_{i=1}^{N} X_{i}^{\prime} \mathbf{Q} \mathbf{y}_{i}+\psi \sum_{i=1}^{N}\left(\overline{\mathbf{x}}_{i}-\overline{\mathbf{x}}\right)\left(\bar{y}_{i}-\bar{y}\right)\right]
\end{align*}
$$

where $\sigma_{\mu}^{2}$ and $\sigma_{\varepsilon}^{2}$ denote, respectively, the variance of $\mu_{i}$ and $\varepsilon_{i t}$. $\psi=\sigma_{\varepsilon}^{2} /\left(\sigma_{\varepsilon}^{2}+T \sigma_{\mu}^{2}\right), \mathbf{Q}=\mathbf{I}_{T}-\frac{1}{T} \mathbf{i i}, \mathrm{i}$ is a $T \times 1$ unit vector, and $\mathbf{I}_{T}$ is a $T \times T$ identity matrix.
a) Making reference to (7), evaluate the statement that treating $\mu_{i}$ as a random effect provides a solution intermediate between treating all effects as different and treating them all as equal.
b) Let $\kappa=\widehat{\boldsymbol{\beta}}_{G L S, s}-\widehat{\boldsymbol{\beta}}_{F E, s}$ denote a subvector of differences in parameter estimates for time-varying regressors for the GLS and FE estimator.
State the implied null and alternative hypothesis that follow directly from the use of the test statistic $\widehat{H}=\boldsymbol{\kappa}^{\prime}[\operatorname{Var}(\boldsymbol{\kappa})]^{-1} \boldsymbol{\kappa}$.
Explain why one might want to consider a robust form of this test statistic.
A. 4 Consider the following bivariate linear regression model

$$
Y_{i}=\beta_{1}+\beta_{2} X_{i}+\varepsilon_{i}
$$

where $X_{i}$ is a scalar regressor and $E\left(\varepsilon_{i} \mid X_{i}\right)=\tau\left(X_{i}\right)$. A single instrument $Z_{i}$ is available with $\sigma_{\varepsilon_{i} Z_{i}}=0$ and $\sigma_{X_{i}, Z_{i}} \neq 0$.
a) Find an expression for the IV estimator $\widehat{\beta}_{2}^{I V}$.
b) Show that $\widehat{\beta}_{2}^{I V}$ is consistent provided that $\sigma_{\mathrm{ZX}} \neq 0$.
c) Using the results from $b$ ) or otherwise, show that the IV estimator is preferred to OLS on asymptotic grounds when
$\left(\rho_{z, u} / \rho_{z, x}\right)<\rho_{x, u}$.
$\rho_{z, u}$ denotes the correlation coefficient between $z$ and $u$.
d) Carefully explain why in a just-identified model neither the Hansen nor Sargan test for instrument exogeneity can be computed.
A. 5 Elaborate upon the following statements.
a) Relative to the linear probability model, the advantages of the threshold nonlinear binary response model are less when the design matrix is sparse, and/or when there are endogenous binary regressors.
b) In the context of dynamic panel data models, the use of GMMstyle instruments circumvents the sample size efficiency tradeoff.
c) If in a linear fixed effects (FE) static panel data model, time series variation around regressors $\mathrm{x}_{i t}$ is limited, the evaluation of the FE and competing estimators using mean square error may be appropriate.

## SECTION B

B. 1 (a) Suppose $y_{2} \in\{0,1\}$ denotes an endogenous binary treatment and $y_{1} \in\{0,1\}$ is the outcome of interest. $y_{1}(1)$ and $y_{1}(0)$, denote, respectively, the outcome with and without the treatment. $z \in\{0,1\}$ is a scalar binary instrument, independent of $y_{1}(1)$ and $y_{1}(0)$, and correlated with $y_{2}$.
Suppose you have an i.i.d sample of $N$ individuals.
i. Find an expression for the IV estimator for the average treatment effect.
ii. We now impose the following structure on the model:

$$
\begin{aligned}
& y_{1}=\mathbf{1}\left(y_{1}^{*}>0\right) \\
& y_{1}^{*}=\kappa_{y_{1}}+\gamma y_{2}+\varepsilon_{y_{1}} \\
& y_{2}=\mathbf{1}\left(y_{2}^{*}>0\right) \\
& y_{2}^{*}=\kappa_{y_{2}}+\alpha z+\varepsilon_{y_{2}}
\end{aligned}
$$

where $y_{1}^{*}$ and $y_{2}^{*}$ denote latent variables. $\left(\varepsilon_{y_{2}}, \varepsilon_{y_{1}}\right)$ is distributed standard bivariate normal with correlation $\rho$, and independent of $z .1($.$) denotes the indicator function. \kappa_{y_{1}}$, $\kappa_{y_{2}}, \gamma$, and $\alpha$ are unknown parameters.
Find an expression for the average treatment effect for $y_{2}$.
(b) We now consider an alternative model given by

$$
\begin{align*}
& y_{1}^{*}=\mathbf{z}_{1}^{\prime} \delta_{1}+\alpha_{1} y_{2}+u_{1}  \tag{8}\\
& y_{1}=\mathbf{1}\left(y_{1}^{*}>0\right)  \tag{9}\\
& y_{2}=\mathbf{z}_{1}^{\prime} \delta_{21}+\mathbf{z}_{2}^{\prime} \delta_{22}+v_{2}=\mathbf{z}^{\prime} \delta_{2}+v_{2} \tag{10}
\end{align*}
$$

(8) and (9) denotes the structural equation and (10) is the reduced form for $y_{2}$. $\left(u_{1}, v_{2}\right) \sim N(0, \Sigma)$, and is independent of z. We impose the normalisation $\operatorname{Var}\left(u_{1}\right)=1$. $\alpha_{1}, \delta_{1}, \delta_{21}, \delta_{22}$, and $\delta_{2}$ are unknown parameters, and $1($.$) denotes the indicator$ function.
i. Given the distributional assumptions explain why in this instance $y_{2}$ cannot be a discrete random variable.
ii. Find an expression for $\operatorname{Pr}\left(y_{1}=1 \mid \mathbf{z}\right)$.

Show that this method estimates the parameters of (8) up to a common scale, say $1 / \vartheta$. Find an expression for $\vartheta^{2}$.
iii. Using this method, describe a two-step estimation procedure for the parameters of the structural equation.
iv. Contrast the use of $\operatorname{Pr}\left(y_{1}=1 \mid \mathbf{z}\right)$ versus $\operatorname{Pr}\left(y_{1}=1 \mid \mathbf{z}, v_{2}\right)$ as alternate estimation strategies.
B. $2 U_{i j t}$ denotes the utility for individual $i$ who chooses alternative $j$ at time $t$. This utility may be written as follows

$$
\begin{equation*}
U_{i j t}=\mathbf{x}_{i j t}^{\prime} \beta_{i}+\epsilon_{i j t} \tag{11}
\end{equation*}
$$

where $\mathbf{x}_{i j t}$ is a $k \times 1$ vector of explanatory variables, $\beta_{i}$ is a $k \times 1$ parameter vector and $\epsilon_{i j t}$ is a random shock distributed Type 1 extreme value. Individual $i$ chooses alternative $j$ in period $t$ if

$$
U_{i j t}>U_{i m t} \quad \forall j \neq m
$$

We observe $\mathrm{y}_{i}=\left(y_{i 1}, \ldots, y_{i T}\right)^{\prime}$, where $y_{i t}=j$ if individual $i$ chooses alternative $j$ at time $t$.
(a) For $\beta_{i} \sim g(\beta \mid \theta)$, where $\theta$ is a vector of hyperparameters and $g($.$) is the distribution of \beta$ in the population, find:
i. $\operatorname{Pr}\left(\mathbf{y}_{i} \mid \mathbf{x}_{i}, \beta_{i}\right)$ and
ii. $\operatorname{Pr}\left(\mathbf{y}_{i} \mid \mathbf{x}_{i}, \boldsymbol{\theta}\right)$.
$x_{i}$ collects, for each $i$, variables for all alternatives and choice situations.
(b) Find an expression for $h\left(\beta \mid \mathbf{y}_{i}, \mathbf{x}_{i}, \boldsymbol{\theta}\right)$, the density of $\beta$ in the subpopulation of individuals who would choose $\mathbf{y}_{i}$ when facing $\mathrm{x}_{\mathrm{i}}$.
(c) In what sense does a model with preference heterogeneity, as in (11), circumvent the independence of irrelevant alternatives property.
(d) Writing $g\left(\beta \mid \theta=\left(\bar{\beta}, \Sigma_{\beta}\right)\right)$, where $\bar{\beta}$ and $\Sigma_{\beta}$ denote, respectively mean and covariance parameters, and letting $k\left(\bar{\beta}, \Sigma_{\beta}\right)$ denote the prior distribution, find a general expression for the joint posterior distribution for $\bar{\beta}$ and $\Sigma_{\beta}$.
Comment on any difficulties encountered in taking draws from this posterior and how augmenting the posterior with $\beta_{i}$ might help.

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Friday 23 May 2014 2:00pm to 4:00pm

M330

## Applied Econometrics

Candidates are required to answer all four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Durbin-Watson and Dickey-Fuller Tables
New Cambridge Elementary Statistical Tables
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

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1. We examine a sample of $N$ individuals who are eligible for treatment. Let ( $Y_{i}, X_{i}, D_{i} ; i=1,2 . . N$ ), be a vector of observations on the scalar-valued outcome $Y$, a vector of observable covariates $X$ and a binary indicator of treatment $D$.
a) Discuss, in brief, the assumption of conditional independence for the identification of the average treatment effect (ATE) and the average treatment on the treated effect (ATT). Explain the difference in assumptions and the behavioural implications of these assumptions.
b) Discuss the assumption of support or overlap in identifying the two parameters. Again, explain how the assumption required differs between the ATE and ATT and why?
c) What is the key assumption in identifying the treatment effect in a regression discontinuity design? Is the effect thus identified an average treatment effect? Explain and contrast this with the requirements for identification using instrumental variables or matching estimators.
2. The questions below refer to a large-scale randomised evaluation of the impact of teacher performance pay in schools. The aim was to examine whether offering teachers pay based on student performance on tests improved student achievement. The incentive treatment consisted of an announcement that bonuses would be paid at the beginning of the next school year conditional on average improvements in test scores during the current school year.

The overall design of the project was as follows. The eligible sample consisted of 300 schools. 150 schools were in the control group and there was no intervention while in the remaining 150 schools, teachers received bonuses which were conditional on student test scores. Baseline tests of student learning were conducted in the 300 sampled schools two months before the interventions were announced. The baseline test covered material from the previous school year. At the end of the program a final test was conducted, with similar content.

The following specification examines the impact of the incentive program. The variable Incentives, is a binary variable representing treatment, taking the value 1 if it is a treated school and zero otherwise.

$$
T_{i j k m}^{1}=\alpha+\beta T_{i j k m}^{0}+\gamma(\text { Incentives })+\delta Z_{m}+\epsilon_{k}+\epsilon_{j k}+\epsilon_{i j k}
$$

where $T^{t}$ denotes the normalised test score ( $t=0$ denotes the baseline while $t=1$ refers to the end of the program year), $i, j, k, m$ indicate student, grade, school and region respectively, and $Z_{m}$ is a vector of region dummies or region fixed effects. A further specification also includes school and household level variables. Table 1 summarises the results. (Standard errors in brackets).

Table 1: Impact of Incentives on Student Test Scores Dependent Variable: Normalised End-of-Year Test Score

|  | $(1)$ |  |
| :---: | :---: | :---: |
| Normalised lagged test score | $0.503^{* * *}(0.01)$ | $0.498^{* * *}(0.01)$ |
| Incentive school | $0.149^{* * *}(0.04)$ | $0.152^{* * *}(0.04)$ |
| School and household controls | No | Yes |
| $\mathrm{R}^{2}$ | 0.31 | 0.34 |

( $^{* * *}$ Significant at 1 percent; ${ }^{* *}$ Significant at 5 percent; * Significant at 10 percent)
a) Discuss both the specification and results in brief. What further information would you wish to have to persuade yourself that the impact of the intervention is identified?
b) The study also collected information on teacher characteristics such as education, experience and other individual information. The specification below was estimated and the results are presented in Table 2.

$$
\begin{aligned}
& T_{i j k m}^{1}=\alpha+\beta T_{i j k m}^{0}+\gamma_{1}(\text { Incentives })+\gamma_{2}(\text { Characteristics })+ \\
& \gamma_{3} \text { (Incentives*Characteristics) }+\delta Z_{m}+\epsilon_{k}+\epsilon_{j k}+\epsilon_{i j k},
\end{aligned}
$$

Table 2: Impact of Incentives and Teacher Characteristics Dependent Variable: Normalised End-of-Year Test Score

|  | Education | Training | Years of Experience |
| :---: | :---: | :---: | :---: |
| Incentive | $0.113(.163)$ | $0.224(.176)$ | $0.258^{* * *}(.059)$ |
| Characteristic | $0.003(.032)$ | $0.051(.041)$ | $0.001(.003)$ |
| Interaction | $.086^{*}(.050)$ | $0.138^{* *}(.061)$ | $0.009^{* *}(.004)$ |
| $\mathrm{R}^{2}$ | 0.29 | 0.29 | 0.29 |
| (*** Significant at 1 percent; ${ }^{* *}$ Significant at 5 percent; * Significant at 10 |  |  |  |
| percent) |  |  |  |

Explain the rationale for the second specification and interpret the results. An additional specification that included household characteristics such as parental education and incomes and their interaction with the dummy variable, Incentives, was also estimated but none of the coefficients was found significant. Discuss the rationale for this specification as well.
c) Discuss the error structure specified. What is the implication for the computation of standard errors? What is the econometric rationale for the inclusion of the baseline test score?
d) Discuss two issues that might compromise identification in this context.
3. We are interested in analysing the historical performance of the Phillips curve, i.e. the inverse relation between the rate of unemployment $\left(U_{t}\right)$ and the rate of inflation $\left(\pi_{t}\right)$. We start by obtaining least square estimates of the following equation

$$
\pi_{t}=\alpha+\beta U_{t}+\varepsilon_{t}, \quad t=1, \ldots, T,
$$

where $\varepsilon_{t}$ is assumed to be i.i.d. normal with mean zero and variance $\sigma^{2}$. We are concerned that one of the Gauss-Markov assumptions - i.i.d. disturbances - is not satisfied and entertain the hypothesis of autocorrelated disturbances.
a) What are the consequences of residual autocorrelation for least squares estimates? Describe the Lagrange multiplier test procedure and how you could use it to dispel such concerns.
b) Describe the steps involved in obtaining Cochrane-Orcutt estimates of the Phillips curve under the assumption that $\varepsilon_{t}$ follows an autoregressive process of order one.
c) Show that the original model augmented with the assumption of $\mathrm{AR}(1)$ disturbances can be rewritten as an autoregressive distributed lag (ADL) model. Does the original model impose any parameter restrictions on the ADL model? Ignoring such restrictions, is least squares inference based on the (unrestricted) ADL model valid?
4. A researcher is interested in understanding the impact on food expenditures of an income subsidy program for poor households in a village in the Congo. The researcher collected data on household food expenditure and income for 40 randomly sampled households. The following relationship was estimated by OLS:

$$
\widehat{\log } \widehat{\_} \text {food }=\underset{(0.434)}{0.189}+\underset{(0.115)}{0.694} \log \_ \text {income },
$$

where log_food gives the logarithm of household food expenditure, log_income gives the logarithm of household income and figures in parenthesis give standard errors. The researcher stored the residuals, $\widehat{u}$, and then ran the regression:

$$
\widehat{u}^{2}=\widehat{\alpha}+\widehat{\beta} \log \_ \text {income }+\widehat{\gamma} \log \_ \text {income }^{2}
$$

to obtain the following results:

| F-statistic | 3.809082 | Prob. F(2,37) | 0.0313 |
| :--- | :--- | :--- | :--- |
| Obs*R-squared | 6.829652 | Prob. Chi-Square(2) | 0.0329 |
| Scaled explained SS | 4.115153 | Prob. Chi-Square(2) | 0.1278 |

```
Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date:03/09/14 Time:19:00
Sample: }14
Included observations:40
White heteroskedasticity-consistent standard errors & covariance
```

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| :--- | ---: | :--- | ---: | ---: |
| C | 1.775739 | 0.811223 | 2.188966 | 0.0350 |
| LOG_INCOME^2 | 0.124622 | 0.051554 | 2.417298 | 0.0207 |
| LOG_INCOME | -0.933226 | 0.410481 | -2.273493 | 0.0289 |
| R-squared | 0.170741 | Mean dependent var | 0.066338 |  |
| Adjusted R-squared | 0.125917 | S.D. dependent var | 0.077632 |  |
| S.E. of regression | 0.072580 | Akaike info criterion | -2.336208 |  |
| Sum squared resid | 0.194912 | Schwarz criterion | -2.209542 |  |
| Log likelihood | 49.72415 | Hannan-Quinn criter. | -2.290409 |  |
| F-statistic | 3.809082 | Durbin-Watson stat | 1.918104 |  |
| Prob(F-statistic) | 0.031316 |  |  |  |

Table 3: Squared residuals regression.
a) What test is the researcher performing in the squared residuals regression? Do the results in Table 3 cast doubt on the OLS estimates? If so, why?
b) The researcher then calculated White's robust standard errors. What are the merits and drawbacks of pursuing this strategy compared with reestimating the equation by Generalised Least Squares?
c) Figure 1 shows the fitted relationship between income and food expenditure (in levels) and labels the two highest income households, A and B. Do you think these are high leverage observations? Are they influential observations? Justify your answers and name one measure that would help your reasoning.


Figure 1: Scatter plot of household income vs. food expenditure. Solid line gives OLS fit.
d) Why would a robust regression estimation technique - such as Least Absolute Deviations - be of help in this case?
e) Suppose that the researcher decides to drop household $B$ from the sample and reestimates the regression using OLS. Under what condition(s) would this be equivalent to Least Trimmed Squares?

END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Friday 6 June $2014 \quad$ 2:00pm to 4:00pm

M500
Development Economics
Candidates are required to answer two out of three questions.
Each question carries equal weight.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 2
Rough Work Pad
Tags

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Each landlord in a village needs two workers for his land. One worker performs tasks that can be monitored by the landlord. The other worker performs tasks where the effort cannot be observed during each period, while he is working, but is revealed at the end of the period. Wages are paid at the beginning of each period.
(a) The worker assigned to the tasks where effort cannot be observed will evidently shirk if he is hired for a single period. One solution to this moral hazard problem is for the landlord to hire workers, who are infinitelylived, on a permanent basis. There is now a cost to shirking because the permanent worker will be fired by his employer at the end of the period when it is discovered that he failed to put in effort. Let the market clearing wage for casual workers, who perform the monitored task, be $W_{c}$. Let the wage received by the permanent worker be $W_{p}$. Both types of workers incur the same cost of effort $e$, which the permanent workers avoid when they shirk. Casual workers can be monitored, so they never shirk. The landlords in the village communicate with each other, so any permanent worker who shirks will never be hired again as a permanent worker, but will be hired in the future as a casual worker (assume that there is no unemployment). Derive $W_{p}$ as a function of $W_{c}, e$ and $\delta$ (the discount factor).
(b) Now suppose we introduce imperfect communication among the landlords. In particular, let the probability of being rehired as a permanent worker in the period after shirking be $0<P<1$. If a shirker is rehired in the subsequent period, we assume that he stays honest thereafter. If he is not immediately rehired as a permanent worker he will remain as a casual worker forever after. What happens to $W_{p}$ as $P$ increases? Derive the result mathematically and then explain the result.
(c) Suppose, instead, that we introduce an exogenous probability of separation $\tau$ in the model. This means that a permanent worker who puts in effort could still be asked to leave by his landlord. He will become a casual worker for at least one period, but could be rehired in the future as a permanent worker by another landlord. In contrast, if he shirks and is then fired, he will always remain a casual worker. What happens to $W_{p}$ as $\tau$ increases? Explain without explicitly deriving the result mathematically
(d) We effectively assumed in part (a) that no landlord would hire a previous shirker as a permanent worker, which implies that $P=0$. Assuming an exogenous probability of separation, as above, explain why no landlord would want to deviate and hire a previous shirker.

2 Each firm in an industry consists of a single entrepreneur and the firm's output is described by the function: $Y=\omega K^{1 / 2}$, where $\omega$ is the entrepreneur's ability and $K$ is the amount of capital (in dollars) that he invests in the firm. Assume that capital markets are efficient and that the unit cost of capital (the interest rate) is $r$. Normalize so that the unit price of the product is set to one.
(a) Show that (i) high ability entrepreneurs invest more and (ii) higher ability entrepreneurs have higher output.
(b) Now suppose that capital markets are imperfect and firms must turn to informal sources of credit such as their community networks. Half the entrepreneurs in the industry belong to a wealthy caste who moved recently from farming into business and the other half belong to castes that have been engaged in business for generations.
When the firms in the industry were surveyed by a research team, it was discovered that entrepreneurs from the traditional farming community invested more, but had lower output, on average, than entrepreneurs from the traditional business communities. This appears inconsistent with the result in part (a), where entrepreneurs who invested more had higher output. Reconcile this apparent discrepancy, using the framework set up in part (a) to help you. Derive the precise mathematical condition under which this empirical finding could be obtained and then provide the intuition for your answer.
(c) Apart from capital market imperfections, can you think of other explanations for the empirical finding? [Hint: modify the production function or consider imperfections in other markets].
3. a) Marriage in India is endogamous, which implies that individuals can only marry within their sub-caste. In contrast, marriage in sub-Saharan Africa is exogamous, which implies that individuals must marry outside their clan. This implies that marriage in sub-Saharan Africa is accompanied by membership in a new affine network organized around the wife's family and that men must often bring home their brides from far away. Can you provide an economic explanation for these differences in marriage rules?
b) Traditional clan-based networks in Africa have been transplanted to the city in recent decades, with urban migrants receiving support from both their birth network and their affine network (if married) in finding jobs. In turn, they send home remittances and support members of the network in the city, by providing them with accommodation, credit, and other forms of assistance. A team of researchers conducted a survey of male migrants in Kisumu, the third largest city in Kenya, in 2001. They were interested in estimating the effect of marriage on migrant incomes and so ran the following regression:

$$
y_{i}=\alpha+\beta M_{i}+\gamma X_{i}+\varepsilon_{i}
$$

where $y_{i}$ is migrant $i$ 's income, $M_{i}=1$ if he was married, 0 if he was single, $X_{i}$ is the migrant's age, and $\varepsilon_{i}$ is the error term. All men ultimately marry, so the regression was restricted to men aged 20-30. The researchers ran two regressions: OLS and instrumental variables. They found that $\beta$ was positive in each case, but larger in magnitude with the instrumental variable regression (you can assume that the instrument is valid). When they replaced the migrant's income with the fraction of his income that he sent home to his rural network, they obtained the same pattern; $\beta$ was positive, but larger in magnitude with the instrumental variable regression. Can you interpret this result by taking advantage of findings from other papers that you studied in class? [Hint:- think about selection into and out of networks.]

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Wednesday 4 June $2014 \quad$ 2:00pm to 4:00pm

M610

## British Industrialisation

Candidates are required to answer two out of four questions.
Each question carries equal weight.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Describe the key elements that de Vries identifies as characterising the Industrious Revolution. Does the evidence for Britain indicate that an Industrious Revolution occurred?
2. Do we have a satisfactory explanation of population growth in the long-eighteenth century?
3. Early accounts of the Industrial Revolution emphasised rapid growth in output brought about by the widespread use of new technology. To what extent has the work of Crafts challenged this version of events? How does the recent work of ShawTaylor and Wrigley challenge Crafts' interpretation?
4. Assess the role played by agriculture in Britain's industrialisation.

2005 - M120 - Exam

1. Let $N$ be a set of students, and $X$ a set of schools. School $x$ has $q_{x}$ seats. Each student $i \in N$ has strict preferences $\succsim_{i}$ over $X \cup\{i\}$. And each school $x \in X$ has a strict priority ranking $\succsim_{x}$ over the set $N$ of students.
Mechanism $\beta$, also known as the Boston Mechanism, works as follows:
Step 1: For each school $x$, let $S_{1}(x)$ be the set of students who rank $x$ as their first choice. If $\left|S_{1}(x)\right| \leq q_{x}$, then assign every such student to school $x$, remove those students from the pool, and reduce the number of seats available at school $x$ to $q_{x}^{2}=q_{x}-\left|S_{1}(x)\right|$.
If, the school is over-demanded, i.e., $\left|S_{1}(x)\right|>q_{x}$, then rank the students in $S_{1}(x)$ according to $\succsim_{x}$ and assign the top $q_{x}$ students to school $x$, remove them from the pool, and remove the school from the pool.

For $k \geq 2$ :
Step $k$ : For all remaining students, and schools with $q_{x}^{k} \geq 1$, let $S_{k}(x)$ be the set of students who find $x$ acceptable and rank it as their $k$ th choice. If $\left|S_{k}(x)\right| \leq q_{x}^{k}$, then assign every such student to school $x$, remove those students from the pool, and reduce the number of seats available at school $x$ to $q_{x}^{k+1}=q_{x}^{k}-\left|S_{k}(x)\right|$.
If, the school is over-demanded, i.e., $\left|S_{k}(x)\right|>q_{x}^{k}$, then rank the students in $S_{k}(x)$ according to $\succsim_{x}$ and assign the top $q_{x}^{k}$ students to school $x$, remove them from the pool, and remove the school from the pool.
When there is no student or no school left, the algorithm ends. Otherwise go to Step $k+1$.
(a) Show that if the agents truthfully reveal their preferences, the outcome of the mechanism does not necessarily respect priorities.
(b) Show that the above mechanism is not strategy-proof. (You can simply construct an example of an environment in which there is a preference profile at which one agent strictly prefers to misrepresent her preferences.)
(c) Show that if a matching respects priorities (according to the actual preferences), then there is a Nash equilibrium of the above mechanism which to results in this matching.
(d) Show that any Nash equilibrium outcome of the game induced by the above mechanism respects priorities according to the actual preferences.
2. A Principal $(P)$ designs a contract for an agent. $P$ has three possible decisions: project $A$, project $B$ and the status quo $(S)$. The contract must give the agent the incentive to investigate both projects in the cheapest possible way. Each investigation costs $K>0$. An investigation will find evidence in favour of the project with probability $x>0$, independently of the outcome of the other investigation. A project is optimal for $P$ if and only if there is evidence for that project and not for the other one; if there is evidence for neither, or both, projects then $S$ is optimal. Rewards must be decision-based only. Information cannot be withheld or distorted. A's outside utility is zero. Suppose that there is no limited liability (that is, payments to the agent can be negative).
(a) Show that the problem is feasible only if $x<0.5$.
(b) Find P's optimal contract by writing down the incentive constraints and identifying which two must be binding.
(c) Discuss the form of the solution.
3. Two players are bargaining over a cake of size 1. Let $\Delta$ denote the set of all feasible partitions of the cake:

$$
\Delta=\left\{\left(x_{1}, x_{2}\right) \mid x_{1}, x_{2} \geq 0 \text { and } x_{1}+x_{2}=1\right\}
$$

and assume that each player $i$ 's preferences over the different partitions can be described by a utility function $u_{i}: \Delta \rightarrow \mathbb{R}_{+}$.
(a) Suppose the game is an infinite horizon alternating bargaining game with "breakdown" and no discounting. Thus, the rules are such that in odd periods player 1 makes an offer from set $\Delta$ to player 2 , which player 2 may accept or reject; and in even periods player 2 makes an offer from set $\Delta$ to player 1 , which player 1 may accept or reject. If, at any period, a player accepts an offer, the proposed split is immediately implemented, and the game ends. If she rejects, with probability $p \in(0,1)$ nothing happens, and the game proceeds to the next period of the bargaining, whereas with probability $1-p$ the bargaining breaks down. Thus, if the players reach an agreement, at time $t$, on a partition $x=\left(x_{1}, x_{2}\right) \in \Delta$, then the payoff of player $i$ is $u_{i}(x) \geq 0$ (hence no discounting). Assume that for each player $i$, the payoff is $d_{i} \geq 0$ in the event of a breakdown, and zero if the game continues indefinitely with no agreement. Furthermore, assume that $u_{i}(x)$ is increasing in $x_{i}$ for all $i$ and there exists payoff vectors $y=\left(y_{1}, y_{2}\right) \in \Delta$ and $z=\left(z_{1}, z_{2}\right) \in \Delta$ such that

$$
\begin{aligned}
& u_{2}(y)=p u_{2}(z)+(1-p) d_{2} \\
& u_{1}(z)=p u_{1}(y)+(1-p) d_{1} .
\end{aligned}
$$

i. Show that there exists a subgame perfect equilibrium strategy profile of this game that induces a payoff $y$.
ii. Show that the payoff profile of the equilibrium described in (i) converges to a Nash bargaining solution of some cooperative bargaining game as $p \rightarrow 1$.
(b) Suppose the game is an infinite bargaining game with one sided offer, no breakdown, but with discounting. Thus, the rules are such that in each period player 1 makes an offer from set $\Delta$ to player 2, which player 2 may accept or reject. If, at any period, a player accepts an offer, the proposed split is immediately implemented, and the game ends. If she rejects, the game proceeds to the next period of the bargaining. For every $x=\left(x_{1}, x_{2}\right) \in \Delta$ and for each $i$ suppose that $u_{i}(x)=x_{i}$. Assume that players discount the future payoffs by factor $\delta<1$. Thus if the players agree on a division of $\left(x_{1}, x_{2}\right)$ at period $t$ then $i$ 's payoff will be $\delta x_{i}^{t-1}$. Show that the subgame perfect equilibrium is unique and player 1 always offers $(1,0)$.
4. Consider a game $G=\left(A_{i}, u_{i}\right)_{i=1}^{n}$ with $n$ players, where $A_{i}$ stands for the strategy set of player $i$, and $u_{i}$ denotes his payoff function.
(a) Let $G^{T}$ denote the finitely repeated game where $G$ is played $T$ times.

Show that $G^{T}$ might have a history-dependent subgame perfect equilibrium if $G$ has more than one equilibrium. (For a complete answer, it is sufficient to construct an example of a game $G$ that has multiple equilibria and is played twice repeatedly.)
(b) Suppose, now, that $G$ is played infinitely often, and that the players use pure strategies and discount the future. Let

$$
u(A)=\left\{u^{\prime} \in \mathcal{R}^{n} \mid u_{i}^{\prime}=u_{i}(a) \text { for all } i \text { and for some } a \in A\right\} .
$$

Denote a minmax strategy profile for $i$ by $m^{i}$, that is,

$$
m^{i} \in \arg \min _{a_{-i}} \max _{a_{i}} u_{i}\left(a_{i}, a_{-i}\right)
$$

i. Show that the average discounted payoff of any player $i$ in any subgame perfect equilibrium cannot be less than $u_{i}\left(m^{i}\right)$.
ii. Suppose that for all $i, j=1, \ldots, n$

$$
u_{i}\left(m^{j}\right)>u_{i}\left(m^{i}\right) \text { for all } i \neq j .
$$

Show that for any payoff vector $u^{0}=\left(u_{1}^{0}, \ldots, u_{n}^{0}\right) \in u(A)$ such that $u_{i}^{0}>u_{i}\left(m^{i}\right)$ for all $i$, there exists a subgame perfect equilibrium of $G^{\infty}$ that induces an average discounted payoff vector $u^{0}$ if the players are sufficiently patient.
iii. Suppose the game $G$ is full-dimensional. For any payoff vector $u^{0}=$ $\left(u_{1}^{0}, \ldots, u_{n}^{0}\right) \in u(A)$ such that $u_{i}^{0}>u_{i}\left(m^{i}\right)$ for all $i$, describe briefly and informally a vector of strategies in the repeated game such that it induces a average payoff $u^{0}$ and the strategy vector is a subgame perfect equilibrium when players are sufficiently patient.

## [END OF PAPER]

ECM11
MPhil in Finance and Economics

Tuesday 5 May 2015 2:00pm to 4:00pm

## F100

Finance I
Candidates are required to answer all four questions.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

Question 1. Part A and Part B of this question are totally independent.

Part A - You are prepared to make monthly payments of $\$ 350$, beginning at the end of this month, into an account that pays 10 percent interest compounded monthly. How many payments will you have made when your account balance reaches $\$ 35,000$ ?

Part B - You have invested in two certificates of deposit, $A$ and $B$. Contract $A$ states a rate of interest $r^{A}$ quoted per annum but with semi-annual compounding, whereas contract $B$ states a rate of interest $r^{B}$ quoted per annum but with continuous compounding. The effective annual yield you receive from investments $A$ and $B$ are $y^{A}=9.8 \%$ and $y^{B}=14.2 \%$, respectively.

1. What is the annual rate of interest $r^{A}$ stated in contract $A$ ?
2. What is the annual rate of interest $r^{B}$ stated in contract $B$ ?

Question 2. Part A and Part B of this question are totally independent.

Part A - Spectre is considering a drug project that costs $\$ 2.5$ million today and is expected to generate end-of-year annual cash flows of $\$ 227,000$, forever. At what discount rate would Spectre be indifferent between accepting or rejecting the project?

Part B - Riviera Publishing is trying to decide whether to revise its popular book, Monaco for Business. The company has estimated that the revision will cost $\$ 75,000$. Cash flows from increased sales will be $\$ 21,000$ the first year. These cash flows will increase by 4 percent per year. The book will go out of print five years from now. Assume that the initial cost is paid now and revenues are received at the end of each year. If the company requires a return of 10 percent for such an investment, should it undertake the revision?

Question 3. Part A and Part B of this question are totally independent.

Part A - Suppose an investment offers to quadruple your money in 12 months. What rate of return per quarter are you being offered?

Part B - You need a 30-year, fixed-rate mortgage to buy a new home for $\$ 250,000$.

Your mortgage bank will lend you the money at a 5.3 percent APR for this 360 -month loan. However, you can only afford monthly payments of $\$ 950$, so you offer to pay off any remaining loan balance at the end of the loan in the form of a single balloon payment. How large will this balloon payment have to be for you to keep your monthly payments at $\$ 950$ ?

Question 4. Consider a model with three dates $(t=0,1,2)$, and no discounting. At date $t=0$ an entrepreneur needs $F>0$ to realize a project, but has only resources $W$ available for investment, where $W<F$. At date $t=1$, the entrepreneur who is key to the project can choose an "effort" level $e \in\{0,1\}$, bearing a private cost $c(e)$. Consider $c(0)=0$ and $c(1)=c$. At $t=2$, the project delivers a cash flow $X \in\left\{0, X^{H}\right\}$, where $X^{H}>0$, and $\operatorname{Pr}\left[X=X^{H}\right] \equiv \theta+e \cdot \Delta_{\theta}$.

Assume that exerting the effort is efficient, i.e. $\Delta_{\theta} X^{H}>c$, and that the project's value is positive if $e=1$. To finance the project, the entrepreneur can sell a claim on $X$, with promised repayments at $t=2$

$$
0 \text { if } X=0 \quad \text { and } \quad R^{H} \text { if } X=X^{H}
$$

1. Establish the project's value if $e=0$ and $e=1$. Show that if $c=0$ (effort is not costly) the entrepreneur can always finance the project.
2. Assume now that effort is costly (i.e., $c>0$ ) and not contractible. Establish the circumstances under which the entrepreneur has an incentive to exert effort (choose $e=1$ ). Establish the condition under which the project will find financing. Show that this condition is more likely to be satisfied when $W$ is large.

## END OF PAPER

ECM11
MPhil in Finance and Economics

Monday 18 May 2015 1.30pm to 3.30 pm

F200
Finance II
Candidates are required to answer all four questions.
All questions carry equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

## You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

Question 1. Part A and Part B of this question are totally independent.

Part A - Super-fun fund manages a risky portfolio with an expected rate of return of $18 \%$ and a standard deviation of $27 \%$. The T-bill rate is $8 \%$. Big Mike wishes to have a portfolio with total expected rate of return of $14 \%$, by investing in the Super-fun portfolio a proportion $w$ of his total investment and the rest in the T-bills.
(a) What is the proportion $w$ ?
(b) What is the standard deviation on Big Mike's portfolio?

Part B - Assume the term structure is flat. Then, indicate which and why of the following statements are true and which are false:
(a) When, other things being equal, the yield rate rises, so does the duration of a bond.
(b) Other things being equal, the larger the coupon rate of a bond and/or smaller its time to maturity, the larger its duration.

Question 2. Suppose that the return on the CAC 40 portfolio and the return on the Banji Fund portfolio are respectively:

| State | Probability | Return on CAC 40 | Return on Banji Fund |
| :---: | :---: | :---: | :---: |
| Good | 0.6 | $10 \%$ | $20 \%$ |
| Bad | 0.4 | $7 \%$ | $2 \%$ |

(a) What is the expected return on the CAC 40 and on Banji Fund?
(b) What is the standard deviation of the return on CAC 40 and on Banji Fund?
(c) What is the covariance between the return on Banji Fund and the return on CAC 40? What is the correlation?
(d) What is the beta of Banji Fund on the CAC 40?

Question 3. Part A and Part B of this question are totally independent.

Part A - The term structure of spot interest rate is as follows:

| Year | Spot Rate |
| :---: | :---: |
| 1 | $r_{1}=5.00 \%$ |
| 2 | $r_{2}=5.40 \%$ |
| 3 | $r_{3}=5.70 \%$ |
| 4 | $r_{4}=5.90 \%$ |
| 5 | $r_{5}=6.00 \%$ |

Suppose that someone told you that the six-year spot interest rate was $4.8 \%$.
(a) Why would someone believe him?
(b) How could you make money if he was right?
(c) What is the minimum sensible value of the six-year spot rate?

Part B - Gerald Fitzpatick is interested in finding the appropriate discount rate for its new projected distillery. The average beta of a group of similar Irish distilleries is 1.4 and the average debt-equity ratio is 0.30 . Fitzpatick plans to have a debt-equity ratio of 0.20. Assuming beta for debt is zero, if the risk-free rate is 6 percent and the expected risk premium on the market portfolio is 9 percent, what is
(a) the required return for the project,
(b) the required return on the shares of the company.

Question 4. Consider a manager compensated by a package whose value, $C\left(x_{t}\right)$, depends solely on a fundamental variable, $x_{t}$. This fundamental variable determines the value of the firm he manages and follows a Geometric Brownian Motion

$$
d x_{t}=\mu x_{t} d t+\sigma x_{t} d z_{t},
$$

where $z_{t}$ is a standard Brownian motion.
The manager takes two decisions:

- His first decision is to be taken immediately and affects the fundamental's volatility, once and for all. He can essentially select between two volatility levels $\sigma=\sigma_{1}$ or $\sigma=\sigma_{2}$, where $\sigma_{1}<\sigma_{2}$.
- His second decision is to quit and take an alternative employment. He can take take this last decision at any time. His alternative employment determines his reservation value, $R>0$.

The value of the manager's compensation package, if he selects a volatility $\sigma$, and quits the first time $x_{t}$ reaches a lower boundary $\underline{x}$ (where $x_{t}>\underline{x}$ ), is

$$
C\left(x_{t}\right)=x_{t}+[R-\underline{x}]\left(\frac{x_{t}}{\underline{x}}\right)^{\beta} .
$$

Here, $\rho$ is the constant discount rate and $\beta=-2 \rho / \sigma^{2} .\left(\frac{x_{t}}{\underline{x}}\right)^{\beta}$ is the Laplace transform of probability density function of the first passage time of $x_{t}$ to a lower boundary $\underline{x}$.

1. Assuming the manager starts with selecting a given volatility level $\sigma$, derive the optimal threshold level $\underline{x}$ at which the manager will choose to quit the firm.
2. Establish which one of the two volatility levels $\sigma_{1}$ or $\sigma_{2}$ the manager selects to start with, knowing he will quit when his optimal abandonment threshold level $\underline{x}$ is reached (determined in the previous part).

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Wednesday 20 May 2015 9.00am to 11.00am

F300

## Corporate Finance

Candidates are required to answer all four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

Question 1 An entrepreneur has cash $A>0$ and can undertake a project of scale $I \in[0, \infty]$ yielding return $R I$ in case of success and 0 otherwise. The entrepreneur chooses probability of success $p \in\left\{p_{H}, p_{L}\right\}$ with $p_{H}>p_{L}>$ 0 . In case $p=p_{L}$, the entrepreneur obtains private benefit $B I$. The effort choice is unobservable. The entrepreneur borrows $I-A$ from competitive lenders, and offers a contract (sharing rule) $R I=R_{b}+R_{\ell}$. Last, assume that

A1 $p_{H} R>1$
A2 $p_{L} R+B<1$
A3 $p_{H} R<1+p_{H} B / \Delta p$

1. (a) Briefly interpret assumptions A1 and A2.
(b) Write an expression for the agent's utility and show that he prefers unbounded investment.
(c) Carefully setting out the relevant constraints, show that the equilibrium investment scale is necessarily bounded.
(d) Define the equity multiplier and briefly interpret assumption A3. Determine what happens to the agent's utility when $B$ increases.

Question 2 Salaryman Ventures has earnings per share (EPS) of $\$ 3.00,5$ million shares outstanding, and a share price of $\$ 32$. Salaryman is considering buying Cornerstore Ltd, which has earnings per share of $\$ 2.50,2$ million shares outstanding, and a share price of $\$ 20$. Slaryman will pay for Cornerstore by issuing new shares. There are no expected synergies from the transaction.

1. (a) If Salaryman pays no premium to acquire Cornerstore, what will the earnings per share be after the merger?
(b) Calculate Salaryman's price-earnings (P/E) ratio both pre and post merger.
(c) Explain the changes in EPS and P/E generated by the merger and briefly comment on their significance.

Question 3 Suppose that all capital gains are taxed at a $20 \%$ rate, and that the dividend tax rate is $40 \%$. A company is currently trading for $\$ 40$ per share, and is about to pay a $\$ 5$ special dividend.

1. (a) Determine what the share price will be just after the dividend is paid.
(b) Determine the effective dividend tax rate for an investor in the company.
(c) Suppose that the company made a surprise announcement that it would do a share repurchase rather than pay a special dividend. Determine the net tax savings per share for an investor that would result from this decision.

Question 4 Flagging Fortunes (FFCo) are expected to have free cash flow in the coming year of $\$ 8$ million, and this free cash flow is expected to grow at a rate of $3 \%$ per year thereafter. FFCo has an equity cost of capital of $13 \%$, a debt cost of capital of $7 \%$, and it is in the $35 \%$ corporate tax bracket. Assume that FFCo maintains a .5 debt to equity ratio.

1. (a) Determine FFCo's pre-tax WACC.
(b) Determine the value of FFCo as an all equity firm.
(c) Determine FFCo's after-tax WACC.
(d) Determine the value of FFCo as a levered firm.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Thursday 28 May 2015 1:30pm to $3: 30 \mathrm{pm}$

F500

## Empirical Finance

This paper is in two sections. Section A: Candidates are required to answer all four questions. Section B: Candidates are required to answer two out of three questions.

Each section carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION<br>Campbell, Lo \& Mackinlay Tables<br>Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Thursday 28 May 2015 1:30pm to $3: 30 \mathrm{pm}$

F500

## Empirical Finance

This paper is in two sections. Section A: Candidates are required to answer all four questions. Section B: Candidates are required to answer two out of three questions.

Each section carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION<br>Campbell, Lo \& Mackinlay Tables<br>Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## 1 Section A

1. Suppose that the logarithm of (daily) observed stock prices $p_{t}=\log P_{t}$ satisfies

$$
\begin{gathered}
p_{t}^{*}=\mu+p_{t-1}^{*}+\varepsilon_{t} \\
p_{t}=p_{t}^{*}+\eta_{t},
\end{gathered}
$$

where $\varepsilon_{t} \sim N(0,1)$ and $\eta_{t} \sim N(0,1)$ with $\varepsilon_{t}$ and $\eta_{t}$ mutually independent.
(a) What is the mean, variance, and autocovariance function of the observed daily return series?
(b) What is the relationship of this model with the "Roll model"?
(c) What does this say about the autocorrelation function of weekly returns?
(d) Are the theoretical predictions regarding the autocovariance function of observed returns consistent with the empirical results presented in the table (Table 2.4 from CLM)?
2. Suppose that daily stock returns satisfy

$$
\begin{gathered}
r_{t}=\sigma_{t} \varepsilon_{t} \\
\sigma_{t}^{2}=\omega+\gamma r_{t-5}^{2}
\end{gathered}
$$

where $\varepsilon_{t}$ is iid standard normal.
(a) Is this model consistent with the Semi-Strong Efficient Markets Hypothesis?
(b) Is this model consistent with the Weak Form Efficient Markets Hypothesis?
(c) Is this model consistent with the "stylized empirical fact" that

$$
\operatorname{cov}\left(r_{t}^{2}, r_{t-k}^{2}\right)>0
$$

for all $k=1,2, \ldots$
(d) Is this model consistent with the "stylized empirical fact" that

$$
\operatorname{cov}\left(r_{t}^{2}, r_{t-k}\right)<0
$$

for all $k=1,2, \ldots$
3. The Sharpe-Lintner CAPM predicts that the time-series cross-sectional sample of asset returns $R_{i t}$ will obey the regression model (for $i=1, \ldots, n, t=$ $1, \ldots, T)$

$$
R_{i t}=R_{f t}+\beta_{i}\left(R_{m t}-R_{f t}\right)+\varepsilon_{i t}
$$

where $R_{m t}$ is the return on the observable market portfolio and $R_{f t}$ is the return on an observable risk-free asset. An alternative hypothesis is that
$R_{i t}-R_{f t}=\alpha_{i}+\beta_{i}\left(R_{m t}-R_{f t}\right)+\beta_{i}^{2}\left(R_{m t}-R_{f t}\right)+\beta_{i}^{3}\left(R_{m t}-R_{f t}\right)+\varepsilon_{i t}$.
In both cases it is implicitly assumed that the error term $\varepsilon_{i t}$ is iid with mean zero and independent of $R_{m t}-R_{f t}$. Suppose that you observe a sample of data $\left\{R_{i t}, R_{m t}, R_{f t}, i=1, \ldots, n, t=1, \ldots, T\right\}$.
(a) Describe the Fama-MacBeth two pass approach for testing the CAPM against the alternative model
(b) Suppose now that the risk free rate is not observed. How would your answer to part (a) change?
4. True, False, Explain
(a) If $P_{t}^{*}$ is a rational unbiased forecast of $P_{t}$, i.e., $E_{t} P_{t}^{*}=P_{t}$, for example $P_{t}^{*}=\sum_{i=1}^{\infty}\left(\frac{1}{1+R}\right)^{i} D_{t+i}$, then

$$
\operatorname{var}\left[P_{t}^{*}\right] \leq \operatorname{var}\left[P_{t}\right]
$$

(b) In the absence of informed traders, bid ask spreads would be narrower, and so markets would be less efficient.
(c) The equity premium puzzle is a statistical artifact of considering only United States data from a particularly beneficial period.
(d) The calendar time hypothesis and the trading time hypothesis are mutually incompatible and both incompatible with stock return data.
(e) The consumption CAPM implies both cross-sectional and time series restrictions, neither of which are satisfied in the data

## 2 Section B

1. Describe what is meant by cumulative abnormal return in the context of an event study. Explain how it is used to test the hypothesis that an event had no effect on the stock price. Against what type of alternatives does this test have statistical power? Why is it important in an event study that the event occurrence is exogenous to earlier stock price changes?
2. Describe how you would test the Arbitrage Pricing Theory. In your answer, explain the types of data you would use, the restrictions that you would test, and the methods you would use to accomplish these objectives. In your answer explain the terms Diversification and Pervasive Factors.
3. Describe the non synchronous trading model developed in the Campbell, Lo, and MacKinlay book. Show how it can explain the patterns of daily autocorrelation in individual stocks and the cross autocovariances between stocks. What does the model imply should happen to the cross autocovariances when the mean returns of the true stock returns becomes smaller? What do you think is the relevance of this model in today's financial markets?

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Thursday 4 June 2015 1:30pm to $3: 30 \mathrm{pm}$

F510

## International Finance

Candidates are required to answer four out of five questions.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Choose between the following two questions:

## Question IA

Let $\mathcal{F}_{t}$ and $\mathcal{E}_{t}$ denote forward and spot exchange rates (units of domestic currency per unit of foreign currency). Suppose the foreign currency trades at a forward premium.
(a) Explain how you would set up a carry trade portfolio using forward contracts. In writing the strategy, possibly use bid and ask prices.
(b) Based on the readings discussed during lectures, in what ways you expect the distribution of excess returns from carry trade to differ from a normal distribution?
(c) What is a "Hedged Carry Trade strategy"? How would it differ from your answer to (a) above? Explain.

## Question IB

Let the face value of a country's debt be as high as 100. There is only one period, with two state of nature, $s=1,2$. The cash flow paid by the country is

$$
x=\left\{\begin{array}{l}
x_{1}=100 \text { with probability } 1 / 3 \\
x_{2}=25 \text { with probability } 2 / 3
\end{array}\right.
$$

In the two states of nature, international investors have discount factor $D_{1}=1$ and $D_{2}=2$, that is, they value resources more in 2 .
(a) calculate the expected cash flow and the price international investors are willing to pay for the country's debt.
(b) The government proposes a swap of old debt for new debt. The new debt is senior and "safe:" it will be paid first and entirely. How many units of safe debt can the government issue? What would the equilibrium swap rate be?

## Q3 QUESTION: risk sharing and sovereign risk

Residents can trade assets (enter financial contracts) with international investors who are risk neutral and do not discount the future. Each resident can borrow by selling a claim to (=promise to pay) future cash flow $x_{i, 1}^{H}$ (if she turns out to have high output in period 1) and $x_{i, 1}^{L}$ (if low output). The amount she can borrow coincides with the price of this claim.

Consider a small open economy existing for two periods only, where individuals have identical preference

$$
U\left(c_{i}^{0}\right)+E U\left(c_{i}^{1}\right)
$$

and endowment

$$
y_{i}^{0}=\frac{1}{2} y \quad \text { and } \quad y_{i}^{1}=\left\{\begin{array}{c}
y_{i}^{H}=2 y \text { prob. } 1 / 2 \\
y_{i}^{L}=y \text { prob. } 1 / 2
\end{array}\right.
$$

Residents can borrow from international investors who are risk neutral and do not discount the future. Each resident can borrow by selling a claim to (=promise to pay) a future cash flow that differ across states of nature: $x_{i, 1}^{H}$ (if she turns out to have high output in period 1) and $x_{i, 1}^{L}$ (if low output). Define a financial contract as a cash flow $x_{i, 1}^{L}, x_{i, 1}^{H}$, from which you can derive a price $q=E(D x)$.
(a) Which contract would allow the country's residents to smooth consumption completely across periods and states of nature? Write carefully cash flows and price.
(b) Explain why sovereign risk may prevent this financial contract to traded.
(c) Suppose that if an investor decides to default on her payment, she loses $k=.2 y$, say, she needs to sustain a sunk legal costs. How would your answer to (b) change? Which contracts will the country residents choose to borrow from abroad?
(d) Discuss the optimal contract and risk sharing (i.e. consumption) when the legal costs are $k=.4 y$ and $k=y$.

## Q4 QUESTION: CURRENCY CRISES

1. Suppose the central bank of an oil-producing economy aims to peg its currency to the US dollar so that $S=\bar{S}$, where $S$ is the domestic currency price of US dollars. The central bank balance sheet is given by

$$
M=B+R
$$

where $M$ denotes the money supply, $B$ domestic government bonds held by the central bank and $R$ foreign reserves. If the central bank has depleted its foreign reserves, it adopts a free float. Assume that the government runs a structural budget deficit and forces the central bank to buy government bonds so that $\dot{B} / B=\gamma>0$, where $\dot{X} \equiv \mathrm{~d} X / \mathrm{d} t$ denotes the time derivative of $X$.
Money market equilibrium is given by

$$
\frac{M}{P}=Y e^{-\frac{1}{2} i}
$$

where $P$ is the aggregate price level, which is flexible, $i$ is the nominal interest rate, and $Y$ is aggregate output, which is exogenous and normalized to $Y=1$. Purchasing power parity holds so that

$$
P=S P^{*}
$$

where $P^{*}$ is the US aggregate price level, which is exogenous and normalized to $P^{*}=1$. In addition, assume that

$$
i=i^{*}+\dot{S} / S
$$

where $i^{*}$ is the US nominal interest rate.
(a) Derive the level of the money supply $M$ that is required to keep the exchange rate fixed at $S=\bar{S}$. Explain intuitively why the exchange rate peg is unsustainable.
(b) Derive the time $T$ of the speculative attack (assuming no speculative bubbles). Explain how it depends on the initial level of foreign reserves $R_{0}$.
(c) Suppose that sometime after fixing the exchange rate at $S=$ $\bar{S}$ at $t=0$, there is suddenly a drop in the oil price that worsens the budget deficit so that $\gamma$ increases at time $\tau$, where $0<\tau<T$. Explain whether this could trigger a currency crisis.

## Q5 QUESTION: CURRENT ACCOUNT

The world consists of two countries populated by many identical individuals with preferences

$$
U\left(C_{t}\right)+\beta E_{t} U\left(C_{t+1}\right)
$$

so that their fundamental stochastic discount will be

$$
D=\frac{\beta U^{\prime}\left(C_{t+1}\right)}{U^{\prime}\left(C_{t}\right)}
$$

Denote Foreign variables with a star *, i.e., $D^{*}=\frac{\beta U^{\prime}\left(C_{t+1}^{*}\right)}{U^{\prime}\left(C_{t}^{*}\right)}$
The current output is lower in the Foreign country than in the Home country, that is $Y_{t}^{*}<Y_{t}$. However, the Foreign country is expected to grow faster and reach in the future the same output as Home in expectations, that is $E_{t} Y_{t+1}^{*}=E_{t} Y_{t+1}$. Faster growth comes with higher macroeconomic uncertainty: $\operatorname{Var}\left(Y_{t+1}^{*}\right)>\operatorname{Var} Y_{t+1}$.

Assume that the two can trade in a sure bond $B$, that is exchanged at the price $q_{t}$ today, and promise one unit of goods at $t+1$.

1. Explain how growth and uncertainty determines the current account of a country according to result on the "comparative advantage in trade of bonds" discussed in class. In doing so, (i) define the rate of interest in financial autarky $R^{A}$ and explain its interpretation; (ii) be precise on which assumptions on the utility function you need to support your argument.
2. Assume

$$
U=\ln \left(C_{t}\right)+E_{t} \ln \left(C_{t+1}\right) \text { and } U^{*}=\ln \left(C_{t}^{*}\right)+E_{t} \ln \left(C_{t+1}^{*}\right)
$$

In the Home country $Y_{t}=Y_{t+1}=10$; in the foreign country $E_{t} Y_{t+1}^{*}=10$ with

$$
Y_{t+1}^{*}=\left\{\begin{array}{c}
Y_{H}^{*}=12 \text { with prob. } 2 / 3 \\
Y_{L}^{*}=8 \text { with prob. } 1 / 3
\end{array}\right.
$$

For which level of current Foreign output $Y_{t}^{*}=$ ? will the current account of the Foreign country in period $t$ switch from a deficit to a surplus? Explain.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
Monday 1 June $2015 \quad 1: 30 \mathrm{pm}$ to $3: 30 \mathrm{pm}$

F520

## Behavioural Finance

Candidates are required to answer two out of three questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Overconfidence is an important psychological bias affecting decisions in finance.
(a) Provide a definition of overconfidence. Describe an intuition behind it, different types of overconfidence.
(b) Consider a risky asset paying random liquidating dividends $d$ with zero means and variance $\sigma^{2}$, which is public knowledge. John and Barbara receive identical signals $s=d+e$ at date 1 about future dividend payments, where $e$ is a noise uncorrelated with $d$. The true variance of $e$ is $\sigma_{e}^{2}$. John mistakenly believes that the variance of the signal is smaller $\sigma_{J}^{2}<\sigma_{e}^{2}$. Both Barbara and John use Bayesian rule to update the new information. If half of all investors in the market share John's beliefs about the signal precision and half of investors share Barbara's beliefs, which effect will this have on asset prices? Assume that all investors are mean-variance maximizers. Justify your answer. Describe main differences between John's and Barbara's the posterior expectation of future dividend payoff.
(c) Outline main empirical findings of Odean (1998) regarding trading volume overconfidence in financial markets. Does your model support this statement?
2. Distinction between risk and uncertainty is important in financial markets.
a. Provide the definition of Knightian uncertainty. Describe Ellsberg's Paradox. Provide an example how the Choquet Expected Utility model resolves Ellsberg's Paradox?
b. Suppose that an analyst estimates the value of an oil exploration firm. The analyst knows that in the case of positive drilling results in one of the fields the firm will pay $\$ 2$ per share of dividends with probability 0.5 or $\$ 1$ per share with probability 0.5 . He also knows that in the case of negative drilling results the firm will pay $\$ 0.5$ per share of dividends with probability 0.4 or $\$ 0.1$ per share with probability 0.6 . The analyst does not know, however, the probability $\alpha$ of the positive outcome of the drilling results but estimates it to be between 0.3 and 0.4 . Let $D_{\alpha}$ denote the expected dividend given the probability $\alpha$. Suppose that the analyst is ambiguity averse and evaluates the stock according to the multiple priors model as

$$
P=\min _{\alpha}\left\{D_{\alpha}\right\}
$$

What is the value of his estimate? Suppose that the firm announces that it finds the oil. How would the analyst revise his estimate after the announcement? Provide necessary calculations.
c. Suppose that the analyst instead of public announcement receives only a rumor about positive results which he knows if correct in $80 \%$ cases. What would the analysts' estimate be in this case?
3. Investor sentiments and limits to arbitrage.
(a) Provide theoretical arguments for the efficient market hypothesis (EMH) and Behavioural Finance objections to it. What are the empirical justifications of these objections?
(b) Provide an intuiting for why closed-end fund discounts, trading volume, option implied volatility and IPO first-day returns could serve as proxies for investors' sentiments.
(c) Discuss the intuition behind the noise trader model of De Long et al (1990). Provide necessary analysis.

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Friday 5 June $2015 \quad 1: 30 \mathrm{pm}$ to $3: 30 \mathrm{pm}$

F540

## Topics in Applied Asset Management

This paper is in two sections:
Section A: candidates are required to answer one compulsory question.
Section B: Candidates are required to answer two out of four questions.
Section $A$ is weighted at $40 \%$ and section $B$ is weighted at $60 \%$.
Provide as much mathematical detail as you can to justify your answers.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad
New Cambridge Elementary Statistical Tables

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION <br> New Cambridge Elementary Statistical Tables <br> Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

1. Provide as much mathematical detail as you can when answering these questions.
(a) Diversification is often seen to be a central element in portfolio construction but it was also seen by many in the asset management industry to have failed during the crisis of 2007-8.
i. Explain what is commonly meant by diversification, how it can be measured and discuss any limitations in this usual view of diversification.
ii. Explain how we can measure the additional contribution to diversification from adding an asset to a portfolio and hence judge if an asset is worth including in a portfolio- how would you qualify any judgement based on your measure?
iii. Describe an alternative approach to asset allocation compared to mean variance (not Parametric Portfolio Policies) that was developed in the light of this apparent failure of diversification? Describe the approach in some detail and discuss its properties relative to mean variance.
iv. Explain why a $1 / N$ allocation is so hard to beat and under what two specific conditions do mean variance allocations, when properly computed, start to outperform $1 / N$.
(b) Explain why the Parametric Portfolio Policies can overcome the failures of Mean Variance in practice and the steps needed to carry out their construction.
(c) Ferson and Siegel(2001) have shown how the optimal weight function is changed when you take into account the fact that you can exploit a predictive model (conditional information) rather than use the unconditional moments in mean variance optimisation. Derive and explain the rationale behind their formula.
2. Predictability;
(a) Goyal and Welch (2008) appears to demonstrate that asset returns are largely unpredictable with conditioning information.
i. Explain why we should expect some degree of predictability on theoretical grounds and why this may be consistent with the observation that the evidence for predictability varies over time and in particular is asymmetric with respect to the business cycle.
ii. Provide and discuss a limit to the degree of predictability that would be consistent with theory either in terms of a Stochastic Discount Factor or the Sharpe Ratio.
iii. Apart from Forecast Combination methods describe two other approaches that have delivered significant evidence of predictability in recent research and when is the degree of predictability found to be strongest?
(b) Forecast combination methods have recently been shown to be highly effective in uncovering predictive structure
iv. Why does this result come about and why does combining forecasts rather than information sets models work better? In particular why do the kitchen sink regressions in Goyal and Welch fail to deliver better performance?
v. Bates and Granger(1969) derived restricted least squares weights for forecast combination- explain what these are (do not derive them from first principles) and the connection with mean variance portfolio weights.
(c) Show how the returns from a cross section momentum strategy can be decomposed into three components; terms reflecting time series autocorrelation, cross covariances and a mean term. What results did Moskowitz, Ooi and Pedersen (2012) find as to the relative values of these three components with futures data with what implications?
3. Quantile and Directional Methods in Asset Allocation:
(d) Assume you have an asymmetric linear loss function, LIN-LIN, i.e.you are Loss Averse
i. Derive the optimal predictor and explain its relation to the conditional expectation.
ii. What critical observation on the form of the optimal predictor follows if your loss function is LIN-EXP in place of LIN-LIN ?
(e) Explain what is meant by quantile regression and how it is computed
(f) What implication does the use of quantile regressions have for interpreting the literature on predictability and for asset allocation in general?
(g) Show that the ability to predict the direction of future returns depends on the ability to predict their conditional volatility and explain how this result can impact asset allocation.
4. Performance Measurement;
(a) Explain what is meant by:
i. The Calmar Ratio
ii. The Sortino Ratio
iii. Max Drawdown
iv. The Information Ratio
(b) Explain how standard Backtesting Strategies may result in problems of over-fitting and how k-fold Cross Validation may overcome this problem.
(c) Explain what is meant by the Familywise Error Rate (FWER) and False Discovery Rate (FDR), how they are related and explain the problem in performance evaluation they are attempting to resolve?
(d) Explain the simple adjustment to the Sharpe Ratio (The Haircut Sharpe Ratio) proposed by Harvey and Liu (2014).
i. If we have a $p$-value of 0.05 for a single," independent", $t$ - test ( one sided for testing strategy profitability), derive the corrected $p$-value which recognises that 10 previous independent tests have been carried out prior to this final test. Comment on the result.
ii. Suppose you have a strategy, developed with 10 years of monthly data, that delivers an annualised Sharpe Ratio of 0.92 with a corresponding $t$-statistic of 2.91 . The $p$-value is $0.4 \%(0.0004)$ - would you invest in this strategy?
iii. Suppose now you discover this strategy was developed through a process that involved 200 other strategies or tests, adjust the $p$ value using the Bonferroni method, would you still invest in this strategy?
5. Risk Management;
(a) Define and explain the relationship and relatives advantages of Value at Risk and Expected Shortfall.
(b) What are the principle advantages and disadvantages of Historical Simulation when used as a method of computing Value at Risk and describe one method that may be used to overcome its deficiencies.
(c) Consider 100 independent defaultable bonds each with a current price of 100 and a default probability of $2 \%$ for each bond. When they don't default they deliver 105 at the end of the year. Now consider two portfolios, the first, portfolio A, is concentrated and made up of 100 units of only the first bond and the second portfolio, portfolio B , is completely diversified containing one unit of each bond. Examine the
risk capital required when using a $95 \%$ VaR level for each portfolio and discuss the result you find. Explain your results and why they come about? In general, under what situations are you likely to find VaR fails to satisfy the conditions for coherency?
(d) Describe the CAViaR approach to the measurement of Value at Risk.

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Tuesday 19 May 2015 9:00am to 11:00am

M110

## Microeconomics II

Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Consider a pure exchange economy.
(a) Define excess demand function and explain what it means for an agent's excess demand function to satisfy the gross substitutes (GS) property.
(b) Show that if all agents' excess demand functions satisfy $G S$, then the exchange economy has at most one equilibrium price vector.
(c) Suppose that the economy has 4 commodities and $n$ agents. Each agent is endowed with a positive amount of each commodity. Moreover, these agents $k=1, \ldots, n$, have utility functions of the following form:

$$
u_{k}(x, y, z, t)=k \ln x+2 k \ln y+3 k \ln z+4 k \ln z
$$

Does this economy always have an equilibrium? Does this economy have a unique equilibrium? Explain your answer clearly.
2. Consider a financial economy with a single consumption good and two dates, today and tomorrow, where trade occurs today. There are two possible states of the world tomorrow, 1 and 2 . So this economy has three (contingent) commodities. The economy has two agents, $A$ and $B$. Agent $A$ is endowed with 4 units of consumption today. Tomorrow, this agent has nothing if state 1 occurs, whereas he has 2 units of consumption good if state 2 occurs. In other words, agent $A$ 's stage-contingent endowment vector is $\omega^{A}=(4,0,2)$. Agent $B$ 's endowment is $\omega^{B}=(0,4,2)$. The utility functions of the agents are given by

$$
U^{A}\left(x_{0}, x_{1}, x_{2}\right)=x_{0} x_{1}^{2} x_{2} \quad \text { and } \quad U^{B}\left(x_{0}, x_{1}, x_{2}\right)=x_{0}^{2} x_{1} x_{2}
$$

where $x_{0}$ is consumption today, $x_{1}$ is consumption tomorrow if state 1 occurs, and $x_{2}$ is consumption tomorrow if state 2 occurs.
We fix the price of the consumption good today to be 1 .
Suppose the economy has three assets $\alpha, \beta, \gamma$ with payoffs tomorrow. Asset $\alpha$ has a payoff of 5 if state 1 occurs, and 3 otherwise; asset $\beta$ has a payoff of 2 if state 1 occurs, and 1 otherwise; asset $\gamma$ has a payoff of 5 if state 1 occurs, and 1 otherwise.
(a) Does this financial economy have an equilibrium? Explain briefly.
(b) If you answer above is yes, compute the equilibrium prices of the assets. If your answer is no, show how one can change the set of assets available to make sure there exists an equilibrium?
(c) Does the first welfare theorem of financial economies apply here? If so, what is the conclusion?
(d) By trading these assets available, can the agents reach an efficient allocation of the consumption good? If not, what additional ingredient would help achieve efficiency? Explain carefully.
(e) If asset $\beta$ is the only asset available in this economy, how do your answers to parts (a), (b), (c), and (d) change?
3. $n>2$ bidders have private values for a single object. These values are independently drawn from the uniform distribution on $[0,1]$. Assume that the utility of an agent $i$ with value $v_{i}$, who receives the object and makes a payment of $c \leq v_{i}$ is $\left(v_{i}-c\right)^{a}$, where $a>0$. If she does not receive the object and makes a payment $c>0$, then her utility is $-c^{a}$.
(a) Find a symmetric equilibrium of the second price auction. Explain why this is indeed an equilibrium. Elaborate on the relationship between the value of $a$ and the equilibrium bidding behaviour.
(b) Find a symmetric equilibrium of the first price auction. Elaborate on the relationship between the value of $a$ and the equilibrium bidding behaviour. Compare with your answer above.
4. In an average price auction, the bidders submit sealed non-negative bids, the highest bidder wins, and pays the average bid. That is, if $b_{1}, b_{2}, \ldots, b_{n}$ are the submitted bids, and if $b_{i}$ is the highest of these, then bidder $i$ wins and pays $\left(b_{1}+\cdots+b_{n}\right) / n$. Suppose that the bidders' valuations $x_{1}, \ldots, x_{n}$ are independently drawn from the uniform distribution on $[0,1]$. Assume also quasi-linear utilities.
(a) Elaborate, in words, whether you expect the bidders to bid their true values, below their values, or above their values.
(b) State, clearly, the Revenue Equivalence Theorem.
(c) Find a symmetric, increasing equilibrium of the auction described above. (Hint: look for a linear equilibrium strategy.)
(d) Does your result appear to be in agreement with your intuition in part (a)? Explain clearly.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics

Monday 25 May 2015 9:00am to 11:00am

## M130

## Applied Microeconomics

This paper is in two sections. Candidates are required to answer four out of six questions in Section A; and the one compulsory question in Section $B$.

Questions within Section A carry equal weight. Each section carries equal weighting.

Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Section A

1. Consider the following cost function:

$$
\begin{gathered}
\log c(p, u)=a(p)+b(p) u \\
a(p)=a_{0}+\sum_{k} \alpha_{k} \log p_{k}+\frac{1}{2} \sum_{k} \sum_{l} \gamma_{k l} \log p_{k} \log p_{l} \\
b(p)=\beta_{0} \Pi p_{k}^{\beta_{k}}
\end{gathered}
$$

(a) Derive the indirect utility function
(b) What are Hicksian and Marshallian demands? (Express as quantities or budget shares)
(c) What conditions are necessary for this specification to satisfy adding-up and homogeneity?
2. The effect of disability insurance on labour market participation can be estimated by the regression:

$$
p_{i t}=\alpha+\beta X_{i t}+\gamma D I_{i t}+\varepsilon_{i t}
$$

where $p$ is an indicator of participation status and $D I$ is an indicator of receipt of disability insurance.
(a) In what conditions can estimates of $\gamma$ be interpreted as the effect of disability insurance on labour market participation?
(b) How might these effects be estimated if these conditions do not hold?
3. The government has to provide two goods, $x_{1}$ and $x_{2}$, and charge prices to break even. Production costs

$$
c\left(x_{1}, x_{2}\right)=F+c_{1} * x_{1}+c_{2} * x_{2}
$$

What price should the government charge for each good? What assumption have you made about the form of the utility function?
4. Specify carefully in what circumstances will a cost of living index be the same for all individuals.
5. How would you expect consumption to be affected by a $5 \%$ cut in VAT? In what circumstances will an income tax rebate have the same effect on consumption as a VAT cut?
6. When wages increase, labour supply may rise or fall. Explain why this occurs. Does the deadweight loss of a tax increase depend on whether labour supply rises or falls with the tax increase?

## Section B

1. A researcher estimates a demand system by OLS with each budget share regression given by:

$$
w_{i h}=\alpha_{i 0}+\alpha_{i a d}{a d u l t s_{h}}+\alpha_{i k i d s} k i d s_{h}+\sum_{j=1}^{7} \gamma_{i j} \ln p_{j, h}+\beta_{i} \ln \frac{x_{h}}{P_{h}}+u_{i h}
$$

where $w_{i h}$ is the budget share on good i for household h ; the price index $P_{h}$ is the Stone price index for household h. There are seven goods in this demand system: food at home (ebsfath); food in restaurants (ebsrest); transport (ebstr 1); services (ebsserv); recreation (ebsrecr); alcohol and tobacco (ebsvice); and clothing (ebsclth). The coefficient estimates from each OLS regression are given in the columns in Figure 1.
(a) What can we say about the properties of this demand system?
(b) Calculate the Hicksian, Marshallian and Income elasticities for food at home. The budget share on food at home is $26 \%$. Specify if there is additional information that is needed.
(c) How could these estimates be used to calculate the welfare cost of putting VAT on food at the rate of $20 \%$. Specify if there is additional information that is needed.
(d) What would be the deadweight cost of raising revenue from VAT in this case?
(e) What assumptions on intertemporal choice are implicit in your calculations?

Figure 1: Estimated Demand System

| Variable | ebsfath | ebsrest | ebstr~l | ebsserv | ebsrecr | ebsvice | ebsclth |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| adults | .07862 | -.04017 | .02191 | -.01415 | -.02555 | -.01924 |  |
| kids | .05335 | -.02035 | -.0178 | .01892 | -.0032 | -.01458 | -.01634 |
| lpfath | -.08612 | .07472 | -.11953 | .06402 | .03241 | -.03854 | .07304 |
| lprest | -.01356 | -.02084 | .15008 | -.06528 | .08464 | -.06867 | -.06637 |
| lptranall | .0103 | .00873 | .00611 | -.01605 | -.0089 | -.00444 | .00425 |
| lpserv | .02922 | .00155 | .0167 | -.02833 | -.0886 | .08911 | -.01963 |
| lprecr | .06552 | -.06444 | -.01299 | .0868 | .01423 | -.07892 | -.0102 |
| lpvice | -.0084 | .00683 | -.02874 | -.01914 | -.03534 | .06926 | .01553 |
| lpclth | -.09284 | -.0031 | .00619 | .07004 | .06598 | $3.9 e-05$ | -.04631 |
| lrxtot | -.16047 | .04959 | .02237 | -.01085 | .04597 | .01676 | .03664 |
| _cons | 1.624 | -.30593 | -.04318 | .28313 | -.28405 | -.02999 | -.24402 |

END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 18 May 2015 9:00am to 11:00am

M150

## Economics of Networks

Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

Question 1: Consider network formation with $n$ individuals. Suppose that a player $i$ can create a link with a player $j$ unilaterally by paying a cost $c>0$. The set of player $i$ links is denoted by $g_{i}$; the profile of linking decisions of all players is given by $g=\left(g_{1}, \ldots, g_{n}\right)$. The payoffs to player $i$ under a strategy profile $g=\left(g_{1}, \ldots, g_{n}\right)$ are

$$
\begin{equation*}
\Pi_{i}(g)=1+n_{i}(g)-\eta_{i}(g) c \tag{1}
\end{equation*}
$$

where $n_{i}(g)$ is the number of agents accessed by $i, i \neq j$, in network $g$ and $\eta_{i}(g)$ is the number of links formed by $i$. An agent accesses another agent if there is a (directed) path between them.

1. Define a Nash equilibrium network and provide two examples of (non-empty) Nash networks.
2. Define a strict Nash equilibrium network. Provide a characterization of strict Nash equilibrium networks as a function of $c$ and $n$.
3. A network is efficient if it maximizes the sum of individual utilities. Provide a characterization of efficient networks.
4. Next suppose that information only travels two links. Provide an example of a strict Nash network when $c<1$ and $n \geq 4$.

Question 2: Consider a game in which $n$ players are located on nodes of an undirected network $g$. Players simultaneously choose actions $x_{i} \in \mathcal{R}_{+}$. Let $N_{i}(g)$ be the set of players with whom player $i$ has a link in network $g$. The payoffs of player $i$ faced with a strategy profile $x$ are given by

$$
\begin{equation*}
\Pi_{i}(x)=f\left(x_{i}+\sum_{j \in N_{i}(g)} x_{j}\right)-c x_{i} . \tag{2}
\end{equation*}
$$

where $f(0)=0, f^{\prime}()>$.0 and $f^{\prime \prime}()<$.0 and $c>0$. Suppose there exists a number $\hat{x}>0$ such that $f^{\prime}(\hat{x})=c$.

1. Show that in every network there exists an equilibrium with specialization: some players choose $\hat{x}$ and others choose 0 .
2. Define social welfare as the sum of individual utilities. An action profile is efficient if it maximizes social welfare. Show that every equilibrium in any non-empty network is inefficient.
3. Now consider the first best problem: suppose a planner can choose efforts for every person and also design a network. What is the first best aggregate effort and network?

Question 3 Consider the four-region Allen and Gale (2000) economy, with regional liquidity demands in state $S_{i}, i=1,2$. Each state takes place with equal probability. In state $S_{1}$, regions $A$ and $C$ have liquidity demand of 0.75 and regions B and D have liquidity demand of 0.25 respectively. In
state $S_{2}$ the liquidity demands are reversed across states. There is a liquid asset which represents a storage technology. Investment in a long asset is available at $t=0$. Per unit invested in the long asset, the yield is of $r=0.4$ at $t=1$ (premature liquidation), and of $R>1$ at $t=2$. Assume that the per period utility function is of the form $u\left(c_{t}\right)=\ln \left(c_{t}\right)$.

1. Describe the combination of investment decisions and interbank deposits that achieve the first-best allocation when:
(i) Complete structure: the representative bank of a region holds deposits in the representative banks of all other regions
(ii) Incomplete structure: the representative bank of region $\mathbf{A}$ deposits in the representative bank of region $\mathbf{B}$, the representative bank of region $\mathbf{B}$ deposits in the representative bank of region $\mathbf{C}$, the representative bank of region $\mathbf{C}$ deposits in the representative bank of region $\mathbf{D}$, and the representative bank of region $\mathbf{D}$ deposits in the representative bank of region $\mathbf{A}$

For each case, explain the sequence of withdrawals if state $S_{1}$ takes place.
2. Now suppose that a state $\bar{S}$, which was assigned zero probability at $t=0$, takes place. In this state, regions $\mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ have a liquidity demand of 0.5 , but region $\mathbf{A}$ faces a demand of $0.5+\varepsilon$. Consider the incomplete structure and set $\varepsilon=0.1$ and $R=1.5$. Show that there is no contagion. Would your results change if $\varepsilon$ was larger?
3. Suppose now that $\varepsilon=0.1$ and $R=1.2$. Show that there is contagion when the structure is incomplete but not when it is complete.

Question 4 Consider a network with a single Source and a single Destination and $n$ intermediary agents. There is a demand at Destination given by $Q=1-P$. The network and the demand are common knowledge. Intermediaries post prices simultaneously. Trade proceeds along the cheapest cost route (randomizing across paths if there are multiple lowest cost paths). A trader is said to be critical if she lies on all paths between Source and Destination. Trade takes place if this minimum path cost is at most 1 .

1. Formalize this economic exchange as a game of simultaneous moves and show that there exists a pure strategy Nash equilibrium, in every network.
2. Now consider a line network with 2 intermediaries and suppose that agents set prices sequentially, starting with trader 1 , who sets $p_{1}$. Trader 2 takes as given this price and sets a price $p_{2}$. Destination takes as given this price and sets price $p_{D}$. Construct an equilibrium in which all traders set positive prices and there is trade. Comment on the difference with the simultaneous pricing model.
3. Next suppose that each trader has to process the good before passing it on to the next intermediary. The processing involves production. Consider the line network with 2 intermediaries.

Assume that intermediary 2 needs $a_{1}$ units of good 1 from intermediary 1 to produce 1 unit of his output, and that Destination needs $a_{2}$ units from intermediary 2 to produce one unit of output. Solve for optimal prices as a function of production technology coefficients.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Friday 22 May 2015 9:00am to 11:00am

M180
Labour: Search, Matching and Agglomeration
Candidates are required to answer three out of four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION <br> Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

Make 3 out of 4 questions. Each question carries equal weight.

## 1. Comparative advantage and sorting

Consider a world with workers with different levels of skill $x$ and jobs which produce different types of output $y$, each type of output being sold on commodity markets at a price $P(y)$; higher the output of type $y$, the lower is the price $P(y)$. Other things equal, the higher the supply of type $y$, the lower its price. The productivity of worker type $x$ in job type $y$ is $A(x, y)$.; the function $A(x, y)$ is twice differentiable. Better skilled workers have an absolute advantage: $A_{x}(x, y)>0$. A firm can offer only a single job. New firms will enter in the production of type $c$ until the profit on the output of this job type is zero. The distribution of $x$ among labour supply is exogenously determined. Let $W(x)$ be the wage of a worker of type $x$. Both $P(y)$ and $W(x)$ are determined by the market. Workers only care about their wage, not about the type of job they hold.
(a) Give a verbal interpretation of the condition of absolute advantage. What is the immediate consequence of this condition?
(b) Derive the profit function of a firm offering a job of type $y$.
(c) Derive the first order condition for the optimal worker type $x$ hired by the firm.
(d) What condition should $A(x, y)$ satisfy to make sure that in equilibrium firms offering more complex jobs will hire better skilled workers.
(e) Prove this. Explain each step carefully.
(f) Give a verbal argument why this is the case.
2. Job durations and tenure profiles in wages

Discuss various theoretical models that can explain the following phenomena:

- The distribution of job durations is skewed to right relative to the exponential distribution. Distinghuish carefully between the uncondtional distribution and the distribution conditional on job or match quality and conditional on experience.
- When running a wage regression, we find a strong positive effect of the incomplete job-tenure on wages. Discuss the possibility of reverse causality.


## 3. Cities and land rents

Consider a city with a CBD to which workers living in the surrounding neighbourhoods commute on a daily basis. The monetary cost of commuting is $\tau r$, where $\tau$ is a parameter and where $r$ is the commuting distance.

There are no non-monetary cost of commuting. Workers choose where to live. Let $S$ be the edge of the city. The price of agricultural land is equal to a half. Workers in the CBD earn a wage $W=1+\tau S$.
(a) Suppose each worker consumes one unit of land for living. What would the land rent $R$ be as a function of $r$ ? Explain.
(b) Suppose workers have a Cobb Douglas utility function $U(C, L)=$ $C \cdot L$ where $L$ is the consumption of land and where $C$ is other consumption. The price of other consumption is normalized to unity. Worker can choose to live and work outside the city, which yields utility $U=1$. Derive the land rent function for this case. Where is the slope of this function with respect to $r$ the highest?
(c) How does land use $L$ varies with $r$. Why is this the case? Do land price differentials further efficiency?
(d) Compare the land rent function under (a) and (b). The difference regarding land rents on various locations is similar to Sattinger's argument regarding wages for various skill levels (i.e. the fatness of the right tail of the wage distribution). Explain.

## 4. Cyclical fluctions, hiring, and lay offs

Consider the model by Mortensen and Pissarides (1994). Specify how you model cyclical fluctuations. Discuss the cyclical co-movement of hiring and lay off rates in steady state and explain. Discuss the difference between the immediate response of hiring and lay off rates after an upward or a downward shock. Does the timing of these responses contribute to efficiency?

## END OF PAPER

## ECM9/ECM10/ECM11

MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
Wednesday 6 May $2015 \quad$ 2:00pm to $4: 00 \mathrm{pm}$

M200

## Macroeconomics I

Answer all four questions from section $A$ and one out of two questions from section $B$.

Each section carries equal weight.
Write your candidate number (not your name) on the cover sheet of each answer booklet.

Write your answers legibly.

## STATIONERY REQUIREMENTS

20 Page answer booklet x 1
Rough work pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - candidates are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

## A. 1 Solow Growth Model with Capitalists

Consider the continuous time Solow model without technical progress and without population growth. Firms produce output $Y(t)$ using the production function

$$
Y(t)=K(t)^{\alpha} L(t)^{1-\alpha}, \quad \alpha \in(0,1)
$$

where $K(t)$ denotes the capital stock, which depreciates at constant rate $\delta$, and $L(t)$ is labour. The equation of motion of the capital stock is:

$$
\dot{K}(t)=I(t)-\delta K(t),
$$

where $I(t)$ corresponds to investment and the initial capital stock, $K(0)>0$, is given. The economy is closed and all markets are competitive. Assume that besides firms, there are two types of agents: (i) capitalists, who own the capital stock $K(t)$ that is rented to firms for the production of output; and (ii) workers, who supply labour $L$ to firms and earn a real wage $w$, so their income is $w L$. As in Kaldor (1957), suppose that capitalists save a fraction $s_{K} \in(0,1)$ of their income, while workers save a fraction $s_{L} \in(0,1)$ of their income. Derive the level of capital per worker in the steady state. What is the capitalists' saving rate $s_{K}$ that maximises consumption per worker in the long run? Explain.

## A. 2 Investment and Secular Stagnation

Consider a $q$-theory model with the following dynamics for the capital stock $K_{t}$ and the shadow price of capital $q_{t}$ :

$$
\begin{aligned}
\Delta K_{t+1} & =\gamma\left(q_{t}-1\right) \\
\Delta q_{t+1} & =r q_{t}-A F^{\prime}\left(K_{t}+\gamma\left(q_{t}-1\right)\right)
\end{aligned}
$$

where $r$ denotes the real interest rate, with $1+r>0$; the production function is given by $Y_{t}=A F\left(K_{t}\right)$, with $F^{\prime}(\cdot)>0$ and $F^{\prime \prime}(\cdot)<0$, where $A$ is a positive productivity parameter; and $\gamma$ is a positive parameter related to investment adjustment costs. The initial capital stock $K_{0}>0$ is given and the transversality condition is $\lim _{T \rightarrow \infty}\left(\frac{1}{1+r}\right)^{T} q_{T} K_{T+1}=0$.
Draw the phase diagram and explain the dynamics implied by this model. Suppose that 'secular stagnation' leads to a sudden permanent decline in the productivity parameter $A$. Carefully explain how $K_{t+1}$ and $q_{t}$ are affected over time using a phase diagram and provide an intuitive explanation for the effect on investment.

## A. 3 Wage Bargaining

Suppose a firm chooses employment $L$ to maximise real profits

$$
\Pi=\frac{1}{\alpha} L^{\alpha}-w L
$$

where $w$ denotes the real wage, and the parameter $\alpha \in(0,1)$. The firm and a union bargain over the wage before the firm decides on its level of employment. The union maximises its objective function

$$
U=(w-\bar{w})^{\beta}
$$

where $\bar{w}>0$ denotes the minimum wage, and the parameter $\beta \in(0,1]$. The bargained wage $w$ maximizes $U^{\gamma} \Pi^{1-\gamma}$, with parameter $\gamma \in(0,1)$. Assume that $\alpha>\gamma$.
Derive the level of the bargained wage $w$. Explain how it depends on $\bar{w}$ and $\gamma$.

## A. 4 Fiscal Policy and Austerity

Suppose the government sets the income tax $\tau_{t}$ each period to minimise the expected value of the following loss function:

$$
L_{t}=\sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} \frac{1}{2} \kappa \tau_{s}^{2} Y_{s}
$$

where $Y_{s}$ denotes aggregate output in period $s, r$ is the constant real interest rate, and $\kappa$ is a positive parameter. The intertemporal budget constraint of the government is given by

$$
(1+r) B_{t}+\sum_{s=t}^{\infty} \frac{G_{s}}{(1+r)^{s-t}}=\sum_{s=t}^{\infty} \frac{\tau_{s} Y_{s}}{(1+r)^{s-t}}
$$

where $B_{t}$ denotes government debt at the start of period $t$ and $G_{s}$ denotes government purchases in period $s$. Assume that initially, $B_{t}=0, Y_{s}=\bar{Y}$ and $G_{s}=\bar{G}$ for $s=t, t+1, \ldots$.
Suppose now that the government plans to implement austerity measures in period $t$ that will reduce $\bar{G}$ to $(1-\alpha) \bar{G}$ for $T$ periods, where $\alpha \in(0,1)$. Explain how this would affect the income tax $\tau_{s}$ and the primary government budget deficit $D_{s} \equiv G_{s}-\tau_{s} Y_{s}$ over time.

## SECTION B

## B. 1 Equilibrium with Heterogenous Consumers

Consider an economy in which each agent maximises her lifetime utility:

$$
U=\sum_{t=0}^{\infty} \beta^{t} \ln \left(c_{t}\right),
$$

where $c_{t}$ corresponds to her consumption in period $t$ and $\beta \in(0,1)$ is her subjective discount factor. Assume that the agent can borrow or lend at a given sequence of real interest rates $\left\{r_{t}\right\}_{t=0}^{\infty}$ and that the agent receives a deterministic sequence of perishable endowment $\left\{y_{t}\right\}_{t=0}^{\infty}$. Let $b_{t}$ be the asset position of the agent at the start of period $t$ and assume that $b_{0}=0$. The one-period budget constraint of the agent is

$$
c_{t}+b_{t+1}=y_{t}+\left(1+r_{t}\right) b_{t} .
$$

There is no possibility of a Ponzi scheme.
(a) Set up the maximisation problem of the agent and derive the Euler equation. Give an economic interpretation of your result.
(b) Solve for consumption in period $t=0$. Comment on your result.

Now assume that this economy has a total of $N$ agents and that there are two types of agents, $A$ and $B$, which are indicated by a superscript. Half of agents are of type $A$ and have the following endowment profile:

$$
y_{t}^{A}=\{1,0,1,0,1,0, \ldots\}
$$

The other agents are of type $B$ and have the endowment profile

$$
y_{t}^{B}=\{0,1,0,1,0,1, \ldots\} .
$$

Therefore, type- $A$ agents have one unit of the endowment good in even periods, while type- $B$ agents have one unit of the endowment good in odd periods.
(c) Write down the market clearing condition for the goods market in each period and use it to get an equilibrium condition between $c_{t}^{A}$ and $c_{t}^{B}$ for even periods and for odd periods.
(d) Use the equilibrium condition for the goods market and the Euler equation to derive the equilibrium real interest rate in each period. Solve for the levels of consumption $c_{t}^{A}$ and $c_{t}^{B}$ in equilibrium. Interpret your results.
(e) Explain how the levels of lifetime utility $U^{A}$ and $U^{B}$ compare in equilibrium.

## B. 2 Monetary Policy, Uncertainty and Macroeconomic Volatility

Suppose the central bank maximises the expected value of the objective function

$$
W=-\frac{1}{2} \alpha\left(\pi-\pi^{*}\right)^{2}-\frac{1}{2} y^{2}
$$

where $\pi$ is inflation, $y$ the output gap, $\pi^{*}$ the inflation target, and the parameter $\alpha>0$. The economy is described by the Phillips curve

$$
\pi=\pi^{e}+\beta y+s
$$

where $\pi^{e}$ denotes private sector inflation expectations, $s$ is an i.i.d. white noise cost-push shock with variance $\sigma_{s}^{2}$, and the parameter $\beta>0$. Aggregate demand is given by

$$
y=-\gamma(r-\bar{r})+d
$$

where $r$ is the real interest rate, $\bar{r}$ the natural real interest rate, $d$ an i.i.d. white noise aggregate demand shock with variance $\sigma_{d}^{2}$, and the parameter $\gamma>0$. The timing is as follows. First, the private sector rationally forms its inflation expectations $\pi^{e}$. Next, the shocks $s$ and $d$ are realized and observed by the central bank. Subsequently, the central bank sets its monetary policy instrument $r$.
(a) Derive the policy rate $r$ set by the central bank for a given level of $\pi^{e}$. Give an intuitive explanation of the result.
(b) Derive private sector inflation expectations $\pi^{e}$, and solve for the policy rate $r$, the output gap $y$ and inflation $\pi$. Provide an economic interpretation of their properties.

Now suppose the central bank faces uncertainty about the shocks $s$ and $d$. In particular, it does not observe these shocks when it sets the policy rate $r$, but it has access to staff forecasts, $f_{s}$ for the cost-push shock $s$ and $f_{d}$ for the aggregate demand shock $d$. These forecasts satisfy $s=f_{s}+\varepsilon_{s}$ and $d=f_{d}+\varepsilon_{d}$, where $\varepsilon_{s}$ and $\varepsilon_{d}$ are white noise forecast errors that are unknown to the central bank. Assume that $\varepsilon_{s}, \varepsilon_{d}, f_{s}$ and $f_{d}$ are all uncorrelated with each other. The forecast errors have variance $\operatorname{Var}\left[\varepsilon_{s}\right]=\kappa_{s} \sigma_{s}^{2}$ and $\operatorname{Var}\left[\varepsilon_{d}\right]=\kappa_{d} \sigma_{d}^{2}$, where the parameters $\kappa_{s} \in(0,1)$ and $\kappa_{d} \in(0,1)$ reflect the degree of uncertainty faced by the central bank.
(c) Derive the policy rate $r$ set by the central bank for a given level of $\pi^{e}$. Compare the result to part (a) and explain how greater uncertainty affects the variability of the policy rate, $\operatorname{Var}[r]$.
(d) Derive private sector inflation expectations $\pi^{e}$, and solve for the policy rate $r$, output gap $y$ and inflation $\pi$. Carefully compare the results to part (b).
(e) Explain how the central bank's uncertainty affects macroeconomic volatility, $\operatorname{Var}[y]$ and $\operatorname{Var}[\pi]$. Could greater uncertainty $\kappa_{s}$ or $\kappa_{d}$ be beneficial?

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research
Monday 1 June $2015 \quad$ 1:30pm to 3:30pm

Module 210
MACROECONOMICS II
Candidates are required to answer: Section A three questions and Section B one question.

Each section carries equal weight.
Write your number not your name on the cover sheet of each booklet
STATIONERY REQUIREMENTS SPECIAL REQUIREMENTS

20 Page booklet x 2 Approved calculators allowed
Rough work pads
Tags

> | You may not start to read the questions printed on |
| :--- |
| the subsequent pages of this question paper until |
| instructed that you may do so by the Invigilator |

## SECTION A

## A. 1 Natural Real Interest Rate and Secular Stagnation

Suppose the representative consumer maximizes the expected value of lifetime utility

$$
U_{t}=\sum_{s=t}^{\infty} e^{-\rho(s-t)}\left(\ln C_{s}-\frac{1}{2} N_{s}^{2}\right)
$$

where $C_{s}$ denotes consumption, $N_{s}$ is employment, the subscript $s$ indicates the time period, and $\rho$ is the subjective intertemporal discount rate $(\rho>0)$. The budget constraint of the representative consumer is:

$$
C_{s}+Q_{s} B_{s}=B_{s-1}+W_{s} N_{s}
$$

where $B_{s}$ denotes one-period riskless real discount bonds purchased in period $s$ at price $Q_{s}$, and $W_{s}$ the real wage. The profit-maximizing representative firm produces output $Y_{s}$ under perfect competition using the production function

$$
Y_{s}=A_{s} N_{s}
$$

where $A_{s}$ denotes productivity, which is stochastic. The goods market equilibrium condition is

$$
C_{s}=Y_{s}
$$

Derive the dynamic, optimising IS equation relating $y_{t} \equiv \ln Y_{t}$ to $r_{t} \equiv$ $-\ln Q_{t}$, and use it to solve for the natural real interest rate $r_{t}^{n}$. Suppose that 'secular stagnation' leads to a reduction in expected productivity growth. Explain how this would affect the natural real interest rate.

## A. 2 Value functions of a matching model of the labour market

Time is discrete. The economy is populated by a measure one of infinitely lived and risk neutral firms and workers who discount the future at rate $r>0$. Firms post vacancies at a flow cost equal to $c>0$. The unemployment income of workers is equal to $b \geq 0$. The aggregate matching function is homogenous of degree one. Hence, job seekers meet firms with probability $\theta q(\theta)$, where $\theta=\frac{v}{u}$ is the tightness of the labour market and firms meet workers with probability $q(\theta)$ with $q^{\prime}(\theta)<0$. Variables $v$ and $u$ correspond to the vacancy rate and unemployment rate, respectively. Each firm employs only one worker. Once a firm and a worker are matched, they start to produce. All jobs have the same productivity $y>b$, which remains constant during the whole lifetime of the job. Finally, jobs are destroyed with probability $s \in(0,1)$. Write the stationary value functions for workers and firms and explain intuitively why the decentralised allocation with wages determined by Nash Bargaining is not necessarily optimal.

## A3. Neoclassical Growth Model

Consider an economy with a continuum of firms of measure one and each firm has access to the following technology:

$$
Y(t)=K(t)^{\alpha} N(t)^{1-\alpha}, \alpha \in(0,1) .
$$

$Y$ is output, $K$ is capital, and $N$ is labour. There is no growth in population and all markets are competitive. There is also a continuum of measure one of households who are identical and infinite lived, with preferences

$$
U=\int_{0}^{\infty} e^{-\rho t} u(c(t)) d t, \quad \rho>0
$$

where $c(t)$ denotes consumption of the representative household at period $t$, and

$$
u(c)=-\frac{e^{-\theta c}}{\theta}, \theta>0
$$

Each household has an instantaneous unit of productive time. Households are endowed with the initial capital stock, $K(0)$. Capital evolves according to:

$$
\dot{K}(t)=X(t)-\delta K(t),
$$

where $X$ corresponds to investment and $\delta>0$ is the depreciation rate. Derive the system of equations which describes the equilibrium dynamics of physical capital and consumption per capita. Explain.

## A. 4 Asset Pricing

Consider an economy populated by a continuum of measure one of identical households. There are two assets in this economy. The first is a tree which gives no direct utility in itself but yields perishable dividends $\frac{1}{2} d_{t}$ (fruits). The second kind of a tree gives direct utility to households and also yields a stream of the same kind of the fruit $\frac{1}{2} d_{t}$. Households are initially endowed with some number of shares of the two tree: $s_{10}$ and $s_{20}$ and their preferences are given by $\mathbf{E}_{0}\left[\sum_{t=0}^{\infty} \beta^{t}\left(\ln \left(c_{t}\right)+\gamma \ln \left(s_{2 t}\right)\right)\right]$ where $c_{t}$ is consumption at period $t, s_{2 t}$ is the stock of the second tree, which generates utility for investor, $\beta \in(0,1)$ corresponds to the subjective discount factor, and $\gamma>0$ is a positive constant. Assume that $d_{t}$ is a random variable taking values from the set $\mathbf{D} \equiv\left\{d_{1}, \ldots, d_{N}\right\}$ and is distributed according to a first-order Markov chain with constant transition probability matrix. Households can trade the share of the trees. The price of each tree when the current realization of $d_{t}$ is $d$ is given by the function $p_{i}: \mathbf{D} \rightarrow R_{+}, i \in 1,2$. State the problem of the representative household in recursive form and assuming that the value function is differentiable, derive the stochastic Euler equation for the second tree. Explain.

## SECTION B

## B. 1 OLG Model

Consider a discrete time economy, $t=0,1,2, \ldots$ Agents in this economy live for two periods only. In the first period of life (young period), agents are entrepreneurs and have access to the following technology to produce the consumption good:

$$
y(t)=A k(t)^{\alpha}, \alpha \in(0,1),
$$

where $A$ is a positive constant and $k(t)$ is the capital input used in the production of the consumption good $y(t)$. Entrepreneurs rent capital to maximise profits. Entrepreneurs are price takers.

There are $L(t)$ young agents at period $t$ and population grows at constant rate $n$. Agents have preferences over consumption when young, $c_{1}(t)$, and when old, $c_{2}(t+1)$ and they are represented by the following utility function

$$
U=\ln \left(c_{1}(t)\right)+\beta \ln \left(c_{2}(t+1)\right),
$$

where $\beta \in(0,1)$. Capital fully depreciates after being used, $\delta=1$. At period $t=0$ there exists a unit measure of old agents and each has $k(0)$ of physical capital.
(a) Given profits, derive the optimal allocation of consumption and saving of each young individual at period $t$.
(b) Given the rental price of the capital stock, derive the optimal profit and capital input of each entrepreneur.
(c) State the equilibrium equation of motion of capital per capita (i.e., $k(t+1)=G(k(t)))$ and prove that there is an unique and globally stable steady-state equilibrium of physical capital per capita.
(d) How does the steady-state rental price of capital change with the population growth rate? Explain.

Now, assume that entrepreneurs can run away and avoid their payment obligation of the rented capital stock. However, by breaking their promises entrepreneurs incur a cost proportional to the output they produce, $\phi y(t)$ with $\phi \in(0,1]$.
e. Write the condition which states that it always optimal for entrepreneurs to keep their promises. Does this condition limit the amount of capital entrepreneurs can rent? Explain.

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Tuesday 2 June 2015 1:30pm to 3:30pm

M230
Applied Macroeconomics
Candidates are required to answer three out of seven questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 2
Metric Graph Paper
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Alan Taylor has argued that credit growth is a powerful predictor of financial crises, suggesting that such crises are "credit booms gone wrong" and that policymakers ignore credit at their peril. Discuss this assertion in the light of recent developments in the conduct of macroeconomic policy.

2 (a) Show how the effect of the zero interest floor can be explained in terms of the Taylor Principle.
(b) In response to the zero interest floor what sorts of policy have been adopted by some Central Banks?
(c) What empirical evidence is there for the success of these policies?

3 We do not need to appeal only to aggregate shocks in order to explain business cycle fluctuations. There are many shocks at the firm and the sector level that matter also. Discuss in the context of the recent literature on business cycles.
$4 y_{t}^{d}$ is the desired (target) rate of output and $y_{t}$ is the actual output in time period $t$. Define the difference between target and actual output as $e_{t}=\left(y_{t}-y_{t}^{d}\right)$. The policymaker uses the short term interest rate, $r_{t}$, as the instrument of policy,
(a) Write down a PID controller for output and explain carefully what it means.
(b) Is output controllable in this case?
(c) Suppose now the policymaker cares also about inflation. How does this affect your answer to part (b)?
(d) Under what circumstances might both output and inflation be controllable using only a single instrument?
(e) If they are not both controllable how might the policymaker represent the problem she faced?
(f) What other instruments might the policymaker use?

5 Why in response to the 2008-9 world financial crisis did most parts of the world introduce macroprudential policies?

6 In February 2015 H.M. Treasury published a comparison of independent forecasts for the UK economy for 2014, 2015 and 2016.
(a) Why are they reporting 'forecasts' for last year?
(b) H.M.-Treasury also reports the arithmetic average of these forecasts. Are there other ways in which forecasts can be combined?
(c) H.J. Treasury no longer produces public forecasts, but are there ways in which these independent forecasts could be incorporated into the (internal) forecasts of H.M. Treasury?

7 The growth of Greek national debt as a proportion of national income is the interaction of their debt today, the primary balance, the real interest rate and the rate of growth of real income.
(a) Explain this interaction.
(b) Suppose the Greek government negotiates a reduction in its debt to half of what it was. Will this necessarily improve the position of Greece?
(c) Since the 3rd quarter of 2011, GDP at market prices has fallen from 55,550 billion Euros to 45,103 billion Euros in the second quarter of 2014, the latest quarter for which we have data. Does this mean that the debtincome dynamics we have looked at are misleading?

## END OF PAPER

ECM9/ECM10/ECM11/ MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil in Finance

Friday 8 May $2015 \quad$ 2.00pm to 4:00pm

M300

## Econometric Methods

The paper is in two sections.
Section A: Candidates are required to answer all six questions.
Section B: Candidates are required to answer two out of four questions.

Weighting: Section A as 60\%, Section B as 40\%.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad
New Cambridge Elementary Statistical Tables
Durbin-Watson and Dicky-Fuller Tables

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Section A

A. 1 The joint density of random variables $X$ and $Y$ is given by

$$
f_{X Y}(x, y)= \begin{cases}\frac{1 / x+1 / y}{2 \ln 2} & \text { if } 1 \leq x \leq 2 \text { and } 1 \leq y \leq 2 \\ 0 & \text { otherwise }\end{cases}
$$

You observe $X=1$. What is the value of the minimum mean squared error predictor of $Y$ ?

## A. 2 Consider a regression

$$
y_{i}=\alpha+\beta d_{i}+\varepsilon_{i},
$$

where $E\left(\varepsilon_{i} \mid d_{i}\right)=0, y_{i}$ is a randomly chosen MPhil student's mark on the midterm econometrics exam, and $d_{i}$ is a binary variable, which equals 1 if the student took an advanced econometric course in their undergraduate studies. Using the potential outcome framework, explain the possible selection bias in $\beta$ considered as a measure of the causal effect of $d_{i}$ on $y_{i}$. In your answer, assume that the causal effect does not depend on $i$.
A. 3 In the lectures we considered the problem of estimating the demand for cigarettes using aggregate state-level data from the USA. In the context of that problem, let $q_{i}$ be the logarithm of the number of packs of cigarettes sold per capita in state $i ; p_{i}$ be the logarithm of the average real price per pack of cigarettes in state $i$ including all taxes, and let $\operatorname{SalesTax}_{i}$ be the portion of the tax on cigarettes arising from the general sales tax in state $i$. Discuss the validity of SalesTax ${ }_{i}$ as an instrument for $p_{i}$. The OLS regression of $p_{i}$ on constant and SalesTax ${ }_{i}$ yields

$$
\widehat{p_{i}}=\underset{(0.03)}{4.63}+\underset{(0.005)}{0.031 \text { SalesTax }_{i}},
$$

where the standard errors of the estimates are given in the parentheses. Using these result, determine whether SalesTax ${ }_{i}$ is a weak instrument.
A. 4 A researcher observes i.i.d. draws from normal distribution with mean $\mu$ and variance 1 . The true value of the mean, which is unknown to the researcher, is 2 . As the sample size $n$ grows, what function of the parameter $\mu$ the log likelihood divided by $n$ converges to? Write an explicit form of this function.
A. 5 Consider a linear regression $y_{i}=\alpha+\beta x_{i}+\varepsilon_{i}$, where $E\left(\varepsilon_{i}\right)=0$ and $\operatorname{Cov}\left(\varepsilon_{i}, x_{i}^{2}\right)=0$. Note that the regressor is $x_{i}$, whereas the above covariance is between $\varepsilon_{i}$ and $x_{i}^{2}$. Suppose that you have $n$ i.i.d. observations $\left(y_{i}, x_{i}\right)$. Propose a GMM estimator of $\alpha$ and $\beta$. Is your estimator an optimal GMM? What is the distribution of the minimum value of your GMM criterion function?
A. 6 Suppose that you would like to test the null hypothesis that there is a stochastic trend in the logarithm of the quarterly UK industrial production time series. Your alternative is that the series is stationary around a deterministic trend. Explain how you would perform such a test. Justify the exact form of the regression (or regressions) that you would run.

## Section B

B. 1 Let $\mathrm{Wage}_{i}$ be the wage of a randomly chosen worker. Further, let $\mathrm{Job}_{i}$ be a job status dummy equal to one if the worker has a white collar job, and let College $i_{i}$ be a dummy equal to to one if the worker graduated from a college. Finally, let Wage ${ }_{1 i}$ and Wage ${ }_{0 i}$ be potential wages and $\mathrm{Job}_{1 i}$ and $\mathrm{Job}_{0 i}$ be potential job statuses had College $_{i}$ been equal to 1 and 0 , respectively. Suppose that $\mathrm{Wage}_{1 i}-\mathrm{Wage}_{0 i}=\delta$, which does not depend on $i$, and that

$$
E\left(\text { Wage }_{i} \mid \text { College }_{i}, \mathrm{Job}_{i}\right)=\alpha_{1}+\alpha_{2} \text { College }_{i}+\alpha_{3} \mathrm{Job}_{i}
$$

a) Show that

$$
\begin{aligned}
\alpha_{2}= & E\left(\text { Wage }_{1 i} \mid \text { College }_{i}=1, \mathrm{Job}_{1 i}=1\right) \\
& -E\left(\text { Wage }_{0 i} \mid \text { College }_{i}=0, \mathrm{Job}_{0 i}=1\right)
\end{aligned}
$$

b) Assuming that College ${ }_{i}$ is randomly assigned, show that

$$
\alpha_{2}=\delta+E\left(\mathrm{Wage}_{0 i} \mid \mathrm{Job}_{1 i}=1\right)-E\left(\mathrm{Wage}_{0 i} \mid \mathrm{Job}_{0 i}=1\right)
$$

Is it likely that $\alpha_{2}=\delta$ ? Explain your reasoning.
c) Under the assumption made in b), would the slope of the OLS regression of Wage $_{i}$ on constant and College $i_{i}$ consistently estimate $\delta$ ? Why $\mathrm{Job}_{i}$ can be called a bad control variable? Give a general characteristic of bad control variables.
B. 2 Consider a linear regression

$$
y_{i}=\alpha+\beta x_{i}+u_{i}, i=1, \ldots, n,
$$

where $\left(y_{i}, x_{i}, u_{i}\right), i=1, \ldots, n$, are i.i.d. and the conditional distribution of $u_{i}$ given $x_{i}$ is normal with mean zero and variance $\sigma^{2}$.
a) You observe $\left(y_{i}, x_{i}\right), i=1, \ldots, n$. Write down the conditional log likelihood function of $\alpha, \beta$, and $\sigma^{2}$.
b) Find the maximum likelihood estimators of $\alpha, \beta$, and $\sigma^{2}$. Are these estimators unbiased? Explain.
c) Suppose that it is known that $\alpha=0$ and $\sigma^{2}=1$. However, $\beta$ is unknown. Write the LR test statistic for the test of the hypotehsis that $\beta=0$ in terms of the sum of squared residuals (SSR) in the restricted and unrestricted regressions. Compare with the F-statistic. In this situation, what would you prefer to use, LR test or F test?
B. 3 Consider a linear regression without intercept

$$
y_{i}=\beta x_{i}+u_{i}, i=1, \ldots, n,
$$

where $x_{i}$ is an endogenous variable. Suppose that $z_{1 i}$ is a valid instrument and the i.i.d. data $\left(y_{i}, x_{i}, z_{1 i}\right), i=1, \ldots, n$, are available.
a) Describe the indirect least squares estmator of $\beta$.
b) Suppose that, in addition to $z_{1 i}$, you have $k-1$ other valid instruments, $z_{2 i}, \ldots, z_{k i}$. Suppose further that $\sum_{i=1}^{n} z_{j i} z_{r i}=0$ for $j, r=1, \ldots, k$ such that $j \neq r$ and that $\sum_{i=1}^{n} z_{j i}^{2}=1$ for $j=1, \ldots, k$. Consider the OLS estimates of the reduced form equations

$$
\begin{aligned}
& \hat{y}_{i}=\hat{\pi}_{y 1} z_{1 i}+\ldots+\hat{\pi}_{y k} z_{k i}, \\
& \hat{x}_{i}=\hat{\pi}_{x 1} z_{1 i}+\ldots+\hat{\pi}_{x k} z_{k i} .
\end{aligned}
$$

Is $\hat{\pi}_{y 1} / \hat{\pi}_{x 1}$ different from the value of the indirect least squares estimator that you described in a)? Explain your answer. (Hint: it might be convenient to use matrix notation for the reduced form regressions.)
c) Under assumptions of b), show that the 2SLS estimate of $\beta$ minimizes function $\sum_{j=1}^{k}\left(\hat{\pi}_{y j}-\beta \hat{\pi}_{x j}\right)^{2}$. Hence demonstrate that the 2SLS estimator can be viewed as a generalization of the indirect least squares estimator.
B. 4 Discuss ALL of the following statements
a) If a treatment is randomly assigned, then we can estimate the causal effect of the treatment by running an OLS regression of the variable of interest on the treatment indicator. No control variables are necessary.
b) MLE is a special case of GMM.
c) A $95 \%$ forecast interval is not a confidence interval, and is likely to be invalid.
d) In small samples, the appeal of the heteroskedasticity-robust standard errors diminishes.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Friday 22 May 2015 1:30pm to 3:30pm

M330

## Applied Econometrics

Candidates are required to answer all four questions.
Each question carries equal weight.
Write your number not your name on the cover sheet of each booklet.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet x 1
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION <br> Durbin-Watson and Dickey-Fuller Tables <br> New Cambridge Elementary Statistical Tables <br> Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. You are interested in estimating the effect of whether students do their mathematics homework on mathematics test scores. You have data on all Year 9 students in Cambridge schools in 2014. The data set contains the following variables

- for students: end of year test scores, whether did homework, number of missed classes, gender, age, test scores in 2014.
- for their parents: education
- for their teacher: gender, age, education

About half of the students do their homework.
(a) If you want to abolish homework, what treatment effect would you estimate? If you want to make homework mandatory, what effect would you estimate?
(b)You want to estimate how well students who are currently not doing their homework would do, if they did their homework. You decide to use matching methods to construct an estimator, and will therefore rely on a conditional independence assumption (CIA). Specify the CIA in this and explain why it is likely to hold. Be explicit about the counterfactual outcomes and the variables that you want to control for.
(c) What are the potential reasons why your estimates might be biased upward? Are there reasons why they might be biased downwards? Explain.
(d) Suppose that boys never do their homework. What implications does this have for your analysis?
2. Schools in England are inspected to judge whether they are performing well. We would like to evaluate the effect on test scores in schools, of an inspection judgement of 'unsatisfactory' on poorly performing schools. The aim is to use a regression discontinuity design to evaluate the effect of being judged unsatisfactory on future test scores. We have access to the following data on schools and pupils in Year 7 for the years 2002-2011:

- Schools: Pupil test score average $\left(Y_{s}\right)$, 2002-2011
- Schools: A vector of average pupil characteristics, denoted by $X_{s}^{\prime}$ (proportion of: poor children, white British, English mother tongue, female; and past attainment in Year 4)
- Schools: School rating denoted as $Z_{s}$; A school is rated as failing if the school rating is negative or as fail ${ }_{s}=1$ if $Z_{s}<0$ and as satisfactory otherwise.
(a) Inspectors base their overall view of a school on a series of subjudgements which are ordinal, taking values from 1-10, about different aspects of the school such as pupil outcomes, quality of provision and leadership and management. These judgements are aggregated into a weighted average that is the continuous rating variable $Z_{s}$, where 0 indicates the threshold of a passing or satisfactory school. The idea is that this variable is to be used as the running or assignment variable in the RDD. Explain why this is used as opposed to using the sub-judgements. What is the discontinuity we hope to exploit?
(b) School performance is modelled as below, based on the change in test scores over time, where the year of inspection is $t$ and change is taken between $\mathrm{t}+1$ and $\mathrm{t}-1:, \Delta Y_{s t}=Y_{s t+1}-Y_{s t-1}$. An alternative, longer term change over 3 periods, $\Delta Y_{s t}=Y_{s t+3}-Y_{s t-1}$ is also used.

$$
\begin{equation*}
\Delta Y_{s t}=\alpha+\beta f a i l_{s t}+f\left(Z_{s t}\right)+\gamma \Delta X_{s t}^{\prime}+\varepsilon_{s t} \tag{1}
\end{equation*}
$$

Explain how the RD design helps to identify the treatment effect. Explain why the assignment variable (rating) is treated as a non-linear function.
(c) The specification above is criticized and the following specification is chosen instead:
$\Delta Y_{s t}=\alpha+\beta$ fail $l_{s t}+f\left(Z_{s t}\right)+\gamma \Delta X_{s t}^{\prime}+\lambda\left(Y_{s t-2}-Y_{s t-4}\right)+\delta($ year of inspection $)+\varepsilon_{s t}$
Suggest reasons why this new specification might be preferred.
(d) The graph below provides the density function of the assignment variable. What information does it provide and why is it useful?

(e) The lines in the graph above denote different bandwidths for comparisons of schools. The results below are based on specification (2) and the different bandwidths.

| $\Delta Y_{s t}$ | All observations |  |  | Broad bandwidth |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | estimate | Standard Error | N | estimate | Standard Error |
| $(t+1)-(t-1)$ | 4000 | $0.057^{* * *}$ | 0.012 | 467 | 0.043* | 0.022 |
| $(t+3)-(t-1)$ | 3313 | $0.112^{* * *}$ | 0.016 | 421 | 0.092*** | 0.031 |

${ }^{* * *}$ denotes significance at $1 \%,{ }^{* *}$ significance at $5 \%$ and $*$ significance at $10 \%$.

| $\Delta Y_{s t}$ | Narrow bandwidth |  |  | Very narrow bandwidth |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | estimate | Standard Error | N | estimate | Standard Error |
| $(t+1)-(t-1)$ | 315 | 0.046 | 0.032 | 156 | 0.022 | 0.061 |
| $(t+3)-(t-1)$ | 283 | $0.121^{* *}$ | 0.044 | 139 | 0.135 | 0.084 |
| $* * *$ denotes significance at $1 \%,{ }^{* *}$ significance at $5 \%$ and ${ }^{*}$ significance at $10 \%$. |  |  |  |  |  |  |

Explain both the pattern of results, across bandwidths and that across the two different specifications of the outcome variable, the change in test scores.
(f) Describe two potential threats to identification of the treatment effect above and how you might check whether they matter?
3. You are interested in understanding whether immigrants are discriminated in the labor market. You have access to a random sample of workers with information on their labor market earnings, experience, years of education and whether or not a worker is an immigrant. As a first pass you run the following regression:

Earnings $_{i}=\beta_{0}+\beta_{1}$ Experience $_{i}+\beta_{2}$ Experience $_{i}^{2}+\beta_{3}$ Education $_{i}+\beta_{4}$ Immigrant $_{i}+\varepsilon_{i}$
where $\varepsilon_{i}$ is assumed to be an independent and identically distributed random variable. Based on your least square estimates, you decide to plot the square of your residuals, $\widehat{\varepsilon}_{i}^{2}$, (y-axis) against the level of education (in the x -axis). You obtain the following graph.

(a) Based on the above graph, are ordinary least square estimates and/or inference based on the regression above likely to be correct? Why or why not?
(b) Discuss a formal test procedure that allows you to assess your answer in (a) systematically.
(c) Supposed your proposed test confirms the existence of a problem. Propose a solution and discuss its merits and potential problems.
4. A policy think-tank is interested in understanding whether difficulties in obtaining bank credit affect entrepreneurial earnings. To do this, researchers at the think-tank collect data from a nationally representative survey. Having identified the subset of entrepreneurs in their sample - those with income from "self-employment" activities - they decide to run the following regression:
$\log \left(\right.$ Earnings $\left._{i}\right)=\alpha+\beta_{1}\left(\right.$ Education $_{i}+\beta_{2} \log \left(\right.$ Wealth $\left._{i}\right)+\beta_{3}\left(\right.$ Age $\left._{i}\right)+\beta_{4}\left(\right.$ Age $\left._{i}^{2}\right)+\varepsilon_{i}$
where "Earnings" gives the income of entrepreneur $i$, "Education", giving the number of years of formal schooling of entrepreneur $i$, is included to proxy entrepreneurial ability and "Age" and "Age ${ }^{2}$ " of entrepreneur $i$ are proxies for experience. The variable "Wealth", giving the market value of assets owned by entrepreneur $i$, is included to proxy the effect of credit constrains on the scale of entrepreneurial activity. This is motivated by theories predicting that, were it not for credit constraints, the size of an entrepreneurial project (and hence its earnings) should not be correlated with individual wealth. The disturbance term $\varepsilon_{i}$ is assumed to be a normally distributed random variable with mean zero and variance $\sigma_{\varepsilon}^{2}$.

Additionally, the researchers know that the decision to become an Entrepreneur (i.e. Entrepreneur ${ }_{i}=1$ in the sample) is well modelled by the following system of equations:

$$
\text { Entrepreneur }_{i}=\left\{\begin{array}{lll}
1 & \text { if } & E_{i}^{*}>0  \tag{4}\\
0 & \text { if } & E_{i}^{*} \leq 0
\end{array}\right\}
$$

where $E_{i}^{*}$ is a latent index given by:
$E_{i}^{*}=\eta+\gamma_{1}(\text { Education })_{i}+\gamma_{2} \log \left(\right.$ Wealth $\left._{i}\right)+\gamma_{3}\left(\right.$ Age $\left._{i}\right)+\gamma_{4}\left(\right.$ Age $\left._{i}^{2}\right)+\gamma_{5}\left(\right.$ HH Size $\left._{i}\right)+\nu_{i}$
and $\mathrm{HH} \mathrm{Size}_{i}$ gives the number of persons living in the household of $i$. $\nu_{i}$ is assumed to be a normally distributed random variable with mean zero and variance 1 .

Finally, researchers know the following statistical result on bivariate normal random variables: if x and y have a bivariate normal distribution with
means 0 , standard deviations $\sigma_{x}$ and $\sigma_{y}$ and correlation $\rho$, then

$$
\begin{aligned}
& E(x \mid y>c)=\rho \lambda\left(\frac{a}{\sigma_{y}}\right) \\
& \lambda\left(\frac{a}{\sigma_{y}}\right)=\frac{\phi\left(\frac{a}{\sigma_{y}}\right)}{1-\Phi\left(\frac{a}{\sigma_{y}}\right)}
\end{aligned}
$$

where $a$ is a constant, $\phi($.$) is the probability distribution function of a$ standard normal random variable and $\Phi($.$) is the corresponding cumulative$ distribution function.
(a) The researchers run ordinary least squares on specification (3) above and find a statistically insignificant estimate $\widehat{\beta}_{2}$. Prove that the policy conclusion that "individual wealth has no impact on entrepreneurial earnings" is only justified if the disturbances in equations (3) and (5) are independent, i.e. $\operatorname{Corr}\left(\varepsilon_{i}, \nu_{i}\right)=0$.
(b) Assume that, in general, you cannot ascertain a priori whether $\operatorname{Corr}\left(\varepsilon_{i}, \nu_{i}\right)=$ 0 . Using equations (3) and (5) suggest an estimation procedure that provides consistent estimates of the elasticity of entrepreneurial earnings to entrepreneurial wealth. Discuss how you could then test whether $\operatorname{Corr}\left(\varepsilon_{i}, \nu_{i}\right)=0$.
(c) Discuss why including the variable Household Size ${ }_{i}$ in equation (5) aids in the identification of the elasticity in (b).
(d) Suppose the following is true. First, there is an unobserved variable - call it "Entrepreneurial Grit" - that is positively correlated both with the decision to become an entrepreneur and with entrepreneurial earnings. Second, suppose you have access to independent evidence that credit constraints affect the decision to become an entrepreneur, i.e. the latter decision depends positively on the private wealth of the individual $\left(\gamma_{2}>0\right)$. Based on these two assumptions do you expect the the elasticity of entrepreneurial income to private wealth, as obtained from a least squares regression on (3) alone, to be upwards or downwards biased? Please justify your answer.

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Wednesday 27 May $2015 \quad$ 1:30pm to $3: 30 \mathrm{pm}$

M600
Topics in Macroeconomic History
Candidates are required to answer two out of four questions.
Each question carries equal weight.

## STATIONERY REQUIREMENTS

20 Page Answer Booklet
Rough Work Pad

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Scholars have done a better job at explaining why the Great Depression became great - why the 1930s downturn, once underway, became so severe - than in explaining the onset of the Depression itself. What explains this differential success? What were the main factors, in your view, responsible for the business cycle inflection point?
2. International capital flow reached high levels prior to 1913, or so some comparisons with international capital flows in recent decades suggest. What is the validity of these comparisons? To the extent that they are valid, what historical factors explain the higher level of capital flows before 1913?
3. Evaluate the effects of banking crises drawing on the historical evidence.
4. What are the features of credit cycles? Evaluate the relationship between credit cycles and the real economy.

ECM11
MPhil in Finance \& Economics

Tuesday 03 May $2016 \quad 2.00 \mathrm{pm}-4.00 \mathrm{pm}$

F100
FINANCE I
Candidates are required to answer all four questions.
Each question carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS

## EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1. Part A and Part B of this question are totally independent.

Part A - You are considering leasing a photocopy machine for your business. You are offered three lease options:

| Length of Lease | Price |
| :---: | :---: |
| 1 year | $\$ 1,000$ |
| 2 years | $\$ 2,000$ |
| 3 years | $\$ 2,800$ |

In each case you pay the entire lease cost up front. You are certain that you will need the photocopy machine for 3 years and that the terms of the leases will not change in that time. Assuming a $10 \%$ annual interest rate, which pattern of leases would you choose?

Part B - You are asked to choose one from two new projects, A and B. After a careful analysis, you report to Mr. Stern, the CEO, that A is better than B since it has a higher NPV (both have positive NPVs).

Mr. Stern looks at you and says: "Son, there is one key factor you ignored: $75 \%$ of our shareholders are senior citizens. Project B pays off only in three years, but project A pays off in ten years. Many of the shareholders won't be around to collect the dividends if we take A. They would be happier with project B".

Would you agree with Mr. Stern? Explain your answer.

Question 2. There are three riskless bonds which mature 2 years from today. They all pay $\$ 100$ at maturity and coupons are paid annually. Your junior analyst brings you the following table of coupons and current prices (see below).
(a) Show that there is some inconsistency in the prices of these bonds.
(b) How can you create an immediate arbitrage profit with these bonds?

| Coupon Rate | Price |
| :---: | :---: |
| $0 \%$ | $\$ 82.00$ |
| $6 \%$ | $\$ 92.5$ |
| $12 \%$ | $\$ 102.5$ |

Question 3. Consider a risky portfolio. The end-of-year cash flow derived from the portfolio will be either $\$ 50,000$ or $\$ 150,000$ with equal probabilities of 0.5 . The alternative risk-free investment in T-bills pays $5 \%$ per year.
(a) If you require an expected rate of return of $15 \%$ on the portfolio, how much will you be willing to pay for the portfolio? What is the risk premium you are demanding?
(b) Now suppose that you require an expected rate of return of $20 \%$, what is the price you are willing to pay? What is the risk premium you are demanding?
(c) Compare your answers in (a) and (b), what do you conclude about the relation between the required risk premium on a portfolio and the price at which the portfolio will sell (given the distribution of its payoff)?

Question 4. Consider a manager compensated by a package whose value, $C\left(x_{t}\right)$, depends solely on a fundamental variable, $x_{t}$. This fundamental variable determines the value of the firm he manages and follows a Geometric Brownian Motion

$$
d x_{t}=\mu x_{t} d t+\sigma x_{t} d z_{t}
$$

where $z_{t}$ is a standard Brownian motion.
The manager takes two decisions:

- His first decision is to be taken immediately and affects the fundamental's volatility, once and for all. He can essentially select between two volatility levels $\sigma=\sigma_{1}$ or $\sigma=\sigma_{2}$, where $\sigma_{1}<\sigma_{2}$.
- His second decision is to quit and take an alternative employment. He can take take this last decision at any time. His alternative employment determines his reservation value, $R>0$.

The value of the manager's compensation package, if he selects a volatility $\sigma$, and quits the first time $x_{t}$ reaches a lower boundary $\underline{x}$ (where $x_{t}>\underline{x}$ ), is

$$
C\left(x_{t}\right)=x_{t}+[R-\underline{x}]\left(\frac{x_{t}}{\underline{x}}\right)^{\beta}
$$

Here, $\rho$ is the constant discount rate and $\beta=-2 \rho / \sigma^{2} .\left(\frac{x_{t}}{\underline{x}}\right)^{\beta}$ is the Laplace transform of probability density function of the first passage time of $x_{t}$ to a lower boundary $\underline{x}$.
(a) Assuming the manager starts with selecting a given volatility level $\sigma$, derive the optimal threshold level $\underline{x}$ at which the manager will choose to quit the firm.
(b) Establish which one of the two volatility levels $\sigma_{1}$ or $\sigma_{2}$ the manager selects to start with, knowing he will quit when his optimal abandonment threshold level $\underline{x}$ is reached (determined in part (a)).

ECM11
MPhil in Finance \& Economics

Monday 16 May $2016 \quad 1.30 \mathrm{pm}-3.30 \mathrm{pm}$

F200
FINANCE II

Candidates are required to answer all four questions..

Each question carries equal weight

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1. Part A and Part B of this question are totally independent.

Part A - You are considering moving your money to new bank offering a one-year CD that pays an $r_{m}=8 \%$ APR with monthly compounding. Your current bank's manager offers to match the rate you have been offered. The account at your current bank would pay interest every six months. How much interest will you need to earn every six months to match the CD?

Part B - Suppose that the term structure is flat and that two year bonds with a $\$ 1,000$ face value and a $3.5 \%$ coupon rate are selling at par. Coupons are assumed to be paid semi-annually. What is the price of a ten year bond with a $\$ 1,000$ face value and with:
(a) a $2.0 \%$ coupon?
(b) a $6.0 \%$ coupon?

Question 2. Part A and Part B of this question are totally independent.

Part A - Evaluate the following:
(a) If you identify a stock with expected return greater than that suggested by the CAPM model, should you invest all your money in this stock?
(b) If you identify a call option with price lower than that of the put-call parity replicating portfolio (composed of put option, stock, and bonds), should you invest all your money in this option?

Are your answers to (a) and (b) the same? Explain why?

Part B - Starship Enterprises is currently selling for $\$ 100$ per share. Their dividends are growing at $7 \%$ per year under their policy of plowing back $20 \%$ of earnings into the company. Earnings next year are expected to be $\$ 30$ per share.
(a) Find the return on equity and the appropriate market capitalization rate (the discount rate) for Starship Enterprises.
(b) Due to their high return on equity relative to their market capitalization rate, Starship Enterprises is viewed as a prime takeover target. The hostile takeover genius, Darth Raider, vows to take over the company and raise the plowback ratio to $80 \%$. If the takeover is expected to occur immediately, what is the effect on the price of Starship Enterprises?

Question 3. Leverit Inc. is a firm that produces mechanical equipment. Leverit's only earnings will be received in a year, and will be $\$ 80 \mathrm{M}$ with probability $1 / 6, \$ 160 \mathrm{M}$ with probability $1 / 3, \$ 280 \mathrm{M}$ with probability $1 / 3$, and $\$ 360 \mathrm{M}$ with probability $1 / 6$. Leverit is all-equity financed, but management considers issuing some debt and using the proceeds to buy back shares. Leverit has 100 M shares outstanding. The riskless rate is $5 \%$, the market risk-premium is $6 \%$, and Leverit's beta is 1.2 .
(a) What is Leverit's market value, stock price, return on equity, and earnings per share $\left(E P S_{1}\right)$ ?
(b) Assume that Leverit issues zero coupon debt, with a par value of $\$ 20 \mathrm{M}$, and maturity of 1 year. What is the market value of Leverit's debt?
(c) Assume that there are no market imperfections. What is the new market value of Leverit's equity? What is Leverit's new leverage?
(d) Denote Leverit's new stock price by $P$. How many shares can Leverit buy back? (Your answer should be a function of $P$. Ignore integer constraints.) What is Leverit's new stock price?

Hint: Using the definition of the stock price, i.e.

$$
P=\frac{\text { market value of equity }}{\text { number of shares outstanding }},
$$

determine an equation for $P$.
Explain the intuition for your answer.

Question 4. A stock price is currently 25 . It is known that at the end of two months it will be either 23 or 27 . The risk-free interest rate is $10 \%$ per annum with continuous compounding. Suppose that $S_{T}$ is the stock price at the end of the two months. What is the value of a power derivative that pays of $\left(S_{T}\right)^{3}$ at that time?

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Thursday 19 May 2016 09:00am - 11:00am

F300
CORPORATE FINANCE

Candidates are required to answer all four questions.

Write your candidate number (not your name) on the cover of each booklet.

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If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

GBA Plc, which operates in the car manufacturing industry, has a cost of capital of its assets of $5.9 \%$. The company is expecting a pre-tax free cash flow of $€ 4.46$ million at the end of each year in perpetuity and pays corporate tax at the rate of $30 \%$. GBA has permanent debt outstanding with current value of $€ 25$ million and annual cost of debt of $2 \%$. The personal tax rates on interest payments to debt holders is $20 \%$ and on dividends and capital gains to shareholders is $10 \%$.
a) What would be the expected after-tax cash flow to equity holders each year in perpetuity, if the company were unlevered? What would be the value of GBA if it were unlevered?
b) What is the after-tax cash flow to debt holders every year in perpetuity? What is the after-tax cash flow to (levered) equity holders every year?
c) What is the value of the tax shield from debt financing net of the effect of personal taxes (state your own assumptions, if needed)?
d) What is the total firm value for GBA Plc?

## Question 2

A venture capitalist ( VC ) is considering investing in a start-up, which requires an initial investment of $\$ 1.1$ million to buy property, plant and equipment. These fixed assets will be depreciated via a straight-line method over the next 4 years. However, depreciated fixed asset will be replaced, so that the scale of initial investment will remain unchanged during this period. The project is financed also with $\$ 0.6$ million raised by taking a loan with annual interest rate of $8 \%$ and all principal repaid in one installment at the end of fourth year. In each year, the project is expected to have revenues for $\$ 1$ million, and costs for $\$ 0.475$ million. The VC plans to hold the equity position for 4 years, and sell it right afterward. The continuation value (which corresponds to the selling price) of the firm is computed assuming that after 4 years both the leverage and the investment will be maintained at the initial level by the new owners. The corporate tax rate is $35 \%$.

The business risk of the start-up is comparable to that of a listed firm, whose current equity value is $\$ 100$ million, market value of debt of $\$ 50$ million, and which adjusts the leverage back to the current level once a year. Given the current leverage, for the comparable firm the cost of levered equity is $20 \%$ and the cost of debt is $7 \%$. The corporate tax rate of the comparable firm is also $35 \%$.
(a) Determine the initial unlevered cost of capital of the start-up.
(b) Compute the NPV of the investment project using the APV method (i.e., Unlevered Asset Value plus Present Value of the Tax Shield).
(c) Determine the cost of equity capital and the WACC.
(d) Compute the NPV of the investment project discounting the free cash flows using the WACC.

## Question 3

Consider a mine that contains 100 million lbs. of copper that you can extract in the next two years. Due to environmental regulations, no extraction can take place after the end of the second year. The production cost is $\$ 0.70$ per lb. and will remain constant for the next two years. The spot copper price during each year is projected to be as in the following tree:


On the tree, the conditional risk-neutral probability of an up-jump is given in parentheses (e.g., in Year 2, the probability of an up-jump, conditional on a spot price of 0.5475 at the end of Year 1 , is 0.3 ). Assume that the entire annual production of copper is sold at the spot price prevailing during the year. The annual risk-free rate is $r_{f}=5 \%$ for both years.
The production capacity of the mine is 100 million lbs. per year. However, you can suspend production under unfavorable scenarios for the copper price and defer it to a later year under better conditions. This flexibility comes at no additional costs.
a) What is the value of the mine if you commit (i.e., assuming no flexibility) to extract all the resource in Year 1? What is the value if you commit to extract copper in Year 2 only? What is the better of these two strategies?
b) Assuming you decide to operate only in the first year, what is the value of the real option not to extract if the copper price is too low?
c) What is the value of the mine under the best possible operating policy?
d) As of today, Year 1 forward price is $\$ 0.6619$ and Year 2 forward price is 0.6950 . It can be shown also that, as of Year 1, forward price for delivery in Year 2 conditional on the spot price being 0.8334 is 0.87507 , and forward price for delivery in Year 2 conditional on 0.5475 is 0.5749 . Given this information, what is the maximum value of the mine if you hedge $100 \%$ of the production at the current forward copper prices? How is hedging changing the optimal operating policy?

## Question 4

Consider a two-date $(t=0,1)$ setup. At $t=1$, the state of the overall economy can be either high or low with risk-neutral probability of 0.6 and 0.4 , respectively. The risk free rate is $10 \%$.
The manager of CBS plc is making an investment decision regarding two mutually exclusive projects, denoted A and B. Project A will generate a cash flow of $£ 1.10$ million in the high state, and $£ 0.88$ million in the low state. Project B will generate a cash flow of $£ 1.65$ million in the high state, and $£ 0$ in the low state. Both projects require an initial investment of $£ 0.80$ million. Suppose CBS needs to finance the $£ 0.80$ million investment externally. Assume the manager is making the decision to maximize the value of the existing shareholders and that, although external investors know about the existence of the two projects, they cannot impose their preference on the manager.
a) Assuming the manager finances the project exclusively by issuing new equity, which project will she choose?
b) Assuming the manager finances the project exclusively by issuing new debt (in the form of a zero coupon bond), which project will she choose? What is the value of debt for the chosen project?
c) Comparing (a) to (b), which is the financing alternative that maximizes the value of CBS shareholders?

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research
MPhil in Finance

Friday 20 May 2016 9:00am - 11:00am

## F400

ASSET PRICING

Candidates are required to answer all four questions.
Each question carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS
EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. There is a single investor with utility function

$$
u\left(c_{0}, c_{1}, c_{2}\right)=v\left(c_{0}\right)+\delta\left(\pi_{1} v\left(c_{1}\right)+\pi_{2} v\left(c_{2}\right)\right)
$$

where $c_{0}$ is her consumption at date $0, c_{1}$ is her consumption in state 1 at date 1 and $c_{2}$ is her consumption in state 2 at date 1 . This investor is endowed with $e_{0}$ units of the date- 0 consumption good, $e_{1}$ units of the asset with payoff vector $(1,0)$ and $e_{2}$ units of the asset with payoff vector $(0,1)$.
(a) What is meant by the stochastic discount factor in this framework?
(b) Give a formula for the stochastic discount factor at date 0 . Justify your answer carefullly.
(c) Give a formula for the stochastic discount factor at date 1. Justify your answer carefullly.

Now suppose that $v=\log$ (the natural logarithm), $\delta=\frac{2}{3}, \pi_{1}=\frac{3}{4}, \pi_{2}=\frac{1}{4}$ and $e=(2,3,1)$.
(d) What is the cum-dividend price, at date 0 , of an equity that pays a dividend of 1 at date 0,3 at date 1 in state 1 and 5 at date 1 in state 2 ?
2. There are two assets, a safe asset and a risky asset. The price $B$ of the safe asset evolves according to the s.d.e.

$$
d B=r B d t
$$

Furthermore there is a single dividend date $T \in(0,1)$ such that the price of the risky asset evolves according to the s.d.e.

$$
d S=\mu S d t+\sigma S d Z
$$

on the intervals $(0, T)$ and $(T, 1)$. However, at date $T$, the risky asset pays a lump-sum dividend of $\delta S_{T-}$. Here $Z$ is a standard Wiener process.
(a) How does the price of the risky asset change at date $T$ ? Justify your answer.
(b) What is meant by a European Put Option with strike price $K$ and maturity 1 written on the risky asset?
(c) What is the price, at time 0 , of the put option of part (b)?
(d) How would this price change if there were two dividend dates $T_{1}, T_{2} \in(0,1)$ at which lump-sum dividends of $\delta_{1} S_{T_{1}-}$ and $\delta_{2} S_{T_{2}-}$ were paid respectively?
3. (a) Write down a model of the dynamics of forward rates in the presence of a stochastically evolving economic state. Explain the meaning of the various elements of the model.
(b) What is meant by the term structure of forward rates?
(c) State the condition that must be satisfied by these dynamics if there is to be no arbitrage? Explain this condition briefly.
(d) Assuming that the condition of part (c) is satisfied, how would you set about predicting the future evolution of the term structure of forward rates?
4. An investor has initial wealth $w_{0}>c$ and utility function $u$ given by the formula

$$
u(w)=\left\{\begin{array}{ll}
\frac{(w-c)^{1-\rho}-1}{1-\rho} & \text { if } \rho \neq 1 \\
\log (w-c) & \text { if } \rho=1
\end{array}\right\} .
$$

She can invest in one of two assets. The safe asset has net return $r_{f}$ and the risky asset has net return $\widetilde{r}$.
(a) How does her investment in the risky asset vary with $w_{0}$ ?
(b) Justify your answer to part (a).

## END OF PAPER

CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Tuesday 24 May $2016 \quad 1: 30 \mathrm{pm}$ to $3: 30 \mathrm{pm}$

## F500 <br> EMPIRICAL FINANCE

This paper is in two sections. Section A: Candidates are required to answer all four questions. Section B: Candidates are required to answer two questions.

Each section carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS

 EXAMINATIONCalculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## 1 Section A

A1. Explain what you think of the following statements regarding the Efficient Markets Hypothesis (EMH)
(a) Although the EMH claims investors cannot outperform the market, analysts such as Warren Buffet have done exactly that. Hence the EMH must be incorrect.
(b) According to the weak form of the EMH, technical analysis is useless in predicting future stock returns. Yet financial analysts are not driven out of the market, so their services must be useful. Hence, the EMH must be incorrect.
(c) The EMH must be incorrect because stock prices are constantly fluctuating randomly.
(d) If the EMH holds, then all investors must be able to collect, analyze and interpret new information to correctly adjust stock prices. However, most investors are not trained financial experts. Therefore, the EMH must be false.
A2. Suppose you have a sample of data on stock returns $\left\{R_{i t}, i=1, \ldots, n, t=\right.$ $1, \ldots, T\}$. Suppose that the data were generated by the CAPM with risk free rate of zero, that is

$$
R_{i t}=\beta_{i} R_{m t}+\varepsilon_{i t},
$$

where $R_{m t}$ is the unobserved market return. Here, $R_{m t}, \varepsilon_{i t}$ are i.i.d both with mean zero and finite variance, with both being mutually independent. Suppose that you compute the two sample covariance matrices $\widehat{\Omega}, \widehat{\Sigma}$ defined below with typical elements

$$
\begin{aligned}
& \widehat{\Omega}_{i j}=\frac{1}{T} \sum_{t=1}^{T} R_{i t} R_{j t}, \quad i, j=1, \ldots, n \\
& \widehat{\Sigma}_{t s}=\frac{1}{n} \sum_{i=1}^{n} R_{i t} R_{i s}, \quad t, s=1, \ldots, T
\end{aligned}
$$

Give the limit of $\widehat{\Omega}$ when $T \rightarrow \infty$ and $n$ is fixed, and the limit of $\widehat{\Sigma}$ when $n \rightarrow$ $\infty$ and $T$ is fixed. What can we learn from these covariance matrices about the risk return tradeoff, the market return, and the idiosyncratic component?
A3. Suppose that the dividend/price ratio $x_{t}$ and stock returns $r_{t}$ obey the following predictive regression

$$
\begin{aligned}
& r_{t+1}=\beta x_{t}+\varepsilon_{t+1} \\
& x_{t+1}=\rho x_{t}+\eta_{t+1}
\end{aligned}
$$

where the error terms are iid with mean zero and coviarance matrix given below

$$
\binom{\varepsilon_{t}}{\eta_{t}} \overbrace{\sim}^{i i d} 0,\left(\begin{array}{ll}
\sigma_{\varepsilon \varepsilon} & \sigma_{\varepsilon \eta} \\
\sigma_{\varepsilon \eta} & \sigma_{\eta \eta}
\end{array}\right) .
$$

Empirically we find that $\beta$ is quite large and statistically significant but returns are almost uncorrelated over time. How can that be?
A4. True, False, Explain
(a) If $P_{t}^{*}$ is a rational unbiased forecast of $P_{t}$, i.e., $E_{t} P_{t}^{*}=P_{t}$, for example $P_{t}^{*}=\sum_{i=1}^{\infty}\left(\frac{1}{1+R}\right)^{i} D_{t+i}$, then

$$
\operatorname{var}\left[P_{t}^{*}\right] \geq \operatorname{var}\left[P_{t}\right]
$$

(b) In the presence of informed traders, stock prices fail to be martingales, and prices will not converge to true values.
(c) One explanation for the apparent excess volatility of stock returns is that expected returns are time varying
(d) The consumption CAPM has more both cross-sectional and time series predictions, i.e., more predictions than the CAPM. Since the CAPM is widely rejected, then it is pointless to test the CCAPM.
(e) Factor mimicking portfolios are arbitrage portfolios that mimick the return on factor.

## 2 Section B

B1. The government wants to introduce minimum resting times into stock market trading, and so carries out an experiment to determine whether it should proceed. Specifically, for any stock whose name begins with $\{A, C, E, \ldots\}$, they decree that any limit order that is entered into the market order book has to remain in place at the same price and same quantity for the whole trading day. For any stock whose name begins with $\{B, D, F, \ldots\}$ nothing changes from the current system, where there is no restriction. Explain what effect this proposed policy might have on the market? Given a sample of data on the market before the regulatory change and after it has taken place, explain how you might determine the effects of the regulatory change.
B2. The Sharpe-Lintner CAPM predicts that the time-series cross-sectional sample of asset returns $R_{i t}$ will obey the regression model (for $i=1, \ldots, n, t=$ $1, \ldots, T$ )

$$
R_{i t}=R_{f t}+\beta_{i}\left(R_{m t}-R_{f t}\right)+\varepsilon_{i t},
$$

where $R_{m t}$ is the return on the observable market portfolio and $R_{f t}$ is the return on an observable risk-free asset. An alternative hypothesis is that

$$
R_{i t}-R_{f t}=\alpha_{i}+\beta_{i}\left(R_{m t}-R_{f t}\right)+\gamma_{i}\left(R_{m t}-R_{f t}\right)^{2}+\delta_{i}\left(R_{m t}-R_{f t}\right)^{3}+\varepsilon_{i t} .
$$

In both cases it is implicitly assumed that the error term $\varepsilon_{i t}$ is iid with mean zero and independent of $R_{m t}-R_{f t}$. Suppose that you observe a sample of data $\left\{R_{i t}, R_{m t}, R_{f t}, i=1, \ldots, n, t=1, \ldots, T\right\}$. Describe the Fama-MacBeth two pass approach for testing the CAPM against the alternative model. Pay particular attention to explaining how the portfolio grouping method can be used to eliminate measurement errors.

B3. Describe the three main methods of estimating volatility. Describe the different sources of data that are needed for each method, and the logic behind each approach. What are the main facts about volatility that have been established?

END OF PAPER CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Tuesday 31 May 2016 9:00am - 11:00am

F510
INTERNATIONAL FINANCE

Candidates are required to answer four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## QUESTION 1: Covered and uncovered interest parity with macroeconomic and political risk

Denote with $\mathcal{E}_{t}$ the exchange rate in units of Home currency in terms of Foreign currency, with $i$ and $i^{*}$ the Home and Foreign interest rates respectively, with $\mathcal{F}_{t}$ the forward rate.

1. Define and contrast covered and uncovered interest parity
2. Consider a two-country world, Home and Foreign. Suppose that, based on new information, investors attribute a small probability $(\psi)$ to a global crisis at $t+1$. Investors come to believe that
(i) the crisis will lead to a large drop in wealth and consumption, everywhere
(ii) the government of the Foreign country will suffer a large depreciation, i.e. $\mathcal{E}_{t+1}$ will be low, denote: $\mathcal{E}_{t+1}^{L}$
(iii) the government of the Foreign country will respond to the crisis by imposing capital controls in the form of a tax $\tau$ on capital outflows from the country.
Using the pricing equation analyzed during the lectures, discuss the implications for
(a) the covered interest parity
(b) the uncovered interest parity

## QUESTION 2: The returns from hedged carry trade

Let $\mathcal{F}_{t}$ and $\mathcal{E}_{t}$ denote forward and spot exchange rates (units of Home currency per unit of Foreign currency). Suppose the Foreign currency trade at a forward premium.

1. Write a "hedged carry trade" strategy combining forward contracts and options (if possible using bid and ask spread). Please define all the terms you use and explain briefly the specific rational for hedging carry trade.
2. Based on the reading and the evidence discussed during lectures, briefly explain the effects of hedging carry trade on the returns and the Sharpe ratio of a well-diversified strategy.
3. What is the "peso problem" in international finance?
4. Briefly discuss how the evidence on returns from hedged carry trade strategies may help us understand the different channels through which the "peso problem" influences asset pricing in international finance?

## QUESTION 3: Uncertainty and the current account

Consider two small open economies operating for two periods only, populated by identical agents with expected utility:

$$
\mathcal{W}=U\left(C_{0}\right)+E_{t} U\left(C_{1}\right)
$$

and preferences such that marginal utility $U^{\prime}()$ is strictly convex.
Output in both periods Y is exogenous. In the initial period (period 0 ), both have identical output $Y_{0}=8$. In period 1, according to available forecasts, one economy has

$$
Y_{1}=\left\{\begin{array}{cc}
Y_{1 H}=4 & \text { prob. . } 4 \\
Y_{1 L}=14 & \text { prob. . } 6
\end{array}\right.
$$

the other economy (denoted with a tilde) has:

$$
\widetilde{Y}_{1}=\left\{\begin{array}{cc}
\widetilde{Y}_{1 H}=2.5 & \text { prob. . } 4 \\
\widetilde{Y}_{1 L}=15 & \text { prob. . } 6
\end{array}\right.
$$

where $\widetilde{Y}_{1}$ denotes output in the second economy.

1. Define the real interest rate under financial autarky $\mathcal{R}^{F A}$.
2. How would the real interest rate in financial autarky differ across the two economy? I.e., how does $\widetilde{\mathcal{R}}^{F A}$ compare with $\mathcal{R}^{F A}$ ?
3. Suppose the two economies can borrow and lend in international financial market. What are the implications for the current account of the small open economies in the initial period? Explain.
4. According to the former chairman of the Federal Reserve Board, Ben Bernanke, the US current account deficit in the last two decades is determined by a 'saving glut' at world level. In light of the evidence on 'global imbalances,' briefly explain how the mechanism underlying your answers to the questions above can help understanding the 'saving glut' hypothesis by Bernanke.
5. Suppose that the residents of the two countries have the following preferences:

$$
U(C)=30 C-C^{2}
$$

Using these preferences, can you verify numerically your answers to the point 2 and 3 above? Why/why not?

## QUESTION 4: Debt relief and debt overhang

Consider a country with an external debt equal to 100. During the period, the country may be either in an expansion (a good state of nature) or in a contraction (a bad state of nature), with probability $1 / 3$ and $2 / 3$ respectively. In the 'good state of nature', the country is able to repay the full amount 100. In the bad state of nature, the country will only be able to pay 25 .

The government, the creditors and an international agency are discussing the possibility of reducing the stock of debt from 100 to 80 through some form of relief, under the assumption that reducing the debt will improve the macroeconomic performance of the country. Namely, reducing debt will raise the probability of the good state from $1 / 3$ to $1 / 2$.

1. Under risk neutrality: calculate the initial secondary market price of debt, and then contrast the effects of the following strategies
a. the creditors forgive 20 units of debt
b. an international agency buys back 20 units of debt

Please compute the changes in the equilibrium expected repayment and debt pricing, and discuss the costs and benefits of each strategy for the government, the creditors and the international agency.
2. The government is considering a "debt swap" by which investors are given the option to exchange the debt they hold with new "safe" debt at some predetermined rate. A swap is usually detrimental to creditors, but the government claims that the swap will raise the probability of the good state from $1 / 3$ to $1 / 2$.
(a) Assuming that investors believe the macroeconomic forecast of the government, calculate the swap rate, the stock of old debt that remains in the market after the swap, and the gains/costs for the debtor and the creditors.
(b) Do creditors have reason to oppose the debt swap?

## QUESTION 5: Asset Pricing, Uncertainty and Risk

Consider the following asset pricing model for a risky asset (e.g. foreign currency) with a public signal and rational expectations. Each trader $i \in[0,1]$ maximizes mean-variance utility

$$
u\left(\mu_{i}, \sigma_{i}^{2}\right)=\mu_{i}-\frac{1}{2} \theta \sigma_{i}^{2}
$$

where $\theta$ is the coefficient of absolute risk aversion $(\theta>0)$, and $\mu_{i}$ and $\sigma_{i}^{2}$ denote the mean and variance of wealth $W_{i}$, which is normally distributed: $W_{i} \sim N\left(\mu_{i}, \sigma_{i}^{2}\right)$. Each agent $i$ faces the budget constraint

$$
W_{i}=\left(X_{i}-D_{i}\right) P+D_{i} V
$$

where $X_{i}$ is the quantity of the risky asset that trader $i$ is endowed with, $X_{i} \sim$ $N\left(\mu_{X}, \sigma_{X}^{2}\right) ; D_{i}$ the demand for the risky asset by trader $i ; P$ the price of the risky asset; and $V$ the payoff of the risky asset, $V \sim N\left(\mu_{V}, \sigma_{V}^{2}\right)$. There is perfect competition in the market for the risky asset and the equilibrium price $P$ is determined by an implicit auction such that

$$
\int_{0}^{1} D_{i}(P) d i=\int_{0}^{1} X_{i} d i \equiv X
$$

where $D_{i}(P)$ is the conditional demand order of trader $i$. The timing in the model is as follows. First, trader $i$ observes her own endowment $X_{i}$ and a public signal $S$ of the payoff to the risky asset:

$$
S=V+\varepsilon
$$

where $\varepsilon \sim N\left(0, \sigma_{\varepsilon}^{2}\right)$. Second, the risky asset is traded at equilibrium price $P$. Finally, the risky asset pays off $V$. Assume that $\mu_{V}, \mu_{X}, \sigma_{V}^{2}, \sigma_{X}^{2}$ and $\sigma_{\varepsilon}^{2}$ are strictly positive, and that the stochastic shocks $X_{i}, V$ and $\varepsilon$ are mutually independent.
(a) Show that demand for the risky asset by trader $i$ equals

$$
D_{i}=\delta \frac{\mathrm{E}\left[V \mid \Omega_{i}\right]-P}{\operatorname{Var}\left[V \mid \Omega_{i}\right]}
$$

where $\delta$ denotes a positive coefficient and $\Omega_{i}$ is the information set available to trader $i$ at the time of trading. Give an intuitive explanation of the result.
(b) Show that the equilibrium price is described by

$$
P=\alpha S-\beta X+\gamma
$$

and compute the coefficients $\alpha, \beta$ and $\gamma$. Explain intuitively how $\mu_{V}, S$ and $X$ affect $P$.
(c) Explain how the asset pricing coefficients $\alpha$ and $\beta$ are affected by an increase in (i) uncertainty about the public signal $\sigma_{\varepsilon}^{2}$, (ii) the riskiness of the asset $\sigma_{V}^{2}$, and (iii) traders' risk aversion $\theta$.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Thursday 26 May 2016 09:00am - 11:00am

F520
BEHAVIOURAL FINANCE

Candidates are required to answer two questions

Each question carries equal weight

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. (a) Can the fact that investors are proven to the disposition effect explain existence of momentum in stock prices? - Fully justify your answer.
(b) Beatrix behaves consistently with Prospect theory, with value function equal to:

$$
v(x)=\left\{\begin{array}{cl}
x, & x \geq 0 \\
2 x, & x<0
\end{array}\right.
$$

and the probability weighting function

$$
\pi(p)=\frac{p^{\delta}}{\left(p^{\delta}+(1-p)^{\delta}\right)^{1 / \delta}} \quad \text { with } \delta=0.9
$$

Beatrix faces only two investment opportunities: either investing her entire wealth $\mathrm{W}=£ 50,000$ in bonds at $2.75 \%$ risk-free return, or investing her entire wealth in the stock of company Hanzo Plc, with an expected return of $8 \%$. The stock return is however risky: $8 \%$ with probability $70 \%,-34 \%$ with probability $15 \%$ and $50 \%$ with probability $15 \%$. Assuming that Beatrix’s reference point is $£ 50,000$, which is the best investment for Beatrix’s savings, stocks or bonds? How the results change if Beatrix does not engage in the probability weighting? Show all relevant calculations.
(c) Provide intuition on how Prospect Theory can explain Equity Premium Puzzle. What would happen to the equity premium if investors become less loss averse? What would happen to equity premium if the investors' evaluation period becomes longer? Explain why.
2. (a) Describe the bias of "conservatism", and discuss whether this bias can be employed to explain stock market anomalies, providing examples from theoretical and/or empirical studies to support your arguments. Describe characteristics of new information when the "conservatism" bias is more likely to emerge as compared to the "representativeness" bias.
(b) Discuss whether CEO's can enhance the value of their companies by selecting corporate policies that exploit market inefficiencies, providing examples from theoretical and/or empirical studies to support your arguments.
(c) You are a divisional manager. Currently you are a member of a committee that is considering two product investments proposed by two other divisional managers, Joe and John. While walking over to the presentations, Joe seems rather arrogant. He mentions that he golfs with the CEO, is a key player in the firm, and that you could really learn from him. In thinking over the projects after the presentations, you find that you are really leaning toward John's proposal even though the projects are quite similar in terms of estimated cash flows and risks. How can you explain this?
3. (a) Describe how emotions (sentiment) can bias expectations and affect stock prices. Do you expect stock markets to be more efficient in optimistic or pessimistic sentiment periods? Discuss, providing examples from theoretical and/or empirical studies to support your arguments.
(b) In their paper, Lee, Shleifer and Thaler (1991) (Investor Sentiment and the Closed-End Fund Puzzle, Journal of Finance) suggest that the closed-end fund discount is a good proxy for investor sentiment. Provide the rationale and intuition for this statement.
(c) Discuss the intuition behind the positive feedback trader model of De Long, Shleifer, Summers and Waldmann (1990) (Positive Feedback Investment Strategies and Destabilizing Rational Speculation, Journal of Finance). Consider noiseless signal case only. Provide necessary analysis.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Thursday 02 June $2016 \quad 1: 30 \mathrm{pm}-3: 30 \mathrm{pm}$

F540
TOPICS IN APPLIED ASSET MANAGEMENT

This paper is in two sections:
Section A: candidates are required to answer one compulsory question.
Section B: Candidates are required to answer two questions.

Section A is weighted at $40 \%$ and section B is weighted at $60 \%$.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator New Cambridge Elementary Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A

1. General question:
(a) Portfolio Construction
i. Derive Mean Variance weights ( without cash/risk free rate) under the sole constraint that the portfolio volatility is targeted at $10 \%$
ii. Derive the Mean Variance weights (without cash) when they are constrained to sum to one, i.e. the "budget constraint" is binding.
iii. Interpret the formula you have derived in part (ii)- under what two conditions would these weights correspond to the Minimum Variance weights?
iv. Consider a portfolio of a set of risky assets and the risk free rate, derive an equation linking the return on this portfolio and the quantity of risk on this portfolio, interpret this equation.
v. Show how this relation leads directly to the CAPM model by considering a portfolio with a proportion $x$ invested in an asset $i$ and $(1-x)$ invested in the tangency portfolio
vi. What insight does this give you as to why the CAPM model may fail empirically?
vii. Consider the design of an active portfolio which is managed against a benchmark -say FTSE100, We are then interested in the active return and the active risk measured relative to the benchmark and we want to design the active mean variance portfolio which has the maximum expected active return for a given level of active risk. We can decompose the portfolio weights into benchmark weights and active weights, $w=b+a$. Because both the benchmark and portfolio weights satisfy the "budget constraint", the active weights must be dollar neutral,i.e. $a^{T} i=0$. In other words the overweights $\left(a_{i}>0\right)$ must perfectly offset or finance the underweights ( $a_{i}<0$ ). Given an expected return forecast, $\mu$ and covariance matrix, $\Sigma$, find an expression for the active weights that optimise the active mean variance trade off.
viii. Comment on the form of these active weights and what can you say about the Mean Variance weights you derived in (ii) above?
(b) Prediction:
i. Prove that the conditional expectation provides the optimal predictor when the loss function is quadratic.
ii. Under what more general conditions does the conditional expectation retain optimality?
(c) Risk Management:
i. Chebyshev's inequality provides some guidance as to how to employ standard deviation as a risk measure compare this with standard rules of thumb which state that 3 sigma event corresponds to $99.7 \%$, and a 2 sigma event ..... etc. for a standard normal distribution
A. - what probability is implied for a 2 sigma event by Chebyshev's inequality and why might you use it?
B. - what are the relative advantages and disadvantages of using standard deviation as a risk measure.
ii. Suppose that the change in the value of a portfolio over a 1 day time period is Normal with a mean of zero and a standard deviation of $\$ 2$ million
A. What is the (a) 1-day $97.5 \% \mathrm{VaR}$, (b) the 5-day $97.5 \%$ VaR and (c) the 5-day $99 \%$ VaR

## SECTION B

2. Problems with Mean Variance:
(a) Explain briefly why the approach to constructing mean variance weights used by De Miguel, Garlappi and Uppal exacerbated numerical errors and under what conditions we might expect to find the Mean Variance weights dominate $\frac{1}{N}$ as shown in their simulation and also their empirical results.
(b) Describe two numerical approaches (i.e not volatility timing nor Parametric Portfolio Policies) that have been employed to counter the numerical instability of mean variance weights.
(c) Briefly explain two ways in which Volatility Timing strategies can be computed and discuss if they can outperform $\frac{1}{N}$
(d) Briefly explain how Parametric Portfolio Polices (PPP) can be constructed and what their advantages are relative to Mean Variance
(e) Explain what is meant by Risk Parity and Maximum Diversified strategies, derive their structure mathematically and discuss how they differ from Mean Variance
3. Predictability:
(a) Why should we not be surprised to see periods of predictability in efficient asset prices- provide a brief mathematical explanation.
(b) Derive a formula that relates the economic value ( Sharpe Ratio) of a conditional forecast of expected returns to the fit in a predictive regression- as used by Campbell and Thompson and if you can relate this to a more general expression for the degree of predictability we should expect to see in an efficient market.
(c) Briefly describe four methods have been employed to extract predictability in conditional predictive regressions by increasing the signal to noise ratio
(d) Provide a brief mathematical explanation why the "Kitchen Sink" regressions of Goyal and Welch perform so badly.
(e) Explain what is meant by Time Series Momentum as opposed to Cross Section Momentum and briefly how they are related and why the returns to a trend following CTA appear to mimic the returns to a Straddle.
4. Quantile and Copula Methods:
(a) Define how a Quantile regression can be estimated and provide a brief example of how it might be important in asset allocation
(b) A loss averse investor might want to employ an asymmetric loss function when when computing the optimal predictor- compute the formula for a solution in the case of a LIN-LIN loss when the investor values a loss of size $x$ as three times more serious than an equivalent profit of size $x$ and explain how this is related to quantile regressions
(c) Explain why the presence of Conditional Heteroskedasticty in returns could imply directional or sign predictability.
(d) Describe what is meant by a Copula Function
(e) Describe the problems of using Correlation as a measure of association or dependence in non-elliptic distributions we often face with financial returns and describe one alternative copula based measure of association that might be better able to describe joint risks in a portfolio.
5. Risk Management:
(a) What is meant by a Coherent risk measure.
(b) Define and compare the properties of Value at Risk, Expected Shortfall and Expectiles as risk measures
(c) Explain what is meant by Filtered Historical Simulation as a means to compute VaR and why it is likely to perform better than simple Historical Simulation.
(d) Suppose that each of two investments has a $4 \%$ chance of loss of $\$ 10$ million, a $2 \%$ chance of a loss of $\$ 1$ million and a $94 \%$ chance of a profit of $\$ 1$ million. They are independent of each other.
i. What is the VaR for one of the investments when the confidence interval is $95 \%$
ii. What is the expected shortfall when the confidence level is $95 \%$
iii. What is the VaR for a portfolio consisting of the two investments when the confidence interval is $95 \%$
iv. What is the expected shortfall for a portfolio consisting of the two investments when the confidence level is $95 \%$
v. Show that, in this example, VaR does not satisfy the sub-additivity condition whereas expected shortfall does?.

## END OF PAPER

## UNIVERSITY OF

 CAMBRIDGEECM9/ECM10/ECM11/MGM3<br>MPhil in Economics<br>MPhil in Economic Research<br>MPhil in Finance and Economics<br>MPhil Finance

Monday 02 May $2016 \quad 2: 00 \mathrm{pm}-4: 00 \mathrm{pm}$

M100
MICROECONOMICS I

Candidates are required to answer three questions

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION <br> Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. There are 5 goods. Good 1 is in category $A$, goods 2 and 3 are in category $B$ and goods 4 and 5 are in category $C$. The consumer must choose a bundle of goods $\mathbf{x} \in \mathbb{R} \times \mathbb{R}_{+}^{4}$, such that consumption of all goods except good 1 must be weakly positive. The consumer's utility function is given by:

$$
u(\mathbf{x})=x_{1}+\phi_{B}\left(x_{2}, x_{3}\right)+\phi_{C}\left(x_{4}, x_{5}\right),
$$

where $\phi_{B}: \mathbb{R}_{+}^{2} \rightarrow \mathbb{R}$ and $\phi_{C}: \mathbb{R}_{+}^{2} \rightarrow \mathbb{R}$. Assume that $p_{1}=1$ throughout the question.
(a) Letting $w$ be the wealth (in pounds sterling) of the consumer and $p_{i}$ be the price of good $i$, formulate the consumer's utility maximization problem.
(b) Show that the Marshallian demand of the consumer for the goods in category $B$ is given by the solution to:

$$
\max _{x_{2} \geq 0, x_{3} \geq 0}\left\{\phi_{B}\left(x_{2}, x_{3}\right)-p_{2} x_{2}-p_{3} x_{3}\right\} .
$$

(c) How does the demand of goods in category $B$ depend on the consumer's wealth and the prices of goods in category $C$ ?
(d) What is the intuition for this result? (Hint: Think about how it relates to the assumption that $\mathbf{x} \in \mathbb{R} \times \mathbb{R}_{+}^{4}$ ?)
(e) Formulate the consumer's expenditure minimization problem.
(f) Show that the consumer's Hicksian demand for goods in categories $B$ and $C$ does not depend on the consumer's wealth or the price of goods in different categories.
(g) When is the utility formulation in this question reasonable/unreasonable?
2. Let higher payouts be strictly preferred to lower payouts. Suppose the prize space (in dollar payouts) is:

$$
\mathcal{X}=\{1,2,4,8,16\}
$$

and consider the lotteries with the following probability distributions over these prizes

$$
p_{1}=\left(0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right), \quad p_{2}=\left(\frac{1}{8}, \frac{1}{8}, \frac{3}{8}, \frac{1}{8}, \frac{1}{4}\right), \quad p_{3}=\left(\frac{1}{8}+\frac{3}{10}, \frac{1}{8}, 0, \frac{1}{8}, \frac{1}{4}+\frac{3}{40}\right) .
$$

(a) Find the expected value of each lottery.
(b) Do all expected utility maximizers prefer lottery $p_{1}$ to $p_{2}$ ?
(c) Do all risk-averse expected utility maximizers prefer lottery $p_{1}$ to $p_{2}$ ?
(d) Do all risk-averse expected utility maximizers prefer lottery $p_{2}$ to $p_{3}$ ?
(e) Do all expected utility maximizer prefer lottery $p_{2}$ to $p_{3}$ ?
(f) Using your previous results argue whether or not all risk-averse expected utility maximizer prefer lottery $p_{1}$ to $p_{3}$ ?
(g) If a lottery $p$ first order stochastically dominates $p^{\prime}$ and $p^{\prime}$ first order stochastically dominates $p^{\prime \prime}$, does that imply that $p$ first order stochastically dominates $p^{\prime \prime}$ ?
3. Consider an economy with two agents 1,2 and two goods $a, b$. Suppose the agents have the utility functions:

$$
\begin{align*}
u^{1}\left(a^{1}, b^{1}\right) & =\min \left(a^{1}, b^{1}\right)  \tag{1}\\
u^{2}\left(a^{2}, b^{2}\right) & =\min \left(a^{2}, b^{2}\right) \tag{2}
\end{align*}
$$

Individual endowments are given by $e^{1}=\left(\frac{1}{4}, \frac{3}{4}\right)$ and $e^{2}=\left(\frac{3}{4}, \frac{1}{4}\right)$.
(a) Using a full page, draw the Edgeworth box for this economy and illustrate the endowment point and at least two indifference curves for agent 1.
(b) In your diagram identify the set of allocations preferred by both agents to their endowments.
(c) Draw the contract curves for the two agents and show that there exists a continuum of Walrasian equilibria.
(d) Normalizing $p_{b}=1$, find the range of $p_{a}$ for which there is a Walrasian equilibrium.
(e) Suppose we perturbed the utility function of agent 2 by setting $u^{2}\left(a^{2}, b^{2}\right)=\min \left(a^{2}, \gamma b^{2}\right)$ for $\gamma$ close to, but not equal to 1 . Normalizing $p_{b}=1$, is there a continuum of prices $p_{a}$ for which there is a Walrasian equilibrium?
4. There is a manager $m$ and two workers 1 and 2 who must jointly complete a task. Each worker chooses an effort level $e_{i}$. The cost of effort for worker $i$ is $c\left(e_{i}\right)=\frac{e_{i}^{2}}{2}$. The manager does not observe the workers' efforts, but does observe the (joint) output $y=e_{i}+e_{j}+\varepsilon$, where $\varepsilon$ is a noise term drawn from a normal distribution with mean 0 , variance $\sigma^{2}$. First the manager offers both workers a contract $w(y)$. Each worker then chooses whether to accept the contract, and if so how much effort to exert. By rejecting the contract the workers guarantee themselves a payoff of 0 . If the contracts are accepted, outputs are then realized and payments made from the manager to the workers in accordance with their contracts. The firm is risk neutral and the worker is risk averse. Their respective utility functions are:

$$
U_{M}=\mathbb{E}\left[y-w_{1}-w_{2}\right] \quad \quad U_{i}=\mathbb{E}\left[w-c\left(e_{i}\right)\right]-(\lambda / 2) \operatorname{Var}[w]
$$

Suppose that the manager offers the workers a linear contract:

$$
w(y)=\alpha+\beta y
$$

where $\beta \geq 0$ (but the manager may choose $\alpha<0$ ).
(a) Suppose worker $i$ has accepted a contract $(\alpha, \beta)$. What level of effort will the worker choose?
(b) If the contract $(\alpha, \beta)$ is offered to the workers, when will worker $i$ accept it?
(c) Using the response of the workers found above, find the contract parameters $(\alpha, \beta)$ the manager will choose.
(d) Comment on whether you think effort would be higher, lower or the same in equilibrium if the manager was constrained to set $\alpha \geq 0$.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Tuesday 17 May 2016 09:00am -11:00am

M110
MICROECONOMICS II

Candidates are required to answer three questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. (a) An object is being auctioned, and there are two risk-neutral bidders. Each bidder $i$ has a private valuation $v_{i}$ for the object. These valuations are independently drawn from a uniform distribution on the interval $[0,1]$. Each bidder knows his own valuation and the distribution which draws the valuations of others.
i. Suppose that the auction is all-pay: each player submits a sealed bid, the bidder who has the highest bid gets the object, and every bidder pays the auctioneer the amount of his bid irrespective of whether he gets the objects or not (if the bidders submit the same bid, each gets the object with equal probability). Show that this game has a symmetric Bayesian Nash equilibrium in which bidder with valuation $x$ bids $x^{2} / 2$. What is the expected revenue of the seller in this case?
ii. Show that the expected revenue of the seller remains the same if the auctioneer were to run a second price auction instead.
(b) Two players play the following simultaneous-move game. There are two states $\omega^{1}$ and $\omega^{2}$. If the state is $\omega^{1}$, then the payoff matrix is

|  | Player 2 |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $A_{2}$ | $B_{2}$ | $C_{2}$ |  |
| Player 1 | $A_{1}$ | $(1,2 \alpha)$ | $(1,0)$ | $(1,3 \alpha)$ |
|  | $B_{1}$ | $(2,2)$ | $(0,0)$ | $(0,3)$ |

If the state is $\omega^{2}$, then the payoff matrix is

$$
\begin{array}{lllll} 
& & \text { Player } & 2 & \\
& & A_{2} & B_{2} & C_{2} \\
\text { Player 1 } & A_{1} & (1,2 \alpha) & (1,3 \alpha) & (1,0) \\
& B_{1} & (2,2) & (0,3) & (0,0)
\end{array}
$$

where $0<\alpha<1 / 2$.
i. Suppose each state is equally likely and neither player knows the state. Calculate the Bayesian Nash equilibria (there may be one or more).
ii. Now suppose that player 2 knows the state, but player 1 believes them to be equally likely. Formulate the game as a (Harsanyi) Bayesian game and compute the Bayesian Nash equilibria (there may be one or more).
iii. Briefly comment on the light your analysis sheds on the value of having more information.
2. (a) Consider the following one-shot Bertrand game. Two identical firms produce an identical product at zero cost. The aggregate market demand curve is given by $6-p$, where $p$ is the price facing the consumers. The two firms simultaneously choose prices once. Suppose further that the firm that charges the lower price gets the entire market and if both charge the same price they share the market equally. Assume that prices can only be quoted in integer units (only prices of $0,1,2, \ldots$ are allowed).
i. Show that the monopoly price is 3 .
ii. Show that both firms charging price 0 or both firms charging price 1 are Nash equilibria of the above game. Is either of the Nash equilibria weakly dominated?
iii. Show that choosing a price $p \geq 6$ is a weakly dominated strategy. Is $p=4$ or $p=5$ weakly dominated?
iv. Show that if the firms are restricted to choosing prices 1,2 , or 3 only, then the monopoly price is a dominated strategy.
v. Show that the set of prices that survive iterative elimination of weakly dominated strategies is unique.
(b) Which of the following statements are true and which are false? Explain your answer carefully (to show a statement is false it suffices to provide a counter-example).
i. Consider the following game

| $1 / 2$ | $A_{2}$ | $B_{2}$ |
| :--- | :--- | :--- |
| $A_{1}$ | 200,30 | 0,0 |
| $B_{1}$ | 0,0 | 30,200 |

The pure strategy profile $\left(B_{1}, A_{2}\right)$ cannot occur if the players are rational and rationality is common knowledge.
ii. A normal form game has a Nash equilibrium in pure strategies if it is finite.
iii. A normal form game has a Nash equilibrium in pure strategies if the strategy sets are convex and compact and the payoff function of each player is quasi-concave in his own strategy.
iv. A normal form game has a Nash equilibrium in pure strategies if the strategy sets are convex and compact and the payoff functions are continuous.
3. (a) Suppose that a game $G=\left(A_{i}, u_{i}\right)_{i=1}^{n}$, where $A_{i}$ and $u_{i}$ refer to the strategy set and the payoff function, respectively, of player $i$. Denote by $G^{T}$ the finitely repeated game which is the game $G$ played $T$ times.
i. Show that if $G$ has a unique Nash equilibrium and if players do not discount the future, then $G^{T}$ has a unique history-independent subgame perfect equilibrium.
ii. Show also that $G^{T}$ might have a history-dependent subgame perfect equilibrium if $G$ has more than one equilibrium (for a complete answer, it is sufficient to construct an example of a game $G$ that has multiple equilibria and is played twice repeatedly).
(b) Suppose that game $G=\left(A_{i}, u_{i}\right)_{i=1}^{n}$ is played infinitely often, and the players use pure strategies and discount the future. Denote the infinitely repeated game by $G^{\infty}$. Let

$$
u(A)=\left\{u^{\prime} \in \mathbb{R}^{n} \mid u_{i}^{\prime}=u_{i}(a) \text { for all } i \text { and for some } a \in A\right\}
$$

be the set of feasible payoffs in $G$. Suppose $G$ has a Nash equilibrium $a^{*} \in A$. Also, denote a minmax strategy profile for $i$ by $m^{i}$, thus $m^{i} \in$ $\arg \min _{a_{-i}} \max _{a_{i}} u_{i}\left(a_{i}, a_{-i}\right)$.
i. Show that if a subgame perfect equilibrium of $G^{\infty}$ induces an average payoff vector $u^{0}=\left(u_{1}^{0}, \ldots, u_{n}^{0}\right) \in \mathbb{R}^{n}$, then $u_{i}^{0} \geq u_{i}\left(m^{i}\right)$ for all $i=$ $1, \ldots, n$.
ii. Show that for any payoff vector $u^{0}=\left(u_{1}^{0}, \ldots, u_{n}^{0}\right) \in u(A)$ such that $u_{i}^{0}>u_{i}\left(a^{*}\right)$ for all $i$, there exists a subgame perfect equilibrium of $G^{\infty}$ that induces an average payoff vector of $u^{0}$ if the players are sufficiently patient.
iii. Suppose that for all $i, j=1, \ldots, n$

$$
u_{i}\left(m^{j}\right)>u_{i}\left(m^{i}\right) \text { for all } i \neq j .
$$

Show that for any payoff vector $u^{0}=\left(u_{1}^{0}, \ldots, u_{n}^{0}\right) \in u(A)$ such that $u_{i}^{0}>u_{i}\left(m^{i}\right)$ for all $i$, there exists a subgame perfect equilibrium of $G^{\infty}$ that induces an average payoff vector of $u^{0}$ if the players are sufficiently patient.


Figure 1: Question 4 (a)
4. (a) Consider the game in Figure 1. Compute all the subgame perfect equilibria (both pure and mixed strategies) of the game.
(b) Define the concept of perfect Bayesian equilibrium for finite extensive form games with imperfect information. (In this question, perfect Bayesian equilibrium refers to the concept of weak perfect Bayesian equilibrium).
(c) Consider the game in Figure 2.
i. Compute all pure strategy perfect Bayesian equilibria of this game.
ii. Discuss the reasonableness of the beliefs of the player for all the perfect Bayesian equilibria obtained in part i. (For example are the beliefs robust to trembles or to forward induction type reasoning?)


Figure 2: Question 4 (c)
[END OF PAPER] CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Wednesday 01 June 2016 1:30pm to $3: 30 \mathrm{pm}$

M120
TOPICS IN ECONOMIC THEORY

Candidates are required to answer three questions

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Two firms, $A$ and B, are engaged in a commercial dispute. Firm B owes money to firm A, but there is a disagreement about the right amount. Firms can either go to court or use mediation as an alternative resolution. Mediation involves a neutral third party that assists firms to work towards a negotiated settlement.
The outcome of the dispute depends on how strong each case is. The table shows how much firm B will have to pay firm A as a function of the strengths of both cases (numbers are expressed in thousand pound sterling).

| $\mathrm{A} \backslash \mathrm{B}$ | strong | weak |
| ---: | :---: | :---: |
| strong | $£ 100$ | $£ 164$ |
| weak | $£ 36$ | $£ 100$ |

Each firm knows whether its case is strong or weak. The probability that the other's case is strong is $\frac{1}{2}$. A mediation plan involves asking each firm to report whether the case is strong $(s)$ or weak $(w)$, and specifies, for each pair of reports $(a, b)$ a payment $P(a, b)$ from B to A made for sure, and a probability $Q(a, b)$ that, despite the payment, the case goes to court anyway with the same outcomes (i.e., in such cases, B makes both a mediation mandated payment and a court mandated payment to A). There is no cost of mediation, but going to court costs a total of $£ 30$ for each firm.
(i) Write down the participation constraints and incentive compatibility constraints that the mediation plan must satisfy.
(ii) Show that there is no mediation plan if the probability of going to court is zero. Interpret the result in a mechanism design framework.
(iii) Assume that the probability of going to court is zero if A reports "strong." Is there now an incentive compatible and individually rational mediation plan?
2. Consider Spence's signalling model in which the worker has productivity equal to $\theta \in$ $\left\{\theta_{L}, \theta_{H}\right\}$, that is, $\theta$ represents the productivity of the $\theta$-type worker. Suppose that $\theta_{H}=10,000$ for the high-productivity worker, and $\theta_{L}=5,000$ for the low-productivity worker. The worker is privately informed about his productivity, and before entering the job market the worker chooses a level of education $e \in[0, \infty)$. The firm cannot observe the worker's productivity but knows that the proportion of high-productivity workers is $\lambda=\frac{4}{5}$. For any belief that the firm may hold about the worker's productivity, assume that the firm offers a wage $w$ equal to the worker's expected productivity. The worker's utility function is $u(\theta, w, e)=w-c(\theta, e)$. The cost of acquiring education level $e$ is given by $c\left(\theta_{H}, e\right)=e^{2}$ for the high-productivity worker and $c\left(\theta_{L}, e\right)=2 e^{2}$ for the low-productivity worker.
(a) Define perfect Bayesian equilibrium in this set-up.
(b) Find all education levels that the high type worker and the low type worker can obtain in a separating perfect Bayesian equilibrium of this game.
(c) Show that the separating equilibria can be Pareto ranked.
(d) Find all education levels that the workers can obtain in a pooling perfect Bayesian equilibrium of this game. Identify the most efficient pooling equilibrium.
(e) Define briefly the Intuitive Criterion. Find an equilibrium that satisfies the Intuitive Criterion.
3. Consider a standard sequential herding model. Each agent $t=1,2, \ldots$ has to make a binary decision, either to invest in a new technology or to reject it. Agents make their decisions sequentially with agent $t$ making his decision at date $t$. There is a fixed cost of adoption equal to $1 / 2$. The return to investment is random and is given by $\theta$ which takes values 1 or 0 with equal probability. Not investing results in zero payoff. Agents maximize their net return. Assume also that when an agent is indifferent between the two options he randomises between them with equal probability. Suppose that when agent $t$ makes his decision, he observes the actions chosen by all the previous agents $i=1,2, \ldots, t-1$ and a private symmetrical binary signal $x \in\{H, L\}$ about $\theta$ with precision $q>1 / 2$; thus $\operatorname{prob}(x=H \mid \theta=1)=\operatorname{prob}(x=L \mid \theta=0)=q$.
(a) Define an informational cascade for this model.
(b) Show that there exist $\mu^{*}$ and $\mu^{* *}$ such that there is no cascade at period $t$ if and only if the public belief at $t, \mu_{t}$, is such that $\mu^{*}<\mu_{t}<\mu^{* *}$.
(c) Show that the probability of a cascade on investing (Up Cascade) after two periods is $\frac{1-q+q^{2}}{2}$.
(d) What is the probability that there is a cascade on investment by the end of period $t$, for an even number $t$ ?
(e) Briefly outline the difficulties of applying a simple sequential herding model of the above type to explain cascades and herding amongst traders in an efficient financial market.
4. Consider the set of 2-person bargaining problems $\mathcal{B}$ where the set of possible utility agreements is some subset of of $\mathbb{R}_{+}^{2}$ and disagreement results in a payoff vector $d=$ $\left(d_{1}, d_{2}\right) \in \mathbb{R}_{+}^{2}$. Thus $\mathcal{B}=\left\{(U, d) \mid U \subseteq \mathbb{R}_{+}^{2}\right.$ and $\left.d \in U\right\}$. Let $f: \mathcal{B} \rightarrow \mathbb{R}^{2}$ be the bargainining solution. Thus, $f(U, d) \in U$ describes the utility vector the players will receive if $U \subseteq \mathbb{R}_{+}^{2}$ is the set of possible utility agreements and $d$ is the disagreement payoff vector.
(a) i. What does it mean to say that $f$ satisfies Symmetry, Pareto, Independence of utility units and Independence of irrelevant alternatives axioms?
ii. Suppose that $f$ satisfies the following axioms: symmetry, Pareto and Independence of utility units. Let $U^{\prime}=\left\{\left(u_{1}, u_{2}\right) \in \mathbb{R}_{+}^{2} \mid a u_{1}+b u_{2} \leq c\right\}$ for some constant positive real numbers $a, b$ and $c$. Show that $f\left(U^{\prime}, d\right)=\left(\frac{c}{2 a}, \frac{c}{2 b}\right)$ if $d=0$.
iii. Suppose that $f$ satisfies the following axioms: symmetry, Pareto and Independent of irrelvant alternatives. Let $U^{\prime \prime}=\left\{\left(u_{1}, u_{2}\right) \in \mathbb{R}_{+}^{2} \mid u_{1}+u_{2} \leq c\right.$ and $\left.\frac{u_{1}}{2}+u_{2} \leq \frac{3 c}{4}\right\}$ for some constant positive real number $c$. Show that $f\left(U^{\prime \prime}, d\right)=\left(\frac{c}{2}, \frac{c}{2}\right)$ if $d=0$.
(b) Consider Rubinstein and Wolinsky (Review of Economic Studies 1990) random matching model of a market with two identical buyers $b_{1}$ and $b_{2}$ and a single seller. The seller has one unit of an indivisible good. Each buyer wants to buy at most one unit of the good. The valuations of the buyers and the seller for one unit of the good are one and zero respectively. Time is discrete and each player has a discount factor $\delta=1$. At each period $t \geq 1$, the agents in the market are randomly matched in pairs of one seller and one buyer (all possible matches are equally likely). One member of each matched pair is then randomly chosen (with probability $1 / 2$ ) to propose a price $p \in[0,1]$. Then the other agent accepts or rejects the offer. If a proposal is accepted by the responder, the parties implement it and leave the market and the game ends. Rejection dissolves the match, in which case the agents proceed to the next period. Assume perfect observability of all the past plays.
i. Describe the competitive price. What is the monopoly price?
ii. Sketch a proof showing for any price $p^{*} \in[0,1]$ and any buyer $b^{*}$ there exists a subgame perfect equilibrium in which the seller sells one unit of the good to buyer $b^{*}$ for a price $p^{*}$.

## END OF PAPER

CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Wednesday 25 May $2016 \quad$ 1:30pm to $3: 30 \mathrm{pm}$

M130
APPLIED MICROECONOMICS

This paper is in two sections. Candidates are required to answer four in Section A; and the one compulsory question in Section B.

Questions within Section A carry equal weight. Each section carries equal weighting.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Section A

A1. What is meant by the "True Cost of Living Index"? When will the True Cost of Living Index depend on the level of utility?

A2. If a government preannounces a tax cut, would this improve or weaken the effect of the tax cut as a stimulus mechanism?

A3. Explain the Diamond-Mirrlees Production Effciency result. Why is the ability of the government to set producer and consumer prices separately necessary for this result?

A4. How will an unexpected increase in an individual's income change the individual's consumption over their lifetime?

A5. What should the marginal rate of tax be on the highest income band?

A6. In estimating the effect of disability insurance on labour supply, why can an econometrician not simply regress hours worked on the amout of disabiltiy insurance received? What could the econometrician do to resolve the issues?

## Section B

B1. A researcher estimates a demand system by OLS with each budget share regression given by:

$$
w_{i h}=\alpha_{i 0}+\alpha_{i a d} \text { adults }_{h}+\alpha_{i k i d s} k i d s_{h}+\sum_{j=1}^{7} \gamma_{i j} \ln p_{j, h}+\beta_{i} \ln \frac{x_{h}}{P_{h}}+u_{i h}
$$

where $w_{i h}$ is the budget share on good i for household h ; the price index $P_{h}$ is the Stone price index for household h. There are seven goods in this demand system: food at home (fath); food in restaurants (rest); transport (tranall); services (serv); recreation (recr); alcohol and tobacco (vice); and clothing (clth). The coefficient estimates from each OLS regression are given in the columns in Figure 1.
(a) What are the advantages of estimating this equation using budget shares, $w_{i h}$, on the left hand side rather than an equation where the left hand side variable is the the quantity of the good, $q_{i} h$ (where $w_{i h}=\frac{q_{i h} p_{i h}}{x_{h}}$ ), i.e.:

$$
q_{i h}=\alpha_{i 0}+\alpha_{i a d} a^{2} u l t s_{h}+\alpha_{i k i d s} k i d s_{h}+\sum_{j=1}^{7} \gamma_{i j} \ln p_{j, h}+\beta_{i} \ln \frac{x_{h}}{P_{h}}+u_{i h}
$$

(b) Calculate the Hicksian, Marshallian and Income elasticities for vices (vice), transport (tranall) and food at home (fath). The budget share on vices is $8 \%$, on transport is $18 \%$ and on food at home is $26 \%$. Which good should be subject to the highest tax rate?
(c) The government is considering raising revenue from taxing these three goods. What assumptions are necessary for the tax rate on each good to depend only on their own price elasticity? Do these assumptions hold with these estimates?
(d) What would be the welfare loss to the individual of a $5 \%$ increase in the price of each good of these three goods?
(e) One of the purchasers of "tranall" (transport goods) is a firm that uses transport services to bring other goods to consumers. What rate of tax should these firms pay on the transport costs?
(f) The government realises that some of the workers for this firm spend their wages on food at home. Should this fact affect the tax rate on food at home?

| Variable | BS fath | BS rest | BS tranall | BS serv | BS recr | $B S$ vice | BS clth |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No of adults | 0.079 | -0.040 | 0.022 | -0.014 | -0.026 | -0.019 | -0.001 |
|  | (0.002) | (0.0015) | (0.002) | (0.002) | (0.002) | (0.002) | (0.001) |
| No of kids | 0.053 | -0.020 | -0.018 | 0.019 | -0.003 | -0.015 | -0.016 |
|  | (0.0008) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| $\log p$ (fath) | -0.086 | 0.075 | -0.120 | 0.064 | 0.032 | -0.039 | 0.073 |
|  | (0.026) | (0.019) | (0.026) | (0.021) | (0.021) | (0.023) | (0.018) |
| $\log p$ (rest) | -0.014 | -0.021 | 0.150 | -0.065 | 0.085 | -0.069 | -0.066 |
|  | (0.022) | (0.012) | (0.022) | (0.018) | (0.018) | (0.020) | (0.015) |
| $\log p$ (tranall) | 0.010 | 0.009 | 0.006 | -0.016 | -0.009 | -0.004 | 0.004 |
|  | (0.0095) | (0.007) | (0.003) | (0.008) | (0.008) | (0.008) | (0.007) |
| $\log \mathrm{p}($ serv) | 0.029 | 0.002 | 0.017 | -0.028 | -0.089 | 0.089 | -0.020 |
|  | (0.026) | (0.019) | (0.027) | (0.015) | (0.021) | (0.023) | (0.018) |
| $\log p(r e c r)$ | 0.066 | -0.064 | -0.013 | 0.087 | 0.014 | -0.079 | -0.010 |
|  | (0.031) | (0.023) | (0.031) | (0.025) | (0.025) | (0.028) | (0.021) |
| $\log p($ vice $)$ | -0.008 | 0.007 | -0.029 | -0.019 | -0.035 | 0.069 | 0.016 |
|  | (0.010) | (0.008) | (0.010) | (0.008) | (0.008) | (0.009) | (0.007) |
| $\log p$ (clth) | -0.093 | -0.003 | 0.006 | 0.070 | 0.066 | 0.000 | -0.046 |
|  | (0.023) | (0.017) | (0.024) | (0.019) | (0.019) | (0.021) | (0.016) |
| log real expend | -0.160 | 0.050 | 0.022 | -0.011 | 0.046 | 0.017 | 0.037 |
|  | (0.002) | (0.001) | (0.002) | (0.001) | (0.001) | (0.002) | (0.001) |
| Constant | 1.624 | -0.306 | -0.043 | 0.283 | -0.284 | -0.030 | -0.244 |
|  | (0.016) | (0.012) | (0.016) | (0.013) | (0.013) | (0.014) | (0.011) |

Standard errors are given in brackets.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Thursday 26 May 2016 1:30pm-3:30pm

M140
BEHAVIOURAL ECONOMICS

Candidates are required to answer all four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. (a) Why might an individual find it difficult to save for retirement?
(b) Describe a formal model that captures these difficulties.
(c) To what extent can the difficulties be overcome?
2. (a) Describe an experimental procedure that can be used to identify the utility function of a monkey.
(b) In what sense is this a utility function?
(c) Can it be used to make out-of-sample predictions? Justify your answer.
3. First consider the following experiment of Kahneman and Tversky:

Imagine that you face the following pair of concurrent decisions. First examine both Decision (i) and Decision (ii), then indicate the option that you prefer in each Decision.
Decision (i) Choose between:
A. a sure gain of $\$ 240$
B. $25 \%$ chance to gain $\$ 1,000$, and $75 \%$ chance to gain nothing

Decision (ii) Choose between:
A. a sure loss of $\$ 750$
B. $75 \%$ chance to lose $\$ 1,000$, and $25 \%$ chance to lose nothing

Now answer the following questions:
(a) Describe the results of the experiment.
(b) How did Kahneman and Tversky explain the results?
(c) Give a formal definition of framing.
(d) Explain how framing leads to biases in insurance purchase.
(e) Give one example of how framing matters in labour markets.
4. (a) List the four components of behavioural design according to Datta and Mullainathan (2012).
(b) Datta and Mullainathan (2012) identify 7 principles to follow to design an intervention that takes into account findings from behavioural economics. List at least 5 of these principles. For each principle:
(i) clearly state the issue/bias that it addresses;
(ii) give an example of a practical solution to the issue/bias that can be implemented in an intervention.
(c) Pick two of the biases you listed in (b)(i). For each of these two biases, present an experiment that illustrates the existence of the bias. Explain carefully the design and the results of each experiment.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

## 

M150
Economics of Networks

Candidates are required to answer three out of four questions.
Each question carries equal weight

Write your candidate number (not your name) on the cover of each booklet.

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If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1: Consider network formation with $n$ individuals. Suppose that two players $i$ and $j$ can form a link if they both pay a cost $c>0$. The network created by this bilateral linking is denoted by $g$. The payoffs to player $i$ under network $g$ are

$$
\begin{equation*}
\Pi_{i}(g)=1+\sum_{j \neq i} \delta^{d(i, j ; g)}-\eta_{i}(g) c, \tag{1}
\end{equation*}
$$

where $d(i, j ; g)$ is (geodesic) distance between $i$ and $j$ in network $g, \eta_{i}(g)$ is the number of links formed by $i$, and $\delta \in(0,1)$ is decay factor.
(A) Define a pairwise stable network. Provide the range of parameter values, $c$ and $\delta$, for which the empty, the complete and star network are pairwise stable.
(B) An efficient network maximizes sum of players' payoffs across all networks. Provide a characterization of efficient networks.
(C) Now consider an alternative link formation protocol: an individual can unilaterally create a link with any other player by paying a cost $c>0$ (per link). Let $g$ be the network thus formed. Define $\hat{g}_{i j}=\max \left\{g_{i j}, g_{j i}\right\}$ and let $\hat{g}$ be the corresponding undirected network. Suppose that the payoffs are given by

$$
\begin{equation*}
\Pi_{i}(g)=1+\sum_{j \neq i} \delta^{d(i, j ; \hat{g})}-\eta_{i}(g) c, \tag{2}
\end{equation*}
$$

where $d(i, j ; \hat{g})$ is the (geodesic) distance between $i$ an $j$ in network $\hat{g}, \eta_{i}(g)$ is the number of links formed by $i$, and $\delta \in(0,1)$ is the decay factor.
(i) Suppose decay is small ( $\delta$ is close to 1 ). Give three examples of Nash equilibrium networks for $c<1$.
(ii) Show that if $c>1$ then any non-empty equilibrium network is a peripherysponsored star.
(D) Comment on the role of link formation protocol in shaping network architecture.

Question 2: There are $n \geq 2$ firms located in a collaboration network, $g$, facing a linear demand, given by $Q=1-p$. The initial marginal cost of production in a firm is $\bar{c}$ and assume that $n \bar{c}<1$. Each firm $i$ chooses a level of research effort given by $s_{i}$. The marginal costs of production of a firm $i$, in an (undirected) network $g$, facing a profile of efforts $s=\left(s_{1}, s_{2}, . ., s_{n}\right)$, are given by:

$$
\begin{equation*}
c_{i}(s \mid g)=\bar{c}-\left(s_{i}+\sum_{j \in N_{i}(g)} s_{j}\right), \tag{3}
\end{equation*}
$$

where $N_{i}(g)$ refers to the neighbors in network $g$. Let $\eta_{i}(g)=\left|N_{i}(g)\right|$. Research effort is costly: $Z\left(s_{i}\right)=\alpha s_{i}^{2} / 2$, where $\alpha>0$. Given costs $c=\left\{c_{1}, c_{2}, \ldots c_{n}\right\}$, firms choose quantities $\left(\left\{q_{i}\right\}_{i \in N}\right)$, with $Q=\sum_{i \in N} q_{i}$.
(A) Given costs $c=\left(c_{1}, c_{2}, . ., c_{n}\right)$, compute the Cournot equilibrium quantities and profits.
(B) Taken as given this market equilibrium, compute the payoffs of firm $i$, located in network $g$ given the research efforts, $s$.
(C) Define positive and negative externalities and strategic complements and strategic substitutes. Show that this game exhibits a positive externality across neighbors and negative externality across non-neighbors actions. And show that actions of neighbors are strategic complements, while the actions of non-neighbors are strategic substitutes.
(D) Show that in a regular network, research effort is decreasing in degree.

Question 3: There is a single Source and a single Destination and a collection of $n$ intermediary traders in a connected network. The value of exchange is 1 . The network of traders and the valuations are common knowledge. Traders post prices simultaneously; they have zero costs. The Source and Destination compute the cost of every path between them. They pick the lowest cost path (if it costs less than 1) and randomize uniformly across least costs paths; if every path costs more than 1 , then there is no trade. Source and Destination divide the residual surplus, after paying for the cost of the path.
(A) Formalize this economic exchange as a game of simultaneous moves.
(B) An equilibrium outcome is said to be efficient if trade takes place with probability 1. Prove that for any network there exists an efficient pure strategy Nash equilibrium.
(C) Show that in any equilibrium with trade, the distribution of surplus is extremal: intermediate traders extract either all or none of the surplus.

Question 4: This question studies the role of social networks in labor markets. Consider a 2 period model with the following features. There is an equal measure of 1 ability and 0 ability workers. Workers know their ability while firms do not know it. In each period a firm hires one worker. The output of a firm is equal to the ability of the worker who works for the firm. In period 1, all firms have same average quality of worker, and pay wages corresponding to this average. During period 1, a firm learns the ability of its worker. At the start of period 2 , it has a choice between asking the period 1 worker for the name of contact and offering a referral wage or simply posting a wage in the market. Competition between firms means that posted wages equal expected ability of workers. And that expected profits of firms are equal to zero over two periods. Each period 1 worker knows at most one period 2 worker, possessing a social tie with probability $r \in[0,1]$. Conditional upon holding a tie, a period 1 worker knows a period 2 worker of his own type with probability $\alpha>1 / 2$. The social structure is thus defined by $r$ and $\alpha$.
(A) Explain why a firm will offer a referral wages in period 2 only if its current worker is of high ability. Then show that the optimal referral wage offer must involve randomization.
(B) Show that period 2 wages are characterized by a lemons effect: the market wage is below $1 / 2$.
(C) Social connections create inequality in the labour market. Comment.

## End of Paper

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Friday 20 May $2016 \quad$ 1:30pm - 3:30pm

M180
LABOUR: SEARCH, MATCHING AND AGGLOMERATION

Candidates are required to answer three questions. Each question carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. The internal structure of a city

Consider the model of Lucas and Rossi-Hansberg (2002) on the internal structure of a city. Following their notation, let $\delta$ be the exponential decay rate of knowledge spill overs and let $\kappa$ be the exponential decay rate of labour input, both with respect to the physical distance between two locations. The production technology per unit of land satisfies:

$$
x(r)=z(r)^{\gamma} n(r)^{\alpha}
$$

where $x(r)$ is output per unit of land, $z(r)$ measures the knowledge spill over of people working in the proximity and $n(r)$ is the number of workers per unit of land, both at locations at a distance $r$ from the city centre; $\alpha$ is the labour share in production; $\gamma$ is the knowledge spill over elasticity. Let $W(r)$ be wages at location $r$. $W(r)$ is gross pay at the firm when $r$ is the job location. $W(r)$ is the net pay after paying the commuting cost when $r$ is the home location. Let $\Pi(r)$ be the profits/land rent per unit of land.
(a) Discuss the slope of $W(r)$ for 3 types of locations: commercial, residential and mixed.
(b) Derive $W(r)$ as a function of $n(r)$ and $z(r)$ and return $\Pi(r)$ per unit of land as a function of $W(r)$ and $z(r)$, both for locations with commercial activity.
(c) Claim: the decay of knowledge spill overs with respect to the distance to the city centre is smaller than $\delta$

$$
-\frac{d[\ln z(r)]}{d r}<\delta .
$$

Provide a verbal proof for this claim.
(d) Suppose $\gamma \delta<\alpha \kappa$. What is the implication of this inequality for spatial organization of commercial activity?
(e) The commercial centra of Los Angeles and Houston are spread out over a wide area. Both cities do not have a subway system. San Francisco and Boston have concentrated Central Business Districts and an extensive subway system used for commuting. Can you related this difference in outcome to your conclusion under d.?
2. Agglomeration of human capital.

Lucas and Rossi-Hansberg (2002) show how agglomeration benefits drive land rents. Desmet and Rossi-Hansberg $(2009,2014)$ show that city size might be mean diverting, deviating from Gibrats' law. Teulings and Van Rens (2008) report that the short run private and social return to human capital measured at the country level are about equal at a level consistent with the standard human capital model (that is: about equal to return to physical capital), while the long run social return to human capital exceeds the private return by a wide margin. Gennaioli, La Porta, Lopez-de-Silanes, and Shleifer (2013) report that within each country, human capital agglomerates in a small number of regions and that the social return to human capital exceeds by far the private return to human capital. Their evidence is based on cross section data. Their model accounts for the role of house prices.
(a) Provide the outlines of a model that explains these phenomena. Pay attention to the drivers of the spatial equilibrium of the supply and demand for labour and to the dynamics of the model, the role of wages, knowledge spill overs and land rents/house prices.
(b) Is the outcome efficient? Explain why.
(c) The present value of knowledge spill overs is captured by land rents. Explain.
3. Minimum wages
(a) Give a summary of some stylized evidence on the effect of minimum wages on employment and the wage distribution.
We seek to explain this evidence by two alternative models, a neoclassical model with heterogeneity on the supply and demand side of the market. On the demand side, tasks differ in their complexity $z$. Tasks are combined in the final consumption good in fixed proportions. Hence, the distribution of $z$ among labour demand is fixed. Prices for each task clear the market for tasks. On the supply side, workers differs by their skill level $x$. The distribution of $x$ among labour supply is also fixed. The production of tasks occurs by a constant returns to scale technology with labour as the only input. The productivity of a worker with skill $x$ in a task of complexity $z$ reads:

$$
\exp \left(-e^{\gamma(z-x)}\right)
$$

(b) Show that this production function exhibits absolute advantage of better skilled workers in any task and comparative advantage of better skilled workers in more complex tasks. What are the implications of both forms advantage of better skilled workers?
(c) Let $W(x)$ be the equilibrium locus of wages for workers with skill level $x$. Derive the return to a unit of skill in market equilibrium, that is: $d[\ln W(x)] / d x$, as a function of the market allocation of workers-types to tasks $z(x)$.
(d) Document graphically how the assignment of workers to tasks changes due to the introduction of a minimum wage. Explain the graph.
(e) Use this graph and the result derived under 3. to show graphically how the locus of $\ln W(x)$ changes by the introduction of a minimum wage. Explain the changes in both the slope and in the level of $W(x)$. Explain how this contributes to an explanation of the stylized facts mentioned under 1., both for employment and the wage distribution.
The model of Bontemps, Robin, and Van den Berg (1999) gives an alternative explanation in terms of search frictions. Workers differ by their value of leisure in that model, while firms differ by their productivity. The productivity of the least productive firm is higher than the value of leisure of the worker with the lowest value of leisure but lower than the value of leisure of the worker with the highest value of leisure.
(f) Explain the model and discuss the shape of the wage function (no more than 200 words). How does the introduction of a minimum wage affect wages and employment. How would the minimum wage affect the employment rate of workers with different values of leisure. How would it affect employment in firms with different levels of productivity.
(g) What is the key difference in the predictions of both models?
4. Efficiency in job search models

Consider a general class of job search models where firms post vacancies at a fixed cost and where workers (either employed or unemployed) look for jobs. There is free entry of firms. The productivity of a job is known to the worker and the firm at the start of the job. Moreover, this productivity is constant over the duration of the job. The cost of job change is zero. Provide intuitions, also when the answer is unclear.
(a) The efficiency of this type of economies depends on two issues, vacancy creation and job transition. Describe both issues.
(b) Consider the model of Burdett and Mortensen (1998), where workers and firms(=jobs) are ex ante homogeneous and where firms post a fixed wage before entering the market. Consider the efficiency of the market equilibrium along both dimensions discussed under a.. Are both issue relevant in this model?
(c) Consider a slightly more general model than under b., where firms differ by their productivity. They can choose their productivity on entry. However, the higher their productivity, the higher the cost of creating a vacancy. What difference does this make for your answer.
(d) Postel-Vinay and Robin (2002) consider a largely similar model as under c., however with a different mechanism for wage setting. When an employed worker gets an outside job offer, his current employer and the firm making the outside offer engage in Bertrand competition. What is the implication of this assumption for both efficiency issues? What would be the social cost of closing down the labour market in this model. Explain.
(e) Finally, consider the model by Burdett and Coles (2003), where firms post wage-tenure contracts rather than a fixed wage, as under b. and c.. A worker's starting wage is low, but gets gradually increased over the duration of the job. Consider the efficiency of this set up on the issue of job transition.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research
Wednesday 04 May $2016 \quad 2: 00 \mathrm{pm}-4: 00 \mathrm{pm}$

M200
MACROECONOMICS I

The paper is in two sections: You should answer all four questions in Section A and one question from Section B. Each Section carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

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Calculator - students are permitted to bring an approved calculator

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## SECTION A

## A. 1 Solow Growth Model with Land

Consider the following continuous-time Solow model with land. The production function is:

$$
Y(t)=K(t)^{\alpha} T(t)^{\beta}(A(t) L(t))^{1-\alpha-\beta}
$$

where $K(t)$ denotes the capital stock, which depreciates at constant rate $\delta \in(0,1)$; $L(t)$ is labour, which grows at rate $n ; A(t)$ corresponds to a productivity factor, which grows at rate $g$; and $T(t)$ is land, which has a fixed supply of $\bar{T}<\infty$. The parameters $\alpha$ and $\beta$ satisfy $\alpha, \beta \in(0,1)$ and $\alpha+\beta<1$. The equation of motion of the capital stock is:

$$
\dot{K}(t)=I(t)-\delta K(t),
$$

where $I(t)$ corresponds to investment, and the initial capital stock, $K(0)>0$, is given. Agents save a fraction $s \in(0,1)$ of income. The economy is closed, which implies that investment equals saving.
Derive the long-run growth rate in capital per worker along the balanced growth path. Explain whether capital per worker has a positive growth rate along the balanced growth path.
Suppose that there is perfect competition in the market for land such that its rental price equals its marginal product. Derive the growth rate of the rental price of land along the balanced growth path. What is the land share in total income? Explain.

## A. 2 Investment and $q$-theory

Suppose that a firm facing a gross real interest rate $1+r>0$ produces output $Y_{t}$ according to the production function $Y_{t}=A K_{t}^{\alpha}$, where $A>0$ is a productivity parameter and $\alpha \in(0,1)$. The firm's objective is to maximise the present discounted value of profits over an infinite horizon. In addition to the cost of investment $I_{t}$, the firm faces adjustment costs equal to $\frac{\chi I_{t}^{2}}{2 K_{t}}($ with $\chi>0)$ in any period in which it invests (or disinvests) $I_{t}$. Capital $K_{t}$ accumulates according to :

$$
K_{t+1}=K_{t}+I_{t}
$$

where $K_{0}>0$ is given.
Set up the firm's optimisation problem and find the first-order conditions characterising efficient investment. Define marginal- $q$ and explain the difference between Tobin's average $Q$ and marginal- $q$.

## A. 3 Equilibrium with Imperfect Competition

Suppose a representative firm $i \in[0,1]$ maximizes real profits

$$
\Pi_{i}=\frac{P_{i}}{P} Q_{i}-\frac{W}{P} L_{i}
$$

where $Q_{i}$ denotes the production of good $i, P_{i}$ the price of good $i, P \equiv \int_{0}^{1} P_{i} d i$ the aggregate price level, $W$ the nominal wage, and $L_{i}$ the firm's labour input. The production technology is described by

$$
Q_{i}=A L_{i}
$$

where $A$ denotes productivity. The firm operates under monopolistic competition and faces demand

$$
Q_{i}=Y\left(\frac{P_{i}}{P}\right)^{-\eta}
$$

where $Y \equiv \int_{0}^{1} Q_{i} d i$ denotes aggregate output and the parameter $\eta>1$. In the labour market, the firm faces the following real wage function:

$$
\frac{W}{P}=B L^{\beta}
$$

where $L \equiv \int_{0}^{1} L_{i} d i$ denotes aggregate employment, and $B$ and $\beta$ are positive parameters. The aggregate demand relation is described by

$$
Y=\frac{M}{P}
$$

where $M$ denotes aggregate demand.
Derive the level of the real wage $\frac{W}{P}$, aggregate employment $L$ and aggregate output $Y$ in the flexible price equilibrium. Explain how they depend on $A$.

## A. 4 Dornbusch Model with Negative Aggregate Demand Shock

Consider the following Dornbusch model with short run dynamics for the (log) nominal exchange rate $s$ (defined as the domestic price of foreign currency) and the (log) aggregate price level $p$ :

$$
\begin{aligned}
& \dot{s}=(s-\bar{s})+(p-\bar{p}) \\
& \dot{p}=\frac{1}{4}(s-\bar{s})-\frac{1}{4}(p-\bar{p})
\end{aligned}
$$

where the steady state (indicated by an upper bar) equals $\bar{s}=\bar{q}+\bar{k}$ and $\bar{p}=\bar{p}^{*}+\bar{k}$, with (log) real exchange rate $\bar{q}=2(\bar{y}-\bar{u})$ and macroeconomic fundamentals $\bar{k} \equiv$ $\left(\bar{m}-\bar{m}^{*}\right)-\left(\bar{y}-\bar{y}^{*}\right)$. In addition, $\bar{m}$ denotes the money supply, $\bar{y}$ the natural rate of output and $\bar{u}$ is a measure of exogenous aggregate demand. Foreign variables are indicated by an asterisk.
Carefully illustrate the dynamics of this model in a phase diagram. Explain the effects of a sudden permanent decrease in aggregate demand $\bar{u}$ on the nominal exchange rate $s$, real exchange rate $q$ and aggregate price level $p$ over time.

## SECTION B

## B. 1 Consumer Behaviour

Consider a consumer whose goal is to maximise her lifetime utility:

$$
U=\sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right),
$$

where $c_{t}$ corresponds to consumption in period $t$, and $\beta \in(0,1)$ is the subjective discount factor. The one-period utility function $u(c)$ is twice continuously differentiable and strictly concave. Assume that this consumer can borrow or lend at a given real interest rate $r \in(0,1)$, and that the consumer faces a deterministic sequence of income $\left\{y_{t}\right\}_{t=0}^{\infty}$. Let $b_{0}<\infty$ be the asset position of this consumer in period 0 . The one-period budget constraint of the consumer is

$$
c_{t}+b_{t+1}=y_{t}+(1+r) b_{t} .
$$

There is no possibility of a Ponzi scheme.
(a) Define the optimisation problem of this consumer, take the first-order conditions and derive the Euler equation. Comment on your results.
(b) Assume that $u(c)=\frac{c^{1-\sigma}-1}{1-\sigma}$ with $\sigma>0$ and $\frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}<1$.

Derive the intertemporal budget constraint and use the Euler equation to solve for the optimal level of consumption in period $t=0$. Comment on your results.

Now assume instead that

$$
u(c)=\frac{\left(\frac{c}{c}\right)^{1-\sigma}-1}{1-\sigma}
$$

where $\bar{c}$ corresponds to average consumption in society, which the consumer takes as given, with $0<\bar{c}<\infty$ and $\sigma>0$.
(c) What is the intuition behind this utility function? Derive the Euler equation for this problem and comment on your results.
(d) Assume that $\bar{c}_{t+1}=g \bar{c}_{t}$, where $g \geq 1$ and $\bar{c}_{0}=1$. Explain how the growth rate $g$ of average consumption in society affects the consumption growth $\frac{c_{t+1}}{c_{t}}$ of this consumer between period $t$ and $t+1$. Solve for the optimal level of consumption $c_{t}$ in period $t=0$. Comment on your results.

## B. 2 Optimal Tax Rates and Seignorage

Suppose the government sets the income tax $\tau_{t}$ each period to minimize the expected value of the following loss function:

$$
L_{t}=\sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} \frac{1}{2} \kappa \tau_{s}^{2} Y_{s}
$$

where $Y_{s}$ denotes aggregate output in period $s, r$ is the constant real interest rate, and $\kappa$ is a positive parameter. The government's budget constraint equals

$$
G_{s}+r B_{s}=\tau_{s} Y_{s}+S_{s}+D_{s}
$$

where $G_{s}$ denotes government purchases in period $s, B_{s}$ government debt at the start of period $s, S_{s}$ seignorage in period $s$, and $D_{s} \equiv B_{s+1}-B_{s}$ the government's budget deficit. Assume money is superneutral and $Y_{s}=\bar{Y}$ for $s=t, t+1, \ldots$. In addition, $B_{t}>0$ is given and $\lim _{T \rightarrow \infty} \frac{1}{(1+r)^{T+1-t}} B_{T+1} \leq 0$.
(a) Give an economic interpretation for the latter inequality, and derive the intertemporal budget constraint of the government.
(b) Assume that initially, $G_{s}=\bar{G}$ and $S_{s}=\bar{S}$ for $s=t, t+1, \ldots$ Derive the optimal tax rate $\tau_{t}$ and the government's primary budget deficit $\tilde{D}_{t} \equiv$ $D_{t}-r B_{t}$. Give an intuitive explanation of the results.
(c) Suppose that a rise in the growth rate of the money supply due to 'quantitative easing' leads to a permanent increase in seignorage $\bar{S}$ by $\Delta S$ in period $t$. Carefully explain how this would affect the income $\operatorname{tax} \tau_{s}$, the primary deficit $\tilde{D}_{s}$ and the level of government debt $B_{s}$ over time.
(d) Now suppose instead that 'quantitative easing' leads to a temporary increase in seignorage $\bar{S}$ by $\Delta S$ in period $t$ for $T$ periods. Carefully explain how this would affect the income tax $\tau_{s}$, the primary deficit $\tilde{D}_{s}$ and the level of government debt $B_{s}$ over time. Compare the results to part (c).

## END OF PAPER

ECM9/ECM10<br>MPhil in Economics<br>MPhil in Economic Research

Tuesday 31 May $2016 \quad 1: 30 \mathrm{pm}-3: 30 \mathrm{pm}$

M210
MACROECONOMICS II

This paper is in two sections. Section A: Candidates are required to answer three questions. Section B: Candidates are required to answer one compulsory question.

Each section carries equal weighting.

Write your candidate number (not your name) on the cover of each booklet.

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## STATIONERY REQUIREMENTS

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## SECTION A

## A. 1 New Keynesian Model

Suppose the representative consumer maximizes the expected value of lifetime utility

$$
U_{t}=\sum_{s=t}^{\infty} e^{-\rho(s-t)} \frac{C_{s}^{1-\sigma}-1}{1-\sigma}
$$

where $C_{t}$ denotes consumption in period $t, \rho$ the subjective intertemporal discount rate $(\rho>0)$, and $\sigma$ the coefficient of relative risk aversion $(\sigma>0)$. The budget constraint of the representative consumer is:

$$
C_{t}+T_{t}+Q_{t} B_{t}=B_{t-1}+Y_{t}
$$

where $T_{t}$ denotes lump-sum taxes in period $t, B_{t}$ one-period riskless real discount bonds purchased in period $t$ at price $Q_{t}$, and $Y_{t}$ the output endowment in period $t$, which is stochastic. The goods market equilibrium condition is

$$
Y_{t}=C_{t}+G_{t}
$$

where $G_{t}$ denotes government purchases in period $t$, which are stochastic and funded by lump-sum taxes, so $T_{t}=G_{t}$.
Let $y_{t} \equiv \ln Y_{t}, c_{t} \equiv \ln C_{t}$ and $\gamma_{t} \equiv-\ln \left(1-G_{t} / Y_{t}\right)$. Show that $y_{t}=c_{t}+\gamma_{t}$ and derive the dynamic optimising IS equation relating $y_{t}$ to $\gamma_{t}$ and $r_{t} \equiv$ $-\ln Q_{t}$. Give a brief economic interpretation of the result.

## A. 2 Search Model with Nonhuman Wealth

An agent receives every period an offer to work forever at a wage $w$, where $w$ is drawn from the distribution $F(w)$ with support $[0, B]$ where $B$ is finite. Offers are independently and identically distributed. The agent has another source of income, which we denote by $\epsilon_{t}$, and that may be regarded as nonhuman wealth. In every period the agent, regardless of whether they are employed or unemployed gets a realization of $\epsilon_{t}$, which is independently and identically distributed over time, with distribution $G(\epsilon)$ with support $[0, E]$ with $E$ finite. We also assume that $w$ and $\epsilon$ are independent. The objective of the agent is to maximise $\sum_{t=0}^{\infty} \beta^{t} y_{t}$, where $y_{t}=w+\phi \epsilon_{t}$ if the worker has accepted a job that pays $w$, and $y_{t}=b+\epsilon_{t}$ if the agent remains unemployed. We assume that $0<\phi<1$ to reflect the fact that an employed agent has less time to engage in the collection of nonhuman wealth. Assume $1>\operatorname{prob}\{w \geq b+(1-\phi) \epsilon\}>0$, such that there is a positive probability that the income from being employed will be higher than the income of being unemployed.
(i) Write down Bellman's Functional Equation for an unemployed agent.
(ii) Show that the optimal policy takes the form of a reservation wage, and that this reservation wage is increasing in non-human wealth.

## A3. Contraction Mapping

Consider the problem of a social planner that seeks to maximise the utility of the representative agent $U=\sum_{t=0}^{\infty} \beta^{t} \ln \left(c_{t}\right)$ subject to the resource constraint $c_{t}+k_{t+1} \leq A k_{t}$, where $\beta \in(0,1)$ and $A>0$. Variable $c_{t}$ corresponds to consumption at period $t$ and $k_{t}$ is the capital stock at period $t$. Assume that $k_{0}>0$ is given.
(i) Write down Bellman's Functional Equation associated with the planner's problem.
(ii) Is the Bellman's Functional Equation a Contraction Mapping? Explain.

## A. 4 Asset Pricing

Consider an economy populated by a continuum of measure one of identical households, each with preferences given by $\mathbf{E}_{0}\left[\sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right)\right]$ where $c_{t}$ is consumption at period $t, u\left(c_{t}\right)$ is a function that is strictly increasing, strictly concave and twice differentiable, and $\beta \in(0,1)$ corresponds to the subjective discount factor. There is a single asset tree which yields perishable dividends $d_{t}$ (fruits). Households are initially endowed with some number of shares of the tree: $s_{0}$. Assume that $d_{t}$ is a random variable taking values from the set $\mathbf{D} \equiv\left\{d_{1}, \ldots, d_{N}\right\}$ and is distributed according to a first-order Markov chain with constant transition probability matrix. Households can trade the share of the tree. The price of the tree when the current realization of $d_{t}$ is $d$ is given by the function $p: \mathbf{D} \rightarrow R_{+}$. There are no other assets in this economy. State the problem of the representative household in recursive form and define a Recursive Competitive Equilibrium for this economy.

## SECTION B

## B. 1 Neoclassical Growth Model

Consider an economy with a representative firm which has access to the following technology:

$$
Y(t)=K(t)^{\alpha} N(t)^{1-\alpha}, \alpha \in(0,1) .
$$

$Y$ is output, $K$ is capital, and $N$ is labour. There is neither population growth nor technological progress and all markets are competitive.
There is also a representative agent who is infinitely lived, with preferences given by

$$
U=\int_{0}^{\infty} e^{-\rho t} u(c(t)) d t, \quad \rho>0
$$

where $c(t)$ denotes consumption of the representative household at $t$, and

$$
u(c)=\frac{c^{1-\theta}-1}{1-\theta}, \theta>0
$$

The representative household has an instantaneous unit of productive time and owns $k(0)$ units of capital at $t=0$. Capital evolves according to:

$$
\dot{k}(t)=x(t)-\delta k(t), \delta>0
$$

where $x$ corresponds to investment in capital and $\delta$ is the depreciation rate.
(a) Taking prices as given, set up the Hamiltonian of the representative household, and derive the necessary and sufficient conditions for the allocation of consumption over time. Comment on your results.
(b) Taking prices as given, derive the first-order conditions of the representative firm, and define the market clearing conditions. Comment on your results.
(c) Derive the steady-state level for the capital stock $k^{*}$ and consumption $c^{*}$ and draw the phase diagram which characterises the evolution of capital and consumption toward their steady-state levels. Comment on your results.

Let $k(0)<k^{*}$ and assume that at $t=0$ a giant oilfield is discovered which will generate a fixed and permanent increase in output $O>0$ at period $\bar{t}>0$ without the need of any capital and labour, so that:

$$
Y(t)=K(t)^{\alpha} N(t)^{1-\alpha}+O, \text { for } t \geq \bar{t} .
$$

(d) What are the effects of this giant oilfield discovery on the long-run level of consumption $c^{*}$, capital $k^{*}$ and the interest rate $r^{*}$. Explain.
(e) From $t=0$ show the dynamics of consumption and capital toward their long-run levels when the representative agent knows that at $t \geq \bar{t}$, there will be a permanent increase in output by $O>0$.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Wednesday 18 May 2016 09:00am-11:00am

M220
MACROECONOMICS III

This paper is in two sections:
Section A - candidates are required to answer three questions.
Section B -candidates are required to answer one compulsory question.
Each section carries equal weight.

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## STATIONERY REQUIREMENTS

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Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Section A

## QUESTION 1: The current account

Consider the following small open economy with investment and government. The representative agent maximizes lifetime utility

$$
U_{t}=\sum_{s=t}^{\infty} \beta^{s-t} \ln \left(C_{s}\right)
$$

where $C_{t}$ is consumption in period $t$, and the parameter $\beta$ satisfies $0<\beta<1$. The intertemporal budget constraint is given by

$$
W_{t} \equiv(1+r) B_{t}+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(Y_{s}-I_{s}-T_{s}\right)=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_{s}
$$

where $W_{t}$ denotes lifetime wealth, which is positive, $B_{t+1}$ net foreign assets at the end of period $t$, with $B_{t}$ given, and $r$ the world real interest rate, which is positive. Output $Y_{s}$ is produced according to

$$
Y_{s}=2 \sqrt{K_{s}}
$$

where $A_{s}$ is a positive productivity parameter and $K_{s}$ capital, which satisfies

$$
K_{s+1}=K_{s}+I_{s}
$$

where investment $I_{s} \in \mathbb{R}$, and $K_{t}>0$ is given. Government purchases $G_{s}$ are financed by lump-sum taxes $T_{s}$, so $G_{s}=T_{s}$ for all $s=t, t+1, t+2, \ldots$. Assume free trade with the rest of the world and no international capital market imperfections. Assume that initially, $G_{s}=\bar{G}$ and the economy is in a steady state with $K_{s}=\bar{K}$ for all $s$.
(a) Derive the fundamental equation for the current account $C A_{t}$.
(b) Suppose that fiscal consolidation leads to a sudden permanent decline in government purchases $\bar{G}$ in period $t$. Explain the effect on consumption $C_{t}$, investment $I_{t}$, output $Y_{t}$, and the current account $C A_{t}$. Does this fiscal consolidation help to reduce current account imbalances?

## QUESTION 2: Risk sharing

Define a "valuation wedge" $\mathcal{W}_{t}$ as follows:

$$
\mathcal{W}_{t}=\frac{U^{\prime}\left(C_{t}^{*}, \zeta_{C, t}^{*}\right) \frac{1}{\mathcal{E}_{t} \mathbb{P}_{t}^{*}}}{U^{\prime}\left(C_{t}, \zeta_{C, t}\right) \frac{1}{\overline{\mathbb{P}_{t}}}}
$$

where $U$ is utility, $C\left(C^{*}\right)$ is consumption at Home (Foreign), $\mathbb{P}_{t}\left(\mathbb{P}_{t}^{*}\right)$ is the price level at Home (Foreign), $\zeta_{C, t}\left(\zeta_{C, t}^{*}\right)$ a taste shock, and $\mathcal{E}_{t}$ is the exchange rate of the Home country - Foreign analogs are denoted with a *.
(a) Discuss the wedge under complete and incomplete markets.
(b) Can you give the wedge a macroeconomic interpretation?
(c) Discuss how to derive a regression test of "market completeness" at international level assuming that utility is exponential $U(C)=\zeta_{C, t}\left(\frac{1-e^{-a C}}{a}\right)$ and $U\left(C^{*}\right)=\zeta_{C, t}^{*}\left(\frac{1-e^{-a^{*} C^{*}}}{a^{*}}\right)$ where $a$ is the "absolute risk aversion" $\frac{U^{\prime \prime}}{U^{\prime}}=a$.

## QUESTION 3: Exchange rate pass through

In the Home country, there are many imperfectly competitive firms, each producing one differentiated intermediate input $h$ with the following technology, linear in labor $\ell_{t}$

$$
y_{t}(h)=Z_{\mathrm{H}, t} \ell_{t}
$$

where $Z_{\mathrm{H}, t}$ is productivity, identical across firms. Hence each firm has marginal costs $\frac{W_{t}}{Z_{\mathrm{H}, t}}$ where $W_{t}$ is the nominal wage rate. Each firm sells exclusive in a Foreign country, setting its price $p_{t}(h)$, or equivalently $\mathcal{E}_{t} p_{t}^{*}(h)$, where $\mathcal{E}_{t}$ is the exchange rate, and Foreign currency prices and variables are denoted with a *. In the Foreign country, each intermediate input $y(h)$ is bought by competitive firms, which produce a final good $y_{t}^{*}(f h)$ using a Leontief technology requiring $\eta^{*}$ units of Foreign labor per unit of Home input. Hence the price of the final good $f h$ is

$$
p_{t}^{*}(f h)=\bar{p}_{t}^{*}(h)+\eta W_{t}^{*}
$$

where $W_{t}^{*}$ is the Foreign nominal wage in Foreign currency. Note that $p_{t}^{*}(f h)$ is the consumer price of the final good $f h$ produced by Foreign firms using the intermediate good $h$ (purchased at the price $p_{t}^{*}(h)$. The demand for each $f h$ good is

$$
D_{t}^{*}(f h)=\left(\frac{p_{t}^{*}(f h)}{P_{\mathrm{FH}, t}^{*}}\right)^{-\theta} D_{\mathrm{FH}, t}^{*}
$$

where $\theta$ is the elasticity of substitution across the $f h$ final goods, $D_{\mathrm{FH}, t}^{*}$ is the aggregate Foreign demand for these goods, $P_{\mathrm{FH}, t}^{*}$ is the corresponding price index
(a) Write the profit maximization problem of the Home firm and derive the optimal markup.
(b) Assume that wages and a monetary shock causes a Home currency devaluation: what is the exchange rate pass? Explain.
(c) Does the demand elasticity faced by the Home firm satisfy the sufficient condition for incomplete pass through defined by Marston (1990)?

## QUESTION 4: Debt Maturity and Default



Consider the 3 -period economy presented above. The economy has a representative consumer with preferences

$$
U=E\left[c_{0}+\beta c_{1}+\beta^{2} c_{2}\right]
$$

where $E$ is the expectation operator, $c_{i}$ is consumption in period $i=0,1,2$ and $\beta$ is the discount factor. The endowment process is stochastic. The consumer receives nothing in the first period, and with probability $\alpha$ and $1-\alpha$ receives endowment $y_{H}$ and $y_{L}$, respectively. If the $H$ state is realized in $t=1$, the consumer receives endowment $y_{H}$ with probability $p_{H}$ in the next period. On the other hand, if the $L$ state is realized in $t=1$, the consumer receives the high endowment $y_{H}$ in $t=2$ with probability $p_{L}$.
The picture above from Arellano and Ramanarayanan (2012) depicts the case where the consumer has access to both long-term (two-period) and short-term (one-period) bonds, and repays in all states of the world (notation is standard: $q$ stands for bond prices, supercripts $H$ and $L$ stands for states and combines to form histories, e.g. $H L$ in $t=2$ ). Below, we will consider three environments with different debt instruments available, where we also allow for default. If the consumer defaults, it does not repay her debt, and her consumption falls to $y_{d e f}=0$. Assume that $\frac{p_{H} y_{H}-y_{L}}{p_{H}\left(y_{H}-y_{L}\right)}<\beta<\left[\frac{p_{H} y_{H}-y_{L}}{p_{H}\left(y_{H}-y_{L}\right)}\right]^{1 / 2}$.
(a) First consider an economy in which the consumer has access to short-term 1 -period bonds only, both in time $t=0$ and $t=1$. Write the consumer's maximization problem. Characterize the optimal consumption and borrowing decisions in all states of the world. Does the economy default in this case? In what state of the world?
(b) Consider instead an economy with access to only long-term bonds in $t=0$ that come due in $t=2$. How does the consumer's maximization problem differ in this case, relative to part (i)? What are the consumption, borrowing, and default decisions in this case?
(c) Now consider the case where the consumer has access to both short- and long-term bonds, and can default. What does the consumer's maximization problem look like in this case? Characterize the consumption, borrowing, and default decisions. What are the benefits to having both types of bonds available? Explain.

## Section B

## QUESTION 5: Cole and Obstfeld economies

Consider a two-country ( $H$ and $F$ ) world economy where endowment output is stochastic. A social planner solves:

$$
\begin{aligned}
& \operatorname{Max} \mu\left(\frac{C_{H}^{\gamma} C_{F}^{1-\gamma}}{1-\sigma}\right)^{1-\sigma}+(1-\mu)\left(\frac{C_{H}^{* 1-\gamma} C_{F}^{* \gamma}}{1-\sigma}\right)^{1-\sigma} \\
& \text { s.t. }: C_{H}+C_{H}^{*}=Y_{H} \\
&: C_{F}+C_{F}^{*}=Y_{F}
\end{aligned}
$$

where $\mu$ is the planner welfare weight in favor of the Home consumers, $C_{J}\left(C_{J}^{*}\right)$ is the consumption of good $J=H, F$ by the Home (Foreign) consumers, $\sigma$ is relative risk aversion, and agents' consumption preferences may be biased towards home goods, i.e. $\gamma \geq 1 / 2$.
(a) Derive the Cole and Obstfeld result - that the social planner allocation can be supported by a market allocation under financial autarky - in the special case $\gamma=1 / 2$.
(b) Under what restrictions on consumption preferences would the same result holds when $\gamma>1 / 2$. Explain.

## END OF PAPER

 CAMBRIDGEECM9/ECM10<br>MPhil in Economics<br>MPhil in Economic Research

Friday 3 June $2016 \quad 9.00 \mathrm{am}$ to 11.00am

M230
APPLIED MACROECONOMICS

Candidates are required to answer three questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 2
Metric Graph Paper
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 A government's fiscal debt and borrowing is captured by the following relationship.

$$
\Delta b_{t}=d_{t}+\left(r_{t}-g_{t}\right) b_{t}
$$

(a) explain carefully what each of the terms in the expression mean.
(b) Show in a diagram how debt evolves when $r_{t}>g_{t}$, for
(i) a high level of initial debt.
(ii) a low level of initial debt.
(c) Do the same as in (b) but for the case in which $r_{t}<g_{t}$.
(d) How can you use this relationship to understand the sovereign debt crisis in the Euro Area?

2 A policymaker faces a forward looking New Keynesian model with a zero lower bound on interest rates:

$$
\begin{gathered}
\pi_{t}=\kappa y_{t}+\beta \pi_{t+1}+m_{t} \\
y_{t}=-\frac{1}{\sigma}\left(i_{t}-\pi_{t+1}-r_{t}^{n}\right)+y_{t+1} \\
i_{t} \succeq 0 \\
L=\sum_{i=0}^{\infty} \beta^{i} \frac{1}{2}\left(\pi_{t}^{2}+\lambda y_{t}^{2}\right)
\end{gathered}
$$

$\pi_{t}$ and $\pi_{t+1}$ are inflation in periods $t$ and $t+1$ and $y_{t}$ and $y_{t+1}$ are the output gaps in periods $t$ and $t+1$. $i_{t}$ is the nominal interest rate. $L$ is the loss function for the policymaker. When $t=0$ the economy is subjected to a positive shock $m_{0}$.
(a) Explain briefly what $\kappa, \beta, \sigma$ and $\lambda$ capture.
(b) When there is no zero lower bound, what is the optimal policy under discretion? Show diagramatically.
(c) Suppose the Central Bank introduces forward guidance? What does this do to the optimal policy with no zero lower bound? Show diagramatically.
(d) Suppose now there is a zero interest rate bound. Show diagrammatically what the discretionary policy is in these circumstances.
(e) The Central Bank introduces forward guidance with the ZLB. How does this affect the optimal policy?

3 Point and density forecasts are published by many central banks.
(a) Explain what these are. Why publish both?
(b) There is considerable uncertainty associated with macroeconomic forecasting. Why is this?
(c) Does nowcasting have a role to play in forecasting?
(d) How might a central bank improve its forecasts by making use of the forecasts of others?

4 We have a structural vector autoregresive model with four variables $\left[\begin{array}{c}p \\ y \\ s \\ x\end{array}\right]$,where $p$ is the price level, $y$ is output, $s$ is the short term interest rate and $x$ is the exchange rate. The reduced form errors and the structural shocks are related by $A e_{t}=B u_{t}$. The matrix $B$ is an identity matrix and:

$$
\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
a_{21} & 1 & 0 & 0 \\
a_{31} & a_{32} & 1 & 0 \\
a_{41} & a_{42} & a_{43} & 1
\end{array}\right]\left[\begin{array}{c}
\varepsilon^{p} \\
\varepsilon_{t}^{y} \\
\varepsilon_{t}^{s} \\
\varepsilon_{t}^{x}
\end{array}\right]=\left[\begin{array}{c}
u_{t}^{p} \\
u_{t}^{y} \\
u_{t}^{s} \\
u_{t}^{x}
\end{array}\right]
$$

(a) What is the relationship between the structural equation and the reduced form.
(b) Is this model identified?
(c) Is there an economic interpretation of the restrictions placed on the $A$ matrix?
(d) Do these restrictions seem plausible?
(e) What other forms of identifying restrictions could be used?

5 What role do firm specific and sector specific shocks play in understanding business cycle fluctuations?

6 Why do 'Inflation Targeting' regimes still remain an important form of policy despite their limited success in the last 8 years?

7 A static model for output and inflation can be written as:

$$
\begin{align*}
y_{t} & =-\lambda r r_{t}+\eta_{t}  \tag{1}\\
\pi_{t} & =\pi_{t}^{e}+\beta y_{t}+\varepsilon_{t}  \tag{2}\\
L_{t} & =\delta\left[y_{t}^{2}\right]+\left(\pi_{t}\right)^{2} \tag{3}
\end{align*}
$$

$y_{t}$ is the output gap in time period $\mathrm{t}, r r_{t}$ is defined as $r r_{t}=r_{t}-\pi_{t}-r^{*}$, with $r^{*}$ the long term (natural) real interest rate and $r$ the nominal interest rate. $\pi_{t}$ is the inflation rate. Expected inflation is assumed to be zero and the Central Bank observes current period shocks $\eta_{t}$ and $\varepsilon_{t}$ perfectly. These shocks have zero means and variances $\sigma_{\eta}^{2}$ and $\sigma_{\varepsilon}^{2}$ respectively. The nominal interest rate is given by:

$$
\begin{equation*}
r_{t}=r^{*}+\phi_{1} \eta_{t}+\phi_{2} \varepsilon_{t} \tag{4}
\end{equation*}
$$

where $\phi_{1}=[1 / \lambda]$ and $\phi_{2}=\left[\beta / \lambda\left(\delta+\beta^{2}\right)\right]$.
(a) Explain carefully what these 4 equations capture.
(b) Is equation (4) the 'optimal' feedback rule?
(c) Which variables are policy targets and which variables are instruments of policy?
(d) Define controllability. Is the model above controllable? Explain your answer.
(e) In what sense can we think of expectations of inflation, $\pi_{t}^{e}$, as an 'instrument' of policy?
(f) What is the efficient policy frontier?
(g) If there is a rise in $\sigma_{\eta}^{2}$ and $\sigma_{\varepsilon}^{2}$ how does this effect the efficient policy frontier?

## END OF PAPER

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research
MPhil Finance

Monday 23 May 2016 09:00am - 11:00am

M310
TIME SERIES

Candidates are required to answer five questions from Section A and one question from Section B.

Section A is weighted as $60 \%$ and Section B as $40 \%$.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A

A.1. Show that in the stationary second-order autoregressive $[A R(2)]$ process

$$
y_{t}=\phi_{1} y_{t-1}+\phi_{2} y_{t-2}+\varepsilon_{t}, \quad t=1, . ., T
$$

where $\varepsilon_{t}$ is serially uncorrelated with mean zero and variance $\sigma^{2}$, the autocorrelation at lag one is $\rho(1)=\phi_{1} /\left(1-\phi_{2}\right)$. Hence obtain an expression for $\rho(2)$ in terms of $\phi_{1}$ and $\phi_{2}$.
A.2. What restrictions, if any, need to be placed on $d$ and $\theta$ to ensure that the fractionally integrated process

$$
\Delta^{d} y_{t}=\varepsilon_{t}+\theta \varepsilon_{t-1}, \quad \varepsilon_{t} \sim \operatorname{NID}\left(0, \sigma^{2}\right),
$$

is stationary and invertible? Assuming that $y_{t}$ is stationary, find its the power spectrum, $f(\lambda)$, and sketch it against frequency, $\lambda$.
A.3. Consider the random walk plus noise:

$$
y_{t}=\mu_{t}+\varepsilon_{t}, \quad \mu_{t}=\mu_{t-1}+\eta_{t}, \quad t=1, \ldots, T
$$

where $\varepsilon_{t}$ and $\eta_{t}$ are serially and mutually uncorrelated Gaussian disturbances with variances $\sigma_{\varepsilon}^{2}$ and $\sigma_{\eta}^{2}$ respectively. How would you test the null hypothesis that $\sigma_{\eta}^{2}=0$ against the alternative $\sigma_{\eta}^{2}>0$ ? How is the test modified when (i) the model contains a time trend, that is $y_{t}=\mu_{t}+\beta t+\varepsilon_{t}$, (ii) the white noise disturbance $\varepsilon_{t}$ is replaced by any stationary process?
A.4. Write down the local linear trend structural time series model and briefly explain how it is to be interpreted. Show how the model is put in state space form. How would you initialize the Kalman filter?
A.5. In the model

$$
\begin{aligned}
y_{t} & =\mu_{t}+\lambda w_{t}+\varepsilon_{t}, \quad t=1, . ., T \\
\mu_{t} & =\mu_{t-1}+\eta_{t},
\end{aligned}
$$

where $\varepsilon_{t}$ and $\eta_{t}$ are serially and mutually uncorrelated Gaussian disturbances with variances $\sigma_{\varepsilon}^{2}$ and $\sigma_{\eta}^{2}$ respectively, the intervention variable $w_{t}$ takes the value zero for $t<\tau, 1<\tau<T$, and unity for $t \geq \tau$. (i) If $y_{t}$ is in logarithms, what is the interpretation of $\lambda$ ? (ii) Suppose observations on a related variable become available. What time series properties of this variable will make it a good control group ?
A.6. Suppose that in the $V A R(1)$ model,

$$
\begin{aligned}
& y_{1 t}=\phi_{11} y_{1, t-1}+\phi_{12} y_{2, t-1}+\varepsilon_{1 t}, \quad t=1, \ldots, T, \\
& y_{2 t}=\phi_{21} y_{1, t-1}+\phi_{22} y_{2, t-1}+\varepsilon_{2 t},
\end{aligned}
$$

where $\varepsilon_{1 t}$ and $\varepsilon_{2 t}$ are serially uncorrelated disturbances with mean zero, it is known that $\phi_{21}=0$. What restrictions are needed on the other autoregressive parameters in order to ensure that $y_{1 t}$ and $y_{2 t}$ are (jointly) stationary? What can be said about Granger causality in this model?
A.7. The probability density function (pdf) of the Weibull distribution is

$$
f(y ; \alpha, v)=\frac{v}{\alpha}\left(\frac{y}{\alpha}\right)^{v-1} \exp \left(-(y / \alpha)^{v}\right), \quad 0 \leq y<\infty, \quad \alpha, v>0
$$

where $\alpha$ is the scale and $v$ is the shape parameter. Write down a dynamic model in which the scale, or some transformation of it, changes over time. How would you estimate the parameters?

## SECTION B

B.1. (a) Consider the $\operatorname{GARCH}(1,1)$ model

$$
\begin{array}{rlr}
y_{t} & =\sigma_{t \mid t-1} \varepsilon_{t}, & t=1, \ldots, T \\
\sigma_{t \mid t-1}^{2} & =\gamma+\beta \sigma_{t-1 \mid t-2}^{2}+\alpha y_{t-1}^{2}
\end{array}
$$

where $\varepsilon_{t} \sim \operatorname{NID}(0,1)$. (i) Find the autocorrelation function (ACF) of $y_{t}$ and show that the ACF of $y_{t}^{2}$ has the same form as that of an $A R M A(1,1)$ process. (ii) Based on information at $T$, find an expression for the variance of $y_{T+3}$ in terms of $\gamma, \alpha, \beta$ and $\sigma_{T+1 \mid T}^{2}$. (iii) What happens to the GARCH model when $\alpha+\beta=1$ and $\gamma=0$ ?
(b) Compare and contrast the GARCH and Dynamic Conditional Score (DCS) models for a bivariate time series in which the observations at time $t$ are assumed to have a conditionally Gaussian distribution with mean zero and covariance matrix $\Omega_{t \mid t-1}$. (You do not have to derive formulae for conditional scores.)
B.2. (a) How would you construct a consistent estimator of the spectral density of an (indeterministic) stationary process? Explain the terms lag length, kernel, bandwidth and spectral window in your answer.
(b) Suppose that a finite one-sided moving average with weights summing to one is applied to a series that is integrated of order one, that is $y_{t}$ is $I(1)$. Show that the corresponding detrending filter will produce a stationary series. Now consider an exponentially weighted moving average (EWMA) filter, that is

$$
m_{t}=\lambda \sum_{j=0}^{\infty}(1-\lambda)^{j} y_{t-j},
$$

where $0<\lambda<1$. Assuming, for mathematical convenience, that observations go back into the infinite past, show that when the corresponding detrending filter, $y_{t}-m_{t}$, is applied to a random walk it produces an $A R(1)$ process. What is the variance of this process?
(c) Obtain the spectrum of $\Delta_{4} y_{t}$ when $y_{t}$ is a stationary $A R(1)$ process. Find the value of the spectrum of $\Delta_{4} y_{t}$ at frequency zero if $y_{t}$ is a random walk. Compare this value with the value of the spectrum at zero frequency for the stationary $A R(1)$ process and briefly comment.

## END OF PAPER

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research
MPhil in Finance

Monday 30 May 2016 09:00am - 11:00am

## M320 <br> CROSS SECTION AND PANEL DATA ECONOMETRICS

Candidates are required to answer three questions from Section A and one question from Section B.

Section A is weighted as $60 \%$ and Section B as $40 \%$

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR <br> THIS EXAMINATION

Durbin-Watson and Dickey-Fuller Tables
New Cambridge Elementary Statistical Tables
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A

A1 Consider the following linear unobserved effects dynamic panel data model

$$
\begin{equation*}
y_{i t}=\alpha y_{i t-1}+x_{i t}^{\prime} \beta+u_{i t} \quad i=1, \ldots, N \quad t=1, \ldots, T . \tag{1}
\end{equation*}
$$

a) Discuss the properties of the OLS estimator under the following assumptions:
i) $u_{i t}=\eta+v_{i t} \quad$ where $v_{i t}$ is i.i.d. $N\left(0, \sigma^{2}\right)$ across $i$ and $t$. $\eta$ is a fixed parameter.
ii) $u_{i t}=\eta_{i}+v_{i t} \quad$ where $v_{i t}$ is i.i.d. $N\left(0, \sigma^{2}\right)$ across $i$ and $t$. $\eta_{i}, i=1, \ldots, N$, are fixed parameters. Under (ii) discuss how unbiased estimates of the parameters may be obtained.
b) Given $u_{i t}=\eta_{i}+v_{i t}$ and the assumption

$$
\begin{equation*}
E\left(v_{i t} \mid x_{i 1}, \ldots, x_{i T}, \eta_{i}\right)=0 \quad(t=1, \ldots, T) \tag{2}
\end{equation*}
$$

write down a set of moment conditions that, subject to a rank condition, will ensure identification given the endogeneity of lagged $y$ in (1) and serial correlation in $v_{i t}$ of unspecified form.

A2 Suppose you have a control group $A$ and a treatment group $B$, and two periods of data. Between the two periods a new policy is implemented that affects group $B$.
a) If your outcome variable is binary and you have no covariates, how would you estimate the effect of the policy?
b) If you have covariates, write down a probit model that allows you to estimate the effect of the policy change. Explain in detail how you would estimate the effect of the policy.
c) How would you get an asymptotic 95 per cent confidence interval for the estimate in part b)?
d) Assume that the binary outcome variable follows a linear probability model (LPM). Write down the log-likelihood function for observation $i$. Why might maximum likelihood estimation of the LPM be problematic?

A3 A sample is comprised of $G$ mutually exclusive groups such that total sample size $N=\sum_{g=1}^{G} N_{g} ; N_{g}$ denotes the number of observations for the $g^{t h}$ group. For the $i^{\text {th }}$ observation in group $g$ we write

$$
\begin{equation*}
y_{i g}=x_{i g}^{\prime} \omega+u_{i g}, \quad i=1, \ldots, N_{g}, \quad g=1, \ldots, G \tag{3}
\end{equation*}
$$

where $x_{i g}$ is a $K \times 1$ vector. We make the following assumptions:
s. $1 E\left[u_{i g} \mid x_{i g}\right]=0$
s. $2 E\left[u_{i g} u_{j g^{\prime}} \mid x_{i g}, x_{j g^{\prime}}\right]=0 \quad \forall g \neq g^{\prime}$.
a) Given (3) and assumptions s. 1 and s.2, write down an expression for the Ordinary Least Squares estimator.
b) i) Given s. 2 suggest a model for $\boldsymbol{\Omega}_{g}=E\left[\boldsymbol{u}_{g} \boldsymbol{u}_{g}^{\prime} \mid \boldsymbol{X}_{g}\right]$ and write down the corresponding Feasible Generalised Least Squares (FGLS) estimator. $\boldsymbol{X}_{g}$ is $N_{g} \times K$ and $u_{g}$ is $N_{g} \times 1$.
ii) Explain how you might generate a cluster robust estimate of the asymptotic variance matrix of the FGLS estimator.

A4 Labour market histories for a random sample of 347 men aged between 25 and 50 in 2007 contains information on labour market status (employed or not employed), education (in five categories, indicating increasing levels of education) and age (in years).
Using a probit model, the probability that an individual was employed in 2007, is estimated using the following specification:

$$
\begin{aligned}
\operatorname{Pr}[L=1 \mid A, E] & =\Phi\left(x^{\prime} \boldsymbol{\theta}\right) \\
& =\Phi\left[\theta_{0}+\theta_{1} \cdot E+\theta_{2} \cdot(A-35)+\theta_{3} \cdot(A-35)^{2}\right]
\end{aligned}
$$

where $L=1$ if someone is employed in January 2007 and 0 if not. Education is denoted by $E$, and age by $A . \Phi(\cdot)$ denotes the cumulative distribution function for the standard normal distribution.
a) A GMM estimator incorporates knowledge of the conditional employment probabilities using the following moment conditions

$$
\begin{equation*}
\psi_{j}(L, x, \theta)=\frac{L-\Phi\left(x^{\prime} \theta\right)}{\Phi\left(x^{\prime} \theta\right)\left(1-\Phi\left(x^{\prime} \theta\right)\right)} \phi\left(x^{\prime} \theta\right) \cdot x_{j} \quad j=1, \ldots, 4 \tag{4}
\end{equation*}
$$

where $\boldsymbol{x}=\left(1, E, A-35,(A-35)^{2}\right)$ and $\phi($.$) denotes the probability density$ function of the standard normal distribution.

Explain the rationale for these moment conditions

Table 1: Size of male population in thousands by age category

|  |  |  |  |
| :--- | :--- | :--- | :--- |
| Age category | Working | Total Population | $p_{i}$ |
|  |  |  |  |
| $25-29$ | 555 | 609.0 | 0.911 |
| $30-34$ | 499 | 534.8 | 0.933 |
| $35-39$ | 407 | 436.7 | 0.932 |
| $40-44$ | 370 | 396.8 | 0.932 |
| $45-49$ | 337 | 378.0 | 0.891 |
|  |  |  |  |
| Total | 2168 | 2355.3 | 0.921 |

b) Using the individual-level data the hypothesis that $\theta_{1}=\theta_{2}=\theta_{3}=0$ could not be rejected. In Table 1 we present information on the actual size of the male population in various age categories, with $p_{i}$ denoting the age related employment probabilities.
In what sense does the aggregate data in Table 1 provide a basis for additional moment conditions relative to $\psi_{j}(L, \boldsymbol{x}, \boldsymbol{\theta}), j=1, \ldots, 4$. Briefly justify your answer.
A. 5 Elaborate upon the distinction between the following:
a) random effects, fixed effects and the correlated random effects estimator;
b) strict versus weak exogeneity assumption;
c) control function versus instrumental variable estimators in linear and nonlinear models.

## SECTION B

B. 1 The relationship between female labour force participation $(y)$ and fertility $\left(d_{1}\right)$ is written as

$$
y_{i t}=\mathbf{1}\left(\alpha+\delta_{1} d_{1, i t}+\delta_{2} d_{2, i t}+\boldsymbol{x}_{i}^{\prime} \boldsymbol{\beta}+\xi_{i t}>0\right)
$$

where $\mathbf{1}($.$) denotes the indicator function, y_{i t}$ is a binary indicator of participation in year $t$ (defined as positive annual hours of work), fertility $\left(d_{1, i t}\right)$ is a dummy variable which takes the value one if the age of the youngest child in $t+1$ is 1 , and $d_{2, i t}$ is a dummy indicator equal to one if the woman has a child between 2 and $6 . x_{i}$ is vector of additional controls.

In addition $\xi_{i t}=\alpha_{i}+u_{i t}$ is a standard composite panel data error term in which $\alpha_{i}$ represents an individual-specific effect and $u_{i t}$ denotes time varying errors including idiosyncratic labour market shocks.
a) Briefly discuss why fertility is most likely an endogenous variable. Provide justification for the use of an indicator of whether a woman has two children of the same sex as an external instrument for fertility.
b) In Table 2 we report results based on the application of a number of linear estimators. Columns 2, 3, and 4, report, respectively, results based upon the use of an OLS, 2SLS and within-group (WG) estimator.

Columns 5 and 6 present results for two first-difference GMM estimators, that exploit, respectively, strict and weak exogeneity assumptions.
i) For each estimator outline the identification assumptions and in particular discuss how each estimator accounts for the endogeneity of fertility.
ii) In what sense does the use of the WG estimator represent a tradeoff between accounting for unobserved heterogeneity and bias?
c) Write down the moment equations which underly the results for the GMM-1 GMM-2 estimators.

For each estimator state the degree of overidentification
d) Using the results in columns 5 and 6 test for strict exogeneity. What do you conclude?

Table 2: Female Labour Force Participation
Linear Probability Models ( $\mathrm{N}=1442,1986-1989$ )

| Linear Probability |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Variable | OLS | 2 SLS $^{1}$ | WG | GMM $^{2}$ | GMM-2 $^{3}$ |
| Fertility | -0.15 | -1.01 | -0.06 | -0.08 | -0.13 |
|  | $(8.2)$ | $(2.1)$ | $(3.8)$ | $(2.8)$ | $(2.2)$ |
| Kids2-6 | -0.08 | -0.24 | 0.001 | -0.005 | -0.09 |
|  | $(5.2)$ | $(2.6)$ | $(0.04)$ | $(0.4)$ | $(2.7)$ |
|  |  |  |  | 48.0 | 18.0 |
| Sargan <br> (d.f.) |  |  |  | $(22)$ | $(10)$ |

## Notes

- Heteroskedasticity robust $t$-ratios shown in parentheses
$-{ }^{1}$ External instrument: whether a woman has two children of the same sex.
- The variable Kids2-6 is equal to one if a woman has a child between 2 and 6 .
$-{ }^{2}$ Treats fertility as endogenous and uses all lags and leads of Kids2-6 and the external instrument same sex.
$-{ }^{3}$ Treats fertility as endogenous and uses lags of Kids 2-6 and same sex up to t-1.
- Estimators which do not account for individual effects also include a constant, age, race, and education dummies (not reported).
- Sargan denotes the Sargan test statistic for testing over-identifying restrictions.

The sample consists of 1442 women aged between 18-55 in 1986 .

B 2 Data on sales of over-the-counter pain medication for 3 brands (Tylenol, Advil and Bayer) in Chicago supermarkets have been collected for a number of stores over a 52 week period. Data on a generic store brand are also available.

Available product information includes attributes (brand dummy, retail prices, and a promotion dummy) and market share, recorded for each brand and generic product for each store over 52 weeks.
The utility function for consumer $i$ for product $j$ in store-week $t$ is given by

$$
\begin{equation*}
u_{i j t}=\delta_{j t}+\epsilon_{i j t} \tag{5}
\end{equation*}
$$

where

$$
\begin{equation*}
\delta_{j t}=\boldsymbol{v}_{j t} \boldsymbol{\omega}+\omega_{p} p_{j t}+\xi_{j t} . \tag{6}
\end{equation*}
$$

$p_{j t}$ denotes price, $\boldsymbol{v}_{j t}$ are observed non-price attributes (here brand and a promotion dummy) and $\xi_{j t}$ are unobserved product attributes. $\omega$ and $\omega_{p}$, denote unknown parameters and $\epsilon_{i j t}$ is an i.i.d. draw from a Type 1 extreme value distribution.
a) You are told that price is endogenous. By locating the problem of endogeneity within a linear regression, and assuming that the distribution of consumer unobservables $\epsilon_{i j t}$ is distributed type 1 extreme value, carefully outline the form of the GMM estimator for this model.
b) Table 3 presents results for a number of specifications based upon estimating the parameters of (6) using an OLS and an IV estimator.
For each estimator three sets of results are reported:

1. Using price and promotion as product characteristics.
2. Using price and promotion as product characteristics and brand dummies.
3. Using price and promotion as product characteristics and store-brand (the interaction of brand and store) dummies.
Interpret the results.
c) As an extension to (5) consider the random coefficient model

$$
\begin{equation*}
u_{i j t}=\delta_{j t}+\omega_{i, B} B_{j t}+\omega_{i, p} p_{j t}+\epsilon_{i j t}, \tag{7}
\end{equation*}
$$

where $B_{j t}$ is a dummy variable indicating if $j$ is a branded or generic product. $\omega_{i, B}=\sigma_{B} \zeta_{i}$, and $\omega_{i, p}=\sigma_{p} \zeta_{i}$, with $\zeta_{i}$ a draw from a standard normal distribution. $\sigma_{B}$ and $\sigma_{p}$ are unknown parameters.
In what sense does such an extension represent an improvement over the logit model given in (5).

# Table 3: Results from Logit Regression 

|  |  |  | ESTIMATOR: OLS |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (i) |  | (ii) |  | (iii) |  |
| Price | -0.0514 | $(0.0025)$ | -0.3413 | $(0.0101)$ | -0.3302 | $(0.0096)$ |
| Promotion | 0.2132 | $(0.0163)$ | 0.3295 | $(0.0126)$ | 0.3290 | $(0.0115)$ |
| Dummies | - |  | Brand | Store-Brand |  |  |

ESTIMATOR: IV

|  | (iv) |  | $(\mathrm{v})$ | $(\mathrm{vi})$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Price | -0.0511 | $(0.0026)$ | -0.5469 | $(0.0135)$ | -0.5480 | $(0.0122)$ |
| Promotion | 0.2133 | $(0.0163)$ | 0.2669 | $(0.0130)$ | 0.2624 | $(0.0118)$ |
| Dummies | - |  | Brand |  | Store-Brand |  |

## Note: - Standard errors in parentheses

- For the IV estimator, Hausman instruments are the average price in other markets
d) Given (7), find an expression for $\varsigma_{j t}\left(\boldsymbol{\delta}_{t}, \boldsymbol{v}_{t}, \boldsymbol{p}_{t} ; \boldsymbol{\eta}\right)$, the predicted market share for good $j$ in market $t . \delta_{t}, v_{t}$, and $p_{t}$ are all $J \times 1$ vectors. $\eta$ collects unknown parameters for the mean and deviation from mean component of utility.
Explain how would you compute $\varsigma_{j t}\left(\boldsymbol{\delta}_{t}, \boldsymbol{v}_{t}, \boldsymbol{p}_{t} ; \boldsymbol{\eta}\right)$.
e) You are told that data is available on the income distributions in the store regions. Explain how you would incorporate this information, and how you would expect this to impact your results.


## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 16 May 2016 09:00am-11:00am

M330
APPLIED ECONOMETRICS

The paper is in two sections: Candidate are require to answer two questions from Section A and two questions from section B.

Each question carries equal weight.
Write your candidate number (not your name) on the cover of each booklet.
Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Section A

1. The table below describes a hypothetical experiment with 2,400 participants, where Z is a pre-treatment variable, D , the treatment status, and $\left(\mathrm{Y}^{0}, \mathrm{Y}^{1}\right)$ denote the pair of potential outcomes. The categories group the different combinations of variables and outcomes that occur in this experiment. Note that we do observe both potential outcomes in this hypothetical experiment.

| Category | number of participants | Z | D | $\mathrm{Y}^{0}$ | $\mathrm{Y}^{1}$ | Y |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 300 | 0 | 0 | 4 | 6 |  |
| 2 | 300 | 1 | 0 | 4 | 6 |  |
| 3 | 500 | 0 | 1 | 4 | 6 |  |
| 4 | 500 | 1 | 1 | 4 | 6 |  |
| 5 | 200 | 0 | 0 | 10 | 12 |  |
| 6 | 200 | 1 | 0 | 10 | 12 |  |
| 7 | 200 | 0 | 1 | 10 | 12 |  |
| 8 | 200 | 1 | 1 | 10 | 12 |  |

(a) Complete the last column in the table and calculate the average treatment effect (ATE)?
(b) What is the relationship between i) the difference in the average outcomes of treated and controls and ii) the ATE?
(c) Is it plausible that these data come from a randomised control trial (RCT)? Explain your answer.
2. A manager of a large automotive firm is interested in understanding whether the introduction of Information Technology (IT) tools improves the productivity of individual workers. All workers at the firm perform the same car assembly task and the factory floor is organized into equal-sized teams of assemblers, each assembly team working in parallel and independently from others. There are many such teams. The manager is proposing the following experiment to the CEO of the firm: (i) randomly assign some of the teams of workers to an IT-"treatment", consisting of a tailored computer-aided design (CAD) technology and an IT engineer that helps workers interact with the CAD system; (ii) collect information of the output per week of all workers on the factory floor $\left(Y_{i}\right)$ and test whether an individual worker's labor productivity is improved by the introduction of CAD technology by the following regression:

$$
Y_{i}=\alpha+\beta D_{i}+X_{i}^{\prime} \delta+\varepsilon_{i}
$$

where $D_{i}=1$ if worker $i$ was in an assembly team assigned to the IT treatment (and $D_{i}=0$ otherwise) and $X_{i}$ are individual workers' characteristics. The manager proposes to obtain Ordinary Least Squares estimates under the assumption that $\varepsilon_{i}$ is independent and identically distributed across workers.
(a) The CEO objects saying that $\varepsilon_{i}$ is unlikely to be independent across workers. Why is the CEO likely to be right?
(b) The CEO proposes instead that the disturbance on the regression has the following form:

$$
\varepsilon_{i j}=\eta_{j}+\theta_{i j}
$$

where $\eta_{j}$ is a team-specific mean zero disturbance (with variance $\sigma_{\eta}^{2}$ ) and $\theta_{i j}$ is the mean-zero disturbance (with variance $\sigma_{\theta}^{2}$ ) corresponding to worker $i$ in team $j$. Assume that while this is the true disturbance term, the manager goes ahead and assumes disturbances are independent across workers. Do you expect the standard errors obtained by the manager to be upwards or downward biased? Please justify. Does the bias change as the number of teams increases (keeping the total number of workers fixed)? Does it change when $\sigma_{\eta}^{2}$ increases relative to $\sigma_{\theta}^{2}$ ?.
(c) If the CEO is correct, and $\varepsilon_{i}$ is not independent across workers, please describe two strategies to obtain the correct standard errors for the OLS regression with worker-level outcomes and covariates.
(d) A member of the board of the firm points out that the different teams on the factory floor do not perform the exact same assembly task. Rather, groups of teams work in particular car models being assembled on the factory floor. She additionally adds that, historically, different car models face different sources of disruptions during the car assembly process, due to a difference in the car parts each model requires. How does this affect the strategy proposed by the CEO? Does your answer depend on the number of different car models being assembled on the factory floor?

## Section B

3. We are interested in how the minimum wage affects (full-time) employment decisions. In April 1992, New Jersey increased the minimum wage from $\$ 4.25$ to $\$ 5.05$.In Pennsylvania, the minimum wage was unchanged at $\$ 3.80$. The following regression was estimated on the sample of four brands of fast food restaurants in Feb-Mar and Nov-Dec 1992. Let Empft denote full-time employment, minwage, the minimum wage, nregs, the number of cash registers in the restaurant, hrsopen denotes the number of hours open of the restaurant, while the following dummy variables indicate the type of restaurant, where $d_{1}$ denotes Burger King ; $d_{2}$ is KFC; $d_{3}$ is Roy Rogers ; $d_{4}$ is Wendy 's . Also, $i$ denotes restaurant, $k$ denotes state, and $t=0$ if the observation is from Feb-Mar and $t=1$ if the observation is from Nov-Dec

Empft $_{i k t}=\alpha+\gamma$ minwage $_{k t}+\beta_{1}$ nregs $_{i k t}+\beta_{2}$ hrsopen $_{i k t}+\beta_{3} \sum_{j=2}^{j=4} \eta_{j} d_{j}+\epsilon_{i k t}$

| Empft | Coefficient | Standard error | $\mathrm{P}>[\mathrm{t}]$ |
| :---: | :---: | :---: | :---: |
| minwage | -5.17 | 2.28 | 0.02 |
| nregs | 1.25 | 0.23 | 0.00 |
| hrsopen | 0.41 | 0.44 | 0.35 |
| $\mathrm{~d}_{2}$ | 1.12 | 1.67 | 0.50 |
| $\mathrm{~d}_{3}$ | -1.17 | 1.18 | 0.32 |
| $\mathrm{~d}_{4}$ | 4.41 | 1.57 | 0.00 |
| Constant | 9.73 | 10.20 | 0.34 |
| Observations | 396 | $\mathrm{~F}(6,390)=10.22$ | 0.00 |

(a) Interpret the coefficient on minwage. Is this estimate likely to be unbiased? Explain.
(b) Assume that $\epsilon_{i k t}=\mu_{k}+\nu_{t}+u_{i k t}$ and that $E\left[u_{i k t} \mid x_{i k t}\right]=0$, where $\mathrm{x}_{i k t}$ is the vector of RHS variables in the regression except for minwage. How would you identify the impact of the minimum wage in this case? Explain, with a discussion of the assumptions required to identify the effect.
(c) Offer an interpretation of the coefficients $\eta_{2}-\eta_{4}$. What might explain the relatively large coefficient on $d_{4}$ ?
(d) How might you test the key identifying assumptions underlying your estimation procedure in this application, and in general?
4. In 1996, the Chicago Public Schools instituted an accountability policy that tied summer school and promotional decisions to performance on standardised tests. Third graders must obtain a minimum score of 2.8 grade equivalents in both reading and maths achievement on the Iowa Test of Basic Skills (ITBS) in order to advance. Someone who received 2.8 on reading but 2.7 on maths had to go to remedial summer school. Assume that the data are available on the cohort of students who were in the third grade from the 1993-1994 school year to the 1998-1999 school years. We are interested in estimating the treatment effect of remedial summer school using the specification below:

$$
Y_{i t+1}=\delta^{\prime} X_{i t}+\beta_{1} \text { Treat }_{i t}+u_{i}+\epsilon_{i t+1}
$$

where $Y$ is the outcome, $X$ is a vector of demographic and past performance variables, Treat is a binary variable that takes a value of one if the student went to summer school and zero otherwise, $u$ represents student motivation, unobserved by the researcher, $\epsilon$ is an error term, and $t$ and $i$ are time and individual subscripts respectively.
(a) Discuss the potential biases in using OLS to estimate the regression.
(b) Explain how you might use the nature of the Chicago accountability policy to design a regression discontinuity evaluation of the impact of the policy.
(c) You discover that 3 percent of students who scored below the cutoffs in June received waivers from summer school and about 14 percent of students in summer school received waivers in August. What are the implications of this for your design and identification?
(d) Outline and discuss the assumptions under which the regression discontinuity design is valid in this example. What are the key tests that you would implement to examine the validity of the design?
5. The neoclassical theory of investment ( $q$-theory of investment) predicts that the investment rate $i_{t} \equiv \frac{I_{t}}{K_{t}}$, where $I_{t}$ is investment and $K_{t}$ is a measure of the capital stock, should be equal to:

$$
\begin{equation*}
i_{t}=\alpha+\beta q_{t}+\varepsilon_{t} \tag{1}
\end{equation*}
$$

where $q_{t}$ is Tobin's $q_{t}$ at time $t$ (for completeness, $q_{t}$ is the ratio the market value of the capital stock over its replacement value) and $\varepsilon_{t}$ is a random variable with mean zero and constant variance (more on which below). However, recently, new research has shown that under general forms of capital adjustment costs, the above equation should instead read:

$$
\begin{equation*}
i_{t}=\gamma+\delta i_{t-1}+\lambda q_{t}+\eta_{t} \tag{2}
\end{equation*}
$$

where $\eta_{t}$ is an independent and identically distributed random variable with zero mean and constant variance.
(a) Suppose that researchers estimate specification (2) by OLS and find a statistically significant estimate of $\delta$. Show that whenever $\varepsilon_{t}$ is allowed to follow an autoregressive process of order one,

$$
\varepsilon_{t}=\rho \varepsilon_{t-1}+v_{t}, \text { with } v_{t} \text { being i.i.d. }
$$

the neoclassical specification (1) is also consistent with a significant coefficient on the lagged value of the dependent variable $i_{t-1}$. Detail a test procedure which would allow the neoclassical researcher to understand whether the latter specification is valid.
(b) Prove that in the neoclassical model, the long run response of $i$ to $q$ is equal to the response on impact while specification (2) derived by new research, implies that the long run response is larger than the impact response if $\delta \in(0,1)$.
(c) Suppose that, in specification (2), the true disturbance is autocorrelated, i.e.

$$
\eta_{t}=\varrho \eta_{t-1}+\nu_{t}, \text { with } \nu_{t} \text { being i.i.d. }
$$

but researchers erroneously assume that $\eta_{t}$ is i.i.d. What are the consequences for OLS estimation of specification (2)?
6. You are interested in understanding whether immigrants are discriminated in the labor market. You have access to a random sample of 40 workers with information on their labor market earnings, experience, years of education and whether or not a worker is an immigrant. As a first pass you run the following regression:

Earnings $_{i}=\beta_{0}+\beta_{1}$ Experience $_{i}+\beta_{2}$ Experience $_{i}^{2}+\beta_{3}$ Education $_{i}+\beta_{4}$ Immigrant $_{i}+\varepsilon_{i}$
where $\varepsilon_{i}$ is assumed to be an independent and identically distributed random variable. Based on your least square estimates, you decide to plot the square of your residuals, $\widehat{\varepsilon}_{i}^{2}$, (y-axis) against the level of education (in the x-axis). You obtain the following graph.

(a) Based on the above graph, are ordinary least squares estimates and/or inference based on the regression above likely to be correct? Please justify your answer formally.
(b) Discuss a formal test procedure that allows you to assess your answer in (a) systematically.
(c) Supposed your proposed test confirms the existence of a problem. Propose a solution and discuss its merits and potential problems.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Thursday 02 June 2016 09:00am - 11:00am

M500
DEVELOPMENT ECONOMICS

Answer two questions. Each question carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1(a) There are two education levels: low (L) and high (H). Less educated workers are channelled into low-skill occupations, while more educated workers are channelled into high-skill occupations. There are two locations: an origin (O) and a destination (D), with wages at these destinations denoted by $W_{e}^{O}, W_{e}^{D}$, $e \in\{L, H\}$. A worker will migrate if his wage at the destination net of moving costs exceeds his wage at the origin. The distribution of moving costs, $c$, is independent of education and is characterized by the function $F(c)$. Show that migrants will be positively selected on education; i.e. more educated individuals are more likely to migrate than less educated individuals, if there is greater wage inequality at the destination than at the origin; $W_{H}^{D}-W_{L}^{D}>$ $W_{H}^{O}-W_{L}^{O}$.
(b) The simple model described above does not explain the dynamics of migrant selection. In particular, it does not explain why there is often positive selection initially, which is replaced by negative selection later in time. Add networks to the model to explain this pattern, suitably justifying your modelling choice

2(a) There are two types of occupations, skilled and unskilled, and two levels of associated education, high (H) and low (L). Individuals are distinguished by their level of ability. It costs $\bar{C}$ for low-ability individuals to attain higher education, whereas the corresponding cost for high-ability individuals is $C<\bar{C}$. There is no cost to attaining low education. Although the skilled wage, $W_{H}$, is exogenously determined, the unskilled wage, $W_{L}$, is increasing in the number of members from the individual's community who selected into that occupation in the previous generation. In the first generation, all $N$ members of each community are assigned to either the skilled or the unskilled occupation. Derive conditions under which the $N$ members of each subsequent generation end up in their community's initial occupation, despite the fact that the time-stationary ability distribution is the same in all communities.
(b) Now suppose that the economy restructures, resulting in an increase in $W_{H}$. Explain why the blue-collar communities that were locked into the low-skill occupation would want to instill a sense of community identity among their members to discourage them from shifting to the high-skill occupation. While this community identity could be welfare enhancing to begin with, show that it could result in a dynamic inefficiency if it persisted and $W_{H}$ became sufficiently large.

3(a) The population in a developing country is divided into ethnic communities with different levels of social connectedness. The lifetime payoff for individual $i$ from community $j$ who starts a career in business in period $t$ is $\omega_{i}^{j}+\beta^{j} \sum_{\tau=0}^{t-1} \Delta \omega_{\tau}^{j}$, where $\omega_{i}^{j}$ is his ability, $\beta^{j}$ is a parameter that is increasing in his community's social connectedness, and $\Delta \omega_{\tau}^{j}$ is the flow of individuals from his community into business in previous periods. Ability is distributed uniformly on the unit interval. In addition to business, individuals in this economy can select into a traditional occupation, which provides a common payoff $u>1$. In period $0, \Delta \omega_{0}$ individuals are moved exogenously into business in each community. Derive an expression for $\Delta \omega_{t}^{j}$ as a function of $\beta^{j}$ and $t$, and then show that this expression implies that the number of entrants into business grows over time, more steeply in more connected communities as long as $\Delta \omega_{0}$ is sufficiently large.
(b) Now suppose that there are two business activities, $A$ and $B$. Each individual has activity-specific abilities, $\omega_{i}^{j A}$ and $\omega_{i}^{j B}$. Activity-specific abilities are independent and distributed uniformly on the unit interval. In period $0, \Delta \omega_{0}^{A}$ and $\Delta \omega_{0}^{B}$ individuals are moved exogenously into activities $A$ and $B$, respectively, where $\Delta \omega_{o}^{A}$ is slightly larger than $\Delta \omega_{o}^{B}$. Describe how the flow of entrants into the two activities, measured by $\Delta \omega_{t}^{A} / \Delta \omega_{t}^{B}$, will change over time and across communities.

## END OF PAPER

ECM9/ECM10<br>MPhil in Economics<br>MPhil in Economic Research

Monday 23 May $2016 \quad 1: 30 \mathrm{pm}-3: 30 \mathrm{pm}$

## M600

TOPICS IN MACROECONOMIC HISTORY

Candidates are required to answer two out of four questions. Each question carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. What are credit cycles? Evaluate the relationship between credit cycles, long swings in economic growth and financial crises.
2. Describe the features of international business cycles across different historical periods? What factors affect the balance between global and national economic fluctuations?
3. 'The international gold standard that prevailed from 1880-1914 has been viewed by many as a successful and credible monetary regime; the international gold exchange standard that prevailed in the 1920s lacked credibility and collapsed in the 1930s'. What are the key elements in the gold standard's success before 1914 and its collapse in the inter-war period.
4. Do the causes and effects of Systemic Banking crises differ across long historical time-periods?

## END OF PAPER

## UNIVERSITY OF

 CAMBRIDGEECM9/ECM10<br>MPhil in Economics<br>MPhil in Economic Research

Thursday 02 June 2016 09:00am - 11:00am

## M610 <br> BRITISH INDUSTRIALISATION

Candidates are required to answer two questions. Each question carries equal weight

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Greg Clark has recently argued that there was no improvement in living standards between 1250 and 1800 while Steve Broadberry et al have argued that the pre-industrial economy was not Malthusian. Assess the ways in which the pre-industrial economy was and was not Malthusian and how this changed across the eighteenth and nineteenth centuries.
2. What role did agricultural changes play in economic development 1700-1850?
3. 'Accounts of increasing consumption for ordinary households from 1650 to 1850 are difficult to reconcile with estimates of food availability from agricultural accounts which suggest poor nutrition and with other evidence that suggests static real wages.'
(a) Discuss some of the evidence that underlies this statement.
(b) Consider whether these various aspects can be reconciled and what they indicate about the living standards for the majority of Britons over industrialisation.
4. Central to Jan de Vries' 'industrious revolution' is increased work effort and market participation, exhibited particularly by women and children. To what extent has the available evidence supported this aspect of de Vries' thesis?

## END OF PAPER

ECM11
MPhil in Finance \& Economics

Tuesday 9 May $2017 \quad$ 2:00pm - 4:00pm

F100
FINANCE I

Candidates are required to answer all four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS

## EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1. A 4 percent coupon paying bond with maturity of 10 years has a price of $\$ 855$ whereas a 8 percent coupon paying bond with same maturity has a price of $\$ 1,150$. Coupon payments are semi-annual. What is the semi-annually compounded 10 year spot rate?

Question 2. Blast-Rock Corporation has a capital structure consisting (only) of common stock and corporate debt. The market value of Blast-Rock Corporation's common stock is $\$ 40$ million and the market value of its debt is $\$ 60$ million. The probability of default on debt obligations is essentially zero. The beta of the company's common stock is 0.8 , the expected market risk premium is 10 percent and the Treasury bill rate is 6 percent. Ignoring corporate taxes, what is the firm's cost of capital?

Question 3. Today is Fabio's $5^{\text {th }}$ birthday. His father intends to start saving a fixed amount every month for his university. He will start saving at the end of this month. He will each time place these savings in a bank account that pays an interest of 4 percent per annum with monthly compounding. Will there be 200 times as much as one monthly payment accumulated on the bank account by Fabio's $18^{\text {th }}$ birthday?

Question 4. The CAPM provides the expected return on an asset:

$$
E\left[\tilde{r}_{i}\right]=r_{f}+\beta_{i, m}\left(E\left[\tilde{r}_{m}\right]-r_{f}\right)
$$

Formally derive that the expression of $\beta_{i, m}$ is

$$
\beta_{i, m}=\frac{\sigma_{i, m}}{\sigma_{m}^{2}}
$$

## END OF PAPER

ECM11
MPhil in Finance \& Economics

Wednesday 24 May $2017 \quad$ 1:30pm $-3: 30 \mathrm{pm}$

F200
FINANCE II

Candidates are required to answer all four questions..

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS

## EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1. A stock price is currently 50. Over each of the next two three-month periods it is expected to go up by $6 \%$ or down by $5 \%$. The risk-free interest rate is $5 \%$ per annum with continuous compounding.
(a) What is the value of a six-month European call option with a strike price of 51 ?
(b) What is the value of a six-month European put option with a strike price of 51 ?
(c) Verify that the European call and European put satisfy put-call parity.
(d) If the put option were American, would it ever be optimal to exercise it early at any of the nodes on the tree.

Question 2. The standard deviation of annual returns on the market portfolio is $\sigma_{m}=$ 0.25. Consider a rather risky asset $i$ with a beta equal to $\beta_{p}=1.3$ and a standard deviation of annual returns which is double that of the market portfolio, i.e. $\sigma_{i}=0.5$. What percentage of the total risk of the risky asset $i$ is diversifiable?

Question 3. Consider two firms, $U$ and $L$ :

- The unlevered firm $U$ is financed by equity only. Denote $E_{U}$ the market value of firm $U$ 's equity. The market value of firm $U$ is $V_{U}=E_{U}$.
- The levered firm $L$ is financed by debt and equity. It maintains a debt level $D_{L}$ and pays a perpetual expected interest rate $r$ on this debt. Denote $D_{L}$ and $E_{L}$ the market value of firm $L$ 's debt and equity. The market value of firm $L$ is $V_{L}=D_{L}+E_{L}$.
- At date $t=0$, the two firms only differ in their capital structure, and are expected to generate an identical series of cash flows $C_{t}$ in year $t$, for $t \geq 1$.

Provide a formal arbitrage proof of the Miller-Modigliani Proposition, which states that $V_{U}=V_{L}$.

Question 4. Consider a firm whose balance sheet is as follows:

| Assets | Liabilities |
| :--- | :--- |
| Liquid Assets: $X_{0}$ | Bank Debt: $B_{0}$ due at $t=0$ |
| Fixed Assets: $A$ | Public Debt: $\lambda P$ due at $t=0$ |
|  | Public Debt: $(1-\lambda) P$ due at $t=1$ |

A fraction $\phi_{0}$ of the public debt is as senior as the bank. The other $\left(1-\phi_{0}\right)$ is junior to the bank.

The firm has the following investment opportunity:

- At $t=0$, investment outlay of $F$ required.
- At $t=1$, project generates $X \in[0,+\infty)$ with c.d.f. $H(X)$.
- Value (possibly negative): $V=\mathrm{E}[X]-F$.

The firm is however (i) in financial distress, i.e. $X_{0}<B_{0}+\lambda P+F$, and (ii) insolvent (assets $<$ liabilities), i.e. $X_{0}+A<B_{0}+P$. As a consequence, in liquidation, public debtholders would receive

$$
L_{0}^{P}= \begin{cases}\left(\frac{\phi_{0} P}{B_{0}+\phi_{0} P}\right)\left(A+X_{0}\right) & \text { if } A+X_{0}<B_{0}+\phi_{0} P \\ A+X_{0}-B_{0} & \text { otherwise } .\end{cases}
$$

We are going to examine restructuring the bank debt, considering (i) that public debtholders are passive in restructuring and (ii) frictionless bargaining between the bank and equityholders:

In continuation, the bank issues an amount of new debt due at $t=1$

$$
B_{1}=F-X_{0}+B_{0}+\lambda P .
$$

A fraction $\phi_{1}$ of the public debt is as senior as the new bank debt. The firm can undertake the investment (investing liquid assets $X_{0}$ ), therefore, total debt due at $t=1$ has face value

$$
D_{1}=B_{1}+(1-\lambda) P=F-X_{0}+B_{0}+P,
$$

and the face value of senior debt is

$$
S_{1}=B_{1}+\phi_{1}(1-\lambda) P=D_{1}-\left(1-\phi_{1}\right)(1-\lambda) P .
$$

Assuming that the new debt is risky, i.e., $A<D_{1}$, public debtholders therefore receive (at date $t=0$ plus at date $t=1$ ):

$$
\lambda P+ \begin{cases}(1-\lambda) P & \text { if } X+A \geq D_{1} \\ L_{1}^{P}(X) & \text { if } X+A<D_{1}\end{cases}
$$

where

$$
L_{1}^{P}(X)= \begin{cases}\left(\frac{\phi_{1}(1-\lambda) P}{S_{1}}\right)(A+X) & \text { if } A+X<S_{1} \\ A+X-B_{1} & \text { otherwise }\end{cases}
$$

(a) Determine the condition under which restructuring will occur.
(b) Describe the possible investment distortions it would lead to.
(c) Establish the impact of the priority of existing bank debt on investment incentives.

## END OF PAPER

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research
MPhil in Finance

## F400

ASSET PRICING

Candidates are required to answer all four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS
EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Suppose that there are several investors and several assets. Suppose further that investors have constant absolute risk aversion, and that the returns on the assets are jointly normally distributed.
(a) Find a formula for the asset demands of an individual investor.
(b) Explain the main features of this formula.
(c) Taking the formula from part (a) as your starting point, derive the securitymarket line (SML).
(d) Explain the main features of the SML.
2. Suppose that there are two risky assets: a safe asset and a risky asset. The initial price of the risky asset is 20 . In each period there is a shock $s \in\{d, u\}$. If $s=d$ then the price falls by $10 \%$. If $s=u$ then the price rises by $10 \%$. The initial price of the safe asset is 1 . In each period the price rises by $5 \%$.
(a) What is meant by a put option with strike price 22 and maturity 2 in this setting?
(b) Assuming that there is no arbitrage, what is the price of this option at time 0 ?
3. (a) Write down a model of the dynamics of spot rates. Explain carefully the meaning of the various elements of the model.
(b) In the context of your model, what does it means to say that the term structure of bond prices is affine? Why is it considered to be "affine"?
(c) Assuming that the term structure of bond prices is indeed affine in this sense, find the stochastic differential equation followed by the price of a $T$-bond.
4. There are three assets: a safe asset and two risky assets. The prices $B, R$ and $S$ of the safe asset, risky asset 1 and risky asset 2 evolve according to the s.d.e.s

$$
\begin{aligned}
d B & =r B d t, \\
d R & =\mu_{R} R d t+\sigma_{R} R d Z, \\
d S & =\mu_{S} S d t+\sigma_{S} S d Z
\end{aligned}
$$

respectively, where $Z$ is a one-dimensional standard Wiener process. There is also a derivative written on the first asset that pays $\Phi\left(R_{T}\right)$ at maturity $T>0$. There is no arbitrage.
(a) Find the equation satisfied by the price of the derivative. Be careful to explain any assumptions that you make, and to set out the main steps of your derivation carefully.
(b) What conditions must $\mu_{R}, \mu_{S}, \sigma_{R}$ and $\sigma_{S}$ satisfy? Explain carefully how your arrived at these conditions.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Monday 29 May 2017 1:30pm - 3.30pm

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F500
EMPIRICAL FINANCE
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This paper is in two sections. Section A: Candidates are required to answer all four questions. Section B: Candidates are required to answer two out of three questions.

Each section carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Section A

1. Suppose that observed closing (log) stock prices follow the random walk

$$
\begin{gathered}
p_{t}=\mu+p_{t-1}+u_{t} \\
u_{t}=\varepsilon_{t}+\theta \varepsilon_{t-1}
\end{gathered}
$$

where $\varepsilon_{t} \sim N\left(0, \sigma_{\varepsilon}^{2}\right)$.
(a) For what values of $\theta$ is this model consistent with the Efficient Market Hypothesis?
(b) For what values of $\theta$ is this model consistent with the empirical evidence on daily returns as presented in Table 2.4 of Campbell, Lo and Mackinlay, attached?
(c) What are some market microstructure explanations for this finding?
(d) Define the variance ratio

$$
V R(k)=\frac{\operatorname{var}\left(p_{t+k}-p_{t}\right)}{k \times \operatorname{var}\left(p_{t+1}-p_{t}\right)} .
$$

What is $V R(k)$ as a function of $\theta$ ?
2. True, False, Explain
(a) A portfolio hedged against factor risk always has zero risk
(b) In the absence of noise traders, bid ask spreads would be narrower, and so markets would be more efficient.
(c) The long run risks model says that at plausible values for the preference parameters (IES and RRA), a reduction in economic uncertainty or better long-run growth prospects leads to a rise in the wealth-consumption and the price-dividend ratios.
(d) Stock returns are more volatile during Wednesdays than Mondays. This is incompatible with market efficiency
3. Assume that fundamental price $P^{*}$ is a random walk unrelated to order flow

$$
P_{t}^{*}=P_{t-1}^{*}+\varepsilon_{t} .
$$

Suppose that buy and sell orders arrive randomly each period, and let $I_{t}$ denote the trade direction indicator, +1 for buy and -1 if customer is selling. Assume that $I_{t}$ is iid with equal probability of +1 and -1 and unrelated to $P_{t}^{*}$. Suppose that the bid ask spread is $s$ so that the transaction price each period $P_{t}$ satisfies

$$
P_{t}=P_{t}^{*}+I_{t} \frac{s}{2}
$$

Then derive an expression for cov $\left[\Delta P_{t-1}, \Delta P_{t}\right]$ in terms of $s$. What is Bid ask bounce? How could one obtain an estimate of the bid ask spread based on transactions data only?
4. The linear factor model for assets returns $R_{i}$ says that

$$
R_{i}=\alpha_{i}+\beta_{i}^{\top} f+\varepsilon_{i},
$$

where $f \in \mathbb{R}^{K}$ are common factors and $\varepsilon_{i}$ are idiosyncratic shocks with mean zero, variance $\sigma_{i}^{2}$ and uncorrelated across $i$, i.e., $\operatorname{cov}\left(\varepsilon_{i}, \varepsilon_{j}\right)=0$ for $i \neq j$. Here, $\beta_{i}^{\top}=\left(\beta_{i 1}, \ldots, \beta_{i K}\right)$. True false, explain:
(a) The covariance between observed returns under the linear factor model is for all $i \neq j$

$$
\operatorname{cov}\left(R_{i}, R_{j}\right)=\beta_{i}^{\top} \Sigma_{f} \beta_{j}
$$

where $\Sigma_{f}$ is the covariance matrix of the factors.
(b) The Arbitrage Pricing theory restricts the covariance matrix of observed returns to not depend on the vector of $\alpha_{i}^{\prime} s$.
(c) The Arbitrage Pricing theory restricts the covariance matrix of observed returns to not depend on the vector of $\sigma_{i}^{2 \prime} s$.
(d) If the factors are unobservable, the APT is untestable

## Section B

1. The consumption CAPM (when utility is CRRA) says that

$$
E_{t}\left[\left(1+R_{i t+1}\right) \delta\left(\frac{C_{t+1}}{C_{t}}\right)^{-\gamma}-1\right]=0
$$

where $C_{t}$ is aggregate consumption at time $t$ and $R_{i t}$ are the returns on asset $i$ at time $t, E_{t}$ denotes expectation at time $t$ based on all available information. What are the parameters $\gamma, \delta$ ? Explain how you might construct a test of the CAPM based on this equation. Under the additional assumption that log consumption growth and log equity market return are jointly normal we have

$$
0=E_{t}\left[r_{i, t+1}\right]+\log \delta-\gamma E_{t}\left[\Delta c_{t+1}\right]+\frac{1}{2}\left[\sigma_{i}^{2}+\gamma^{2} \sigma_{c}^{2}-2 \gamma \sigma_{i c}\right]
$$

where $r_{i t}$ are logarithmic returns and $c_{t}=\log C_{t}$ and: $\sigma_{i c}=\operatorname{cov}_{t}\left(r_{i, t+1}, c_{t+1}\right)$, $\sigma_{i}^{2}=\operatorname{var}_{t}\left(r_{i, t+1}\right), \sigma_{c}^{2}=\operatorname{var}_{t}\left(c_{t+1}\right)$. Describe what is the Equity premium puzzle. What are the possible explanations for this puzzle.
2. The Sharpe-Lintner CAPM predicts that stock returns obey

$$
\begin{gathered}
E\left(R_{i}-R_{f}\right)=\beta_{i} E\left(R_{m}-R_{f}\right) \\
\beta_{i}=\frac{\operatorname{cov}\left(R_{i}, R_{m}\right)}{\operatorname{var}\left(R_{m}\right)}=\frac{\operatorname{cov}\left(R_{i}-R_{f}, R_{m}-R_{f}\right)}{\operatorname{var}\left(R_{m}-R_{f}\right)},
\end{gathered}
$$

where $R_{m}$ is the random return on the observable market portfolio and $R_{f}$ is the return on an observable risk-free asset. In practice, we have sample of data $\left\{R_{i t}, R_{f t}, R_{m t}, i=1, \ldots, n, t=1, \ldots, T\right\}$. Describe several methods for
testing this model. In your account be careful to say whether you are working with individual stocks or portfolios, what the null and alternative hypotheses are, and what are the test statistics. How is this model useful in conducting event studies?
3. Suppose that daily stock returns satisfy

$$
\begin{gathered}
r_{t}=\mu+\sigma_{t} \varepsilon_{t} \\
\sigma_{t}^{2}=\omega+\beta \sigma_{t-1}^{2}+\gamma r_{t-1}^{2},
\end{gathered}
$$

where $\varepsilon_{t}$ is iid standard normal and independent of all past information. What properties of daily stock returns does the GARCH model capture? Describe how you would estimate the parameters $\mu, \omega, \beta, \gamma$ given a sample of data $\left\{r_{1}, \ldots, r_{T}\right\}$. Compare the resulting estimate of volatility $\sigma_{t}^{2}$ with the realized volatility estimator and with the implied volatility estimator. Describe in each case how the estimates are constructed, their interpretations, and their properties.

END OF PAPER

Candidates are required to answer four out of five questions.

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20 Page booklet x 1
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 EXAMINATIONCalculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

QUESTION 1 Uncovered Interest Parity
(a) Define the "Uncovered Interest Parity" (UIP). State briefly but precisely how it is tested using regression models, and in which way it fails to hold empirically.
(b) The empirical failure of the UIP is often referred to as the "UIP puzzle". Consider the hypothesis that this puzzle can be attributed entirely to omitting a risk-premium from the regression model.

- Under what conditions would the risk premium help explain the UIP puzzle? Discuss the results by Fama 1984.
- If Fama 1984 is right, what other empirical puzzle would arise? Explain carefully.


## QUESTION 2 Covered Interest Parity

Suppose that international investors attribute a small probability to the event of another large global crisis. They also believe that, if and when this new global crisis occurs, the US would tax interest income on US financial assets owned by foreigners, at a constant tax rate $\tau$.
(a) Define the cross-currency basis of a foreign currency vis-à-vis the dollar. Derive and discuss the implications of the prospective taxation for the crosscurrency basis. In answering this question, make use of the asset pricing equation analyzed in class and explain carefully your results.
(b) Would anticipation of solvency problems (credit risk) among US financial intermediaries make a difference to your answer? (Hint: in addressing this question, it is analytically convenient to ignore the prospective tax discussed in (a)).

## QUESTION 3 The Peso problem

Explain briefly but rigorously the "peso problem" and its applications to the main puzzles in international finance.

## QUESTION 4 Risk sharing and sovereign risk

Consider a stylized two-period model of a small open economy populated by many individuals $i$, each with life-time utility and endowments

$$
\begin{gathered}
\ln \left(c_{i, 0}\right)+E \ln \left(c_{i, 1}\right) \\
y_{i, 0}=0.5, \quad \text { and } \quad y_{i, 1}=\left\{\begin{array}{l}
1.80 \quad \text { with probability } 1 / 2 \\
1.20
\end{array} \text { with probability } 1 / 2\right.
\end{gathered}
$$

Individuals are ex-ante identical, but face individual income risk ex-post. In period 0 , all individuals have the same income $y_{i, 0}=0.5$; in period 1 , an individual $i$ can be 'lucky' and get $y_{i, 1}=1.80$, or 'unlucky' and get $y_{i, 1}=1.20$, each with probability $1 / 2$.
In period 0 , individuals are able to issue liabilities $\widetilde{b}_{i, 0}$ promising a cash flow that is contingent on the realization of their income. Namely, an individual debt contract stipulates that, in period 1 , she will have to pay back either $x_{i}^{H}$ if she turns out to be lucky or $x_{i}^{L}$ if she turns out to be unlucky. International investors are risk neutral and do not discount the future, so that $\widetilde{b}_{i, 0}$ will be equal to the expected payoff:

$$
\widetilde{b}_{i, 0}=E_{0}\left(x_{i}\right)
$$

Answer the following:
(a) Define the real rate in financial autarky and compare it with the world market rate: will the country be a net importer or a net exporter of financial assets in period 0 ?
(b) Write down the individual $i$ consumption-saving problem. What debt contracts would you expect to observe in equilibrium if
(i) international contracts are enforceable by some supranational authority, and individual incomes are verifiable?
(ii) international contracts are not enforceable but incomes are verifiable. However, if an individual defaults she/he has to endure costs for $k=0.5$.
(iii) contracts are not enforceable and income not verifiable, so that international investors are not willing to lend against contingent payment (hint, set $x_{i}^{H}=x_{i}^{L}$.

For each case (i) (ii) (iii) above, calculate $\widetilde{b}_{i, 0}$, the cash flow $x_{i}^{H}$ and $x_{i}^{L}$ and the allocation of consumption. Explain you answer discussing briefly the efficiency of the consumption allocation.
(d) Suppose individuals residing in the country revise their income expectations in the future- $\overleftrightarrow{y}_{i, 1}$ will be a mean preserving spread of $y_{i, 1}$

$$
\overleftrightarrow{y}_{i, 1}=\left\{\begin{array}{c}
2 \\
\text { with probability } 1 / 2 \\
1
\end{array} \text { with probability } 1 / 2\right.
$$

Compare the cases (i) and (iii) above: In which case will agents change the amount of liabilities they issue in period 0 ? Carefully explain your answer.
(e) How would you answer to $\mathrm{b}(i i)$ above change if $k=0.4$ ?

QUESTION 5 US Monetary Tightening, Financial Fragility and Currency Crises
Suppose a small open emerging economy is described by the following Aghion, Bacchetta and Banerjee (2000) model. Money market equilibrium is given by

$$
\frac{M_{t}}{P_{t}}=L\left(Y_{t}, i_{t}\right)
$$

where $M_{t}$ denotes the nominal money supply, $P_{t}$ the aggregate price level, $Y_{t}$ the level of aggregate real output, $i_{t}$ the nominal interest rate, and the subscript $t$ the time period. Real money demand $L($.$) satisfies \partial L / \partial Y>0, \partial L / \partial i<0$ and $L(0, i)>0$. Equilibrium in the foreign exchange market is determined by uncovered interest parity:

$$
1+i_{t}=\left(1+i^{*}\right) \frac{\mathrm{E}_{t}\left[S_{t+1}\right]}{S_{t}}
$$

where $i^{*}$ is the US nominal interest rate, and $S_{t}$ the nominal exchange rate, defined as the domestic price of US dollars. The aggregate price level $P_{t}$ is preset at the end of period $t-1$. Purchasing power parity holds ex ante so that

$$
P_{t}=\mathrm{E}_{t-1}\left[S_{t}\right]
$$

where the foreign price level has been normalized to $P^{*}=1$.
Output is produced with capital $K_{t}$ according to the linear production technology

$$
Y_{t}=A K_{t}
$$

where $A>0$ denotes productivity. Capital accumulation is determined by

$$
K_{t}=B_{t}+F_{t}
$$

where $B_{t}$ is the domestic firm's short term debt, and $F_{t}$ retained cash flow. The firm faces external financing constraints in the form of a borrowing limit

$$
B_{t}=e^{-\beta i_{t-1}} F_{t}
$$

where $\beta>0$. Furthermore, the size of the domestic credit market is limited to $\bar{B}>0$ and any additional borrowing has to take place in US dollars. Assume that $B_{1}>\bar{B}$. Retained cash flow depends on past real profits and equals

$$
F_{t}=\max \left\{0,(1-\alpha) \Pi_{t-1} / P_{t-1}\right\}
$$

where $0<\alpha<1$ and $\Pi_{t}$ denotes nominal profits after debt repayments:

$$
\Pi_{t}=P_{t} Y_{t}-\left(1+i_{t-1}\right) P_{t-1} \bar{B}-\left(1+i^{*}\right) \frac{S_{t}}{S_{t-1}} P_{t-1}\left(B_{t}-\bar{B}\right)
$$

The economy starts from a long run equilibrium in period $t=0$. Assume that initially, $\frac{1+i^{*}}{1+i_{1}} \frac{M_{2}}{L\left(0, i_{2}\right)}<\frac{P_{1} A K_{1}-\left(1+i_{0}\right) P_{0} \bar{B}}{\left(1+i^{*}\right)\left(B_{1}-\bar{B}\right)}$.
(a) Show that asset market equilibrium implies

$$
\begin{equation*}
S_{1}=\frac{1+i^{*}}{1+i_{1}} \frac{M_{2}}{L\left(Y_{2}, i_{2}\right)} \tag{IPLM}
\end{equation*}
$$

Explain intuitively how the domestic nominal interest $i_{1}$, US nominal interest rate $i^{*}$ and future domestic output $Y_{2}$ affect the nominal exchange rate $S_{1}$.
(b) Show that future output $Y_{2}$ satisfies
$Y_{2}=\max \left\{0, A\left(e^{-\beta i_{1}}+1\right)(1-\alpha)\left[A K_{1}-\left(1+i_{0}\right) \frac{P_{0}}{P_{1}} \bar{B}-\left(1+i^{*}\right) \frac{S_{1}}{P_{1}}\left(B_{1}-\bar{B}\right)\right]\right\}$
(FC)
Give an intuitive explanation for the effect of the domestic nominal interest rate $i_{1}$, US nominal interest rate $i^{*}$ and the nominal exchange rate $S_{1}$ on future output $Y_{2}$.
(c) Suppose the economy faces a higher US nominal interest rate $i^{*}$. Show graphically and explain intuitively how this affects the nominal exchange rate $S_{1}$ and future output $Y_{2}$. Could the higher US interest rate lead to a currency crisis in the emerging economy?

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Tuesday 30 May 2017 1:30pm - 3:30pm

F520
BEHAVIOURAL FINANCE

Candidates are required to answer two out of three questions

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Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## QUESTION 1

a. Describe differences and similarities of decision making under Expected utility model and Prospect Theory.
b. Explain what are the main theoretical challenges in testing the Market Efficiency Hypothesis? Fully elaborate your answer.
c. Describe the concept of Adaptive Market Hypothesis.

## QUESTION 2

a. Can herding behaviour be rational? Discuss. Provide some examples to support your answer.
b. Describe main intuition why herding can lead to market inefficiency. Provide some examples to support your answer.
c. Explain how risk-shifting behavior could cause occurrences and subsequently bursts of financial bubbles. Fully justify your arguments. Provide some examples that support your arguments.

## QUESTION 3

a. Large uninformed trades in the stock market typically cause large price impacts (an example of uninformed trades would be fire sales or trades of index tracing fund due to inclusion of a new stock into a stock index). Explain the role of limits to arbitrage in existence of those price impacts. Should we observe the price impacts in markets with active arbitrageurs who face no constraints or risks in executing arbitrage strategies?
b. The closed-end fund discount is typically used as a proxy for noise traders' risk. Provide the rationale and intuition for this argument. Given that we typically observe closed-end fund discounts rather than premiums, does it mean that noise traders are mostly pessimistic?
c. Demonstrate how ambiguity aversion can lead to excess volatility. Consider a case of the model developed in Dow and Werlang (1992) (Excess volatility of stock prices and Knightian uncertainty, European Economic Review). Specifically, consider a two-period model with a single stock traded in the market and risk-free rate is equal to zero. At the end of the second period, the stock pays a liquidating dividends V . There are there are 3 states of nature; the value of $V$ in states 1,2 and 3 are $1,0.5$ or 0 respectively. All agents are identical and risk neutral and makes the decisions according to the multiple prior model (Choquet Expected utility). In period $\mathrm{t}=1$ agents receive public information about the value of the asset represented by a partition $I=\{\{1\},\{2 ; 3\}\}$ of the set of states. Assume that the true objective probability on the space of states is $P(\{1\})=0.5$ and $P(\{2\})=P(\{3\})=0.25$. Assume agents do not know the exact probability and their beliefs and attitudes towards uncertainty are represented by the lower envelope of the following three probability measures: $P, Q$ and $O$, where $Q(\{1\})=Q(\{2\})=0.25$ and $Q(\{3\})=0.5$ and $O(\{1\})=O(\{3\})=0.25$ and $O(\{2\})=0.5$. Provide necessary analysis.

## UNIVERSITY OF

 CAMBRIDGEECM9/ECM10/ECM11/MGM3<br>MPhil in Economics<br>MPhil in Economic Research<br>MPhil in Finance and Economics<br>MPhil Finance

Monday 8 May 2017 02:00pm - 04:00pm

M100
MICROECONOMICS I

Candidates are required to answer three out of four questions

Write your candidate number (not your name) on the cover of each booklet.

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## STATIONERY REQUIREMENTS

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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Consider an economy in which consumers face uncertainty over $S$ possible states of the world. The economy has $I$ consumers, and each consumer $i$ maximizes her expected utility given her own subjective assessment of the probability of each state $s$ being realized. Let $q_{i s}$ be the probability $i$ places on state $s$ being realized. (Assume that these subjective probabilities are NOT caused by different information and so consumers would not update them when learning each others' beliefs directly or indirectly, e.g. through their market behavior.) The economy has one consumption good, of which each consumer $i$ has an endowment $e_{i}>0$, which does NOT depend on the state. Before state $s$ is realized, consumers can buy and sell $S$ different contracts, that respectively each pay out 1 in a different state $s$ and 0 otherwise. This question asks you to study the Walrasian equilibrium of this economy, with the prices $p_{1}, \ldots, p_{S}$ of the contracts. You may assume that agents have constant relative risk aversion utility functions given by $u(c)=\frac{1}{1-r} c^{1-r}$, where $r \neq 1$ is a parameter.
(i) Write down an expression for $r$ in terms of just the shape of then Bernoulli utility function $\left(u^{\prime}(c)\right.$ and $u^{\prime \prime}(c)$ ) and level of consumption $c$.
(ii) Letting $p_{e}$ be the price of the endowment good, argue that in any Walrasian equilibrium $p_{e}=$ $\sum_{s=1}^{S} p_{s}$.
(iii) Letting $x_{i s} \in \mathbb{R}$ be the quantity that consumer $i$ buys of the contract that pays out in state $s$, argue that it is without loss of generality to look for Walrasian equilibria in which $c_{i s}=x_{i s}$ for all $s$.
(iv) Formulate the expected utility maximization problem faced by each price-taking consumer in this economy assuming that $c_{i s}=x_{i s}$ for all $i$ and all $s$.
(v) Solve this problem to obtain an expression for the demands $x_{i s}$ in terms of the prices, subjective probabilities and $i$ 's endowment.
(vi) Write down any additional conditions required for a Walrasian equilibrium in this economy.
(vii) When is there a unique Walrasian equilibrium of this economy?
2. This question asks you to apply the concept of rationalizability to production choices. A price-taking firm can choose any nonnegative input vector $z \in \mathbb{R}_{+}^{m}$ and a nonnegative output vector $y \in \mathbb{R}_{+}^{n}$ subject to the constraint that the vector $(-z, y)$ lies in the firm's production possibility set $Y \subseteq \mathbb{R}_{+}^{m+n}$. Suppose that we can observe the firm's input choices $z$ for different nonnegative input price vectors $w \in \mathbb{R}_{+}^{m}$, but cannot observe the output choices $y$ or output prices $p$ (and may not even know how many outputs $n$ it has). We do know that the output prices are non-negative, such that $p \in \mathbb{R}_{+}^{n}$, and that they do not change between observations.
(i) Suppose that $m=2$, and that at input prices $\widehat{w}=(1,1)$ the firm bought inputs $\widehat{z}=(10,15)$ and at prices $\widetilde{w}=(2,3)$ it bought inputs $\widetilde{z}=(13,14)$. Show that this pair of observations is not rationalizable (i.e. consistent with profit-maximizing behavior for some production set and output prices)? (Hint: What does the firm's choices of $\widehat{z}$ instead of $\widetilde{z}$ at prices $\widehat{w}$ imply?)
(ii) Find a necessary and sufficient condition for two input price-demand observations $(\widehat{z}, \widehat{w})$ and $(\widetilde{z}, \widetilde{w})$ to be rationalizable. Show that your condition is necessary and sufficient. (Hint: Consider how your approach to answering part (i) can be generalized.)
(iii) Suppose we now have the following three observations:

- At prices $\widehat{w}=(1,1)$ the firm bought inputs $\widehat{z}=(10,15)$.
- At prices $\widetilde{w}=(2,3)$ the firm bought inputs $\widetilde{z}=(13,13)$.
- At prices $\bar{w}=(4,1)$ the firm bought inputs $\bar{z}=(8,9)$.

Are these three observations jointly rationalizable?
3. A risk neutral student wants to sell her car. Her value for keeping the car is 0 but dealers have value $v$ for it which they privately observe. From the student's perspective, $v$ is drawn from a cumulative distribution function $F$, with associated probability density function $f$, on the unit interval. Suppose the student offers to sell to the dealer at a price $p$, and the dealer buys if and only if $p<v$.
(i) When does this market yield a Pareto efficient outcome? Do inefficiencies vary with $p$ ?
(ii) Find an expression for the price $p$ that maximizes the student's expected utility. You may assume that $F$ is differentiable.
(iii) Suppose instead that the value is $\widehat{v}$ is drawn from $\widehat{F}$ and that $\widehat{F}$ first order stochastically dominates $F$. Show that under $\widehat{F}$ the student's expected utility is higher when she chooses an optimal $p$.
(iv) Suppose now that instead of quoting an acceptable price to the dealer the student can ask 2 dealers to offer her a price for her car. From the student's perspective, the price each dealer offers $(p)$ is drawn independently from the same cumulative distribution function $G(p)$. The cost of obtaining each quote is $k$ where $0<k<\mathbb{E}[p]$. The student can only visit one dealer at a time and when an offer is made it must be accepted or else it is withdrawn and the dealer will not make another offer. If the student wishes to maximize the expected price she receives net of the cost of obtaining quotes what is her optimal decision rule?
(v) Suppose instead that the distribution quotes are drawn form is $\widehat{G}(p)$ instead of $G(p)$. How will the student's decision rule change if $\widehat{G}(p)$ first order stochastically dominates $G(p)$ ?
(vi) Suppose $\widetilde{G}(p)$ and $G(p)$ have the same mean, but $\widetilde{G}(p)$ second order stochastically dominates $G(p)$. Does the student prefer the dealers' quotes to be drawn from $\widetilde{G}(p)$ instead of $G(p)$ ? Discuss.
4. Consider an economy with two agents indexed 1,2 and two goods indexed $a, b$. Let the agents have the utility functions:

$$
\begin{align*}
& u^{1}\left(a^{1}, b^{1}\right)=a^{1}+b^{1}+\alpha \max \left\{a^{1}, b^{1}\right\}  \tag{1}\\
& u^{2}\left(a^{2}, b^{2}\right)=a^{2}+b^{2}+\alpha \max \left\{a^{2}, b^{2}\right\} \tag{2}
\end{align*}
$$

Individual endowments are given by $e^{1}=\left(\frac{1}{5}, \frac{4}{5}\right)$ and $e^{2}=\left(\frac{4}{5}, \frac{1}{5}\right)$. Except where otherwise stated, assume that $-1<\alpha<0$.
(i) Define a Walrasian equilibrium for this exchange economy.
(ii) Illustrate the rough shape of the indifference curves and the contract curve in an Edgeworth box diagram.
(iii) Provide a formal argument showing that $a^{1}=b^{1}$ on the contract curve.
(iv) Using your Edgeworth box diagram from part (ii) consider an arbitrary point on the contract curve, and argue that there are prices for which this allocation is a Walrasian equilibrium.
(v) Does the contract curve coincide with the set of Walrasian equilibria?
(vi) Find the set of allocations that form the contract curve.
(vii) Find the range of prices that form part of a Walrasian equilibrium.
(viii) Does a Walrasian equilibrium exist if $\alpha=1$ ? Provide a formal argument to support your claim.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Tuesday 23 May 2017 9:00am - 11:00am

M110
MICROECONOMICS II

Candidates are required to answer three out of four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. (a) Consider the following two-player game

| $1 / 2$ | A | B | C |
| :--- | :--- | :--- | :--- |
| A | $a, b$ | 1,1 | $c, d$ |
| B | 1,1 | 0,0 | 1,1 |
| C | $e, f$ | 1,1 | $g, h$ |

where $a, b, c, d, e, f, g, h$ are arbitrary payoffs such that $a+e>2, c+g>2, b+3 d>4$ and $f+3 h>4$.
i. Show that for each player strategy $B$ is strictly dominated by another strategy (possibly a mixed one).
ii. Show that the above game has a pure or a mixed strategy Nash equilibrium. (Hint: First, consider cases in which at least one player has a dominant strategy. Then consider the case in which no player has a dominant strategy.)
(b) Consider a game in which each player $i=1, . ., n$ chooses a real number $s_{i}$ in the interval $[0,1]$ once and simultaneously. Let $\pi_{i}\left(s_{1}, . ., s_{n}\right)$ be the payoff of player $i$ if the players choose a strategy profile $\left(s_{1}, \ldots, s_{n}\right)$. Suppose $\pi_{i}\left(s_{1}, . ., s_{n}\right)$ is differentiable. Explain why, in general, any non-interior Nash equilibrium of this game is Pareto inefficient.
(c) By means of an example or otherwise show that rational players do not necessarily choose a Nash equilibrium in complete information normal form games.
(d) Discuss the assumptions needed to justify the concept of Nash equilibrium (both pure and mixed) in games of complete information.
2. (a) Consider the following linear Cournot model. Two firms, 1 and 2, simultaneously choose the quantities they will sell on the market. If $q_{1}$ and $q_{2}$ are respectively the quantities firms 1 and 2 sell, then the price each receives for each unit given these quantities is $P\left(q_{1}, q_{2}\right)=a-b\left(q_{1}+q_{2}\right)$, where $a$ and $b$ are two positive constants. Each firm $i=1,2$ has a probability $\mu$ of having a cost function given by $c_{L} q_{i}^{2}$ and $(1-\mu)$ of having a cost finction given by $c_{H} q_{i}^{2}$, where $c_{H}$ and $c_{L}$ are two positive constants such that $c_{H}>c_{L}$. Solve for the Bayesian Nash Equilibrium.
(b) Suppose that an object is being auctioned and that there are $n$ bidders. Each bidder $i$ has a private valuation for the object $v_{i}$. The valuation of each bidder is private and independently drawn from a continuously differentiable distribution $F$ on the interval $[0, w]$ where $0<w<\infty$. Each bidder knows his own valuation and the distribution from which the valuations of others are drawn. Bidders are risk-neutral.
i. Suppose that the auction is first-price. Show that the following strategy profile is a symmetric Bayesian Nash equilibrium: each bidder $i$ with valuation $v_{i} \in[0, w]$ bids

$$
v_{i}-\int_{0}^{v_{i}} \frac{(F(x))^{n-1}}{\left(F\left(v_{i}\right)\right)^{n-1}} d x
$$

ii. Suppose that the auction is second-price. Show that the expected revenue of the seller in a symmetric Bayesian Nash equilibrium of this auction is the same as that obtained in the equilibrium of the first-price auction in part i .
3. (a) Show that any game with a finite number of pure strategies has a mixed Nash equilibrium. (You may assume the following result: any normal form game has a Nash equilibrium if the set of strategies for every player is convex and compact and the payoff function for each player $i$ is quasi-concave in $i^{\prime} s$ strategy and continuous in the strategies of all players). Hence, explain briefly why every finite extensive form game has a mixed subgame perfect equilibrium strategy.
(b) Consider a 2-player game $G$ described by the following payoff matrix:

|  | $A$ | $B$ | $C$ |
| :--- | :--- | :--- | :--- |
| $A$ | 3,3 | 0,4 | 3,0 |
| $B$ | 4,0 | 0,0 | $c, d$ |
| $C$ | 0,3 | $a, b$ | 0,0 |

for some paramters $a, b, c$ and $d$. Assume that the players are restricted to using pure strategies.
i. Assume that $a>c+1, c>3, b>3$ and $1>d>0$. Compute the set of Nash equilibria of $G$. Suppose that $G$ in part (b) is played twice, the players do not discount the future and there is perfect monitoring. Show that there exists a subgame perfect equilibrium such that both players choose A in period 1 . Are there any subgame perfect equilibrium that results in outcome $(B, A)$ or $(A, B)$ in period 1 for these paramater values?
ii. Assume that $a=d=1$ and $b=c=4$. Compute the set of Nash equilibrium of $G$. Suppose that $G$ is infinitely repeated and the players discount the future by a factor $\delta<1$. Show that there exists a subgame perfect equilibrium that induces the outcome $(A, A)$ at every period on the equilibrium path if $\delta$ is sufficiently close to 1. Are there any subgame perfect equilibrium that results in the following cyclical equilibrium path: play $(B, A)$ at odd periods and $(A, B)$ at even periods?
iii. Assume that $a=d=1$ and $b=c=2$. Does the game $G$ have a Nash equilibrium? What are the minmax payoffs for each player? Suppose that $G$ is infinitely repeated and the players discount the future by a factor $\delta<1$. Does there exist a subgame perfect equilibrium that induces the outcome $(A, A)$ at every period on the equilibrium path for $\delta$ sufficiently close to 1 ? Explain your answer.
4. (a) Consider the following ultimatum 2-player bargaining game over a pie of size 2 . First, Player 1 makes an offer of 0,1 or 2 . Next player 2 responds to the offer of player 1 by either accepting or rejecting the offer. If an offer $x=0,1,2$ is made and Player 2 accepts then the game ends and the payoffs of 1 and 2 are respectively $x$ and $2-x$. If Player 2 rejects an offer the game ends and each players payoff will be zero.
Describe both the extensive and the normal form representations of the above game. What are the Nash equilibria and the subgame perfect equilibria of this game? Is there a Nash equilibrium that is not credible? If so, explain why this is the case.
(b) Consider the following extensive form game.

i. Show that there does not exist a pure-strategy perfect Bayesian equilibrium in this extensive form game.
ii. Describe the mixed-strategy perfect Bayesian equilibria of the game.
(c) Explain why perfect Bayesian equilibrium may not be always a reasonable solution concept in dynamic games of imperfect information. Discuss briefly how one may refine the concept of perfect Baysian equilibrium to make it a more reasonable solution concept.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Thursday 1 June 2017 1:30pm-3:30pm

M120
TOPICS IN ECONOMIC THEORY

Candidates are required to answer three out of four questions

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. a. Consider the infinite horizon alternating bargaining game with 2 players (Rubinstein model). The size of the pie is 1 . The rules are in odd periods player 1 makes an offer of a split $x \in[0,1]$ to player 2 , which player 2 may accept or reject and in even periods player 2 makes an offer of a split to player 1, which player 1 may accept or reject. If at any period a player accepts an offer, the proposed split is immediately implemented and the game ends. If she rejects, nothing happens until the next period. Assume that each player $i$ discounts future payoffs by a common factor $\delta<1$.
i. Show there exists a subgame perfect equilibrium with the following outcome: there is agreement at the first date and the players receive a unique payoff described by the following split: player 1 receives $\frac{1}{1+\delta}$ and player 2 receives $\frac{\delta}{1+\delta}$.
ii. Explain why the equilibrium payoff described in part (a) is the unique subgame perfect equilibrium outcome (A sketch of the arguments will suffice).
iii. Modify the above bargaining game by giving player 2 the following option: at the end of any even period player 2 can opt out and obtain a payoff of $s \in \mathbb{R}_{+}$ after rejecting the offer of player 1 . Assume that $s \leq \frac{\delta}{1+\delta}$. Describe the subgame perfect equilibrium of this modified game.
b. Consider the set of 2-person bargaining problems $\mathcal{B}$ where the set of possible utility agreements is some subset of $\mathbb{R}_{+}^{2}$ and disagreement results in a payoff vector $d=\left(d_{1}, d_{2}\right) \in \mathbb{R}_{+}^{2}$. Thus $\mathcal{B}=\left\{(U, d) \mid U \subseteq \mathbb{R}_{+}^{2}\right.$ and $\left.d \in U\right\}$. Let $f: \mathcal{B} \rightarrow \mathbb{R}^{2}$ be the bargaining solution. Thus, $f(U, d) \in U$ describes the utility vector the players will receive if $U \subseteq \mathbb{R}_{+}^{2}$ is the set of possible utility agreements and $d$ is the disagreement payoff vector.
i. Suppose that $f$ is such that, for any positive real numbers $a, b$ and $c$,
$f(U, d)=\left(\frac{c}{2 a}, \frac{c}{2 b}\right)$ if $U=\left\{\left(u_{1}, u_{2}\right) \in \mathbb{R}_{+}^{2} \mid a u_{1}+b u_{2} \leq c\right\}$ and if $d=0$.
Assume that $f$ satisfies Independent of irrelvant alternatives. Show that $f(U, d)=\arg \max _{\left(u_{1}, u_{2}\right) \in U} u_{1} u_{2}$ if $U$ is convex, $d=0$.
ii. Suppose that $f$ is egalitarian; that is $f(U, d)=\arg \max _{\left(u_{1}, u_{2}\right) \in U} \min _{i=1,2} u_{i}$. Show that $f$ satisfies Pareto and independence of irrelevant alternatives axioms but it violates independent of utility units.
2. a. Suppose that there exists a mechanism that implements the social choice function $f(\cdot)$ in Bayesian Nash equilibrium. Show that $f(\cdot)$ is truthfully implementable in Bayesian Nash equilibrium.
b. Consider the following mechanism design problem of implementing a project. There is a finite number of agents $I$ and a finite set of projects denoted by $K$. An alternative is given by a vector $x=\left(k, t_{1}, \ldots, t_{I}\right)$ where $k \in K$ is the project choice and $t_{i} \in \mathbb{R}$ is a transfer to agent $i$. Denote the set of private signals each agent $i$ receives by $\Theta_{i}$. Assume $\Theta_{i}$ is finite. Agent $i$ 's utility is given by

$$
u_{i}\left(x, \theta_{i}\right)=w_{i}\left(v_{i}\left(k, \theta_{i}\right)+t_{i}\right)
$$

where $v_{i}: K \times \Theta_{i} \rightarrow \mathbb{R}_{+}$and $w_{i}: \mathbb{R} \rightarrow \mathbb{R}$. Assume that $w_{i}($.$) is an increasing$ function. For any $\theta=\left(\theta_{1}, \ldots, \theta_{I}\right) \in \times_{i=1}^{I} \Theta_{i}$, let $k^{*}(\theta)$ be an expost efficient project; thus

$$
\begin{equation*}
\sum_{i=1}^{I} v_{i}\left(k^{*}(\theta), \theta_{i}\right) \geq \sum_{i=1}^{I} v_{i}\left(k, \theta_{i}\right) \text { for all } k \in K \tag{1}
\end{equation*}
$$

Show that the social choice function $f(\cdot)=\left(k^{*}(\cdot), t_{1}(\cdot), \ldots, t_{I}(\cdot)\right)$ is truthfully implementable in dominant strategies if, for all $i=1, \ldots, I$ and all $\theta=\left(\theta_{i}, \theta_{-i}\right) \in$ $\times_{i=1}^{I} \Theta_{i}$,

$$
t_{i}(\theta)=\left[\sum_{j \neq i} v_{j}\left(k^{*}(\theta), \theta_{j}\right)\right]+h_{i}\left(\theta_{-i}\right)
$$

where $h_{i}(\cdot)$ is an arbitrary function of $\theta_{-i}$.
c. Consider the mechanism design problem described in part b. Suppose that $w_{i}($.$) is$ the identity function for all $i$; thus each agent $i$ 's utility is given by

$$
u_{i}\left(x, \theta_{i}\right)=v_{i}\left(k, \theta_{i}\right)+t_{i} .
$$

Also assume that each agent $i$ 's signal $\theta_{i} \in \Theta_{i}$ is independently distributed according to some probability distribution. Show that for any expost efficient project $k^{*}($.$) ,$ there exists transfer functions $\left(t_{1}(\cdot), \ldots, t_{I}(\cdot)\right)$ such that $\sum_{i=1}^{I} t_{i}(\theta)=0$ and the social choice function $f(\cdot)=\left(k^{*}(\cdot), t_{1}(\cdot), \ldots, t_{I}(\cdot)\right)$ is truthfully implementable in Bayesian Nash equilibrium.
3. Consider the Crawford-Sobel model of strategic information transmission. An expert privately observes the state $\theta \in \Theta=[0,1]$ and then sends a message $m \in M=[0,1]$ to a decision maker. The decision maker does not observe $\theta$ but (possibly) obtains information about it from the expert's message. Before receiving a message from the expert, the decision maker has a prior belief about the state $\theta$ that is uniformly distributed on $[0,1]$. The decision maker observes the message and then chooses an action $y \in \mathbb{R}$. The expert's payoff function is $u^{S}(y, \theta)=-(y-(\theta+b))^{2}$, where $b$ is some positive constant, and that of the decision maker is $u^{R}(y, \theta)=-(y-\theta)^{2}$.
a. Define a pure strategy perfect Bayesian equilibrium in this setup. Explain why the restriction to pure strategy equilibria is without loss of generality in this context.
b. Describe a "babbling" equilibrium of this game and explain why this kind of equilibrium always exists.
c. Find a three-message perfect Bayesian equilibrium for this game in which there is information transmission.
d. Compute the decision maker's ex ante expected payoff in the three-message perfect Bayesian equilibrium.
4. Two agents $(i=1,2)$ are engaged in a productive relationship. There are two physical assets, $a_{1}$ and $a_{2}$. There are two stages. At stage 1 each simultaneously chooses a level of investment, $i$ 's investment being $e_{i} \in[0, \infty)$. At stage 2 the owner of asset $j(j=1,2)$ chooses an action $y_{j} \in\{h, l\}$. The two actions $y_{1}$ and $y_{2}$ are chosen simultaneously. The agents can also make money transfers to each other. The payoff for agent $i(i=1,2)$ is

$$
V_{i}\left(e_{1}, e_{2}, y_{1}, y_{2}, t_{i}\right)=\left(e_{1}+e_{2}\right) \phi_{i}\left(y_{1}, y_{2}\right)+t_{i}-e_{i}^{2}
$$

where $t_{i}$ is the net money transfer to $i$ and $\phi_{i}$ is given by the following matrix, in which the row corresponds to $y_{1}$, the column to $y_{2}$, and the first entry in a cell gives $\phi_{1}\left(y_{1}, y_{2}\right)$ and the second entry gives $\phi_{2}\left(y_{1}, y_{2}\right)$

|  | $h$ | $l$ |
| :---: | :---: | :---: |
| $h$ | 2,2 | $-1,4$ |
| $l$ | $4,-1$ | 0,0 |

At the outset, before stage 1, the agents contract. The only contracts permitted are ownership contracts. At stage 2 they engage in Nash bargaining and can negotiate a binding contract on $\left(y_{1}, y_{2}, t_{1}, t_{2}\right)$. It is not possible to leave the relationship.
a. Find the first-best investments and the maximum total surplus.
b. Solve for the subgame-perfect equilibrium in the case (i) in which 1 owns both assets, and (ii) in which 1 owns asset 1 and 2 owns asset 2 . Which ownership structure is better? Is either structure efficient? If not, discuss the intuition for this.
c. If agent 1 initially owns both assets and initially has all the bargaining power, what would you expect to happen at the initial contracting stage (i) if both are already committed to the relationship, and (ii) if they are not?
d. Do the two assets satisfy Grossman and Hart's definitions of complementary and independent assets?

## END OF PAPER

ECM9/ECM10/ECM11<br>MPhil in Economics<br>MPhil in Finance \& Economics<br>MPhil in Economic Research

Wednesday 31 May 2017 9:00am - 11:00am

M130
APPLIED MICROECONOMICS

This paper is in two sections. Candidates are required to answer four out of six questions in Section A; and the one compulsory question in Section B.

Questions within Section A carry equal weight. Each section carries equal weighting.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Section A

1. Consider an economy with only two individuals who differ in terms of their ability. How does cutting the marginal tax rate of the lower ability person affect the possible incentive compatible tax rates for the high ability?
2. Suppose there is one input, labour $z$, and one output, $y$, in th economy. The government wants to raise revenue, $R$, but there are no lump sum taxes. What prices should the producers face? What prices should consumers face?
3. What is the difference between the Marshallian and Hicksian wage elasticity of labour supply? What is the Firsch elasticity? If wages are known to be high this year, but falling next year, how would you evaluate the labour supply response?
4. Assuming preferences are quadratic,

$$
u(c)=-\frac{1}{2}(b-c)^{2}
$$

what would the effect on consumption be of (i) a permanent increase in income and (ii) a transitory increase in income? How would your answer change if preferences had the CRRA form:

$$
u(c)=\frac{c^{1-\gamma}}{1-\gamma}
$$

5. The government believes uncertainty about the economic situation after Brexit will cause individuals to spend less for the next two years. It is considering announcing now that VAT will be cut to $15 \%$ when Brexit actually happens. Evaluate this policy.
6. An econometrician observes that $30 \%$ of recipients of disability insurance are not actually limited in how much work they can do. She also observes that only $43 \%$ of those who are work limited are in receipt of disability insurance. What can be infered from these observations about the disability insurance program?

## Section B

1. A researcher estimates a demand system by OLS with each budget share regression given by:

$$
w_{i h}=\alpha_{i 0}+\alpha_{i a d} \text { adults }_{h}+\alpha_{i k i d s} k i d s_{h}+\sum_{j=1}^{7} \gamma_{i j} \ln p_{j, h}+\beta_{i} \ln \frac{x_{h}}{P_{h}}+u_{i h}
$$

where $w_{i h}$ is the budget share on good i for household h ; the price index $P_{h}$ is the Stone price index for household $h$. There are seven goods in this demand system: food at home (fath); food in restaurants (rest); transport (tranall); services (serv); recreation (recr); alcohol and tobacco (vice); and clothing (clth). The coefficient estimates from each OLS regression are given in the columns in Figure 1 (overleaf).
(a) What conditions have to hold for the adding-up condition to hold? What would it mean if the adding up condition were violated?
(b) Calculate the Hicksian, Marshallian and Income elasticities for services (serv), clothing (clth) and food in restaurants (rest). The budget share on services is $17 \%$, on clothing is $10.5 \%$ and on food in restaurants is $9 \%$.
(c) Explain whether the estimates are consistent with a utility function of the form:

$$
u=\left(q_{\text {fath }}^{\alpha_{1}} \cdot q_{\text {rest }}^{\alpha_{2}} \cdot q_{\text {tranall }}^{\alpha_{3}} \cdot q_{\text {serv }}^{\alpha_{4}} \cdot q_{\text {recr }}^{\alpha_{5}} \cdot q_{\text {vice }}^{\alpha_{6}} \cdot q_{c l t h}^{\alpha_{7}}\right)
$$

where $\alpha_{i}$ are preference parameters.
(d) Two concepts that can be used to measure the change in welfare assoicated with a price change are the equivalent variation and compensating variation. How do these measures differ? When are the two measures going to give identical answers?
(e) Using either the compensating variation or the equivalent variation, calculate the deadweight loss of imposing a $5 \%$ tax on each of the three goods: clothing, food in restaurants, services.
(f) Assume that initially there is no VAT on clothing but food in restaurants and services are both subject to a $20 \%$ rate of VAT. Is there a way to change the tax schedule to make it more efficient at raising a given amount of revenue?

Figure 1: Estimated Demand System

| Variable | BS fath | BS rest | BS tranall | BS serv | BS recr | BS vice | BS clth |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |
| No of adults | 0.079 | -0.040 | 0.022 | -0.014 | -0.026 | -0.019 | -0.001 |
|  | $(0.002)$ | $(0.0015)$ | $(0.002)$ | $(0.002)$ | $(0.002)$ | $(0.002)$ | $(0.001)$ |
| No of kids | 0.053 | -0.020 | -0.018 | 0.019 | -0.003 | -0.015 | -0.016 |
|  | $(0.0008)$ | $(0.001)$ | $(0.001)$ | $(0.001)$ | $(0.001)$ | $(0.001)$ | $(0.001)$ |
| log p(fath) | -0.086 | 0.075 | -0.120 | 0.064 | 0.032 | -0.039 | 0.073 |
|  | $(0.026)$ | $(0.019)$ | $(0.026)$ | $(0.021)$ | $(0.021)$ | $(0.023)$ | $(0.018)$ |
| log p(rest) | -0.014 | -0.021 | 0.150 | -0.065 | 0.085 | -0.069 | -0.066 |
|  | $(0.022)$ | $(0.012)$ | $(0.022)$ | $(0.018)$ | $(0.018)$ | $(0.020)$ | $(0.015)$ |
| $\log p$ (tranall) | 0.010 | 0.009 | 0.006 | -0.016 | -0.009 | -0.004 | 0.004 |
|  | $(0.0095)$ | $(0.007)$ | $(0.003)$ | $(0.008)$ | $(0.008)$ | $(0.008)$ | $(0.007)$ |
| $\log p$ perv) | 0.029 | 0.002 | 0.017 | -0.028 | -0.089 | 0.089 | -0.020 |
|  | $(0.026)$ | $(0.019)$ | $(0.027)$ | $(0.015)$ | $(0.021)$ | $(0.023)$ | $(0.018)$ |
| $\log p$ (recr) | 0.066 | -0.064 | -0.013 | 0.087 | 0.014 | -0.079 | -0.010 |
|  | $(0.031)$ | $(0.023)$ | $(0.031)$ | $(0.025)$ | $(0.025)$ | $(0.028)$ | $(0.021)$ |
| log p(vice) | -0.008 | 0.007 | -0.029 | -0.019 | -0.035 | 0.069 | 0.016 |
|  | $(0.010)$ | $(0.008)$ | $(0.010)$ | $(0.008)$ | $(0.008)$ | $(0.009)$ | $(0.007)$ |
| log p(clth) | -0.093 | -0.003 | 0.006 | 0.070 | 0.066 | 0.000 | -0.046 |
|  | $(0.023)$ | $(0.017)$ | $(0.024)$ | $(0.019)$ | $(0.019)$ | $(0.021)$ | $(0.016)$ |
| log real expend | -0.160 | 0.050 | 0.022 | -0.011 | 0.046 | 0.017 | 0.037 |
|  | $(0.002)$ | $(0.001)$ | $(0.002)$ | $(0.001)$ | $(0.001)$ | $(0.002)$ | $(0.001)$ |
| Constant | 1.624 | -0.306 | -0.043 | 0.283 | -0.284 | -0.030 | -0.244 |
|  | $(0.016)$ | $(0.012)$ | $(0.016)$ | $(0.013)$ | $(0.013)$ | $(0.014)$ | $(0.011)$ |

Standard errors are given in brackets.

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Thursday 25 May 2017 1:30pm - 3:30pm

M150
ECONOMICS OF NETWORKS

Candidates are required to answer three out of four questions.
Each question carries equal weight

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1: Consider a $n$ player network formation game. Suppose that two players $i$ and $j$ can form a link if they both agree, and pay a cost $c>0$. The network created by this bilateral linking is denoted by $\mathbf{g}$. The payoffs to player $i$ under network $\mathbf{g}$ are

$$
\begin{equation*}
\Pi_{i}(\mathbf{g})=1+\sum_{j \neq i} \delta^{d(i, j ; \mathbf{g})}-\eta_{i}^{d}(\mathbf{g}) c \tag{1}
\end{equation*}
$$

where $d(i, j ; \mathbf{g})$ is the (geodesic) distance between $i$ and $j$ in network $\mathbf{g}$ and $\delta \in(0,1)$ is the decay factor.
a. Define a pairwise stable network. Provide the range of parameter values, $c$ and $\delta$, for which the empty, the complete and star network are pairwise stable.
b. Now consider a variant of the model in which link creation and benefit flow are both directed (in the spirit of Twitter). So only directed paths matter for flow of benefits. Suppose that there is no decay. Write out the payoffs carefully. Show that the cycle containing all nodes is the unique strict (non-empty) Nash network.

Question 2: Consider a $n$ player network formation game. Link formation is two-sided. Every every pair of players who have a path between them create a total surplus of 1. Assume that each player pays $c>0$ for every link he forms. Suppose that the surplus is independent of the length of the path. The payoffs are shared with players who are critical for the exchange. Define player $i$ to be critical for players $j$ and $k$ if she lies on every path between them in the network. Let $C_{j k}(\mathbf{g})$ be the set of critical players for $j$ and $k$ in network $\mathbf{g}$. The surplus of 1 is equally divided between $j$ and $k$ and all the critical players $C_{j k}(\mathbf{g})$.
a. Consider a game with 5 players and write down all the critical players in a cycle and the star network.
b. Write down the strategies of linking and the payoffs. Discuss the different incentives to create links.
c. Show that a star and a cycle containing all players are both pairwise stable.
d. Discuss the strategic robustness of the cycle and the star network. In particular, examine the possibility of sustaining large intermediation rents if pairs of players are able to coordinate their linking activity.

Question 3: Consider a $n$ player game on a network g. Players simultaneously chooses a network action $x_{i} \in\{0,1\}$ and a market action $y_{i} \in\{0,1\}$. Define $a_{i}=\left(x_{i}, y_{i}\right)$. Suppose that in a network $\mathbf{g}$ faced with a action profile $a=\left(a_{1}, a_{2}, . ., a_{n}\right)$, the payoff function for player $i$ is given by

$$
\begin{equation*}
\Phi_{i}(a \mid \mathbf{g})=\left(1+\theta y_{i}\right) x_{i} \chi_{i}(a \mid \mathbf{g})+y_{i}-x_{i} p_{x}-y_{i} p_{y} \tag{2}
\end{equation*}
$$

where $\chi_{i}(a \mid \mathbf{g})$ is the number of neighbors in network $\mathbf{g}$ who choose the network action, and $p_{x} \geq 0$ and $p_{y} \geq 0$, respectively, are the prices of actions $x$ and $y$. We say that the actions $x$ and $y$ are substitutes if $\theta \in[-1,0]$ and complements if $\theta \geq 0$. A Nash equilibrium is said to be maximal if there does not exist another equilibrium that Pareto dominates it.
a. Suppose that $\theta=-1, p_{x}=4$ and $p_{y}=0.5$. Describe how the network shapes behavior in the maximal equilibrium.
b. Suppose that $\theta=1, p_{x}=7$ and $p_{y}=2$. Describe how the network shapes behavior in the maximal equilibrium.
c. Assess the impact of markets on aggregate welfare (measured as the sum of individual payoffs) and inequality (measured as a ratio of highest versus the lowest income) in the two above settings.

Question 4: Consider a four-region economy, with regional liquidity demands in state $S_{i}, i=1,2$, as specified in the table below. Each state takes place with equal probability.

| Regional liquidity demand |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Region |  |  |  |  |  |  |  |  |  |  |
| State |  |  |  |  |  |  | A | B | C | D |
| $S_{1}$ | 0.75 | 0.25 | 0.75 | 0.25 |  |  |  |  |  |  |
| $S_{2}$ | 0.25 | 0.75 | 0.25 | 0.75 |  |  |  |  |  |  |

There is a liquid asset which represents a storage technology. Investment in a long asset is available at $t=0$. Per unit invested in the long asset, the yield is of $r=0.4$ at $t=1$ (premature liquidation), and of $R>1$ at $t=2$. Assume that the period utility function is of the form $u\left(c_{t}\right)=\ln \left(c_{t}\right)$.
a. Denote as $y$ and $x$ the per capita amounts that the social planner invests in the short and long assets, respectively. The feasibility constraint is thus $y+x \leq 1$. Noting that at $t=0$ the probability of being an early or a late consumer equals 0.5 , derive the first-best allocation.
b. Describe the combination of investment decisions and interbank deposits that achieve the firstbest allocation when (i) the representative bank of a region holds deposits in the representative banks of all other regions (complete structure); and (ii) the representative bank of region $\mathbf{A}$ deposits in the representative bank of region $\mathbf{B}$, the latter deposits in the representative bank of region $\mathbf{C}$, and so on (a cycle network). For each case, explain the sequence of withdrawals if state $S_{1}$ takes place.
c. Next suppose that an unanticipated state $\bar{S}$ (assigned zero probability at $t=0$ ), takes place. In this state, regions $\mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ have a liquidity demand of 0.5 , but region $\mathbf{A}$ faces a demand of $0.5+\varepsilon$. Consider the cycle network and set $\varepsilon=0.1$ and $R=1.5$. Show that there is no contagion. Would your results change if $\varepsilon$ was larger?

END OF PAPER CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Tuesday 30 May 2017 9:00am - 11:00am

M170
INDUSTRIAL ORGANISATION

Candidates are required to answer 3 compulsory questions

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

Consider three firms $i=1,2,3$ located on a circular city with distance 1 between them (so the total length of the city is 3 ). Consumers with unit demand for a product are uniformly distributed on the circle with density one. Firms produce at constant and symmetric marginal costs $c>0$ and offer uniform prices to consumers. A consumer's utility is given by

$$
u(x, p)=\left\{\begin{array}{c}
s-d-p \text { if } \quad p \leq s-d \\
0 \text { if } \quad p>s-d
\end{array}\right.
$$

where $d \geq 0$ denotes the distance of the consumer's location to that of the firm and $p$ is the price charged for the product. Assume throughout that there is full market coverage.
a. Consider the case in which all firms remain independent and compete non-cooperatively in prices. Find the equilibrium prices $p_{i}^{*}$, quantities $q_{i}^{*}$ and profits $\pi_{i}^{*}, i=1,2,3$.
b. Now consider a merger between firms 1 and 2 to form. This merged company retains the product varieties (locations) of firms 1 and 2 and competes against the remaining independent firm 3. You may assume that the two firms that merge behave symmetrically, so that $p_{1}=p_{2}=\hat{p}, q_{1}=q_{2}=\hat{q}$ and $\pi_{1}=\pi_{2}=\hat{\pi}$. Find the equilibrium prices $\hat{p}^{*}$ and $p_{3}^{*}$, quantities $\hat{q}^{*}$ and $q_{3}^{*}$ and profits $\hat{\pi}^{*}$ and $\pi_{3}^{*}$, noting that quantities and profits for the merged firm are $2 \hat{q}^{*}$ and $2 \hat{\pi}^{*}$, respectively.
c. Compare the equilibrium prices, quantities and profits before and after the merger. Who benefits/loses from the merger, in absolute and in relative term?

## Question 2

Consider two downstream firms who produce horizontally differentiated goods and who compete in quantities. To produce one unit of the final good, the downstream firm need one unit of an intermediate good. It can either produce the intermediate good in-house (In), our outsource it to a perfectly competitive upstream firm (Out). In-house production involves marginal cost $c>0$, whereas the marginal cost of outsourcing depends on how much the upstream firm sells. Assume that the upstream firm has economies of scale such that if only one downstream firm outsources, it can offer the intermediate good at marginal cost $\bar{k}>c$ whereas if both downstream firms outsource, it can offer a reduces marginal cost $\underline{k}<c$. The two firms simultaneously and non-cooperatively make outsourcing decisions.

At the market stage, the firms face inverse demand functions

$$
\begin{aligned}
& p_{1}=a-b q_{1}-d q_{2} \\
& p_{2}=a-b q_{2}-d q_{1}
\end{aligned}
$$

a. Illustrate the normal form of the outsourcing game, denoting by $\pi_{i}\left(c_{i}, c_{j}\right)$ the profits accruing to firm $i=1,2$ when it has $\operatorname{cost} c_{i} \in\{c, \underline{k}, \bar{k}\}$ and the rival has $\operatorname{costs} c_{j} \in\{c, \underline{k}, \bar{k}\}$.
b. Solve for the pure strategy Nash equilibria of the outsourcing game, paying special attention to the parameter restrictions that must be imposed in each case.
c. Interpret your findings, with reference to the restrictions imposed in sub-question 2. Are equilibrium outsourcing decisions always efficient? Does efficient in-house production (measured as a low $c$ ) make efficient outsourcing decisions more or less likely?

## Question 3

Consider two firms producing horizontally differentiated goods. The firms are located at the end-points of a linear city of unit length. Consumers have unit demand and are uniformly distributed along the city with density one. Firms produce at constant and symmetric marginal costs $c>0$ and offer uniform prices to consumers. A consumer's utility is given by

$$
u(x, p)=\left\{\begin{array}{rl}
s-\tau d-p & \text { if } \quad p \leq s-\tau d \\
0 & \text { if }
\end{array} \quad p>s-\tau d .\right.
$$

where $d \geq 0$ denotes the distance of the consumer's location to that of the firm and $p$ is the price charged for the product. The constant $\tau>0$ is the travel cost borne by the consumers. Assume throughout that there is full market coverage.
a. Consider an infinite period game in which the firms play the market game in each period $t=1,2, \ldots$ The future is discounted by a factor $\delta \in(0,1)$. Carefully define grim trigger strategies in this setting.
b. Derive explicit expressions $\pi^{C}, \pi^{D}$ and $\pi^{N}$, i.e. for the payoffs to cooperating, deviating and playing according to the (stage-game) Nash equilibrium respectively. Derive a condition on the discount factor $\delta$ that determines whether tacit collusion in grim trigger strategies is feasible.
c. Carefully determine how $\pi^{C}, \pi^{D}$ and $\pi^{N}$ vary with the transport cost $\tau$. Explain the tradeoffs involved and discuss how these findings influence the sustainability of tacit collusion via grim trigger strategies.

## UNIVERSITY OF

 CAMBRIDGEECM9/ECM10<br>MPhil in Economics<br>MPhil in Economic Research

Friday 26 May 2017 1:30pm - 3:30pm

M180
LABOUR: SEARCH, MATCHING AND AGGLOMERATION

Candidates are required to answer three out of four questions. Each question carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Consider the Postel-Vinay \& Robin (2002) model. We assume that workers and firms are homogeneous. The productivity of a match is $p$. Parameters are as they were in the lecture notice that is: $\rho$ is the discount rate, $b$ is the flow value of unemployment, $\lambda_{0}$ is the arrival rate of jobs to unemployed, $\lambda_{1}$ is the arrival rate of jobs to employed, $\delta$ is the job destruction rate. Let $U$ be the value of unemployment and $W(w)$ the value of employment at a wage $w$.
(a) Write the Bellman equation for the value of unemployment.
(b) Let $w_{0}$ be the wage that makes a worker indifferent between remaining in unemployment and beginning work $\left(U=W\left(w_{0}\right)\right)$. Use this relation to simplify the expression under (a) and provide an intuition for the result.
(c) What wage offer will an employed worker get when receiving an outside offer from another employer? Is there any indeterminacy about whether he will move/stay?
(d) Write the Bellman equation for a worker at his maximum wage and simplify this expression to obtain the maximum value a worker can get.
(e) Show that

$$
w_{0}=b-\frac{\lambda_{1}}{\rho+\delta}(p-b)
$$

(f) What happens to $w_{0}$ if (i) as $\lambda_{1} \rightarrow 0$ and (ii) as $\rho \rightarrow \infty$ ? Explain your results.
(g) What is the distribution of wages amongst employed?
(h) Derive the mean wage paid in the economy.
(i) What is the mean wage wage when workers don't discount future earnings? Give intuition for your answer and discuss the implications for firms vacancy posting decisions.
2. Minimum wages and job search models
(a) Give a short overview of the stylized facts regarding the introduction of a minimum wage.
(b) David Lee regresses the $10-50 \%$ log wage differential on inter-state variation in minimum wages. He finds a strong negative effect: the higher the minimum wage, the smaller the $10-50 \%$ differential. Can this result be interpreted as evidence that the wage differentials between workers with different human capital are compressed by an increase in the minimum wage?
(c) To what extend can these facts be explained by job search models?
(d) Some studies find that a higher minimum wage might increase employment. What do you have to assume on wage setting, labour supply, vacancy creation, and the matching function to explain this outcome?
3. Public transport in cities
(a) Consider a circular city as in Lucas \& Rossi Hansberg (2002). Suppose that the city would develop a public transport railsystem with a stop at the city center that extends from there in all directions with a single stop at all locations at distance $R$ from the center (so every point on the ring $R$ has a stop that connects it to the city center). Assume that the city autorities have chosen a proper location for these stops such that the system is useful for commuting. Furthermore, this city turns out to have a mixed neighbourhood where residential and commercial use of land coincide. Commuting cost by any other mode of transport are $\kappa$ per unit of distance, commuting by public transport occurs at cost $\psi<\kappa$ per unit of distance. Depict a possible profile of land rents and depict the locations of the various zones (residential, commercial, mixed). Explain your answer, in particular the location of the zones and the wage function that goes with this pattern.
(b) Consider the density of land use in the neighbourhood of the stations, both at the center and at location $R$. Is this density is lower or higher than at locations further away from the station? Is this efficient?
(c) Public transport is a means of transport with high fixed cost (construction of tracks etc.) and low variable cost per trip. Suppose you are the mayor of the city. Design an efficient system for funding the public transport system. Explain your design. What are the drawbacks of alternative designs?
4. Agglomeration of human capital
(a) The agglomeration of people in cities has played an important role in the evolution of the world economy since the rise of agriculture. Give a short historical overview in at most 10 sentences.
(b) Gennaio et.al. (2013) provide a model for the agglomeration of human capital within countries. Describe the main features of that model in at most 10 sentences. Pay attention to the role of economies of scale, knowledge spill-overs, and entrepreneurship. Are there externalities in their model?
(c) Is the equilibrium efficient in their model? Why or why not? If not, what policies could help to alleviate the problem?
(d) Who captures the benefits of agglomeration in their model?
(e) Teulings \& Van Rens (2008) show that an increase in the average level of education in a country reduces the return to human capital. Is this consistent with the analysis of Gennaio et.al.? Explain your answer.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research
Wednesday 10 May $2017 \quad 2: 00 \mathrm{pm}-4: 00 \mathrm{pm}$

M200
MACROECONOMICS I

The paper is in two sections: You should answer all four questions in Section A and one question from Section B. Each Section carries equal weight.

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## STATIONERY REQUIREMENTS

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SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS
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## SECTION A

## A. 1 Solow Growth Model

Consider the following continuous time Solow growth model. The production function is:

$$
Y(t)=\left(A_{K}(t) K(t)\right)^{\alpha}\left(A_{L}(t) L(t)\right)^{1-\alpha}, \quad \alpha \in(0,1),
$$

where $Y(t)$ denotes aggregate output, $K(t)$ aggregate capital, and $L(t)$ aggregate labour force; $\dot{A}_{K}(t)=\gamma_{K} A_{K}(t)$ and $\dot{A}_{L}(t)=\gamma_{L} A_{L}(t)$, where $\gamma_{K}$ and $\gamma_{L}$ are positive and exogenous; $t$ indicates time. The initial values of $A_{K}(0), A_{L}(0)$ and $K(0)$ are given and they are positive. The labour force grows at a constant rate $n$ and agents save a fraction $s \in(0,1)$ of output. The economy is closed and capital evolves according to the following equation:

$$
\dot{K}(t)=I(t)-\delta K(t),
$$

where $I(t)$ denotes investment and $\delta$ the depreciation rate with $\delta>0$. Derive the growth rate of capital, $K(t)$, on the balanced growth path. Is $\frac{K(t)}{A_{L}(t) L(t)}$ constant on the balanced growth path? Explain how your results depend on $\gamma_{K}$.

## A. 2 Investment Theory and Unanticipated Permanent Change in Interest Rates

Consider a $q$-theory model of investment with the following dynamics for the capital stock $K_{t}$ and the shadow price of capital $q_{t}$ :

$$
\begin{align*}
\Delta K_{t+1} & =\frac{1}{\chi}\left(q_{t}-1\right)  \tag{1}\\
\Delta q_{t+1} & =r q_{t}-A F^{\prime}\left(K_{t}+\frac{1}{\chi}\left(q_{t}-1\right)\right),
\end{align*}
$$

where $r$ denotes the real interest rate, with $1+r>0$; the production function is given by $Y_{t}=A F\left(K_{t}\right)$ with $F^{\prime}(\cdot)>0$ and $F^{\prime \prime}(\cdot)<0$, where $A$ is a positive productivity parameter; and $\chi$ is a positive parameter related to the investment adjustment costs. The initial capital stock $K_{0}>0$ is given and the transversality condition is $\lim _{T \rightarrow \infty}\left(\frac{1}{1+r}\right)^{T} q_{T} K_{T+1}=0$.
Draw the phase diagram from the system of equations (1) and explain the dynamics implied by this model. Suppose that the interest rate $r$ unexpectedly and permanently increases from $r$ to $r^{\prime}$, with $r<r^{\prime}$. Carefully explain how $K_{t+1}$ and $q_{t}$ are affected over time using a phase diagram and provide an intuitive explanation for the effect on investment.

## A. 3 Central Bank Contract with Output Growth Bonus

Suppose the economy of Americana is described by the following Phillips equation:

$$
\pi=\pi^{e}+\left(g_{Y}-\bar{g}_{Y}\right)+\varepsilon
$$

where $\pi$ denotes inflation, $g_{Y}$ the growth rate of real output, $\bar{g}_{Y}$ the long run growth rate of real output, and $\varepsilon$ a white noise cost push shock. Private sector inflation expectations $\pi^{e}$ are formed using rational expectations at the beginning of the period, before the shock $\varepsilon$ has been realized. Monetary policy is delegated to a central banker who sets real output growth $g_{Y}$ after the shock $\varepsilon$ has been observed. The central banker minimizes the following loss function:

$$
L=\frac{1}{2}\left(\pi-\pi^{*}\right)^{2}+\frac{1}{2}\left(g_{Y}-g_{Y}^{*}\right)^{2}-\beta\left(g_{Y}-g_{Y}^{*}\right),
$$

where $\pi^{*}$ denotes the central banker's inflation target, $g_{Y}^{*}$ the central banker's real output growth target, and $\beta$ a non-negative parameter. Suppose that before the beginning of the period, the new president of Americana appoints a central banker with an inflation target of $2 \%$ and a real output growth target of $4 \%$, while long run real output growth equals $2 \%$. In addition, the president decides to provide the central banker a personal incentive to achieve the real output growth target through a 'bonus' payment that is increasing in real output growth $g_{Y}$, so that $\beta>0$.
Derive the equilibrium outcome of private sector inflation expectations $\pi^{e}$, real output growth $g_{Y}$ and inflation $\pi$. Explain intuitively how these outcomes are affected by the real output growth bonus.

## A. 4 Anticipated Negative Aggregate Demand Shock in Dornbusch Model

Suppose the economy of Britannia can be described by a Dornbusch model with the following short-run dynamics for the (log) nominal exchange rate $s$ (defined as the domestic price of foreign currency) and the (log) aggregate price level $p$ :

$$
\begin{aligned}
& \dot{s}=(s-\bar{s})+(p-\bar{p}) \\
& \dot{p}=(s-\bar{s})-(p-\bar{p})
\end{aligned}
$$

where the steady state (indicated by an upper bar) equals $\bar{s}=\bar{q}+\bar{k}$ and $\bar{p}=\bar{p}^{*}+\bar{k}$, with macroeconomic fundamentals $\bar{k} \equiv\left(\bar{m}-\bar{m}^{*}\right)-\frac{1}{2}\left(\bar{y}-\bar{y}^{*}\right)$ and (log) real exchange rate $\bar{q}=\bar{y}-\bar{u}$. In addition, $\bar{m}$ denotes the (log) money supply, $\bar{y}$ the (log) natural rate of output, and $\bar{u}$ is a measure of exogenous aggregate demand. Foreign variables are indicated by an asterisk.
Suppose the government of Britannia suddenly announces that it will abandon a single market with Europia in $T$ years, which will increase barriers to international trade and lead to a permanent decrease in $\bar{u}$ in $T$ years. Explain how this affects the nominal exchange rate $s$ and aggregate price level $p$ over time, and carefully illustrate the dynamics in a phase diagram. How do the effects on the nominal exchange rate $s$ and aggregate price level $p$ depend on $T$ ?

## B. 1 Consumption Theory and a Risky Asset

Consider a household whose preferences are described by:

$$
U=E_{t} \sum_{s=t}^{\infty} \frac{u\left(C_{s}\right)}{(1+r)^{s}},
$$

where $C$ is consumption and $u^{\prime}(C)>0$ and $u^{\prime \prime}(C)<0$ and $r>0$ is the discount rate. Assume that the household's unique source of income is the dividends, $D_{t}$, and capital gains from holding equity $A_{t}$ which has price $P_{t}$. Assume that equity holdings could be sold after dividends have been paid. Moreover assume $A_{0}>0$. The budget constraint of the household is:

$$
C_{t}+P_{t} A_{t+1}=\left(P_{t}+D_{t}\right) A_{t}
$$

(a) Provide an economic interpretation of the household's budget constraint.
(b) Set up the household's optimization problem, derive the intertemporal Euler equation for consumption and interpret it.
For the remainder of the question assume that $u\left(C_{t}\right)=C_{t}$.
(c) Using the Euler equation obtained in part (b) derive $P_{t}$ in terms of future dividends. Which condition do you have to impose? Give an economic intuition.
(d) How does an increase of dividends in period $t+s, D_{t+s}$, affect the equity price in period $t$ ? How is this price affected as $s$ increases? How does a permanent increase of all future dividends by $\varepsilon>0$ from period $t$ onwards affect the price in $t$ ? Give an economic intuition of your findings.
(e) Assume that $u\left(C_{t}\right)=C_{t}$ and $P_{t}$ equals the expression derived in part (c) plus a deterministic bubble, $(1+r)^{t} b$, and $b>0$. Carefully show whether the intertemporal Euler equation for consumption derived in part (b) is still satisfied. Could $b$ be negative? Give an economic intuition.

## B. 2 Optimal Tax Rates, Seignorage and Inflation Target

Suppose social welfare losses are described by

$$
L_{t}=\sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}}\left\{\frac{1}{2} \kappa \tau_{s}^{2} Y_{s}+\frac{1}{2} \pi_{s}^{2}\right\},
$$

where $\tau_{s}$ denotes the income tax rate, $Y_{s}$ aggregate output, $\pi_{s}$ inflation, and the subscript $s$ indicates the time period. The real interest rate $r$ is constant and $\kappa$ is a positive parameter. The government sets the income tax rate $\tau_{t}$ in each period to minimize social welfare losses $L_{t}$. The government's intertemporal budget constraint equals

$$
(1+r) B_{t}+\sum_{s=t}^{\infty} \frac{G_{s}}{(1+r)^{s-t}}=\sum_{s=t}^{\infty} \frac{\tau_{s} Y_{s}}{(1+r)^{s-t}}+\sum_{s=t}^{\infty} \frac{S_{s}}{(1+r)^{s-t}},
$$

where $B_{t}>0$ is government debt at the start of period $t$ and $G_{s}$ government purchases, which equal $G_{s}=\bar{G}$ for $s=t, t+1, \ldots$. In addition, seignorage $S_{s}$ is determined by $S_{s}=\mu_{s} \frac{M_{s}}{P_{s}}$, where $M_{s}$ denotes the money supply, $\mu_{s}$ the (approximate) growth rate of $M_{s}$, and $\stackrel{S}{s}_{s}$ the aggregate price level. Real money demand is described by

$$
\frac{M_{s}}{P_{s}}=e^{-\theta i_{s}} Y_{s},
$$

where $i_{s}$ is the nominal interest rate, which satisfies $i_{s}=r+\pi_{s}^{e}$, where $\pi_{s}^{e}$ denotes inflation expectations and $\theta$ is a positive parameter. Assume that money is superneutral, with $Y_{s}=\bar{Y}$ for $s=t, t+1, \ldots$, and that perfect foresight holds, so that $\pi_{s}^{e}=\pi_{s}$. Also assume that the economy is in a steady state with constant real money balances $M_{s} / P_{s}$, so that $\mu_{s}=\pi_{s}$. The money growth rate $\mu_{s}$ is set in each period by an independent central bank that minimizes the loss function

$$
L_{t}^{C B}=\sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} \frac{1}{2}\left(\pi_{s}-\pi^{*}\right)^{2},
$$

where $\pi^{*}$ denotes the central bank's inflation target.
(a) Compute the money growth rate $\mu_{s}$ that the central bank sets to minimize its loss function $L_{t}^{C B}$. In addition, compute the resulting inflation expectations $\pi_{s}^{e}$, nominal interest rate $i_{s}$, real money holdings $M_{s} / P_{s}$ and seignorage $S_{s}$ in terms of the central bank's inflation target $\pi^{*}$.
(b) Derive the money growth rate $\mu$ that maximizes steady-state seignorage $S$. Compute the corresponding level of seignorage $\bar{S}_{\text {max }}$. What level of the inflation target $\pi^{*}$ would generate the maximum steady-state seignorage $\bar{S}_{\text {max }}$ ?
(c) For a given level of $\pi_{s}$ and $S_{s}=\bar{S}$ for $s=t, t+1, \ldots$, derive the optimal tax rate $\tau_{t}$ set by the government in terms of $B_{t}, \bar{G}, \bar{Y}$ and $\bar{S}$. Give an intuitive explanation of the results.
(d) Use the results from parts (a) and (c) to write social welfare losses $L_{t}$ in terms of the inflation target $\pi^{*}$, and derive the optimality condition for the socially optimal inflation target $\pi_{S O}^{*}$ that minimizes social welfare losses $L_{t}$. Determine how $\pi_{S O}^{*}$ compares to $\pi^{*}=0$ and to the inflation target derived in part (b). Provide an intuitive explanation for the results.

ECM9/ECM10<br>MPhil in Economics<br>MPhil in Economic Research

Monday 5 June 2017 1:30pm - 3:30pm

M210
MACROECONOMICS II

This paper is in two sections. Section A: Candidates are required to answer three out of four questions. Section B: Candidates are required to answer one compulsory question.

Each section carries equal weighting.

Write your candidate number (not your name) on the cover of each booklet.

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## STATIONERY REQUIREMENTS

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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A

## A. 1 Neoclassical Growth Model

Consider an economy with aggregate production function

$$
Y(t)=K(t)^{\alpha} E(t)^{\beta}(A(t) N(t))^{1-\alpha-\beta}, \alpha, \beta \in(0,1) \text { and } \beta+\alpha<1
$$

$Y$ is output, $K$ is capital, $N$ is labour, $E$ is energy, and $A$ is labour productivity. Technological progress grows at rate $g$ and the initial technology, $A(0)>0$, is given. Capital evolves according to:

$$
\dot{K}(t)=I(t)-\delta K(t),
$$

where $I$ corresponds to investment and $\delta>0$ is the depreciation rate, and $K(0)$ is given. There is a continuum of measure one of households who are infinite lived, with preferences

$$
U=\int_{0}^{\infty} e^{-\rho t} L(t) u(c(t)) d t, \quad \rho>0, u(c)=\frac{c^{1-\theta}-1}{1-\theta}, \theta>0
$$

where $c(t)$ denotes consumption of each household member. $L(t)$ is the number of household members, which grows at rate $n$. Assume that $L(0)=1$ and each household member has one unit of productive time. Suppose there is a fixed stock of nonrenewable resources, $R(0)$, and that energy use depletes this stock, so that

$$
\dot{R}(t)=-s_{R}(t) R(t),
$$

where $s_{R}(t) \in(0,1)$ is the endogenously determined extraction rate. Set up the Social Planner's problem for this economy and derive the optimal growth rate of output per capita along the balanced growth path equilibrium. Would output per capita be growing along the balanced growth path equilibrium?

## A. 2 Search Model with Probability of Dying

An agent receives every period an offer to work forever at a wage $w$, where $w$ is drawn from the distribution $F(w)$ with support $[0, B]$ where $B$ is finite. Offers are independently and identically distributed. The objective of the agent is to maximise $E_{t=0}\left\{\sum_{t=0}^{\infty}(\beta(1-s))^{t} y_{t}\right\}$, where $y_{t}=w$ if the worker has accepted a job that pays $w$, and $y_{t}=b$ if the agent remains unemployed, $\beta \in(0,1)$ is a subjective discount factor, and $s \in(0,1)$ is the probability that the agent will die in each period. Assume that $0<b<B<\infty$.
(i) Write down Bellman's Functional Equation for an unemployed agent and argue (intuitively) that the optimal policy function is a reservation wage $\bar{w}$ such that the agent will accept a job offer if $w>\bar{w}$.
(ii) Suppose that there is now a continuum 1 of identical individuals sampling jobs from the same stationary distribution $F(w)$. There is a mass of $s$ workers born every period, so that population is constant, and these workers start out as unemployed. What would be the steadystate unemployment rate for this economy?

## A3. Contraction Mapping

Consider the following cake-eating problem with infinite horizon: an agent has an initial cake of size $x_{0}$. She can eat away $c_{t} \geq 0$ from the cake each period, so $x_{t+1}=x_{t}-c_{t}$. The size of the cake can never become negative. The agent has per-period utility $u\left(c_{t}\right)=\sqrt{c_{t}}$ and discount factor $\beta$. Write the Bellman equation for this problem. Is this Bellman equation a Contraction? Explain.

## A. 4 Asset Pricing

Consider an economy populated by a continuum of measure one of identical households, each with preferences given by $\mathbf{E}_{0}\left[\sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right)\right]$ where $c_{t}$ is consumption at period $t, u\left(c_{t}\right)=\frac{c_{t}^{1-\theta}-1}{1-\theta}$ with $\theta \geq 0$, and $\beta \in(0,1)$ corresponds to the subjective discount factor. There is a single asset tree which yields perishable dividends $d_{t}$ (fruits). Households are initially endowed with some number of shares of the tree: $s_{0}$. Assume that $d_{t}$ is a random variable taking values from the set $\mathbf{D} \equiv\left\{d_{1}, \ldots, d_{N}\right\}$ and is distributed according to a firstorder Markov chain with constant transition probability matrix. Households can trade the share of the tree. The price of the tree when the current realization of $d_{t}$ is $d$ is given by the function $p: \mathbf{D} \rightarrow R_{+}$. There are no other assets in this economy. State the problem of the representative household in recursive form and assuming that the value function is differentiable, derive the the optimal price of share of the tree as a function of present and future dividends. Does 'good news' about future output (dividends) always produce an increase in stock prices? Explain.

## SECTION B

## B. 1 Neoclassical Growth Model

Consider a Dynamic New Keynesian model, with flexible prices and sticky wages. Each household $\iota$ is the monopolistic supplier of a specific variety of labour service and consumes the final good. Households maximize expected lifetime utility, discounted by a factor $0<\beta<1$, and their instantaneous utility is:

$$
u\left(C_{t}\right)-v\left(N_{t}(\iota)\right)=\frac{C_{t}^{1-\sigma}}{1-\sigma}-\frac{N_{t}(\iota)^{1+\eta}}{1+\eta}, \quad \sigma>0, \eta>0,0<\beta<1
$$

Households consume all their income, so that $C_{t}=Y_{t}^{d}$ in equilibrium. They are allowed to set nominal wages optimally in every period with probability $1-\psi_{w}$, or they keep their nominal wage from the previous period with probability $\psi_{w}$ (i.e. there is no indexation), where $0 \leq \psi_{w}<1$. The different labour varieties are combined into final labour by an intermediary, according to a Dixit-Stiglitz aggregator with parameter of elasticity of substitution between labour types $\theta_{w}>1$. The final labour is then supplied to the intermediate good firms that have a production function which is linear in labour. Intermediate goods firms set their prices optimally, i.e. there are no price rigidities.
Let $W_{t}$ and $\Delta_{w, t}$ be the aggregate wage index and wage dispersion in period $t$. Also let $M_{t, t+i}=\beta^{i} u^{\prime}\left(C_{t+i}\right) / u^{\prime}\left(C_{t}\right)$ be the equilibrium stochastic discount factor, $N_{t}$ be the aggregate labour supply, $S_{t}=v^{\prime}\left(N_{t}\right) / u^{\prime}\left(C_{t}\right)$ be the average leisure-consumption marginal rate of substitution, $\Omega_{t}=W_{t} / P_{t}$ be the real wage index, $W_{t}^{*}$ be the optimally set wage of households that are allowed to reset their wage in period $t$ and $G_{t}=W_{t}^{*} / W_{t}$. Finally let $\Pi_{w, t}=W_{t} / W_{t-1}$ be the gross wage inflation in this economy. The equilibrium conditions that describe wage setting are:

$$
\begin{align*}
\frac{\Gamma_{t}}{G_{t}} & =\frac{\Omega_{t} N_{t}}{\Delta_{w, t}}+\psi_{w} \mathbb{E}_{t}\left[M_{t, t+1} \frac{\Gamma_{t+1}}{G_{t+1}} \Pi_{w, t+1}^{\theta_{w}-1}\right]  \tag{1}\\
\Upsilon_{t} G_{t}^{\eta \theta_{w}} & =\frac{\theta_{w}}{\theta_{w}-1} \frac{S_{t} N_{t}}{\Delta_{w, t}^{1+\eta}}+\psi_{w} \mathbb{E}_{t}\left[M_{t, t+1} \Pi_{w, t+1}^{(1+\eta) \theta_{w}} \Upsilon_{t+1} G_{t+1}^{\eta \theta_{w}}\right]  \tag{2}\\
\Gamma_{t} & =\Upsilon_{t}  \tag{3}\\
1 & =\left(1-\psi_{w}\right) G_{t}^{1-\theta_{w}}+\psi_{w} \Pi_{w, t}^{\theta_{w}-1} \tag{4}
\end{align*}
$$

(a) Derive the real wage index $\Omega_{t}^{f}$ in the case of fully flexible wages and interpret the expression.
(b) Consider the steady state at which gross wage inflation is $\Pi_{w}=1$ and assume that the dispersion term is very small at the first order and can be safely ignored. Log-linearise the above system of equations around this deterministic steady state and derive the wage Phillips curve:

$$
\pi_{w, t}=\beta \mathbb{E}_{t} \pi_{w, t+1}+\kappa\left(s_{t}-\omega_{t}\right)
$$

where $\kappa>0$ depends on the model's parameters and lower case letters denote log-deviations of variables from their steady state. Provide an interpretation of the wage Phillips curve.
(c) Explain how the slope of the wage Phillips curve $\kappa$ changes as
i. households become more patient,
ii. wages become less sticky,
iii. the labour supply becomes more inelastic and
iv. as different types of labour become more substitutable.

M220
MACROECONOMICS III

This paper is in two sections:
Section A - candidates are required to answer three out of four questions.
Section B - candidates are required to answer one compulsory question.
Each section carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

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$$
1 \text { of } 6
$$

## Section A

## QUESTION 1: The current account

Suppose the world economy consists of two countries, Europia and Americana (indicated by an asterisk), that are initially completely identical, and only operates for two period $(t=1,2)$. In each country, the representative agent maximizes lifetime utility

$$
U=\ln C_{1}+\beta \ln C_{2}
$$

where $C_{t}$ denotes consumption, the subscript $t$ the time period, and $0<\beta<1$. The agent's budget constraint is

$$
B_{t+1}=Y_{t}+(1+r) B_{t}-I_{t}-T_{t}-C_{t}
$$

where $Y_{t}$ denotes output, $I_{t}$ investment, $T_{t}$ lump sum taxes, $r$ the world real interest rate, and $B_{t+1}$ net foreign assets at the end of period $t$, with $B_{1}=B_{3}=0$. Output $Y_{t}$ is produced according to

$$
Y_{t}=2 \sqrt{K_{t}}
$$

where $K_{t}$ denotes capital, which satisfies

$$
K_{t+1}=K_{t}+I_{t}
$$

where $I_{t} \in \mathbb{R}, K_{t} \geq 0$ and $K_{1}=1$. The government's budget constraint is

$$
G_{t}=T_{t}
$$

where government purchases $G_{t}$ are given by $G_{1}=\bar{G}$ and $G_{2}=0$. Assume free trade between Europia and Americana, and no international capital market imperfections.
Solve for optimal investment $I_{1}$, consumption $C_{1}$, national saving $S_{1}$ and the current account $C A_{1}$ in terms of $\beta, \bar{G}$ and $r$. Explain the effect of an increase in Americana's government purchases $\bar{G}^{*}$ in period 1 on the equilibrium world real interest rate $r_{W}$ and on investment $I_{1}$, consumption $C_{1}$, national saving $S_{1}$ and the current account $C A_{1}$ in period 1 in Europia and in Americana.

## QUESTION 2: Risk sharing without trade in assets

Consider a two-country ( $H$ and $F$ ) endowment economy. Each country has a stochastic endowment of a country-specific good, $Y_{H}$ and $Y_{F}$. Also, preferences are subject to country-specific taste shocks $\varpi$ and $\varpi^{*}$. The social planner problem is:

$$
\begin{aligned}
& \operatorname{Max} \mu \varpi\left(\frac{C_{H}^{\gamma} C_{F}^{1-\gamma}}{1-\sigma}\right)^{1-\sigma}+(1-\mu) \varpi^{*}\left(\frac{C_{H}^{* \gamma} C_{F}^{* 1-\gamma}}{1-\sigma}\right)^{1-\sigma} \\
& \text { s.t. }: C_{H}+C_{H}^{*}=Y_{H} \\
&: C_{F}+C_{F}^{*}=Y_{F}
\end{aligned}
$$

where $C_{H}$ and $C_{H}^{*}$ denote the consumption of the $H$ good at Home and in Foreign, $C_{F}$ and $C_{F}^{*}$ denote consumption of the $F$ good at Home and in Foreign, $\mu$ denotes the social planner weight attributed to the Home country.
(a) Solve for the social planner allocation.
(b) For the same economy, work out the market equilibrium under financial autarky. Under what conditions is perfect risk sharing achievable without trade in assets across borders?
(c) How does you answer to (b) changes if shocks are anticipated to occur in the future? Explain.

## QUESTION 3: Exchange rate pass through

Consider a firm producing a good $h$ under imperfect competition, and supplying to both the Home and the Foreign markets. The firm's marginal cost is linear in unit labor costs $\frac{W_{t}}{Z_{\mathrm{H}, t}}$, where $W_{t}$ is the wage rate in domestic currency (taken as given by the firm) and $Z_{\mathrm{H}, t}$ denotes productivity. The demand for the individual $h$ in either market is

$$
\begin{aligned}
C_{t}(h) & =\left(\frac{p_{t}(h)}{P_{\mathrm{H}, t}}\right)^{-\theta} C_{\mathrm{H}, t}, \\
C_{t}^{*}(h) & =\left(\frac{p_{t}^{*}(h)}{P_{\mathrm{H}, t}^{*}}\right)^{-\theta} C_{\mathrm{H}, t}^{*} .
\end{aligned}
$$

where $p_{t}(h)$ and $p_{t}^{*}(h)$ denote the price of $h$ at consumer level, $P_{\mathrm{H}, t}$ and $P_{\mathrm{H}, t}^{*}$ denote the price index of the Home country exports (the subscript H refers to the bundle of Home good exports), which are substitute with each other with elasticity $\theta$, and $C_{\mathrm{H}, t}$ and $C_{\mathrm{H}, t}^{*}$ denote the scale of consumption of the basket of Home goods at Home and in Foreign. As in the model analyzed in the lectures, bringing the good $h$ to the consumer requires the use of $\eta$ retail services. However, departing from the lecture notes, posit that the retail services are provided by competitive foreign companies which hire local workers with country-specific productivity $Z_{R}$ and $Z_{R}^{*}$, but pay a contract wage that is preset in foreign currency only, $W^{*}$-the same in either country. The consumer-level price is thus:

$$
\begin{aligned}
& p_{t}(h)=\bar{p}_{t}(h)+\eta \frac{W^{*}}{Z_{R}} \\
& p_{t}^{*}(h)=\bar{p}_{t}^{*}(h)+\eta \frac{\mathcal{E}_{t} W^{*}}{Z_{R}^{*}}
\end{aligned}
$$

where $\mathcal{E}_{t}$ denotes the exchange rate.
(a) Derive the profit maximizing prices. Are the optimal prices identical across markets of destination? Carefully explain.
(b) Assuming that all nominal wages are preset, is the exchange rate pass through complete if a monetary shock causes a home currency devaluation? Explain.
(c) State the sufficient condition for incomplete pass through proposed by Marston (1990). Is this condition satisfied in the problem above?
(d) Suppose retailers in one country have the option to buy the product $h$ directly from retailers from the other country, rather than from the producer. Using the optimal prices you derived under 1, express the range of exchange rate values (as a function of elasticities and productivities) for which the producer is able to charge the optimal markup.
(e) Assume that the exchange rate falls outside the range derived under (d), so that, if firms charged the unconstrained optimal price, retailers could arbitrage across markets. How would firms adjust their prices in the Home and export markets to rule out this arbitrage? Would the law of one price hold at producer level?

## QUESTION 4: Empirical tests of risk sharing

Consider a two-country model. Use the following notation: $U\left(U^{*}\right)$ denotes utility, $C\left(C^{*}\right)$ consumption at Home (abroad), $\mathbb{P}_{t}\left(\mathbb{P}_{t}^{*}\right)$ the price level at Home (abroad), $\zeta_{C, t}\left(\zeta_{C, t}^{*}\right)$ country-specific taste shocks, and $\mathcal{E}_{t}$ is the exchange rate (units of Home currency per unit of Foreign currency).
(a) State and explain the condition for efficient risk sharing.
(b) Show how to derive a regression model to test of "market completeness" at international level. In doing so, posit that the instantaneous utility is

$$
U=\zeta_{C, t} \frac{C^{1-\sigma}}{1-\sigma}, \quad \text { and } \quad U^{*}=\zeta_{C, t}^{*} \frac{\left(C^{*}\right)^{1-\sigma}}{1-\sigma}
$$

Carefully write down the null hypothesis.
(c) Would the regression model be valid if markets are not complete? Explain.

## Section B

## QUESTION 5: Global monetary stabilization with "dollar pricing"

Consider the model of monetary policy stabilization after Corsetti-Pesenti 2005 discussed in the lecture notes. The Home country welfare function can be written

$$
E_{t-1} \ln C_{t}-\text { constant }=E_{t-1} \ln \frac{\mu_{t}}{P_{t}}-\text { constant }
$$

where $\mu_{t}$ is the monetary stance of the country, controlled by monetary authorities, and the CPI is defined as

$$
P_{t}=2 P_{H}^{1 / 2} P_{F}^{1 / 2}
$$

with $P_{H}$ denoting the price index of domestically produced goods (each sold at the individual price $p(h)), P_{F}$ the price index of imported goods (each sold at $p(f)$ ), all denominated in Home currency. Foreign welfare, consumption, monetary stance and prices are similarly defined, with a star referring to foreign variables and foreign-currency denominated prices.

Assume that Home firms preset the price of their product variety $h$ in own currency (according to the hypothesis of Producer Currency Pricing, or PCP):

$$
\mathcal{E}_{t} p_{t}^{*}(h)=p_{t}(h)=m k p \cdot E_{t-1}\left(\frac{W_{t}}{Z_{H t}}\right)
$$

while Foreign preset their export prices of their good variety $f$ in Home currency (according to Local Currency Pricing, or LCP).

$$
p_{t}(f)=m k p \cdot E_{t-1}\left(\frac{\mathcal{E}_{t} W_{t}^{*}}{Z_{F t}}\right) \quad \text { and } \quad p_{t}^{*}(f)=m k p \cdot E_{t-1}\left(\frac{W_{t}^{*}}{Z_{F t}}\right) .
$$

Above: $W_{t}$ and $W_{t}^{*}$ denote wages, denominated in Home and Foreign currency, respectively; $Z_{H t}$ and $Z_{F t}$ denote productivities, $m k p$ denote constant markup; and $\mathcal{E}_{t}$ denote the exchange rate (units of Home currency per unit of Foreign currency).

Assume that risk sharing is perfect, such that $\mathcal{E}_{t}=\frac{\mu_{t}}{\mu_{t}^{*}}$, and that countries chooses $\mu_{t}$ and $\mu_{t}^{*}$ under commitment, i.e., they solve the optimal policy problem ex ante, at $t-1$.
(a) Derive and discuss the optimal policy rules:
(a.1) under Nash, i.e. assuming that each country sets its monetary stance taking the stance abroad as given;
(a.2) under policy cooperation.
(b) Contrast the optimal response to a positive productivity shock in the foreign country under Nash and Cooperation.

## END OF PAPER

 CAMBRIDGEECM9/ECM10<br>MPhil in Economics<br>MPhil in Economic Research

M230
APPLIED MACROECONOMICS

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## Question 1

A representative agent is exogenously endowed with $y_{t}$ in period $t=0,1, \ldots$ The sequence of endowments, $\left\{y_{t}\right\}_{t=0}^{\infty}$ is entirely deterministic and known already at time $t=0$. The agent may buy and sell zero-coupon bonds of different maturities, and spend the remainder of her resources on consumption, $c_{t}$. Denote the price of a bond at time $t$ with redemption date at time $t+n$ as $q_{t}^{t+n}$. Thus, a standard one-period bond is denoted $q_{t}^{t+1}$. All bonds have face value 1 , so that $q_{t}^{t}=1$.
(a) The representative agent's total amount of resources available in period $t$ is given by

$$
y_{t}+\sum_{n=0}^{N-1} q_{t}^{t+n} \times b_{t}^{t+n}, \quad t=0,1,2, \ldots
$$

What is her total amount of expenditures in period $t$ ? Write down the agent's complete budget constraint in period $t$.
(b) The consumer faces the following optimisation problem

$$
\max _{\left.\left\{c_{t},\{ \}_{t+1}^{t+n}\right\}_{n=1}^{N}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right),
$$

subject to the budget constraint in part $(a)$. Here $u(\cdot)$ denotes the instantaneous utility function satisfying $u^{\prime}>0, u^{\prime \prime}<0, c_{t}$ denotes consumption in period $t$, and $\beta \in(0,1)$ represents the discount factor.
Derive the asset prices, $\left\{q_{t}^{t+n}\right\}_{n=1}^{N}$, from the above optimisation problem, and show that they equal

$$
q_{t}^{t+n}=\beta \frac{u^{\prime}\left(c_{t+1}\right)}{u^{\prime}\left(c_{t}\right)} q_{t+1}^{t+n} .
$$

(c) The yield to maturity, $r_{t, n}$, is defined as the average return on an asset, such that

$$
\frac{1}{\left(1+r_{t, n}\right)^{n}}=\frac{1}{\Pi_{s=1}^{n}\left(1+r_{t+s}\right)},
$$

where $r_{t+s}$ denotes the one period return

$$
r_{t+s}=\frac{q_{t+s}^{t+s+n}}{q_{t+s-1}^{t+s+n}}-1
$$

Using the approximation $\ln (1+x) \approx x$ for $x$ close to zero, show that

$$
r_{t, n} \approx \frac{1}{n}\left(r_{t+1}+r_{t+2} \ldots r_{t+n}\right)
$$

and therefore that

$$
r_{t, n} \approx-\ln \beta+\frac{\gamma}{n}\left(g_{t}^{c}+g_{t+1}^{c} \ldots g_{t+n-1}^{c}\right)
$$

where $\gamma$ is defined as $u^{\prime}(c)=c^{-\gamma}$, and $g_{t}^{c}=\ln c_{t+1}-\ln c_{t}$.

## Question 2

A representative agent solves the following optimisation problem

$$
\begin{gathered}
\max _{\left\{c_{t}, b_{t+1}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right), \\
\text { subject to } \quad b_{t}+w_{t-1}-T_{t}=p_{t} c_{t}+q_{t} b_{t+1}, \quad t=0,1,2, \ldots
\end{gathered}
$$

where $b_{t}$ denotes nominal bond holdings, $w_{t-1}$ nominal income, $c_{t}$ real consumption, $p_{t}$ the price level, $q_{t}$ the nominal bond price, $T_{t}$ nominal lump-sum taxes, and $\beta \in(0,1)$ represents the discount factor. The utility function, $u(\cdot)$, satisfies $u^{\prime}>0, u^{\prime \prime}<0$. It is assumed that $w_{t}=p_{t} y_{t}$, where $y_{t}$ is exogenous.
(a) Derive the Euler equation associated with the above optimisation problem.
(b) The public sector's budget constraint is given by

$$
d_{t}=q_{t} d_{t+1}+T_{t}+\left(m_{t}-m_{t-1}\right), \quad t=0,1,2, \ldots
$$

where $d_{t}$ denotes nominal debt, and $m_{t}$ denotes the monetary base. Use the fact that $m_{t}=p_{t} y_{t}$, and that $d_{t}=b_{t}$ and derive the economy's real resource constraint (i.e. the GDP identity).
(c) Now assume that the central bank targets inflation such that

$$
\mu_{t}=(1-\rho) \bar{\mu}+\rho \mu_{t-1},
$$

with $m_{t+1}=\mu_{t} m_{t}$. Assume further that output is constant over time, $y_{t}=y_{t+1}=\ldots=$ $y$. Derive a similar law of motion for the bond price, $q_{t}$.

## Question 3

In period $t$, a representative agent is in possession of $x_{t}$ units of cash and $b_{t}$ units of a riskless bond. She receives $w_{t-1}$ units of labour income, and interest payments on bond holdings are denoted $i_{t}$. These (nominal) resources may be spent on (real) consumption, $c_{t}$; purchases of new bonds, $b_{t+1}$; and future excess cash holdings, $x_{t+1}$. The price level is denoted $p_{t}$. The budget constraint is therefore

$$
x_{t}+b_{t}+y_{t}-T_{t}=p_{t} c_{t}+b_{t+1}+x_{t+1},
$$

where $y_{t}$ denotes total income, $y_{t}=w_{t-1}+b_{t} i_{t}$, and $T_{t}$ denotes some lump-sum taxes. Given a sequence of prices, $\left\{i_{t}, p_{t}\right\}$, and taxes, $\left\{T_{t}\right\}$, the agent's optimisation problem is given as

$$
\max _{\left\{c_{t}, b_{t+1}, x_{t+1}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right)
$$

subject to the budget constraint above. The utility function, $u(\cdot)$, satisfies $u^{\prime}>0, u^{\prime \prime}<0$, and $\beta \in(0,1)$ represents the discount factor.

The government's sequence of (nominal) debt, $d_{t}$, taxes, $T_{t}$, and real government spending, $g_{t}$, satisfies

$$
d_{t+1}+T_{t}=d_{t}\left(1+i_{t}\right)+p_{t} g_{t}, \quad t=0,1,2, \ldots
$$

Money supply is constant and equals $m$. The sequence of aggregate real output, $\left\{Y_{t}\right\}$, is treated as given.
(a) Derive the Euler equations associated with the household's problem. Given an intuitive explanation.
(b) Define a competitive equilibrium and derive the equation of exchange. Do prices, $p_{t}$, succumb to the monetarist view?
(c) Now suppose that a fraction of the contemporaneous labour income, $\gamma_{t} w_{t}$, is received already in period $t$, but that the reminder, $\left(1-\gamma_{t}\right) w_{t}$, is still only received in period $t+1$. The sequence $\left\{\gamma_{t}\right\}$ is treated as given.

Under these new conditions derive the household's budget constraint, and derive the equation of exchange. How is the velocity of money related to $\gamma$ ? Explain intuitively.

## Question 4

A government's period-by-period budget constraint is given by

$$
p_{t} g_{t}+d_{t}=q_{t} d_{t+1}+T_{t}+\left(m_{t}-m_{t-1}\right), \quad t=0,1,2, \ldots
$$

where $g_{t}$ denotes real government spending, $d_{t}$ nominal debt, $T_{t}$ nominal lump-sum taxes, and $m_{t}$ the stock of money. Here $q_{t}$ denotes the price of a one period riskless bond which pays out one unit of nominal resources in period $t+1$. The implied nominal interest rate is given by $q_{t}=\frac{1}{1+i_{t+1}}$. Denote real taxes as $\tau_{t}=T_{t} / p_{t}$, and the real interest rate as $1+r_{t+1}=\left(1+i_{t+1}\right) \frac{p_{t}}{p_{t+1}}$.
(a) Derive the government's budget constraint in real terms.
(b) The equation of exchange states that $p_{t} y_{t}=v_{t} m_{t}$. Assume that $y_{t}=y$, and $v_{t}=1$, and $r_{t}=r$, for $t=0,1, \ldots$ Show that real government debt can be written as a function of the net present value of real government spending, real taxes, real output, and inflation, $\pi_{t}=\frac{p_{t}-p_{t-1}}{p_{t-1}}$. (Hint: Impose the "no-Ponzi condition" $\lim _{s \rightarrow \infty} \frac{1}{(1+r)^{s}} d_{t+s}=0$ )
(c) In addition to the assumption imposed in part (c) above, assume that taxes follow the autoregressive law of motion

$$
\tau_{t+1}=(1-\rho) \bar{\tau}+\rho \tau_{t},
$$

and that real government debt as well as spending are constant over time, $\hat{d_{t}}=d_{t} / p_{t}=d$ and $g_{t}=g, t=0,1, \ldots$
Derive an expression for inflation from the government budget constraint and show that inflation must follow an autoregressive law of motion. What is the long-run inflation rate in this economy? Why did Sargent and Wallace call this "some unpleasant monetarist arithmetics"?

## Question 5

Consider the following two-period cash in advance model

$$
\begin{aligned}
\max _{c_{0}, c_{1}, b_{1}, x_{1}}\left\{u\left(c_{0}\right)\right. & \left.+\beta u\left(c_{1}\right)\right\} \\
\text { subject to } \quad p_{0} c_{0}+b_{1}+x_{1} & =b_{0}\left(1+i_{0}\right)+w_{-1}+x_{0}-T_{0} \\
p_{1} c_{1} & =b_{1}\left(1+i_{1}\right)+w_{0}+x_{1}-T_{1} \\
x_{1} & \geq 0 \\
x_{0}, w_{-1}, b_{0}, i_{0} & , \text { are given, and } w_{-1}+x_{0}=m_{-1}
\end{aligned}
$$

where $c_{t}$ denotes consumption of the output good, $b_{t}$ nominal government bonds, $x_{t}$ excess cash holdings, $w_{t-1}$ nominal income, $i_{t}$ the nominal interest rate, and $T_{t}$ are nominal taxes, $t=0,1$. The utility function, $u(\cdot)$, is given by $u(c)=\frac{c^{1-\gamma}-1}{1-\gamma}$ with $\gamma>1$, and $\beta \in(0,1)$.
(a) The government's budget constraint is

$$
p_{t} g_{t}=T_{t}+\left(m_{t}-m_{t-1}\right)
$$

where $g_{t}$ denotes real government expenditures, and $m_{t}$ money supply. The equilibrium conditions are $b_{t}=0$, and $y_{t}=c_{t}+g_{t}, t=0,1$, where the latter condition defines nominal GDP. The cash in advance structure implies that $w_{0}=y_{0}$.
Derive the equation of exchange in both period 0 and 1 .
(b) Potential output in period 0 is equal to one. Find the value of real output in period 1 , $y^{\prime}$, that brings the economy to the cusp of a liquidity trap. If $y_{1}<y^{\prime}$ and $g_{1}=0$, what is the fiscal multiplier?
(c) Suppose that $y_{1}<y^{\prime}$ and $p_{0}=m_{0}$, such that $y_{0}<1$. Consider implementing two different policies in period $0:(i)$, to increase government spending, $g_{0}$ such that $y_{0}=1$, or (ii) to increase $m_{1}$ such that $y_{0}=1$. Which policy yields the largest increase in private consumption, $c_{0}$ ? What is the intuition for this result?

## Question 6

Consider the cash-in-advance model,

$$
\begin{array}{cc} 
& \max _{\left\{c_{t}, b_{t+1}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right), \\
\text { s.t. } \quad p_{t} c_{t}+b_{t+1}+x_{t+1}=\left(1+i_{t}\right) b_{t}+w_{t-1}+x_{t}-T_{t},
\end{array}
$$

where $\left\{y_{t}\right\}$ is a given sequence of endowments, $c_{t}$ denotes real consumption, $b_{t}$ nominal government bonds, $x_{t}$ excess cash holdings, $w_{t-1}$ nominal income, $i_{t}$ the nominal interest rate, and $T_{t}$ are nominal taxes. The utility function, $u(\cdot)$, is given by $u(c)=\frac{c^{1-\gamma}-1}{1-\gamma}$ with $\gamma>1$, and $\beta \in(0,1)$.

The government's budget constraint is given by

$$
p_{t} g_{t}+i_{t} d_{t}=\left(d_{t+1}-d_{t}\right)+T_{t}+\left(m_{t}-m_{t-1}\right),
$$

where $g_{t}$ denotes real government expenditures, $d_{t}$ nominal debt, and $m_{t}$ money supply.
Assume that $m_{t}=m \forall t$, and $g_{t}=0 \forall t$. Potential output in period $t$ is equal to one.
Suppose that agents unexpectedly receive news in period $t$ that output in period $t+1$ will decline and equal $y^{\prime}<1$. In addition, assume throughout this exercise that the economy is not in a liquidity trap in period $t+1$.
(a) If the fall in period $t+1$ output is not sufficient to bring the economy to a liquidity trap in period $t$, how do these news affect $p_{t+1}, p_{t}$, and $i_{t+1}$ ? How is the real interest rate, $r_{t+1}$, affected? What is the intuition for these price movements?
(b) How big must the fall in output be for the economy to fall into a liquidity trap in period $t$ ? Suppose that $y_{t}$ is still equal to 1 , what is the effect on period $t$ prices, velocity of money, $v_{t}$, and excess cash holdings, $x_{t+1}$ ?
(c) Assume that the price level in period $t, p_{t}$, is pre-set and equal to $m$.

How is output in period $t$ related to output in period $t+1$ ? Illustrate this relationship both mathematically and graphically.

## Question 7

A government's budget constraint in nominal terms is given by

$$
p_{t} g_{t}+i_{t} d_{t}=\left(d_{t+1}-d_{t}\right)+T_{t}+\left(m_{t}-m_{t-1}\right),
$$

where $g_{t}$ denotes real government expenditures, $d_{t}$ nominal debt, $i_{t}$ the nominal interest rate, $T_{t}$ nominal tax revenue, and $m_{t}$ money supply.
(a) What is the government's budget constraint in real terms?
(b) Explain intuitively why the real debt is defined as $d_{t} / p_{t-1}$ instead of $d_{t} / p_{t}$.
(c) Government budgets often report the nominal gross of interest deficit defined as

$$
p_{t} g_{t}-T_{t}+i_{t} d_{t}
$$

and also the ratio to nominal GDP

$$
\begin{equation*}
\frac{p_{t} g_{t}-T_{t}+i_{t} d_{t}}{p_{t} y_{t}} \tag{1}
\end{equation*}
$$

However, the gross of interest deficit in real terms as a fraction of real GDP is instead given by

$$
\frac{g_{t}-\tau_{t}+r_{t} \hat{d}_{t}}{y_{t}}
$$

where $\hat{d}_{t}$ denotes real government debt.
How come the definition in equation (1) may overstate the government's "deficit problem"? How does this bias change with the rate of inflation, $\pi_{t}$ ?

## END OF PAPER

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Economic Research
MPhil in Finance and Economics
MPhil Finance

Friday 12th, May, $2017 \quad 2: 00 \mathrm{pm}-4: 00 \mathrm{pm}$

M300
ECONOMETRIC METHODS

Answer all six questions from Section A and two questions from Section B. Part A is $60 \%$ of total marks.

All questions in Section A carry equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly. Credit will be given for clear presentation of relevant statistics.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator Dickey-Fuller Tables
New Cambridge Statistical Tables
Engle-Granger Tables
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Section A

A1 You have data on the height $Y_{i}, i=1, \ldots, n$, of $n$ individuals. You run the following regression

$$
Y_{i}=\beta_{0}+\beta_{1} D_{<30, i}+\beta_{2} D_{>20, i}+\varepsilon_{i},
$$

where $D_{<30, i}$ is the dummy variable for individuals who are younger than 30 , and $D_{>20, i}$ is the dummy variable for individuals who are older than 20. Interpret the coefficient $\beta_{2}$.

A2 You would like to test the hypothesis that $\beta_{0}=\beta_{1}=\beta_{2}$ in the regression

$$
y_{i}=\beta_{0}+\beta_{1} x_{i}+\beta_{2} z_{i}+\varepsilon_{i} .
$$

Represent this hypothesis in the standard form $R \beta=a$, where $\beta=\left(\beta_{0}, \beta_{1}, \beta_{2}\right)^{\prime}$. Suppose that you know that $R \hat{\beta}-a$ is distributed as $N\left(0, I_{2}\right)$ under the null, and that the value of $R \hat{\beta}-a$ is $(2,-1)^{\prime}$. Test the null hypothesis. (Here $\hat{\beta}$ is the OLS estimate of $\beta$.)

A3 Consider regression

$$
\text { lwage }_{i}=\beta_{0}+\beta_{1} \text { educ }_{i}+\beta_{2} \text { exper }_{i}+\varepsilon_{i},
$$

where lwage $_{i}$ is log wage, educ $_{i}$ is education, and exper $_{i}$ is experience. Suppose that $e d u c_{i}$ is endogenous and that you have two instruments: mother's education $\left(\right.$ meduc $\left._{i}\right)$, and father's education, $\left(f e d u c_{i}\right)$. Let

$$
\text { resid }_{i}=\text { lwage }_{i}-\hat{\beta}_{0,2 S L S}+\hat{\beta}_{1,2 S L S} \text { educ }_{i}+\hat{\beta}_{2,2 S L S} \text { exper }_{i}
$$

be the 2SLS residuals. The OLS regression of resid $_{i}$, on constant, exper ${ }_{i}$, meduc $_{i}$ and $f_{\text {feduc }}$ gives the following results. Using these results, test the overidentifying restrictions. Interpret the outcome of your test.


A4 Consider a very simple fixed effects regression

$$
\begin{equation*}
y_{i t}=c_{i}+\varepsilon_{i t}, \quad i=1, \ldots, n ; t=1, \ldots, T, \tag{1}
\end{equation*}
$$

where $\varepsilon_{i t}$ are i.i.d. $N\left(0, \sigma^{2}\right)$ variables and there are no explanatory variables. A colleague proposes to estimate $\sigma^{2}$ as $\frac{1}{n T}$ times the SSR from the dummy variables regression. Find the bias of such an estimate.

A5 A time series $y_{t}$ satisfies the following $\operatorname{AR}(3)$ equation

$$
y_{t}=\alpha+\rho_{1} y_{t-1}+\rho_{2} y_{t-2}+\rho_{3} y_{t-3}+\varepsilon_{t}, t=1, \ldots, T .
$$

For which values of $\rho_{1}, \rho_{2}$ and $\rho_{3}$ does $y_{t}$ have a unit root? How would you test for the unit root in $y_{t}$ ?

A6 You have $n$ i.i.d. observations $y_{i}, i=1, \ldots, n$ that equal -1 with probability $p$, +1 with probability $q$, and 0 with probability $1-p-q$. Find the maximum likelihood estimator of $\theta=(p, q)^{\prime}$ and the Fisher information matrix.

## Section B

B1 A researcher has data on the logarithms of UK households disposable income, $\log Y_{t}$, and consumption, $\log C_{t}$, from 1955 to 2015 (61 years in total).
(i) To test the hypotheses of unit roots in $\log Y_{t}$ and $\log C_{t}$, the researcher runs the OLS regression of $\Delta \log Y_{t}=\log Y_{t}-\log Y_{t-1}$ on constant, time trend $t, \log Y_{t-1}$, and $\Delta \log Y_{t-1}$ and a similar regression for $\log C_{t}$. The results of these regressions are reported below

$$
\begin{aligned}
\Delta \widehat{\log } Y_{t} & =\underset{(0.052)}{0.077}+\underset{(0.0009)}{0.0008 t}-\underset{(0.010)}{0.012} \log Y_{t-1}+\underset{(0.074)}{0.881 \Delta} \log Y_{t-1}, \\
\Delta \widehat{\log } C_{t} & =\underset{(0.045)}{0.070}+\underset{(0.0008)}{0.0007 t}-\underset{(0.009)}{0.011} \log C_{t-1}+\underset{(0.065)}{0.905} \Delta \log C_{t-1},
\end{aligned}
$$

where the standard errors are given in the parentheses. Test the hypotheses of unit roots in $\log Y_{t}$ and $\log C_{t}$ at $5 \%$ significance level.
(ii) Explain why the researcher included the time trend, and why did she include the lagged changes in the logarithms in the regressions from (i).
(iii) The permanent income theory of consumption predicts that, in the longrun equilibrium, the average propensity to consume $C_{t} / Y_{t}$ is constant. The researcher argues that this prediction can be tested using the following OLS results

$$
\left.\Delta \log \widehat{\left(C_{t}\right.} / Y_{t}\right)=\underset{(0.006)}{0.020}-\underset{(0.076)}{0.280} \log \left(C_{t-1} / Y_{t-1}\right)+\underset{(0.119)}{0.353 \Delta} \Delta \log \left(C_{t-1} / Y_{t-1}\right) .
$$

Explain the researcher's argument, and perform the test.
(iv) To forecast households consumption, the researcher runs the regression of $\Delta \log C_{t}$ on $\Delta \log C_{t-1}, \Delta \log Y_{t-1}$ and $\log \left(C_{t-1} / Y_{t-1}\right)$. She gets the following OLS results

$$
\Delta \widehat{\log } C_{t}=\underset{(0.008)}{0.001}+\underset{(0.176)}{0.760} \Delta \log C_{t-1}+\underset{(0.174)}{0.153} \log Y_{t-1}+\underset{(0.113)}{0.063} \log \left(C_{t-1} / Y_{t-1}\right) .
$$

What is a reason to include $\log \left(C_{t-1} / Y_{t-1}\right)$ in this regression? How would you test for no serial correlation in the errors of this regression?

B2 Consider a regression model

$$
y_{i}=\beta_{0}+\beta_{1} x_{i}+\varepsilon_{i},
$$

where $\varepsilon_{i}=x_{i}\left(1+\eta_{i}\right)$. Assume that $\left(x_{i}, \eta_{i}\right), i=1, \ldots, n$, are i.i.d. with $E x_{i}=E \eta_{i}=0, \operatorname{Var}\left(x_{i}\right)=\sigma_{x}^{2}, \operatorname{Var}\left(\eta_{i}\right)=\sigma_{\eta}^{2}$ and $\eta_{i}$ independent from $x_{i}$.
(i) What is the value of $\operatorname{Cov}\left(x_{i}, \varepsilon_{i}\right)$ ? What does such a value imply for the results of the OLS regression of $y_{i}$ on $x_{i}$ ?
(ii) Show that, if $z_{i}$ is a valid instrument for $x_{i}$, it cannot be independent from $\eta_{i}$.
(iii) Usually, econometricians do not know the structure of regression errors. However, suppose that you know that $\varepsilon_{i}=x_{i}\left(1+\eta_{i}\right)$. How would you consistently estimate $\beta_{0}$ and $\beta_{1}$ without using the instrumental variables technique? Discuss the efficiency of the estimators that you propose.
(iv) Now suppose that you do not know that $\varepsilon_{i}=x_{i}\left(1+\eta_{i}\right)$, but you suspect that $x_{i}$ is an endogenous regressor. Further, suppose that you observe $z_{i}, i=1, \ldots, n$, which is a valid instrument for $x_{i}$. How would you test for exogeneity of $x_{i}$ ?

B3 A researcher studies the relationship between life satisfaction and gender using a random sample $\left(S_{i}, G_{i}\right), i=1, \ldots, n$, where $S_{i}=1$ if person $i$ is satisfied with life and $S_{i}=0$ otherwise, and $G_{i}=1$ for women and 0 for men. He considers the following probit model

$$
\operatorname{Pr}\left(S_{i}=1 \mid G_{i}\right)=\Phi\left(\alpha+\beta G_{i}\right),
$$

where $\Phi$ is the cdf of a standard normal distribution.
(i) Suppose that the observed numbers of life-satisfied men, life-satisfied women, life-dissatisfied men, and life-dissatisfied women are the same and equal $n / 4$. Derive an explicit expression for the conditional log likelihood function.
(ii) Find the values of the maximum likelihood estimates of $\alpha$ and $\beta$.
(iii) How would the predicted probabilities of life satisfaction be different from those obtained from the linear probability model? Does your answer depend on the assumption about the available sample made in (i)?
(iv) Now suppose that the researcher obtains data on another explanatory variable $M_{i}$, equal to 1 if person $i$ is married and 0 otherwise. He considers a new model

$$
\operatorname{Pr}\left(S_{i}=1 \mid G_{i}, M_{i}\right)=\Phi\left(\alpha+\beta G_{i}+\gamma M_{i}\right) .
$$

Suppose that it is known that the expected value of $S_{i}$ for not married men is above $1 / 2$. What does this imply for the coefficients of the above model? For which values of $\alpha, \beta$, and $\gamma$ will the predicted effect of marriage on life satisfaction be positive and larger for women than for men? Will the linear probability model have similar predictions?

B4 Consider the following two-period panel regression model

$$
y_{i t}=\alpha_{i}+\beta x_{i t}+\varepsilon_{i t}, i=1, \ldots, n, t=1,2 .
$$

where $\left(y_{i 1}, y_{i 2}, x_{i 1}, x_{i 2}, \alpha_{i}\right), i=1, \ldots, n$ are i.i.d., $\varepsilon_{i 1}=0, i=1, \ldots, n$, that is, in the first period $\varepsilon_{i 1}$ is identically zero, and $\varepsilon_{i 2}$ is independent from $\left(x_{i 1}, x_{i 2}, \alpha_{i}\right)$, $E \varepsilon_{i 2}=0$, and $\operatorname{Var}\left(\varepsilon_{i 2}\right)=1$.
(i) Which Gauss-Markov assumptions hold for this model and which may fail?
(ii) Will the fixed effects estimator of $\beta$ be unbiased? Will it be consistent? Explain your answers.
(iii) Propose a consistent estimator of the fixed effects $\alpha_{i}$ and show its consistency.
(iv) Suppose that $\alpha_{i}=\eta_{i}$, where $\eta_{i}$ is independent from $\left(x_{i 1}, x_{i 2}\right), E \eta_{i}=0$ and $\operatorname{Var}\left(\eta_{i}\right)=1$. Let $w_{i t}=\eta_{i}+\varepsilon_{i t}$ and

$$
w=\left(w_{11}, w_{12}, w_{21}, w_{22}, \ldots, w_{n 1}, w_{n 2}\right)^{\prime}
$$

Find $\Omega=\operatorname{Var}(w)$ and its inverse $\Omega^{-1}$. Using your findings, describe the GLS estimator of $\beta$ in the regression $y_{i t}=\beta x_{i t}+w_{i t}$.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research
Friday 2nd June $2017 \quad 1: 30 \mathrm{pm}$ to $3: 30 \mathrm{pm}$

M310
TIME SERIES

Candidates are required to answer five questions from Section A and one question from Section B.

Section A is weighted as $60 \%$ and Section B as $40 \%$.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A

A.1. Consider the random walk plus noise:

$$
y_{t}=\mu_{t}+\varepsilon_{t}, \quad \mu_{t}=\mu_{t-1}+\eta_{t}, \quad t=1, \ldots, T,
$$

where $\varepsilon_{t}$ and $\eta_{t}$ are serially and mutually uncorrelated Gaussian disturbances with mean zero and variances $\sigma_{\varepsilon}^{2}$ and $\sigma_{\eta}^{2}$ respectively. Find the distribution of $y_{T+\ell}$, where $\ell$ is a positive integer, conditional on the information that at time $T, \mu_{T}$ is normal with mean $m_{T}$ and variance $p_{T}$.
A.2. Find the autocorrelation function of the airline model,

$$
(1-L)\left(1-L^{s}\right) y_{t}=(1+\theta L)\left(1+\Theta L^{s}\right) \varepsilon_{t}
$$

where $L$ is the lag operator and $\varepsilon_{t}$ is $N I D\left(0, \sigma^{2}\right)$, that is normally and independently distributed with mean zero and constant variance $\sigma^{2}$. If the first five autocorrelations in a quarterly series take the values $-0.4,0,0.2,-0.5$, 0.2 , and are zero thereafter, find the implied values of the parameters $\theta$ and $\Theta$.
A.3. A detrending moving average has the property that the weights sum to zero. Write such a filter in terms of the lag operator, $L$, and hence show that when it is applied to a stationary process, the spectrum of the detrended series is zero at frequency zero. What is the effect on the spectrum at the frequency corresponding to a period of two?
A.4. In the time-varying $A R(2)$ model,

$$
y_{t}=\phi_{1 t} y_{t-1}+\phi_{2 t} y_{t-2}+\varepsilon_{t}, \quad t=1, . ., T,
$$

where $y_{0}=y_{-1}=0, \varepsilon_{t}$ is $\operatorname{NID}\left(0, \sigma^{2}\right)$ and $\phi_{1 t}$ and $\phi_{2 t}$ are assumed to be generated by independent $A R(1)$ processes. How would you estimate this model?
A. 5 In the models

$$
\begin{equation*}
y_{t}=\phi y_{t-1}+\delta_{0} w_{t}+\delta_{1} w_{t-1}+\varepsilon_{t}, \quad t=1, . ., T, \tag{1}
\end{equation*}
$$

and

$$
\begin{align*}
y_{t} & =\lambda_{0} w_{t}+\lambda_{1} w_{t-1}+u_{t}, \quad t=1, . ., T  \tag{2}\\
u_{t} & =\phi u_{t-1}+\epsilon_{t}
\end{align*}
$$

where $\varepsilon_{t}$ and $\epsilon_{t}$ are normally and independently distributed with mean zero and constant variance, the intervention variable is defined as

$$
w_{t}=\left\{\begin{array}{c}
0 \text { for } t<\tau, \\
1 \text { for } t \geq \tau
\end{array}, \quad 1<\tau<T .\right.
$$

Compare and contrast the dynamics of the intervention in the two models. Can the models ever be the same?
A.6. Consider the model

$$
y_{t}=\varepsilon_{t}+\beta \varepsilon_{t-1} \varepsilon_{t-2}, \quad \varepsilon_{t} \sim N I D\left(0, \sigma^{2}\right), \quad t=1, \ldots, T
$$

where $N I D\left(0, \sigma^{2}\right)$ denotes normally and independently distributed with mean zero and constant variance $\sigma^{2}$, and $\varepsilon_{0}$ and $\varepsilon_{-1}$ are fixed and known. Find the minimum mean square error estimator (MMSE) of $y_{T+1}$ and compare its MSE with that of the best linear predictor. How would you estimate $\beta$ ?
A.7. The Clayton copula is defined as

$$
C\left(u_{1}, u_{2}\right)=\left\{\begin{array}{lr}
\left(u_{1}^{-\theta}+u_{2}^{-\theta}-1\right)^{-1 / \theta}, & \theta>0 \\
u_{1} u_{2}, & \theta=0 .
\end{array}\right.
$$

Explain what is meant by tail dependence and obtain expressions for tail dependence in a Clayton copula when $\theta>0$ and when $\theta=0$. Define the coefficient of lower tail dependence (the lower tail index) and derive this coefficient for the Clayton copula.

## SECTION B

B.1. (a) Consider the $\operatorname{VARMA}(1,1)$ model

$$
\mathbf{y}_{t}=\boldsymbol{\Phi} \mathbf{y}_{t-1}+\boldsymbol{\varepsilon}_{\mathbf{t}}+\boldsymbol{\Theta} \varepsilon_{t-1}, \quad t=1, \ldots, T
$$

where $\mathbf{y}_{t}$ is an $N \times 1$ vector, $\boldsymbol{\Phi}$ and $\boldsymbol{\Theta}$ are $N \times N$ matrices and $\varepsilon_{\mathrm{t}}$ is multivariate white noise with mean vector zero and covariance matrix $\boldsymbol{\Omega}$. Derive an expression for the lag one autocovariance matrix, $\boldsymbol{\Gamma}(1)$, in terms of $\boldsymbol{\Phi}, \boldsymbol{\Theta}, \boldsymbol{\Omega}$ and $\boldsymbol{\Gamma}(0)$, where $\boldsymbol{\Gamma}(0)$ is the covariance matrix of $\mathbf{y}_{t}$. Given $\boldsymbol{\Gamma}(1)$, show how the remaining autocovariance matrices can be computed recursively. Write down a set of equations that could, in principle, be solved to yield $\boldsymbol{\Gamma}(0)$. In the special case $\boldsymbol{\Phi}=\phi \mathbf{I}$, where $\phi$ is a scalar such that $|\phi|<1$, find an expression for $\boldsymbol{\Gamma}(0)$ in terms of $\phi, \boldsymbol{\Theta}$ and $\boldsymbol{\Omega}$.
(b) Consider the model

$$
\begin{gathered}
\mathbf{y}_{t}=\boldsymbol{\mu}_{t}+\boldsymbol{\varepsilon}_{t}, \quad \boldsymbol{\varepsilon}_{t} \sim \operatorname{NID}\left(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon}\right), \quad t=1, \ldots, T, \\
\boldsymbol{\mu}_{t}=\boldsymbol{\mu}_{t-1}+\boldsymbol{\beta}_{t-1}+\boldsymbol{\eta}_{t}, \quad \eta_{t} \sim \operatorname{NID}\left(\mathbf{0}, \boldsymbol{\Sigma}_{\eta}\right), \\
\boldsymbol{\beta}_{t}=\boldsymbol{\Phi} \boldsymbol{\beta}_{t-1}+\boldsymbol{\varsigma}_{t}, \quad \boldsymbol{\varsigma}_{t} \sim \operatorname{NID}\left(\mathbf{0}, \boldsymbol{\Sigma}_{\varsigma}\right)
\end{gathered}
$$

where $\mathbf{y}_{t}, \boldsymbol{\mu}_{t}, \varepsilon_{t}, \boldsymbol{\eta}_{t}$ and $\boldsymbol{\varsigma}_{t}$, are all $N \times 1$ vectors and NID denotes multivariate normal. If $\boldsymbol{\Phi}=\phi \mathbf{I}$, where $\phi$ is a scalar such that $|\phi|<1$, explain how you would estimate $\phi$ and the covariance matrices $\boldsymbol{\Sigma}_{\varepsilon}, \boldsymbol{\Sigma}_{\eta}$ and $\boldsymbol{\Sigma}_{\varsigma}$.
(c) Now suppose that $\boldsymbol{\mu}_{t}=\boldsymbol{\mu}_{t-1}+\mathbf{i} \beta_{t-1}+\boldsymbol{\eta}_{t}$, where $\mathbf{i}$ is an $N \times 1$ vector of ones and $\beta_{t}$ is a scalar. How is this model to be interpreted and what can be said about co-integration when (i) $\boldsymbol{\Sigma}_{\eta}$ is positive definite and (ii) $\boldsymbol{\Sigma}_{\eta}=\mathbf{0}$ ?
B.2. (a) The probability density function of a general error distribution (GED) with mean zero is
$f(y ; \varphi, v)=\left[2^{1+1 / v} \varphi \Gamma(1+1 / v)\right]^{-1} \exp \left(-|y / \varphi|^{v} / 2\right), \quad-\infty<y<\infty, \quad \varphi, v>0$,
where $\varphi$ is a scale parameter, $\Gamma($.$) is the gamma function and v$ is a tailthickness parameter. Derive the EGARCH model based on the conditional score (the Gamma-GED-EGARCH model). Compare the impact curves given by the score for $v=2$ and $v=1$. How would you estimate the unknown parameters?
(b) Explain briefly what is meant by the term 'leverage' when applied to a volatility model and adapt the EGARCH model of (a) to handle leverage. Compare your model with the classic EGARCH model of Nelson.
(c) The survival function of a Type II Pareto distribution with scale $\alpha$ is $\bar{F}(y)=$ $[1+y / \alpha]^{-\eta}$ for $y \geq 0$. What is the relevance of the tail index parameter $\eta$ for the existence of moments of the distribution? Derive the quantile function and hence obtain the median when $\eta=1$. Show that the score for the logarithm of the scale is bounded and comment.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Monday 22 May 2017 09:00am - 11:00am

M330
APPLIED ECONOMETRICS

The paper is in two sections: Candidate are require to answer from Section A two compulsory questions and section B two out of four questions.

Each question carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Section A

1. Using cross-country estimates of physical and human capital stocks, Benhabib and Spiegel (1994) ran the following cross-country growth accounting regression implied by a Cobb-Douglas aggregate production function:

$$
\Delta y_{i}=\alpha+\beta_{1} \Delta K_{i}+\beta_{2} \Delta H_{i}+\beta_{2} \Delta L_{i}+\varepsilon_{i}
$$

The dependent variable, $\Delta y_{i}$, is the log difference of output between 1965 and 1985. The explanatory variables are the log differences of physical capital, $\Delta K_{i}, \log$ differences in average years of schooling, $\Delta H_{i}$, and log differences in labour force, $\Delta L_{i}$, all measured between 1965 and 1985. The data consists of a cross-section of 78 countries. Column (1) in the following Table gives their baselines estimates.

|  | $(1)$ | $(2)$ | $(3)$ |
| :---: | :---: | :---: | :---: |
| Constant | 0.269 <br> $(2.98)$ | 0.166 <br> $(2.86)$ | $(1.105)$ <br>  <br> $\Delta K$ |
| $\Delta .457$ | 0.507 | 0.553 |  |
| $(5.36)$ | $(8.41)$ | $(13.16)$ |  |
| $\Delta H$ | 0.063 | 0.111 | 0.165 |
| $(0.80)$ | $(1.66)$ | $(4.00)$ |  |
| $\Delta L$ | 0.209 | 0.217 | 0.241 |
| $(1.01)$ | $(1.39)$ | $(2.15)$ |  |
| $R^{2}$ | 0.52 | 0.77 | 0.83 |
| $N$ | 78 | 72 | 64 |
| Jarque-Bera p-value | 0.00 | 0.13 | 0.55 |

Table: Cross-Country Growth Regressions
Heteroskedasticity-consistent t-ratios in parentheses
(a) Benhabib and Spiegel's baseline results in column (1) suggest that human capital (growth) enters insignificantly in explaining income growth rates. Please interpret the result of the Jarque-Bera test in column (1) and comment on whether this cautions against or supports Benhabib and Spiegel's main result. Describe an alternative graphical procedure which would supplement the Jarques-Bera test.
(b) Columns (2) and (3) are Least Trimmed Squares estimates of the same regression reported by Jonathan Temple (2001). Briefly (i) describe why Temple (2001) may have chosen to deploy robust regression techniques and (ii) describe the Least Trimmed Squares technique.
(c) In column (3) the Least Trimmed Squares procedure is dropping 14 observations. Upon inspection, we learn that these observations stand out from the rest of the sample in that they experienced very little (or even negative) human capital growth relative to the rest of the world during this twenty year period. Does this alone justify the change in point estimates and statistical significance observed when we go from column (1) to column (3)? Explain your answer.
2. Besley and Burgess (2004) consider the following model for outcomes in Indian states (indexed by $s$ ) at time $t$,

$$
\begin{equation*}
y_{s t}=\alpha_{s}+\beta_{t}+\mu r_{s t}+\xi x_{s t}+\epsilon_{s t} \tag{1}
\end{equation*}
$$

where $\alpha_{s}$ and $\beta_{t}$ are fixed effects, $r_{s t}$ is a "labor reform" variable, and $x_{s t}$ are other exogenous variables. ${ }^{1}$
(a) Explain how $\mu$ defines a "difference-in-difference" (DD) effect. Outline the necessary conditions for the identification of DD effects on the specification (1).
(b) Assume that the econometrician Rob is told by a political scientist friend that there are some states where the labour reforms would be a proforma reform only - that is, even though there formally is a reform (and it is recorded in the data as such), it could never be enforced in practice.

- (i) Explain the danger of just recoding these observations as if there were no reforms to begin with.
- (ii) Outline how you could use the information on these problematic states to better formulate a regression model to estimate the effect of labour reforms.
(c) Explain what the DD effects are using Table 5.2.3 of Angrist and Pischke (2009). Discuss what assumption underlying the specification in equation (1) seems to be violated. Explain your reasoning. The Table of Angrist and Pischke is reproduced at the end of the exam (see Figure 1).

[^2]
## Section B

3. A researcher is interested in testing the theory of Purchasing Power Parity (PPP) using UK and US quarterly data. Let $e_{t}, p_{t}^{*}$ and $p_{t}$ denote the logarithms of the US Dollar-Pound Sterling exchange rate, the US price level and the UK's price level in quarter $t$. Define $f_{t}=e_{t}+p_{t}^{*}$, such that $f_{t}$ gives the pound sterling value of the US price level. The researcher considers testing for two distinct versions of PPP theory. The strong version of PPP theory requires $f_{t}-p_{t}$ to be a stationary, zero mean, process. The weak version of PPP requires $f_{t}-\beta p_{t}$ to be a stationary process for some positive constant, $\beta$. Based on her empirical work, the researcher supplies you with three results:

- First, based on unit-root tests, she has concluded that $f_{t}$ is an integrated stochastic process of order 1. $p_{t}$ is also known to be nonstationary.
- Second, a least squares regression of $f_{t}$ on $p_{t}$ and a constant yields the following estimates:

$$
f_{t}=\underset{(0.041)}{0.012}+\underset{(0.026)}{0.988} p_{t}+\widehat{u}_{t}
$$

where $\widehat{u}_{t}$ denotes least square residuals. The figures in parenthesis give the associated standard errors.

- Third, she has additionally estimated the following equation:

$$
\Delta f_{t}=\underset{(0.004)}{0.019}-\underset{(0.032)}{0.106} \widehat{u}_{t-1}+\widehat{\varepsilon}_{t},
$$

where $\Delta f_{t}$ is the first difference of $f_{t}$ and $\widehat{\varepsilon}_{t}$ denotes least squares residuals. $\widehat{u}_{t} 1$ is a regressor formed by the lagged residuals of the least squares regression of $f_{t}$ on $p_{t}$ and a constant.
(a) The researcher is concerned that the regression of $f_{t}$ on $p_{t}$ might be spurious. What is the spurious regression problem? What information would you need regarding $\widehat{u}_{t}$ in order to dispel this problem? What does this imply for $f_{t}$ and $p_{t}$ and the weak version of PPP theory?
(b) Suppose the researcher was only interested in testing the strong version of PPP theory. Suggest an alternative testing strategy that allows the researcher to test whether the strong version of PPP holds in the data.
(c) Assume that the weak version of PPP theory holds. Give an economic interpretation to the slope parameter in the last equation estimated by the researcher.
4. A widely publicized study in Australia found that highly educated unionized workers received lower wages than highly educated non-union workers. Researchers arrived to this conclusion by estimating the following Mincer equation on a sub-sample of unionized workers:
$\ln$ Wage $_{i}=\alpha+\beta_{1}$ Education $_{i}+\beta_{2}$ Age $_{i}+\beta_{3}$ Gender $_{i}+\beta_{4}$ Experience $_{i}+\varepsilon_{i}$
under the assumption that the covariates on the right hand side of the equation are independent of the disturbance $\varepsilon_{i}$. Further it is assumed that $\varepsilon$ is normally distributed with mean zero and variance $\sigma_{\varepsilon}^{2}$. The study found that OLS estimates of $\widehat{\beta}_{1}$ in this sample was smaller than what was typically estimated by other researchers using data on non-unionized workers.
(a) Assume union membership is compulsory and based on the following simple rule: all workers with more than one year of Experience are required to join a union by law. Are the estimates of this study on wages of unionworkers likely to suffer from selection bias?
(b) Now assume that union membership is voluntary and researchers know that experienced workers with lower levels of motivation are more likely to join a union. In particular, they know that whenever

$$
\delta+\gamma_{1} \text { Experience }_{i}+\gamma_{2} \text { Motivation }_{i}+\nu_{i}>0
$$

the worker decides to become a member of the union. $\nu_{i}$ is an independent and normally distributed random disturbance with mean zero and variance $\sigma_{\nu}^{2}$. Further, the researchers know that motivation is also a determinant of compensation. That is, they know that the true model for wages is given by:
$\ln$ Wage $_{i}=\alpha+\beta_{1}$ Education $_{i}+\beta_{2}$ Age $_{i}+\beta_{3}$ Gender $_{i}+\beta_{4}$ Experience $_{i}+\beta_{5}$ Motivation $_{i}+\epsilon_{i}$
where $\epsilon_{i}$ is a normally distributed random variable which is independent of $\nu_{i}$ (and all other covariates). Unfortunately, researchers did not have access to any proxy for motivation and therefore proceeded to estimate the Mincer equation presented above. Prove that the estimates presented by the researchers are biased.
(c) Under the voluntary union membership case, assume further that in the population (i.e. across all workers, unionized and non-unionized), experience and motivation are uncorrelated. In the sample of union workers, do you expect these to remain uncorrelated, positively correlated or negatively correlated? Why is this related to the presence of selection bias?
(d) The arguments in (b) and (c) were noted to the researchers, who were advised to address them with Heckman's two-step procedure. In their reply they indicate that, in the absence of exclusion restrictions, they are skeptical the procedure would be able to deliver unbiased estimates. Discuss the researchers' reply.
5. In many countries, disability insurance (DI) is granted to people with physical or mental conditions limiting their ability to work. Not all applicants to DI are approved, however. The approval of a DI application is based on health conditions and other characteristics of the applicant that are often unobserved to the econometrician. Suppose you are interested in estimating the effect of disability insurance approval on hours of work, for a sample of people who applied for DI. Let $y_{1}$ be the hours of work if an applicant receives the DI, and $y_{0}$ be the hours of work if the applicant does not receive the DI. Let $D I=1$ if the applicant's DI application is approved, and $D I=0$ if the applicant's DI application is denied.
(a) Explain, in words, the economic meaning of $E\left(y_{0} \mid D I=1\right), E\left(y_{0} \mid D I=\right.$ 0 ). Which of these two is easily estimated given your data? Which one of them is more likely to be larger in our context? Explain your answers.
(b) Suppose we are interested in estimating the average treatment effect on the treated: $\alpha=E\left(y_{1} \mid D I=1\right)-E\left(y_{0} \mid D I=1\right)$. Show the bias in using $E\left(y_{1} \mid D I=1\right)-E\left(y_{0} \mid D I=0\right)$ to estimate $\alpha$.
(c) Suppose now you have a panel data of 2 years. In the first year, no one is in the DI program. In the second year, some people are granted the DI and the others' DI applications are rejected. Hours of work are observed in each year. Discuss how the panel structure of the data may help you reduce the bias you showed in (b). Be specific about the assumptions you make.
(d) Suppose you only have data on people who applied for DI in a single year. You are given one additional variable: for each applicant, you observe the type of case worker assigned to the applicant. Case workers are of one of the following two types: some are lenient and some are strict, and the assignment of case workers to the applicant is random (determined by a lottery). Discuss how the additional variable in the data may help you reduce the bias you showed in (b). Be specific about the assumptions you make.
6. An econometrician, Robert, collects data containing total employment (L) and average wages (W) for all the counties in the UK. Robert considers the following model of supply and demand for labor:

$$
\begin{align*}
D_{i} & =\alpha_{0}+\alpha_{1} W_{i}+\epsilon_{i},  \tag{2}\\
S_{i} & =\beta_{0}+\beta_{1} W_{i}+\mu_{i} \tag{3}
\end{align*}
$$

where $D_{i}, S_{i}$ and $W_{i}$ are labor demand, labor supply and wages in county $i$. The parameters of interest are $\alpha_{1}$. Robert assumes that the labor market in each county is independent and competitive, so labor market clears ( $D_{i}=$ $\left.S_{i}\right)$ for any county i and $\left\{\epsilon_{i}, \mu_{i}\right\}$ are i.i.d..
(a) Draw a simple labor demand and supply diagram for a given county i and derive its equilibrium wage, $W_{i}$.
(b) To estimate labor demand (equation (2)), Robert regresses $L_{i}$ on $W_{i}$ and obtains the least square estimator ( $\widehat{\alpha}_{1}$ ) for $\alpha_{1}$. Is $\widehat{\alpha}_{1}$ unbiased? If so, provide a proof. If not, derive the bias of the least square estimator for $\alpha_{1}$.
(c) Suppose that, in addition to variables on employment and wages, Robert observes the number of new migrants in a county $\left(M_{i}\right)$ in his data. Assume that $E\left(\mu_{i} \mid M_{i}\right)=E\left(\epsilon_{i} \mid M_{i}\right)=0$. Can Robert use the additional variable on migrants to improve the previous estimates of $\alpha_{1}$, and if so, how? Please explain your answers carefully.
(d) Is it possible to identify $\beta_{1}$ with Robert's dataset containing information on $M_{i}, L_{i}$, and $W_{i}$ ? Why? How would you advise Robert to estimate $\beta_{1}$ ? Be clear on any assumptions you are making.

Figure 1:
Table 5.2.3: Effect of labor regulation on the performance of firms in Indian states

|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ |
| :--- | :---: | :---: | :---: | :---: |
| Labor regulation (lagged) | -0.186 | -0.185 | -0.104 | 0.0002 |
|  | $(.0641)$ | $(.0507)$ | $(.039)$ | $(.02)$ |
| Log development |  | 0.240 | 0.184 | 0.241 |
| expenditure per capita |  | $(.1277)$ | $(.1187)$ | $(.1057)$ |
| Log installed electricity |  | 0.089 | 0.082 | 0.023 |
| capacity per capita |  | $(.0605)$ | $(.0543)$ | $(.0333)$ |
| Log state population |  | 0.720 | 0.310 | -1.419 |
|  | $(.96)$ | $(1.1923)$ | $(2.3262)$ |  |
| Congress majority |  |  | -0.0009 | 0.020 |
|  |  |  | $(.01)$ | $(.0096)$ |
| Hard left majority |  |  | -0.050 | -0.007 |
|  |  |  | 0.008 | $(.0091)$ |
| Janata majority |  | $(.0235)$ | -0.020 |  |
|  |  | 0.006 | $(.0333)$ |  |
| Regional majority |  |  | $(.0086)$ | $(.026$ |
|  |  | NO | NO | YES |
| State-specific trends | NO |  | 0.94 | 0.95 |
| Adjusted R-squared | 0.93 | 0.93 | 0.94 |  |
| Notes: Adapted from Besley and Burgess $(2004)$, Table IV. The table reports |  |  |  |  | regression-DD estimates of the effects of labor regulation on productivity. The dependent variable is log manufacturing output per capita. All models include state and year effects. Robust standard errors clustered at the state level are reported in parentheses. State amendments to the Industrial Disputes Act are coded $1=$ pro-worker, $0=$ neutral, $-1=$ pro-employer and then cumulated over the period to generate the labor regulation measure. Log of installed electrical capacity is measured in kilowatts, and log development expenditure is real per capita state spending on social and economic services. Congress, hard left, Janata, and regional majority are counts of the number of years for which these political groupings held a majority of the seats in the state legislatures. The data are for the sixteen main states for the period 1958-1992. There are 552 observations.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Thursday 8 June 2017 1.30pm-3.30pm

M500
DEVELOPMENT ECONOMICS

Answer two out of three questions. Each question carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1(a) Ethnic migrant networks drawn from different European origin countries historically competed with each other in the U.S. labor market. There is a positive relationship between ethnic-network competition in 1850 and participation in ethnic churches in 1860 at the county level. The relationship between ethnic-network competition in 1850 and church participation grows stronger as we move forward in time, over the course of the twentieth century, long after the ethnic networks themselves had ceased to be relevant. Provide an explanation for this observation.
(b) A cohort of high school seniors (aged 18) in a representative sample of U.S. counties were surveyed in 1980 and subsequently followed over their working lives. What relationship would you expect between historical ethnic competition in their birth county and their occupational and migration choices? What are the consequences of these choices for economic efficiency and inequality?

2(a) There is a positive relationship between the population density of rural counties in the Punjab in 1931 and the number of individuals that migrated from those counties to the U.K. in the 1950's (when there were no governmental restrictions on immigration). Social connectedness is increasing in spatial proximity, which is increasing in the population density of a rural population. Can we conclude from the observed relationship between population density and migration that migrant networks are active?
(b) Closer inspection of the data indicates that the positive relationship between population density in the origin county and migration is only obtained for counties with population densities that exceed a threshold level. Below the threshold, there is no relationship between population density and migration. Explain this observation with the help of a repeated-game model of migrant networks. How would you use the observed discontinuity to provide additional support for the claim that migrant networks are active?

3(a) Individuals born in rural Chinese counties can either choose a wage occupation in their birth county or move to one of two available urban destinations and set up a business. It is observed that individuals born in higher population density counties are more likely to select into business. Population density is positively associated with spatial proximity, which, in turn, is positively associated with social connectedness. One explanation for the observed occupational choice is thus that higher population density gives rise to stronger business networks. Develop a test based on the relationship between population density in the birth county and the distribution of businesses across the two urban destinations to provide additional support for this claim, being careful to make precise the assumptions underlying the test that you propose.
(b) Hsieh and Klenow in a series of well known papers provide empirical evidence that there is greater dispersion in firms size within industries in China than
in the United States. They also show that in the same industry, firms are smaller in China than in the United States. They argue that this evidence is indicative of a misallocation of resources in China and that output would increase if these distortions were eliminated. Comment on this argument in light of the fact that business networks, organized around different birth counties, are active in China.

ECM9/ECM10/ECM11/MGM3<br>MPhil in Economics<br>MPhil in Economic Research<br>MPhil in Finance and Economics<br>MPhil Finance

Monday 7 May $2018 \quad$ 2:00pm - 4:00pm

E100
MICROECONOMICS I

Candidates are required to answer three out of four questions
Each question carries equal weight

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. The following table contains information on the most preferred choices of six people, indexed $1,2,3,4,5$ and 6 . The first column shows the set of items each person chooses from, the second column shows which subset of these items are most preferred by person 1 (i.e., all the choices that are most preferred by person 1), and so on. Where the most preferred choices of a person are not observed, there is a question mark. We do not show the trivial choices, but you may assume that each person, when choosing from just one item, selects that item.

| Items choosing <br> from | Choices of <br> person 1 | Choices of <br> person 2 | Choices of <br> person 3 | Choices of <br> person 4 | Choices of <br> person 5 | Choices of <br> person 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\{a, b, c, d\}$ | $\{c, d\}$ | $\{c, d\}$ | $\{c, d\}$ | $?$ | $?$ | $?$ |
| $\{a, b, c\}$ | $\{c\}$ | $?$ | $\{c\}$ | $\{a, b\}$ | $?$ | $\{a, b\}$ |
| $\{a, b, d\}$ | $\{d\}$ | $?$ | $\{d\}$ | $?$ | $?$ | $\{a, b, d\}$ |
| $\{a, c, d\}$ | $\{c, d\}$ | $?$ | $\{c, d\}$ | $?$ | $?$ | $?$ |
| $\{b, c, d\}$ | $\{c, d\}$ | $?$ | $\{c, d\}$ | $?$ | $?$ | $\{b, c, d\}$ |
| $\{a, b\}$ | $\{a\}$ | $?$ | $\{a\}$ | $\{a\}$ | $\{a\}$ | $?$ |
| $\{a, c\}$ | $\{c\}$ | $?$ | $\{c\}$ | $\{c\}$ | $\{c\}$ | $?$ |
| $\{a, d\}$ | $\{a d\}$ | $\{a, d\}$ | $?$ | $\{a\}$ | $\{a\}$ | $?$ |
| $\{b, c\}$ | $\{c\}$ | $?$ | $\{c\}$ | $\{b\}$ | $\{b\}$ | $?$ |
| $\{b, d\}$ | $\{d\}$ | $?$ | $\{d\}$ | $\{b\}$ | $\{b\}$ | $?$ |
| $\{c, d\}$ | $\{c, d\}$ | $?$ | $\{c, d\}$ | $\{c\}$ | $\{c\}$ | $?$ |

(i) What does it mean for the observed choices of a person to be rationalizable?
(ii) Which of the people have choices that are rationalizable? Where the choices are rationalizable find a preference relation $\succeq$ that rationalizes them. Where the choices are not rationalizable, prove it.
(iii) For which of people can HARP be used to conclude whether their choices are rationalizable or not?
(iv) For which of people can WARP be used to conclude whehter their choices are rationalizable or not?
2. There are two goods $x$ and $y$ and two consumers 1 and 2. Each consumer's preferences can be represented by a strictly increasing, strictly concave and differentiable utility function. Normalize the price of good $x$ to 1 and let the price of good $y$ be denoted by $p>0$.
(a) First suppose that consumer 1's endowment is $e_{x}^{1}=10$ and $e_{y}^{1}=0$, consumer 2's endowment is $e_{x}^{2}=0$ and $e_{y}^{2}=5$, and there is a Walrasian equilibrium with allocation $x^{1}=7, y^{1}=3, x^{2}=3$ and $y^{2}=2$.
(i) What is Walras law?
(ii) What is the equilibrium price $p$ for the Walrasian equilibrium allocation described above?
(iii) Does player 1 prefer his/her Walrasian equilibrium allocation $(7,3)$, or the bundle $(3,6)$, or is there too little information to determine this?
(b) Now suppose that the same two consumers (with the same preferences) have different endowments. Consumer 1's endowment is now $\hat{e}_{x}^{1}=0$ and $\hat{e}_{y}^{1}=7$ and consumer 2's endowment is now $\hat{e}_{x}^{2}=5$ and $\hat{e}_{y}^{2}=0$. This part of the question asks you to consider whether the allocation $x^{1}=2, y^{1}=6$, $x^{2}=3$ and $y^{2}=1$ can be a Walrasian equilibrium allocation given these new endowments.
(i) Find the price $\hat{p}$ that must hold for the allocation $x^{1}=2, y^{1}=6, x^{2}=3$ and $y^{2}=1$ to be a Walrasian equilibrium allocation.
(ii) Assuming the allocation $x^{1}=2, y^{1}=6, x^{2}=3$ and $y^{2}=1$ is a Walrasian equilibrium allocation, does player 1 prefer the bundle $(2,6)$, or the bundle $(7.5,3)$, or is there too little information to determine this?
(iii) Using your previous answers (to questions in parts (a) and (b)), show whether or not the allocation $x^{1}=2, y^{1}=6, x^{2}=3$ and $y^{2}=1$ can be a Walrasian equilibrium.
(iv) Discuss your answer to this question.
3. Two villagers indexed $i=1,2$ have Bernoulli utility functions

$$
u\left(c_{i}\right)=\frac{1}{\lambda} e^{\lambda c_{i}},
$$

where $\lambda<0$. There are two states of the world, $A, B$ that occur with equal probability. In state $A$ villager 1 receives income $y_{1}(A)=1$ while villager 2 receives income $y_{2}(A)=0$. In state $B$ villager 1 receives income $y_{1}(B)=1$ while villager 2 receives income $y_{2}(B)=2$.
(i) Write down villager 1's expected utility $U_{1}\left(c_{1}(A), c_{1}(B)\right)$ in autarky, such that $c_{1}(s)=y_{1}(s)$ for $s=A, B$.
(ii) Write down villager 2's expected utility $U_{2}\left(c_{2}(A), c_{2}(B)\right)$ in autarky, such that $c_{2}(s)=y_{2}(s)$ for $s=A, B$.
(iii) Consider a social planner who seeks to maximize

$$
\theta U_{1}\left(c_{1}(A), c_{1}(B)\right)+(1-\theta) U_{2}\left(c_{2}(A), c_{2}(B)\right)
$$

by choosing $c_{1}(A), c_{1}(B), c_{2}(A), c_{2}(B)$ subject (only) to the resource constraints $c_{1}(A)+c_{2}(A) \leq$ $y_{1}(A)+y_{2}(A)$ and $c_{1}(B)+c_{2}(B) \leq y_{1}(B)+y_{2}(B)$, and where $0<\theta<1$. Find the values of $c_{1}(A), c_{1}(B), c_{2}(A), c_{2}(B)$ the social planner will choose in terms of the parameters of the problem.
(iv) Find the set of Pareto efficient and Pareto improving risk sharing arrangements (i.e., allocations of $c_{1}(A), c_{1}(B), c_{2}(A), c_{2}(B)$ satisfying the resource constraint that are Pareto efficient, and that both villagers weakly prefer to the autarky outcome).
(v) What happens to the set of Pareto efficient and Pareto improving risk sharing arrangements as the villagers become risk neutral such that $\lambda \rightarrow 0$ (i.e., $\lambda$ approaches 0 without reaching it)?
(vi) Provide intuition for your previous answer.
4. Suppose there is one person and two goods, and let $x \geq 0$ denote the quantity of one good that is consumed and $y \geq 0$ denote the quantity of the other good that is consumed. Suppose this person's preferences can be represented by the utility function

$$
u(x, y)=\left(x^{\alpha}+y^{\alpha}\right)^{1 / \alpha}
$$

where $\alpha \neq 0$ is a parameter.

Letting $p_{x}>0$ be the price of the first good, $p_{y}>0$ be the price of the other good and $w>0$ be the wealth of the consumer, the consumer solves the following utility maximization problem:

$$
\max _{x, y} u(x, y)
$$

subject to: (i) $p_{x} x+p_{y} y \leq w$, (ii) $x \geq 0$ and (iii) $y \geq 0$.
(i) For what values of $\alpha \neq 0$ is the utility function concave in $x$ ? Explain your answer.

For the rest of the question consider only values of $\alpha$ for which the utility function is concave in $x$ and concave in $y$.
(ii) By setting up and solving the Lagrangian find the Marshallian demand.
(iii) Find the indirect utility function.
(iv) Now consider the expenditure minimization problem:

$$
\min _{x, y} p_{x} x+p_{y} y
$$

subject to: (i) $u(x, y) \geq \bar{u}$, (ii) $x \geq 0$ and (iii) $y \geq 0$.

By setting up and solving the Lagrangian find the Hicksian demand.
(v) Find the expenditure function.
(vi) For what values of $\alpha$ are $x$ and $y$ substitutes.
(vii) For what values of $\alpha$ are $x$ and $y$ gross substitutes.

## END OF PAPER

ECM9//ECM11/
MPhil in Economics
MPhil in Finance and Economics

Tuesday 15 May 2018 9:00am-11:00am

## E101 <br> APPLIED MICROECONOMICS

This paper is in two sections. Section A: Candidates are required to answer four out of six questions. Section B: Candidates are required to answer one compulsory question.

Each Section carries equal weight; questions within Section A carry equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Section A

1. Consider an individual who lives for two periods, the preferences of whom are represented by the following, time-separable utility function:

$$
U\left(c_{1}, c_{2}\right)=u\left(c_{1}\right)+\beta u\left(c_{2}\right)
$$

Here, $c_{1}$ stands for consumption in period $1, c_{2}$ for consumption in period 2 , and $\beta$ for the individual's rate of time preference (discount factor). The individual receives income $y_{1}$ in the first period and income $y_{2}$ in the second. Given interest rate $r$, the individual's intertemporal budget constraint can be specified as:

$$
c_{1}+\frac{c_{2}}{1+r} \leq y_{1}+\frac{y_{2}}{1+r}
$$

How does the individual's propensity to save a portion of her/his firstperiod income change as $r$ increases? Is the overall effect of the increase in $r$ on savings unambiguous?
2. What is the prescription of the Ramsey Tax Rule regarding the optimal mix of commodity taxes? How does this prescription fare with respect to equity considerations?
3. Consider an economy with $n \geq 2$ individuals who differ only in terms of their abilities. Suppose that the government is considering increasing the tax rate, $\tau$, that applies to individuals with income above a certain threshold, $\bar{y}$. Describe the total revenue effect from the tax change.
4. Three concepts that can be used to measure the change in welfare associated with a price change are the equivalent variation, the compensating variation, and the change in consumer surplus. How do these measures differ? When are they going to give identical answers?
5. Consider an economy in which labour, $l$, is the only input in production and output $x$ the only good that is being produced. The production technology exhibits constant returns to scale. The government wants to raise revenue, $R$, but it can not levy lump-sum taxes (i.e. it has to attain its revenue requirement through commodity taxation). Which principle should guide its strategy? What prices should producers face? What prices should consumers face?
6. A group of researchers have estimated wage elasticities of the labour supply of New York City cab drivers that are significantly negative.
i. Is this type of behaviour consistent with the life-cycle model of labour supply that we have examined in the lectures?
ii. Does this group of professionals provide good grounds for testing the predictions of the model?

## Section B

1. A researcher estimates a demand system by OLS, with each budget-share regression give by:

$$
\omega_{i h}=\alpha_{i 0}+\alpha_{i a d} \text { adults } s_{h}+\alpha_{i k i d} k i d s_{h}+\sum_{j=1}^{7} \gamma_{i j} \ln \left(p_{j, h}\right)+\beta_{i} \ln \left(\frac{x_{h}}{P_{h}}\right)+u_{i h}
$$

Here $w_{i h}$ is the budget share on good $i$ for household $h$ and the price index $P_{h}$ is the Stone price index for household $h$ :

$$
\ln \left(P_{h}\right)=\sum_{i=1}^{N} \omega_{i h} \ln \left(p_{i h}\right)
$$

There are seven goods in this demand system: food at home (Fath), food in restaurants (Rest), transport (Tranall), services (Serv), recreation (Recr), alcohol and tobacco (Vice), and clothing (Cloth). The coefficient estimates from each OLS regression are given in the columns in Figure 1. For the following questions you may assume that prices do not feature as arguments in the households' direct utility functions.
(a) What properties of the share equations follow from the budget constraint, $\sum_{i} p_{i h} q_{i h}=x_{h}$ ? What restrictions on the parameters of these share equations are implied by those properties?
(b) What is the difference between using the logarithms of prices and income, $\ln \left(p_{i h}\right)$ and $\ln \left(x_{h}\right)$, and using their levels, $p_{i h}$ and $x_{h}$, as independent variables? In other words, how is the equation specified above different from the alternative:

$$
\omega_{i h}=\alpha_{i 0}+\alpha_{i a d} a^{2} u l t s_{h}+\alpha_{i k i d} k i d s_{h}+\sum_{j=1}^{7} \gamma_{i j} p_{j, h}+\beta_{i} \frac{x_{h}}{P_{h}}+u_{i h}
$$

(c) Calculate the Hicksian, Marshallian (own-price), and Income elasticities for food at home (Fath), transport (Tranall), and clothing (Cloth). The budget share of food at home is $26 \%$, the one of transport is $18 \%$, and the one of clothing is $10.5 \%$.
(d) Using the equivalent variation, calculate the deadweight loss of imposing a $10 \%$ tax on each of the three goods, food at home, transportation, and clothing.
(e) Suppose that a VAT currently applies to clothing and transportation at a rate of $15 \%$, while food at home remains untaxed. Is there a way to render the tax schedule more efficient at raising revenue? If so, how should the tax allocation across these three goods change?

Figure 1: Estimated Demand System

| Variable | BS Fath | BS Rest | BS Tranall | BS Serv | BS Recr | BS Vice | BS Cloth |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of Adults | 0.079 | -0.040 | 0.022 | -0.014 | -0.026 | -0.019 | -0.001 |
| Number of Kids | 0.053 | -0.020 | -0.018 | 0.019 | -0.003 | -0.015 | -0.016 |
| $\ln \left(p_{\text {Fath }}\right)$ | -0.086 | 0.075 | -0.120 | 0.064 | 0.032 | -0.039 | 0.073 |
| $\ln \left(p_{\text {Rest }}\right)$ | -0.014 | -0.021 | 0.150 | -0.065 | 0.085 | -0.069 | -0.066 |
| $\ln \left(p_{\text {Tranall }}\right)$ | 0.010 | 0.009 | 0.006 | -0.016 | -0.009 | -0.004 | 0.004 |
| $\ln \left(p_{\text {Serv }}\right)$ | 0.029 | 0.002 | 0.017 | -0.028 | -0.089 | 0.089 | -0.020 |
| $\ln \left(p_{\text {Recr }}\right)$ | 0.066 | -0.064 | -0.013 | 0.087 | 0.014 | -0.079 | -0.010 |
| $\ln \left(p_{\text {Vice }}\right)$ | -0.008 | 0.007 | -0.029 | -0.019 | -0.035 | 0.069 | 0.016 |
| $\ln \left(p_{\text {Cloth }}\right)$ | -0.093 | -0.003 | 0.006 | 0.070 | 0.066 | 0.000 | -0.046 |
| $\ln \left(\frac{x_{h}}{P_{h}}\right)$ | -0.160 | 0.050 | 0.022 | -0.011 | 0.046 | 0.017 | 0.037 |
| Constant | 1.624 | -0.306 | -0.043 | 0.283 | -0.284 | -0.030 | -0.244 |

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research
Wednesday 9 May $2018 \quad 2: 00 \mathrm{pm}-4: 00 \mathrm{pm}$

E200
MACROECONOMICS I

The paper is in two sections: You should answer all four questions in Section A and one question from Section B. Each Section carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS
EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## A. 1 Investment and Tax Credits

A common argument in the economic debate is that temporary tax credits to investment can help fight recessions. Consider the following standard $q$-theory model assuming that investment costs are $(1-\theta) I_{t}$, where $\theta$ is an exogenous tax credit that decreases the cost of investing and $0 \leq \theta<1$. This results in the following dynamics for the capital stock $K_{t}$ and the shadow price of capital $q_{t}$ :

$$
\begin{align*}
q_{t}+\theta & =1+\chi \Delta K_{t+1}  \tag{1}\\
\Delta q_{t+1} & =r q_{t}-A F^{\prime}\left(K_{t}+\frac{1}{\chi}\left(q_{t}-1\right)\right) \tag{2}
\end{align*}
$$

where $r$ denotes the real interest rate, with $1+r>0$; the production function is given by $Y_{t}=A F\left(K_{t}\right)$ with $F^{\prime}(\cdot)>0$ and $F^{\prime \prime}(\cdot)<0$, where $A$ is a positive productivity parameter; and $\chi$ is a positive parameter related to the investment adjustment costs. The initial capital stock $K_{0}>0$ is given and the transversality condition is $\lim _{T \rightarrow \infty}\left(\frac{1}{1+r}\right)^{T} q_{T} K_{T+1}=0$.
Draw the phase diagram for the system of equations (1)-(2) and explain the dynamics implied by this model.
Assume now that the government unexpectedly increases the tax credit, $\theta^{\prime}>\theta$, for $\bar{T}$ periods only. Carefully explain how $K_{t+1}$ and $q_{t}$ are affected over time using a phase diagram and provide an intuitive explanation for the effect on investment.

## A. 2 Solow Growth Model with Government Spending

Consider the following continous-time Solow growth model. The production function is $Y(t)=F(K(t), L(t))$ where $K(t)$ denotes the capital stock, which depreciates at rate $\delta ; L(t)$ is labour that grows at a constant rate $n . F(\cdot, \cdot)$ exhibits positive and diminishing returns for each of the inputs and constant returns to scale. The capital adjustment equation is: $K(t)=I(t)-\delta K(t)$ where $I(t)$ is investment. The economy is closed so all the resources saved are invested, $S(t)=I(t)$, where $S(t)$ is savings.
Assume that in this economy there is a government that just spends resources without contributing to production or capital accumulation. This government purchases $G(t)$ units of output in each period and $G(t)=g L(t)$. The government finances its purchases through lump sum taxes on consumers, $T(t)$, and runs a balanced budget, $G(t)=T(t)$. Consumers invest a constant fraction, $s$, of their disposable income, $Y(t)-T(t)$, and consume the rest. It is assumed that $0<\delta<1,0<s<1, g>0$ and $n>0$.
Derive the fundamental equation of motion of capital per capita, $k=\frac{K}{L}$, and graphically show that there can be two steady states. Select the stable steady state and derive graphically the effects of an increase in $g$ on capital per capita. Explain the short term and long term effects on the growth rate of aggregate output of the increase in $g$.

## A. 3 Wages, Employment and Productivity

Consider an economy in which the real wage $w$ is set by a union that maximizes the objective function

$$
U=(w-\bar{w}) L
$$

where $\bar{w}$ denotes the minimum wage, with $\bar{w}>0$. After the union sets $w$, the level of employment $L$ is chosen by a representative, competitive firm that maximizes real profits

$$
\Pi=\frac{1}{\alpha} A L^{\alpha}-w L
$$

where $A$ is a productivity parameter, and $\alpha$ is a production technology parameter, with $A>0$ and $0<\alpha<1$.
Derive the real wage $w^{*}$ set by the union and the corresponding employment level $L^{*}$ chosen by the firm. Explain how they are affected by a decline in the productivity parameter $A$.

## A. 4 Optimal Tax Rates and Productivity Slowdown

Suppose the government of Britannia sets the income tax rate $\tau_{t}$ in each period to minimize the following loss function:

$$
L_{t}=\sum_{s=0}^{\infty} \frac{1}{(1+r)^{s}} \frac{1}{2} \tau_{t+s}^{2} Y_{t+s}
$$

where $Y_{t+s}$ denotes Britannia's real national income in period $t+s$, and $r$ is the constant real interest rate. The government's intertemporal budget constraint equals

$$
(1+r) B_{t}+\sum_{s=0}^{\infty} \frac{G_{t+s}}{(1+r)^{s}}=\sum_{s=0}^{\infty} \frac{\tau_{t+s} Y_{t+s}}{(1+r)^{s}}
$$

where $G_{t+s}$ denotes real government purchases in period $t+s$, and $B_{t}$ real government debt at the start of period $t$, with $B_{t}>0$. Suppose that government purchases are set at a constant fraction $\gamma$ of national income, so that $G_{t+s}=\gamma Y_{t+s}$ for $s=0,1,2, \ldots$, where $0<\gamma<1$. Assume real national income grows at a constant rate $g$, so $Y_{t+s}=(1+g)^{s} Y_{t}$ for $s=0,1,2, \ldots$, where $r>g$.
Suppose that in period $t$, Britannia's Office for Budget Responsibility predicts a permanent decline in $g$ due to a productivity slowdown, while $Y_{t}$ is not affected. Derive the optimal income tax rate $\tau_{t}$ in terms of the initial debt-income ratio $B_{t} / Y_{t}$ and the parameters $r, g$ and $\gamma$. Explain how the decline in $g$ affects the tax rate $\tau_{t+s}$ and the primary deficit $\tilde{D}_{t+s} \equiv G_{t+s}-\tau_{t+s} Y_{t+s}$ over time.

## B. 1 Real Business Cycle Model

Consider the following two-period economy. There is a continuum of measure one of identical individuals $i \in[0,1]$. A representative individual maximises the following utility function:

$$
U=\sum_{t=0,1} \beta^{t}\left[\ln \left(c_{t}\right)+\ln \left(1-L_{t}\right)\right]
$$

where $c_{t}$ denotes consumption, $L_{t}$ labour supply, the subscript $t=0,1$ the time period, and $\beta \in(0,1)$ is a constant parameter. The time endowment is equal to one in each period.
Assume that individuals start with no initial assets $\left(b_{0}=0\right)$ and cannot die with debt. Individuals supply labour to the firms at a real wage $w_{t}$ per unit of labour. The consumption good is the numeraire.
There is also a continuum of measure one of identical, profit maximising firms. The production technology of the representative firm is represented by the following function:

$$
Y_{t}=A_{t} L_{t}
$$

where $Y_{t}$ is output, $L_{t}$ is labour used in production and $A_{t}>0$ is an exogenous productivity factor. There is perfect competition in the product and labour market. Assume there is no investment, the real interest rate is $r$ and the economy is closed, and there is no government initially.
(a) Formulate the optimization problem of the representative individual and of the representative firm and take the first order conditions associated with the problems. Give an economic interpretation of the conditions which characterise the trade offs for these problems.
(b) Derive the optimal levels of consumption $c_{0}$ and $c_{1}$, labour supply $L_{0}$ and $L_{1}$, and assets $b_{1}$, for given $w_{0}, w_{1}$ and $r$. Give an economic interpretation of your results.
(c) Derive the equilibrium real interest rate $r$. Assume that in period $t=0$ the agents anticipate a recession in the following period, $t=1$, because of a drop in $A_{1}$ to $A_{1}^{\prime}=\frac{A_{1}}{2}$. Explain the effect of this decrease in future productivity $A_{1}$ on consumption $c_{0}$ and $c_{1}$, labour supply $L_{0}$ and $L_{1}$, assets $b_{1}$, interest rate $r$, and total output $Y_{0}$ and $Y_{1}$.
(d) Assume now that the economy remains closed but there is a government. In $t=0$ the government announces that it will impose a tax on labour income equal to $\tau_{1}=50 \%$ in period $t=1$ so that the after-tax real wage for the consumer is $\frac{w_{1}}{2}$ per unit of labour. This tax is used to finance an unproductive level of government consumption $g_{1}$ in period $t=1$. The government runs a balanced budget. In period $t=0, g_{0}=0$ and $\tau_{0}=0$. Explain the effect of this fiscal policy on consumption $c_{0}$ and $c_{1}$, labour supply $L_{0}$ and $L_{1}$, assets $b_{1}$, interest rate $r$ and total output $Y_{0}$ and $Y_{1}$. Compare your results with the ones found in part (c).

## B. 2 Macroeconomic Policy with Electoral Uncertainty

Suppose that each period the government maximizes the objective function

$$
V_{t}=-\frac{1}{2}\left(\pi_{t}-\pi^{*}\right)^{2}+\beta_{t} y_{t}
$$

where $\pi_{t}$ is inflation, $\pi^{*}$ the government's inflation target, $y_{t}$ the output gap, and the subscript $t$ indicates the time period. The parameter $\beta_{t}$ is equal to $\beta_{t}=0$ for a conservative government, but equals $\beta_{t}=\bar{\beta}>0$ for a populist government. The economy is described by the aggregate supply relation

$$
y_{t}=\alpha\left(\pi_{t}-\pi_{t}^{e}\right)
$$

where $\pi_{t}^{e}$ denotes private sector inflation expectations, and the parameter $\alpha>0$. For every period, private sector inflation expectations $\pi_{t}^{e}$ are formed at the end of period $t-1$ using rational expectations, so $\pi_{t}^{e}=\mathrm{E}_{t-1}\left[\pi_{t}\right]$. Assume for simplicity that the government directly sets $\pi_{t}$ in each period $t$ through its macroeconomic policies.
(a) Use the first order condition to derive the level of $\pi_{t}$ set by the government, and explain intuitively how it depends on $\pi^{*}$ and $\beta_{t}$. Compare inflation under a conservative government, $\pi_{t}^{C}$, and under a populist government, $\pi_{t}^{P}$.

Suppose that the private sector knows in period $t=0$ that a new government will be elected in period $t=1$ for two periods. With probability $p$, a populist government will be elected such that $\beta_{1}=\beta>0$; and with probability $1-p$, a conservative government will be elected such that $\beta_{1}=0$, where $0<p<1$.
(b) Compute the level of private sector inflation expectations $\pi_{1}^{e}$ in period $t=1$. Provide an economic interpretation of the result.
(c) Compute the output gap in period $t=1$ under a conservative government, $y_{1}^{C}$, and under a populist government, $y_{1}^{P}$. Provide an economic interpretation of the results.
(d) Now consider period $t=2$ with the same government in power, so $\beta_{2}=$ $\beta_{1}$. Compute private sector inflation expectations $\pi_{2}^{e}$ and the output gap $y_{2}$ under a conservative government and under a populist government. Explain intuitively how the output gap and inflation evolve over time under each government.

ECM9/ECM11
MPhil in Economics
MPhil in Finance and Economics

Thursday 17 May 2018 9:00am - 11:00am

E201
APPLIED MACROECONOMICS I

Candidates are required to answer three out of four questions.
This written exam carries $70 \%$ of the marks for E201

Write your candidate number (not your name) on the cover of each booklet.

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## STATIONERY REQUIREMENTS

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Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

Consider the optimization problem

$$
\max _{\left\{c_{t}, b_{t+1}, M_{t}\right\}}\left\{\sum_{t=0}^{\infty} \beta^{t}\left(u\left(c_{t}\right)+v\left(\frac{M_{t}}{p_{t}}\right)\right)\right\},
$$

subject to $\quad p_{t} c_{t}+b_{t+1}+M_{t}=p_{t} y_{t}+b_{t}\left(1+i_{t}\right)+M_{t-1}, \quad t=0,1, \ldots$
where $p_{t}$ denotes the price level; $i_{t}$ the nominal interest rate; $c_{t}$ real consumption; $b_{t}$ nominal bonds; $y_{t}$ real non-financial income; $M_{t}$ money holdings, and $t=0,1,2, \ldots$ The functions $u(\cdot)$ and $v(\cdot)$ represent the flow utility received from real consumption and real money balances, respectively, and $\beta \in(0,1)$ is a discount factor.
(a) What is the first order condition with respect to money, $M_{t}$, in terms of $c_{t}, c_{t+1}$, $M_{t}, p_{t}$, and $p_{t+1}$ ?
(b) (i) Suppose that a central bank supplies money according to $M_{t+1}=(1+\mu) \times M_{t}$, for $t=0,1, \ldots$ Derive a nonlinear first-order difference equation of real money balances, $m_{t+1}=M_{t+1} / p_{t+1}$ in terms of $c_{t}, c_{t+1}, m_{t}$, and $\mu$.
(ii) Assume that bonds market clear such that $b_{t}=0$, and that $y_{t}=y$ for all $t=0,1, \ldots$ What condition is necessary to rule out $m^{*}=0$, where $m^{*}$ represents the steady state value such that $m_{t+1}=m_{t}=\ldots=m^{*}$.
(c) Assume that

$$
u(c)=\frac{c^{1-\gamma}-1}{(1-\gamma)}, \text { and } \quad v(m)=\ln (m)
$$

with $\gamma>0$. In addition, assume that bond markets clear and that $y_{t}=1$ for all $t$. Characterize $m^{*}$ in terms of $\mu$ and $\beta$. Show that if $1+\mu>\beta, m^{*}$ is positive and $m_{t}$ is uniquely determined and equal $m^{*}$.
[Hint: To show uniqueness, it could be useful, but it is not necessary, to rewrite the difference equation in terms of $\left(m_{t+1}-m^{*}\right)$ and $\left(m_{t}-m^{*}\right)$.]

## Question 2

Consider the optimization problem

$$
\begin{gathered}
\max _{\left\{c_{t}, b_{t+1}, x_{t+1}\right\}} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right) \\
\text { subject to } p_{t} c_{t}+b_{t+1}+x_{t+1}=w_{t-1}+b_{t}\left(1+i_{t}\right)+x_{t}-T_{t} \\
x_{t+1} \geq 0, \quad t=0,1, \ldots
\end{gathered}
$$

where $p_{t}$ denotes the price level; $i_{t}$ the nominal interest rate; $c_{t}$ real consumption; $b_{t}$ nominal bonds; $w_{t-1}$ nominal non-financial income received in period $t ; T_{t}$ lump-sum taxes; $x_{t}$ excess cash holdings; and $t=0,1,2 \ldots$ The function $u(\cdot)$ represent the flow utility received from real consumption, and $\beta \in(0,1)$ is a discount factor.

The government's budget constraint is given by

$$
p_{t} g_{t}+\left(1+i_{t}\right) d_{t}=d_{t+1}+T_{t}+m_{t}-m_{t-1},
$$

where $g_{t}$ denotes (real) government consumption; $d_{t}$ nominal government debt; and $m_{t}$ the money stock.
(a) Derive the intertemporal optimality conditions for both $b_{t+1}$ and $x_{t+1}$. Assume that $p_{-1} y_{-1}+x_{0}=m_{-1}$, use bond and goods market clearing conditions together with $w_{t-1}=p_{t-1} y_{t-1}$ to derive the equation of exchange.
(b) Suppose that potential output in both period $t$ and $t+1$ is equal to $y=1$. Assume further that $g_{t}=g_{t+1}=0$, and that a central bank targets inflation such that $\pi_{t+1}=\pi^{*}$. The utility function is given by $u(c)=c^{1-\gamma} /(1-\gamma)$, with $\gamma>1$. Suppose that suddenly, and unexpectedly, the discount factor $\beta$ increases to $\beta^{*}>\beta$. For which value of $\beta^{*}$ does the economy reach a liquidity trap? Explain intuitively how this value of $\beta^{*}$ relates to the inflation target $\pi^{*}$. Would a higher inflation target lower the risk for the economy to fall into a liquidity trap?
(c) Now suppose that the central bank operates according to the rule $\left(1+\pi_{t+1}\right)=y_{t}^{-\phi}$. And assume that the utility function is equal to the one in part (b) above, $y_{t+1}=1$, $g_{t+1}=0$, and that $\beta^{*}$ is such that $i_{t+1}=0$ and $y_{t}<1$. Derive the fiscal multiplier, $\partial y_{t} / \partial g_{t}$, around $g_{t}=0$.

## Question 3

Consider a government's consolidated budget constraint

$$
p_{t} g_{t}+d_{t}=q_{t} d_{t+1}+T_{t}+m_{t}-m_{t-1}, \text { for } t=0,1,2, \ldots
$$

where $p_{t}$ denotes the price level; $g_{t}$ (real) government consumption; $d_{t}$ nominal government debt; $q_{t}$ is the bond price; $T_{t}$ lump-sum taxes; $m_{t}$ the money stock; and $t=0,1,2, \ldots$
(a) Define $\tilde{d}_{t}=q_{t-1} d_{t}, i_{t}=\left(1 / q_{t-1}-1\right)$, and rewrite the above expression in terms of $\tilde{d}_{t}, \tilde{d}_{t+1}$ and $i_{t}$, instead of $d_{t}, d_{t+1}$ and $q_{t}$. Define $\hat{d}_{t}$ as real government debt, i.e. $\hat{d}_{t}=\tilde{d}_{t} / p_{t-1}$, and rewrite the government's budget constraint in real terms.
(b) Suppose that $m_{t}=p_{t} y_{t}=p_{t} y, p_{t} / p_{t-1}=1+\pi$ and suppose that the government holds $g, \hat{d}$ and $\tau$ constant, for $t=0,1,2, \ldots$ Solve for inflation, $\pi$, in terms of $g$, $r, \hat{d}$ and $y$. Show that a permanent gross-of-interest deficit may lead to inflation. Provide an intuitive explanation. What is the maximum amount of seignorage the government can generate in this situation? What does that imply for the maximum gross-of-interest deficit?
(c) Suppose instead that we are more interested in the evolution the variable $\bar{d}_{t}=$ $\tilde{d}_{t} / p_{t}$, rather than $\hat{d}_{t}$. How, does $\bar{d}_{t}$ evolve if the conditions in part (b) hold (assuming a finite value of $\pi$ )? Moreover, suppose the government held $\bar{d}_{t}=\bar{d}$ constant. How would this alter your answer from part (b)? Explain.

## Question 4

Consider the following optimization problem

$$
\begin{gathered}
\max _{\left\{c_{t}^{1}, c_{t}^{2}, b_{t+1}, x_{t+1}\right\}} \sum_{t=0}^{\infty} \beta^{t}\left(u\left(c_{t}^{1}\right)+u\left(c_{t}^{2}\right)\right), \\
\text { subject to } \quad p_{t} c_{t}^{1}+p_{t-1} c_{t-1}^{2}+b_{t+1}+x_{t+1}=w_{t}+b_{t}\left(1+i_{t}\right)+x_{t}-T_{t},
\end{gathered}
$$

where $c_{t}^{1}$ denotes real consumption of good one; $c_{t}^{2}$ real consumption of good two; $b_{t}$ nominal bonds; $i_{t}$ the nominal interest rate; $T_{t}$ nominal lump-sum taxes; $w_{t}$ nominal income; and $t=0,1,2, \ldots$

The government's budget constraint is given by

$$
T_{t}=m_{t-1}-m_{t}
$$

where $m_{t}$ denotes the money stock.
(a) Derive the first order conditions for $c_{t}^{1}, c_{t}^{2}$, and $b_{t+1}$.
(b) How is consumption of good two relative to good one related to the real interest rate? If equilibrium consumption of good one and two must satisfy $c_{t}^{1}+c_{t}^{2}=y_{t}$, where $y_{t}$ is exogenously given, derive the Friedman rule for the nominal interest rate.
(c) Suppose that $u(\cdot)=\ln (\cdot)$ and that $p_{-1} c_{-1}^{1}+x_{0}=m_{-1}$. If, in equilibrium, $w_{t}=$ $p_{t-1}\left(c_{t-1}^{1}+c_{t-1}^{2}\right)$, and the equation of exchange states $m_{t} v_{t}=p_{t} y_{t}$ where $y_{t}$ refers to the exogenous output in part (b) and $v_{t}$ denotes the velocity of money, how does $v_{t}$ related to $x_{t+1}$ and the real interest rate. Provide an intuitive explanation for this relationship.

## END OF PAPER

ECM9/ECM11/MGM3
MPhil in Economics
MPhil in Finance and Economics
MPhil Finance

Friday 11 May 2018 2:00pm - 5:00pm

## E300 <br> ECONOMETRIC METHODS

Answer all six questions from Section A and two questions from Section B. Part A is $60 \%$ of total marks.

All questions in Section A carry equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly. Credit will be given for clear presentation of relevant statistics.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator Dickey-Fuller and Engle-Granger Table
New Cambridge Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

A1 Suppose that $y$ and $x$ are two discrete random variables, such that

$$
\begin{aligned}
\operatorname{Pr}(x=-1, y=0) & =\operatorname{Pr}(x=0, y=0)=\frac{1}{3}, \text { and } \\
\operatorname{Pr}(x=0, y=1) & =\operatorname{Pr}(x=1, y=1)=\frac{1}{6}
\end{aligned}
$$

In particular, $x$ takes on only three possible values: $-1,0$, and 1 , whereas $y$ takes on only two possible values: 0 and 1 .
(i) (5 points) Verify that the linear regression $y_{i}=\alpha+\beta x_{i}+u_{i}$, where $\left(y_{i}, x_{i}\right)$ with $i=1, \ldots, n$ are i.i.d. draws from the joint distribution of $y$ and $x$ cannot satisfy the strict exogeneity assumption of the Gauss-Markov theorem.
(ii) (5 points) Assuming that $\operatorname{Cov}\left(x_{i}, u_{i}\right)=0$, what is the value of the probability limit of the OLS estimate of $\beta$ from the regression in (i), when the sample size $n$ goes to infinity?

A2 Consider the linear regression model

$$
\text { price }=\beta_{0}+\beta_{1} d i s t+\beta_{2} s q r f t+\beta_{3} b d r m s+u
$$

where price is the house price measured in thousands of pounds, dist is the distance of the house from the center of the city in kilometers, sqrft is size of house in square feet, $b d r m s$ is the number of bedrooms, and $E(u \mid d i s t, s q r f t, b d r m s)=0$.
(i) (5 points) Explain how you would test for homoskedasticity in such a model.
(ii) (5 points) Suppose that you know that Var $(u \mid d i s t$, sqrft, bdrms $)=$ $1+2 \exp \{d i s t\}$. What would this imply for the OLS estimator of the coefficients $\beta_{0}, \beta_{1}, \beta_{2}$, and $\beta_{3}$ ? Describe the main steps of the GLS estimator of these coefficients.

A3 Suppose that $y_{t}, t=\ldots-2,-1,0,1,2, \ldots$ is a strictly stationary process satisfying the following $\mathrm{AR}(1)$ equation

$$
y_{t}=\frac{1}{2} y_{t-1}+u_{t}
$$

where $u_{t}, t=\ldots-2,-1,0,1,2, \ldots$ are i.i.d. standard normal random variables.
(i) (7 points) Find the population (theoretical) correlogram for the process $z_{t}=\left(y_{t}+y_{t-1}\right) / 2$.
(ii) (3 points) You regress $z_{t}$ on $z_{t-1}$ (no constant) using data with $t=$ $1, \ldots, T$. What is the probability limit of the obtained OLS coefficient as $T \rightarrow \infty$ ?

A4 A researcher has monthly data on the nominal exchange rate $e_{t}$ between UK and US (\$ per $£)$ for the period from January 1975 to February 2018 (518 observations). The researcher obtains the following OLS result

$$
\widehat{\Delta \ln e_{t}}=\underset{(0.022)}{0.039}-\underset{(0.00474)}{0.00851} \ln e_{t-1}+\underset{(0.042)}{0.319 \Delta} \ln e_{t-1}, R^{2}=0.1048
$$

where $\Delta \ln e_{t}=\ln e_{t}-\ln e_{t-1}$ and the standard errors are given in the parentheses.
(i) (5 points) Test the hypothesis that the logarithm of the UK-US exchange rate has a unit root.
(ii) (5 points) The Purchasing Power Parity theory suggests that $\ln \left(e_{t} P_{t}^{U S} / P_{t}^{U K}\right)$, where $P_{t}^{U S}$ is a price index in the US and $P_{t}^{U K}$ is a similar price index in the UK, must be constant in the long run. The short run fluctuations around this constant must be stationary. Describe main steps of the procedure that you would use to test this theory.

A5 Consider a regression model in matrix notation $Y=X \beta+u$, where some of the explanatory variables are endogenous. Suppose that $Z$ is a matrix of valid instruments (including the exogenous explanatory variables that can be thought of as "instruments for themselves"). Recall that $\hat{\beta}_{2 S L S}=$ $\left(X^{\prime} P_{Z} X\right)^{-1} X^{\prime} P_{Z} Y$.
(i) (5 points) Explain what $P_{Z}$ is. What is the interpretation of $P_{Z} X$ in terms of the first stage regressions from the two stage least squares procedure?
(ii) (5 points) Consider a new estimator $\tilde{\beta}=\left(X^{\prime} X\right)^{-1} X^{\prime} P_{Z} Y$. Derive the probability limit of $\tilde{\beta}$ as the sample size $n$ goes to infinity $\left(p \lim _{n \rightarrow \infty} \tilde{\beta}\right)$ assuming that $p \lim _{n \rightarrow \infty}\left(X^{\prime} X / n\right)=p \lim _{n \rightarrow \infty}\left(Z^{\prime} Z / n\right)=\left(\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right)$ and that $p \lim _{n \rightarrow \infty}\left(X^{\prime} Z / n\right)=\left(\begin{array}{cc}1 & 0 \\ 0 & 0.5\end{array}\right)$.

A6 Suppose that you have $n$ i.i.d. observations $\left(y_{i}, x_{i}\right)$ that satisfy

$$
y_{i}=\beta x_{i} \varepsilon_{i}
$$

with $\beta>0$. That is, $y_{i}$ is the product of $\beta, x_{i}$, and $\varepsilon_{i}$, where $\varepsilon_{i} \sim N(0,1)$ is independent from $x_{i}$.
(i) (6 points) Find the maximum likelihood estimator of $\beta$.
(ii) (4 points) Find the LM statistic for testing the hypothesis $\beta=1$. (Hint: $\left.L M=S\left(\beta_{0} ; Y, X\right)^{2} /\left|\frac{\mathrm{d}^{2}}{\mathrm{~d} \beta^{2}} \log L\left(\beta_{0} ; Y, X\right)\right|\right)$.

B1 Consider a univariate linear regression model with one explanatory variable,

$$
y_{i}=\alpha+\beta x_{i}+u_{i}, \quad i=1, \ldots, n,
$$

where $\left(y_{i}, x_{i}\right)$ are i.i.d., $\operatorname{Var}\left(x_{i}^{2}\right)=1$, and $u_{i}$ is a standard normal random variable, which is independent from $x_{i}$. Let $\hat{\beta}$ be the OLS estimator of $\beta$.
(i) (5 points) Describe the finite sample distribution of $\hat{\beta}$, conditional on $x_{1}, x_{2}, \ldots, x_{n}$. What is the asymptotic distribution of $\hat{\beta}$ as $n \rightarrow \infty$ ?
(ii) (5 points) Find the asymptotic distribution of $\hat{\beta}^{2}$.
(iii) (5 points) A standard output of the OLS regression command in STATA includes values of the $t$ statistics for testing zero restrictions on the regression coefficients. Let $t_{\beta}$ be such a $t$ statistic corresponding to $\beta$. Find the probability limit of $t_{\beta} / \sqrt{n}$ as $n \rightarrow \infty$, when $\beta \neq 0$.
(iv) (5 points) Suppose that the quality of any estimator $\tilde{\beta}$ of $\beta$ is measured in terms of its conditional expected squared error

$$
E\left((\tilde{\beta}-\beta)^{2} \mid x_{1}, \ldots, x_{n}\right) .
$$

That is, the smaller the expected squared error of the estimator, conditional on $x_{1}, \ldots, x_{n}$, the better. According to this criterion, for what values of $\beta$ is the OLS estimator $\hat{\beta}$ worse than the estimator $\tilde{\beta}=0$, which always equal zero? How can your answer be reconciled with the statement of the Gauss-Markov theorem?

B2 An econometrician assumes that the annual growth rate $g_{t}$ of the UK GDP is an $\mathrm{AR}(2)$ process

$$
g_{t}=\alpha+\rho_{1} g_{t-1}+\rho_{2} g_{t-2}+u_{t}
$$

where $E\left(u_{t} \mid g_{t-1}, g_{t-2}, \ldots\right)=0$.
(i) (5 points) Assuming that $g_{t}$ is a weakly-stationary process, find the mean of $g_{t}$ in terms of $\alpha, \rho_{1}$, and $\rho_{2}$. Write down an $\operatorname{AR}(2)$ model for the demeaned $g_{t}$, that is, for $\tilde{g}_{t}=g_{t}-E g_{t}$.
(ii) (5 points) Represent the $\operatorname{AR}(2)$ model for $\tilde{g}_{t}$ in the form of $\operatorname{VAR}(1)$. Let $\tilde{g}_{t}=\sum_{j=0}^{\infty} \psi_{j} u_{t-j}$ be the $\mathrm{MA}(\infty)$ representation of $\tilde{g}_{t}$. Using the $\operatorname{VAR}(1)$ representation or otherwise, express $\psi_{1}$ and $\psi_{2}$ as functions of $\rho_{1}$ and $\rho_{2}$.
(iii) (5 points) The OLS regression of $g_{t}$ on a constant, $g_{t-1}, g_{t-2}$, using the data on $g_{t}$ from $t=1949$ to $t=2016$, yields the following estimates of $\alpha, \rho_{1}$, and $\rho_{2}$ :

$$
\hat{\alpha}=0.021, \hat{\rho}_{1}=0.403, \text { and } \hat{\rho}_{2}=-0.258
$$

Verify that the estimated $\mathrm{AR}(2)$ model is stable.
(iv) (5 points) In addition to the results in (iii), the econometrician obtains the following estimate of the variance-covariance matrix of the vector of OLS estimates $\left(\hat{\alpha}, \hat{\rho}_{1}, \hat{\rho}_{2}\right)^{\prime}$ :

$$
\hat{V}=\left(\begin{array}{ccc}
0.00001814 & -0.00025341 & -0.00025556 \\
-0.00025341 & 0.01482915 & -0.00476117 \\
-0.00025556 & -0.00476117 & 0.01479607
\end{array}\right)
$$

That is, for example, $\widehat{\operatorname{Var}}(\hat{\alpha})=0.00001814, \widehat{\operatorname{Cov}}\left(\hat{\alpha}, \hat{\rho}_{1}\right)=-0.00025341$, $\widehat{\operatorname{Cov}}\left(\hat{\alpha}, \hat{\rho}_{2}\right)=-0.00025556$, etc. Show that the equation

$$
g_{t}=\alpha+\rho_{1} g_{t-1}+\rho_{2} g_{t-2}+u_{t}
$$

can be rewritten in the form

$$
\Delta g_{t}=\alpha+\gamma g_{t-1}+\delta \Delta g_{t-1}+u_{t}
$$

Using the estimates reported in (iii) and (iv), test (at $5 \%$ significance level) for a unit root in $g_{t}$ against the alternative that $g_{t}$ is stationary around a constant non-zero mean.

B3 Suppose that you have a random sample $y_{i}, i=1, \ldots, n$, from a Poisson distribution with parameter $\lambda>0$. That is, $y_{i}$ takes on non-negative integer values $k=0,1,2, \ldots$ with respective probabilities $\frac{1}{k!} e^{-\lambda} \lambda^{k}$.
(i) (5 points) Write down an explicit expression for the log likelihood function. Derive the first- and second-order conditions for its maximisation. Find the maximum likelihood estimator $\hat{\lambda}_{M L}$ of $\lambda$.
(ii) (5 points) Find the Fisher information in the sample and state the asymptotic distribution as $n \rightarrow \infty$ of the maximum likelihood estimator $\hat{\lambda}_{M L}$. (Hint: When computing the Fisher information, you might need to use the fact that $\sum_{j=0}^{\infty} \frac{x^{j}}{j!}=e^{x}$ ).
(iii) (5 points) Suppose that the model is reparameterized in terms of $\mu=1 / \lambda$. That is, the probability of $y_{i}=k$ equals $\frac{1}{k!} e^{-(1 / \mu)} \mu^{-k}$. Find the ML estimator $\hat{\mu}_{M L}$ and derive its asymptotic distribution.
(iv) (5 points) Find the explicit form of the Wald statistic for testing the null hypothesis that $\lambda=1$ (in the original model, parameterized by $\lambda$ ). Find the explicit form of the Wald statistic for testing the null hypothesis that $\mu=1$ (in the reparameterized model). What do these results tell us about the Wald test?

B4 Discuss the following two topics.
(i) (10 points) Cointegration.
(ii) (10 points) Likelihood ratio, LM, and Wald tests.

## END OF PAPER

ECM11
MPhil in Finance \& Economics

Monday 14 May $2018 \quad 2.00 \mathrm{pm}-4.00 \mathrm{pm}$

F100
FINANCE I

Candidates are required to answer all four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
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## EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1. Part A and Part B of this question are totally independent.

Part A - What is the volatility (standard deviation) of an equally weighted portfolio of stocks within an industry in which the stocks have a volatility of $50 \%$ and a correlation of $40 \%$ as the number of stock in the portfolio becomes arbitrarily large?

Part B - Suppose Intel's stock has an expected return of $26 \%$ and a volatility (standard deviation) of $50 \%$, while Coca-Cola's has an expected return of $6 \%$ and volatility of $25 \%$. If these two stocks were perfectly negatively correlated (i.e., their correlation coefficient is -1 ),

1. Calculate the portfolio weights that remove all risk.
2. If there are no arbitrage opportunities, what is the risk-free rate of interest in this economy?

Question 2. Part A and Part B of this question are totally independent.

Part A - Stock A has a volatility of $65 \%$ and a correlation of $10 \%$ with your current portfolio. Stock B has a volatility of $30 \%$ and a correlation of $25 \%$ with your current portfolio. You currently hold both stocks. Which will increase the volatility of your portfolio: (i) selling a small amount of stock B and investing the proceeds in stock A , or (ii) selling a small amount of stock A and investing the proceeds in stock B?

Part B -WillyCom Ltd. has a debt-equity ratio of 2.5, and its weighted average cost of capital is of $15 \%$. The company (pre-tax) cost of debt is at $10 \%$, and its effective tax rate is of $35 \%$.

1. What is WillyCom expected return on its equity?
2. What is WillCom unlevered cost of capital?
3. What would be WillCom weighted average cost of capital if its debt-equity ratio was of 0.75 ? If it was of 1.5 ?

Question 3. The date today is $t_{0}=2$ Jan 2018 and the prices of the discount bonds for the different maturities are as follows:

|  | $t_{1}$ | $t_{2}$ | $t_{3}$ | $t_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 Jan 2019 | 1 Jan 2020 | 1 Jan 2021 | 1 Jan 2022 |
| $B_{t}$ | $£ 98,717$ | $£ 96,117$ | $£ 92,860$ | $£ 88,849$ |

1. Construct a table showing the term structure of interest rates.
2. What is the one year forward rate starting at date $t_{3}$ ?
3. What is the price today of a bond issued the 1 Jan 2017 for 5 years with a $5 \%$ coupon and a nominal of $£ 100$ ? A single coupon payment is made each year and the next coupon is scheduled for 1 Jan 2019.

Question 4. You are an option trader for the Dark Star Investment Bank. Today on the market, you can find the following data concerning the Yoda company.

- The Yoda stock is currently trading at $£ 110$;
- The risk free rate is $4 \%$;
- The price of a 2-year maturity European put option on Yoda is $£ 28$, with $\mathrm{K}=£ 110$;
- The price of a European call option with the same characteristics is $£ 38$.

Can you (and by how much) profit from the situation?

## END OF PAPER

ECM11
MPhil in Finance \& Economics

Wednesday 16 May $2018 \quad 2.00 \mathrm{pm}-4.00 \mathrm{pm}$

F200
FINANCE II

Candidates are required to answer all four questions..

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Write legibly.

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## STATIONERY REQUIREMENTS

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Rough work pads
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS

## EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1 . The MMC Company has a debt-equity ratio of 0.45 ; its unlevered cost of capital is of $17 \%$, and its pre-tax cost of debt (the pre-tax interest rate) is at $9 \%$. The sale revenues of the company are stable over time, expected to be of $€ 23,500,000$ forever. The variable costs amount to $60 \%$ of sales. The effective tax rate of the company is of $28 \%$, and it is distributing all its earnings as dividends at the end of each year. You have calculated out that the unlevered cash flows - the free cash flows - of the company in each period amount to:

| Sales | $€ 23,500,000$ |
| :---: | :---: |
| Variable costs | $€ 14,100,000$ |
| EBT | $€ 9,400,000$ |
| Tax | $€ 2,632,000$ |
| Net income | $€ 6,768,000$ |

1. If the company was unlevered, how much would it be worth?
2. What is the cost of the levered equity (i.e. the expected return on the equity capital of the company)?
3. We now calculate the value of the leveraged company. What is the value of the company's equity? What is the value of the company's debt? Assume the debt equity ratio to be fixed over time.

Question 2. Part A and Part B of this question are totally independent.

Part A - You currently hold a portfolio of three stocks, Delta, Gamma, and Omega. Delta has a volatility (standard deviation) of $60 \%$, Gamma has a volatility of $30 \%$, and Omega has a volatility of $20 \%$. Suppose you invest $50 \%$ of your money in Delta, and $25 \%$ each in Gamma and Omega.

1. What is the highest possible volatility of your portfolio?
2. If your portfolio has the volatility in (1), what can you conclude about the correlation between Delta and Omega?

Part B - A company has issued a 4-year bond one year ago with a nominal of $£ 1,000$ and a $4 \%$ coupon (the next coupon is scheduled for next year). Its yield-to-Maturity is $3 \%$. The term structure of interest rates is flat.

Compute the bond's duration and modified duration.

Question 3. A stock price is currently 25. It is known that at the end of two months it will be either 23 or 27 . The risk-free interest rate is $10 \%$ per annum with continuous compounding. Suppose that $S_{T}$ is the stock price at the end of the two months. What is the value of a derivative that pays of $S_{T}^{2}$ at that time?

Question 4. Black-Scholes gives the value of a European call with exercise price $K$ maturing at $T$ as
$C_{t}=S_{t} \Phi\left[\frac{\ln \left(S_{t} / K\right)+\left(r+\sigma^{2} / 2\right)(T-t)}{\sigma \sqrt{T-t}}\right]-K e^{-r(T-t)} \Phi\left[\frac{\ln \left(S_{t} / K\right)+\left(r-\sigma^{2} / 2\right)(T-t)}{\sigma \sqrt{T-t}}\right]$.
Show that the Black-Scholes formula for a call option, gives a price which tends to $\max [0, S-K]$ as $t \rightarrow T$.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Tuesday 8 May $2018 \quad 2.00 \mathrm{pm}-4.00 \mathrm{pm}$

F300
CORPORATE FINANCE

Candidates are required to answer all four questions.

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Write legibly.

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## STATIONERY REQUIREMENTS

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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

A risk neutral entrepreneur has cash $A>0$ and wants to undertake a project that costs $I>A$. The agent seeks to raise $I-A$ from outside lenders. On the market for funds, the agent can borrow and lend at rate $\gamma>0$. If financing is obtained, the entrepreneur exerts effort, either high or low, influencing the probability of success. If effort is high, the project succeeds with probability $p_{H}$ while if effort is low, the probability of success is $p_{L}<p_{H}$. Let $\Delta p=p_{H}-p_{L}$. In case of success, the project yields $R>0$ while if there is failure, the project yields nothing. Low effort yields a private benefit $B>0$, while high effort yields no private benefit. The entrepreneur's effort is not observed by the lenders. The parameters are such that $p_{H} R-I>0$ and $p_{L} R-I+B<0$. A sharing rule between the lenders and the entrepreneur is a split such that $R=R_{b}+R_{\ell}$. Last, all parties are protected by limited liability.
(1) Write up an incentive compatibility constraint on the borrower's share $R_{b}$ that ensures that the entrepreneur puts in high effort.
(2) Write up an individual rationality constraint that ensures that the lenders are willing to lend to the borrower and find a condition on $A$ that ensures that the project is financed. Explain briefly the sense in which there is a tradeoff between NPV and pledgeable income.
(3) What is the entrepreneur's net utility if financed and how does it vary when $\gamma$ is changed (supposing that the agent remains financed after the change)?
(4) Now suppose that instead of the private benefit from low effort $B$, the agent has a reduced private benefit $b<B$. How does this reduction influence (i) the agent's incentive compatibility constraint, (ii) the lender's individual rationality constraint, (iii) the financing condition and (iv) the agent's net utility?

## Question 2

Consider two firms, an acquirer $A$ and a target $T$. The initial share prices of these firms are $p_{A}$ and $p_{T}$, respectively. The acquirer then publicly announces its desire to take over the target via a tender offer to the target's shareholders. The offer is to exchange $x$ shares of firm $A$ for each share of firm $T$ tendered. Upon this announcement, the share prices of the acquirer and the target change to $q_{A}$ and $q_{T}$, respectively, with $q_{T}>p_{T}$. Suppose that there are two possible outcomes as follows: with probability $\alpha \in[0,1]$, the deal succeeds on the proposed terms; with probability $(1-\alpha)$, the deal fails and the share prices of the two firms revert to the pre-accouncement levels $p_{A}$ and $p_{T}$, respectively. All investors are risk neutral.
(1) Find the equilibrium share price of the target $q_{T}$ and the implied offer price of the target $q_{T}^{\prime}$. Define the merger-arbitrage spread.
(2) Consider an investor with private information who knows that the deal will succeed. Explain how this investor can profit from its private information.
(3) Consider an investor with private information who knows that the deal will fail. Explain how this investor can profit from its private information.
(4) Briefly discuss why merger arbitrage may not be considered a true arbitrage opportunity.

## Question 3

A company has $\$ 50$ million in excess cash and has no debt. It expects to generate free cash flow of $\$ 40$ million per year in future years (in perpetuity). The company's unlevered cost of capital is $10 \%$ and it has 10 million shares outstanding.
(1) Suppose that the company decides to use the $\$ 50$ million in cash to pay a special dividend today and that it will pay out the future free cash flows as regular dividends. (i) Calculate the amount of the special dividend and of the regular yearly dividend. (ii) Calculate the company's cum-dividend price and its ex-dividend price.
(2) Suppose that the company uses the $\$ 50$ million in cash to repurchase shares and that it will pay out the future free cash flows as regular dividends. Calculate the amount of the regular yearly dividend.
(3) Consider an investor who owns 2500 shares and assume that the company uses the $\$ 50 \mathrm{~m}$ to repurchase shares. Suppose that the investor would have preferred that the company would pay the special dividend. What can the investor do to receive the same amount in cash as if the company had paid the special dividend?
(4) Consider an investor who owns 2500 shares and assume that the company uses the $\$ 50 \mathrm{~m}$ to pay a special dividend. Suppose that the investor would have preferred that the company would repurchase shares. What can the investor do to undo the company's share repurchase?

## Question 4

The company Magnetic Eel Inc. (ME) is considering an acquisition of another firm for $\$ 100 \mathrm{~m}$. This acquisition is expected to increase ME's free cash flow by $\$ 5 m$ the first year and this contribution is expected to grow at a rate of $3 \%$ every year thereafter. ME currently maintains a debt to equity ratio of 1 and pays marginal corporate tax rate of $40 \%$. Its debt and equity costs of capital are $r_{D}=6 \%$ and $r_{E}=10 \%$, respectively. ME will maintain a constant debt to equity ratio for the acquisition.
(1) Calculate ME's unlevered cost of capital and the unlevered value of ME's acquisition.
(2) Suppose ME issues new debt of $\$ 50 \mathrm{~m}$ initially to fund the acquisition. Calculate the present value of the interest tax shield.
(3) Supposing that ME issues new debt of $\$ 50 \mathrm{~m}$ initially to fund the acquisition, calculate the total value of the acquisition.

## END OF PAPER

## UNIVERSITY OF

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Friday 18 May $2018 \quad 2: 00 \mathrm{pm}-4: 00 \mathrm{pm}$

F400
ASSET PRICING

Candidates are required to answer all four questions.

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## STATIONERY REQUIREMENTS

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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. A time 0 , an investor invests $\$ 1$ in an asset with gross return $R_{1}$, where $R_{1}$ is distributed lognormally with parameters $\mu_{1}$ and $\sigma_{1}$. At time 1 , she reinvests the proceeds of her initial investment in an asset with gross return $R_{2}$, where $R_{2}$ is distributed lognormally with parameters $\mu_{2}$ and $\sigma_{2}$. At time 2 , she consumes her final wealth.
(a) What is meant by the investor's certainty-equivalent wealth in this context?
(b) Assuming that the investor has constant relative risk aversion $\rho>0$, what is her certainty-equivalent wealth?
(c) What is the expected rate of return on her portfolio?

Hint: a random variable $Y$ is said to be distributed lognormally with parameters $\mu$ and $\sigma$ iff $Y=\exp (X)$, where $X$ is distributed normally with mean $\mu$ and standard deviation $\sigma$.
2. Suppose that there are three states of the world and one risky asset. This risky asset pays 1,3 and 5 units of consumption in states 1,2 and 3 respectively.
(a) What is meant by a call option with strike price 2 written on the risky asset?
(b) Suppose that an investment bank is considering introducing this call option. Suppose further that the risky asset is currently the only asset traded, and that its price is 2 . In what range would the bank expect the price of the call option to fall?
(c) Suppose now that two assets are traded: the risky asset discussed so far, and a safe asset that pays 1 unit of consumption in each of the three states. Suppose further that the price of the safe asset is 1 . In what range would the bank now expect the price of the call option to fall?
3. There are three assets: a safe asset and two risky assets. The prices $B, S_{1}$ and $S_{2}$ of the safe asset, risky asset 1 and risky asset 2 evolve according to the s.d.e.s

$$
\begin{aligned}
d B & =r B d t, \\
d S_{1} & =\mu_{1} S_{1} d t+\sigma_{1} S_{1} d Z_{1}, \\
d S_{2} & =\mu_{2} S_{2} d t+\sigma_{2} S_{2} d Z_{2}
\end{aligned}
$$

respectively, where $Z=\left(Z_{1}, Z_{2}\right)$ is a two-dimensional standard Wiener process. There is also a derivative that pays $\Phi\left(S_{1}(T), S_{2}(T)\right)$ at maturity $T>0$. There is no arbitrage.
(a) Find the equation satisfied by the price of the derivative. Be careful to explain any assumptions that you make, and to set out the main steps of your derivation carefully.
(b) Suppose that $\Phi\left(S_{1}(T), S_{2}(T)\right)=K+S_{1}(T)-S_{2}(T)$. What is the price of the derivative at time 0 ?
4. Suppose that the dynamics of the spot rate of interest are given by the s.d.e.

$$
d r(t)=\mu(r(t)) d t+\sigma(r(t)) d Z(t)
$$

with initial condition $r(0)=r_{0}$, where the functions $\mu$ and $\sigma$ are exogenously given. Suppose further that the price $B$ of a locally risk-free asset evolves according to the s.d.e.

$$
d B(t)=r(t) B(t) d t
$$

with initial condition $B(0)=1$.
(a) Consider a zero-coupon bond with maturity $T$. Let $p^{T}(t, r)$ be the price of this bond at time $t<T$ when the spot rate is $r$. Write down a p.d.e. that $p^{T}$ must satisfy.
(b) Explain the significance of all the terms in this equation.
(c) Is $p^{T}$ uniquely determined by this equation? Justify your answer carefully.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Monday 28 May $2018 \quad 1.30 \mathrm{pm}-3.30 \mathrm{pm}$

## F500 <br> EMPIRICAL FINANCE

This paper is in two sections. Section A: Candidates are required to answer all four questions. Section B: Candidates are required to answer two out of three questions.

Each section carries equal weight.

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Write legibly.

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## STATIONERY REQUIREMENTS

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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS

 EXAMINATIONCalculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Section A

1. Suppose that the logarithm of (daily) observed stock prices $p_{t}=\log P_{t}$ satisfies

$$
\begin{gather*}
p_{t}^{*}=p_{t-1}^{*}+\varepsilon_{t}  \tag{1}\\
p_{t}=\alpha p_{t}^{*}+(1-\alpha) p_{t-1}^{*} \tag{2}
\end{gather*}
$$

where $\varepsilon_{t}$ are i.i.d. with mean zero and finite variance $\sigma_{\varepsilon}^{2}$, and $\alpha \in[0,1]$.
(a) Does this model imply that $p_{t}^{*}$ lies between $p_{t}$ and $p_{t-1}$ ?
(b) What is the mean, variance, and autocovariance function of the observed daily return series?
(c) Derive an expression for weekly returns, that is, Monday to Monday returns. What is the autocorrelation function of (non-overlapping) weekly returns?
(d) Are the theoretical predictions regarding the autocorrelations of observed returns consistent with the empirical results presented in the table (Table 2.4 from CLM)?
2. True, False, Explain
(a) Tests of market efficiency based on the variance ratio statistic are invalid when returns are not normally distributed so the Cowles-Jones tests and other nonparametric tests are better.
(b) In the absence of informed traders, bid ask spreads would be narrower, and so prices would reflect fundamentals more accurately.
(c) Firms split their stock because they want to maintain the tick size in real terms.
3. True, False, Explain
(a) Stock returns are more volatile during during trading hours than during weekend hours.
(b) The VIX is a forward looking measure of volatility and so is superior to historical methods such as realised volatility.
(c) The market model for daily data implies that the market model for monthly data holds with the same intercept and slope coefficients, although the error variance increases.
4. Describe what is the Equity premium puzzle. What are the possible explanations for this puzzle.

## Section B

1. Suppose that required returns $R$ are constant and

$$
R=E_{t}\left(\frac{P_{t+1}+D_{t+1}}{P_{t}}\right),
$$

where $P$ are prices, $D$ are fundamentals, and $E_{t}$ denotes expectation taken at time $t$. Derive an expression for prices in terms of the future sequence of fundamentals $\left\{D_{t+j}\right\}_{j+=1}^{\infty}$. Some have argued that stock prices are too variable relative to fundamentals. Explain how you might test this claim given a long sample of data on stock prices and fundamentals.
2. Suppose that returns $R_{i t}$ obey a linear K-factor model

$$
R_{i t}=\alpha_{i}+\sum_{j=1}^{K} b_{i j} f_{j t}+\varepsilon_{i t}
$$

where $\varepsilon_{i t}$ is an (iid over $t$ ) idiosyncratic error term with

$$
\operatorname{cov}\left(\varepsilon_{i t}, f_{j s}\right)=0 \quad ; \quad E\left(\varepsilon_{i t} \varepsilon_{j s}^{\top}\right)=\left\{\begin{array}{cc}
\sigma_{i j} & \text { if } t=s \\
0 & \text { else }
\end{array}\right.
$$

Explain the Fama-French methodology for constructing factor returns based on size and value. What are the restrictions on $\alpha$ implied by the Arbitrage Pricing Theory? Suppose that you observe the panel $\left\{R_{i t}, f_{j t}, j=1, \ldots, K\right.$ $i=1, \ldots, N, t=1, \ldots, T\}$. How would you test these restrictions using the observed dataset. Does it matter how big $N$ is? What difference does it make if one can assume that: $\sigma_{i j}=0$ unless $i=j$ ?
3. Suppose that daily stock returns satisfy

$$
\begin{gathered}
r_{t}=\mu+\sigma_{t} \varepsilon_{t} \\
\sigma_{t}^{2}=\omega+\beta \sigma_{t-1}^{2}+\gamma r_{t-1}^{2}+\delta r_{t-1}^{2} 1\left(r_{t-1}<0\right),
\end{gathered}
$$

where $\varepsilon_{t}$ is iid standard normal and independent of all past information. What properties of daily stock returns does the GARCH model capture? What properties of daily stock returns does the parameter $\delta$ capture? Describe how you would estimate the parameters $\mu, \omega, \beta, \gamma, \delta$ given a sample of data $\left\{r_{1}, \ldots, r_{T}\right\}$. Compare the resulting estimate of volatility $\sigma_{t}^{2}$ with the realized volatility estimator and with the implied volatility estimator. Describe in each case how the estimates are constructed, their interpretations, and their properties.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Friday 1 June 2018 9:00am -11:00am

F510
INTERNATIONAL FINANCE

Candidates are required to answer four out of five questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

Between January 2010 and December 2016, in absolute value, deviations from Covered Interest Parity (Libor-based) relative to the dollar averaged 20 points or above for the Swiss franc (CHF) Danish krone (DKK) euro (EUR) Japanese yen (JPY) Norvegian krone (NOK) Swedish krona (SEK).
(a) Define the Covered Interest Parity condition.
(b) Discuss whether the observed deviations in the 2010-2016 periods can be explained by:
(b1) default risk and/or
(b2) counterparty risk in the forward market.
In your answer, use the theoretical framework discussed in class, and draw on main lessons from readings providing empirical evidence.

## Question 2

(a) Define the uncovered interest parity (UIP) and illustrate the so-called 'UIP puzzle' documented by the literature in the period up to the outburst of the Global Financial Crisis (GFC).
(b) What is the "Peso Problem"? Discuss how this problem can contribute to explain the failure of the UIP documented before the GFC.
(c) What happened to the evidence underlying the classical 'UIP puzzle' since the outburst of the GFC?
(d) Can the "Peso Problem" also provide an explanation of the way the UIP condition fails to hold in the data during the years of the GFC?

## Question 3

(a) Explain the Fama (84) result: what restrictions the risk premium (or excess return) must satisfy to rationalize the UIP puzzle?
(b) Discuss the implications for exchange rate determination.
(c) How can you reconcile Fama (84) with the point stressed by Engel (2016), that interest rate hikes can be expected to strengthen the exchange rate?

## Question 4

According to comparative advantage, on average, a country will sell (=export) assets whose autarky prices (returns) are relatively low (high). Consider a world consisting of two endowment economies (Home and Foreign) operating for two periods only. Foreign variables are denoted with a star. Social preferences are identical:

$$
\begin{aligned}
& U\left(C_{t}\right)+\beta E_{t} U\left(C_{t+1}\right) \\
& U\left(C_{t}^{*}\right)+\beta E_{t} U\left(C_{t+1}^{*}\right)
\end{aligned}
$$

Their period t endowment is identical $Y_{t}=Y_{t}^{*}$. Their period $\mathrm{t}+1$ endowment has the same expected value $E_{t} Y_{t+1}=E_{t} Y_{t+1}^{*}$ but different variance: the Home endowment is more variable than the foreign one, i.e. $\operatorname{Var}\left(Y_{t+1}\right)>\operatorname{Var}\left(Y_{t+1}^{*}\right)$.
(a) If the two economies can only trade a bond $B$, exchanged at the price $q_{t}$ in period t , yielding one unit of goods at $t+1$ : which country will sell the bond and import consumption in period $t$ ? Explain carefully and intuitively.
(b) Briefly discuss the implications of output uncertainty for current account imbalances and the world safe interest rates.
Suppose now that two assets are traded, $B^{H}$ and $B^{L}$, at the price $q_{t}^{H}$ and $q_{t}^{L}$. $B^{H}$ pays one unit of goods only when Home output is high; $B^{L}$ pays one unit of goods when Home output is low.
(c) Which country will import $B^{H}$, which country will export $B^{L}$ according to comparative advantage? For simplicity, assume that there are only two states of the world, in which Home output is either High or Low, occurring with probability $\pi(H)$ and $\pi(L)$ respectively, and $\operatorname{Var}\left(Y_{t+1}^{*}\right)=0$.
(d) Briefly discuss again the implications of output uncertainty for current account imbalances and the world interest rates in the case of trade in the two (contingent) bond: how does the introduction of the two bonds change your predictions? Why?

## Question 5

Suppose the central bank of a Latin American country aims to peg its currency to the US dollar so that $S=\bar{S}$, where $S$ is the domestic currency price of US dollars. The central bank balance sheet is given by

$$
M=B+R
$$

where $M$ denotes the money supply, $B$ domestic government bonds held by the central bank and $R$ foreign reserves. If the central bank has depleted its foreign reserves, it adopts a free float. Assume that the government runs a structural budget deficit and forces the central bank to engage in monetary financing such that $\dot{B} / B=\gamma>0$, where $\dot{X} \equiv \mathrm{~d} X / \mathrm{d} t$ denotes the time derivative of $X$.
Money market equilibrium is given by

$$
\frac{M}{P}=Y e^{-\frac{1}{2} i}
$$

where $P$ is the aggregate price level, which is flexible, $i$ is the nominal interest rate, and $Y$ is aggregate output, which is exogenous and normalized to $Y=1$. Purchasing power parity holds so that

$$
P=S P^{*}
$$

where $P^{*}$ is the US aggregate price level, which is exogenous and normalized to $P^{*}=1$. In addition, assume that

$$
i=i^{*}+\dot{S} / S
$$

where $i^{*}$ is the US nominal interest rate, which is exogenous and constant.

1. (a) Derive the level of the money supply $M$ that is required to keep the exchange rate fixed at $S=\bar{S}$. Explain intuitively why the exchange rate peg is unsustainable.
(b) Derive the time $T$ of the speculative attack (assuming no speculative bubbles). Explain how it depends on the initial level of foreign reserves $R_{0}$.
(c) Suppose that after fixing the exchange rate at $S=\bar{S}$ at $t=0$, there is a sudden increase in the degree of monetary financing $\gamma$ by the central bank at time $\tau$, where $0<\tau<T$. Explain how this would affect the timing of the speculative attack.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Tuesday 5 June $2018 \quad 1.30 \mathrm{pm}-3.30 \mathrm{pm}$

F520
BEHAVIOURAL FINANCE

Candidates are required to answer two out of three questions

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## QUESTION 1

a. Mutual fund A invests in stocks, and has accepted investments from thousands of investors. Hedge fund $B$ on the other hand is a lot more selective. It invests in the same type of stocks as fund $A$, but it has accepted investment from few high net-worth individuals. Would you expect these funds to face the same limits to arbitrage?
b. Can you think of any examples where arbitrageurs made more money in conditions of increased noise trading?
c. Derive the Noise trader model of De Long et al (1990). Provide clear intuition behind each component of the equilibrium price. Demonstrate how changes in predictability of noise traders' sentiment (the variance of the sentiment), proportion of noise traders in the market, risk aversion of arbitrageurs affect the mispricing. Explain why irrational traders are not driven away from the market by arbitrageurs.

## QUESTION 2

a. Microsoft co-founder Bill Gates has made a big bet on the recovery of Spain's construction sector by becoming the second-largest shareholder in FCC, a Spanish builder hit hard by the collapse of a decade-long property bubble five years ago. Shares in FCC, whose main shareholder is the heiress and philanthropist Esther Koplowitz, rose almost 10 per cent on Tuesday to $€ 17.20$ after the company said it had sold 6 per cent of its treasury shares to funds connected to Mr Gates for $€ 113.5 \mathrm{~m}$. Does this story tell us anything about the Efficiency Market Hypothesis?
b. Please explain the main insight from the figure by Benartzi and Thaler (1995) shown below. (i) discuss the plausibility of this result.
(ii) provide your evaluation and discussion regarding the following statement: "Myopic loss aversion is only observed at the individual level; it is irrelevant for organizations"

c. A farmer will sell 500,000 bushels of corn in two months from now that took $\$ 1,800,000$ to produce. We assume no time discounting. The farmer is uncertain about the price of corn in two months. The price may be high (event H: $\$ 4$ per bushel) or low (event L: $\$ 3.80$ per bushel). So, there is no certainty in what future profit will be. It is very useful for the farmer to know what his financial position will be in two month time. The farmer can insure with a bank. The bank will guarantee a sure amount in two months but will charge a risk premium. The bank knows the probabilities $P(H)=P(L)=0.5$. The farmer maximizes his Choquet expected utility (with specific form and beliefs to be specified). What is the minimum amount that the farmer wants to be assured of (Certainty equivalent) and the maximum profit (the risk premium) for the bank for the following three cases:
(i) The farmer knows the probabilities too and is risk neutral
(ii) The farmer knows the probabilities too and his utility function $\mathrm{U}(\mathrm{x})=\mathrm{x}^{0.5}$.
(iii) The farmer does not know the probabilities but forms his subjective beliefs about likelihood of future event in the form of capacity with $\mathrm{v}(\mathrm{L})=0.4$ and $\mathrm{v}(\mathrm{H})=0.3$. The farmers utility function is as in (ii).

## QUESTION 3

a. In 1961, Sony began to develop a color television receiver based on the Chromatron picture tube. Ibuka, one of Sonly's founder, led a two-year effort to develop a commercial prototype and process technology. By September 1964, Ibuka's team had succeeded in developing a prototype. However, they had not developed a commercially viable manufacturing process.

Sony invested in a new facility to house the production assembly. Ibuka announced that the color television would be Sony's top priority. He placed 150 people in its assembly line. To lbuka's chargin, the production process yielded only two or three usable picture tubes per thousand produced. The retail price of Sony's color television set was $\$ 550$, but the cost of production was more than double that amount. There was a sharp difference of opinion within the Sony leadership about the appropriate course of action. Another Sony's founder Akio Morita wanted to terminate the project. However, Ibuka refused.

Sony continued to produce and sell Chromatron sets, eventually selling 13,000 sets, each one at a loss. In November 1966, Sony’s financial managers announced that Sony was close to ruin. Only then did Ibuka agree to terminate the project.

Discuss what behavioural biases are reflected in this case.
b. Describe the main differences in decision making process by a corporate manager from traditional (based on rational paradigm) and Behavioural Finance perspectives. Provide full intuition where the differences come from.
c. A company has $1,000,000$ shares outstanding at $\$ 15$ per share. Managers believe that the discount rate appropriate for the risk borne is $15 \%$ and total cash flows, expected to be $\$ 1$ million next year, will rise by $5 \%$ per year indefinitely. Discuss a strategy that is beneficial to the current shareholders. What are the potential negative consequences for the company?

END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Wednesday 6 June 2018 9.00am - 11.00am

F540
TOPICS IN APPLIED ASSET MANAGEMENT

This paper is in two sections:
Section A: Candidates are required to answer one compulsory question.
Section B: Candidates are required to answer two out of four questions.

Section A is weighted at $40 \%$ and section B is weighted at $60 \%$.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator New Cambridge Elementary Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A

Provide as much mathematical detail as you can when answering these questions

1. General Question
(a) Explain clearly how a particular form of OLS regression can be used to compute Mean Variance portfolio weights.
(b) Ferson and Siegel(2001) have shown how the optimal weight function is changed when you take into account the fact that you can exploit a predictive model (conditional information) rather than use the unconditional moments in mean variance optimisation. Derive and explain the rationale behind their formula.
(c) Define what is meant by a coherent risk measure
(d) Campbell and Thompson discuss the relationship between the $R^{2}$ in a predictive regression and the Sharpe Ratio from the resulting investment strategy- derive this relationship and discuss how it relates to the more general theoretical results we have considered which relate the predictive $R^{2}$ to the stochastic discount factor of an asset pricing model.
(e) Explain the Information Flow model for asset prices put forward by Clark; show how it accounts for non-gaussianity in financial returns and how we can check if we have a good measure of information flow.
(f) Suppose that the change in the value of a portfolio over a 1 day time period is normal with a mean of zero and a standard deviation of $\$ 2$ million
i. What is the (a) 1-day $97.5 \%$ VaR, (b) the 5-day $97.5 \% \mathrm{VaR}$ and (c) the 5 -day $99 \% \mathrm{VaR}$
ii. What for questions (b) and (c) would be the answer be if there is first order autocorrelation with the autocorrelation parameter equal to 0.16 ?
2. Mean Variance and Predictability
(a) Derive the Minimum Variance portfolio weights, explain how the Volatility Timing strategies of Kirby and Ostdiek are related and, with some detail, why both may be preferable to the use of Mean Variance weights
(b) Explain the critical differences between the Optimal Unconstrained, the Tangency Portfolio and Optimal Constrained portfolio weights and why Di MIguel, Garlappi and Uppal conclusion regarding the supremacy of $\frac{1}{N}$ weights needs to be corrected. What two issues in particular are required to ensure that utility based portfolio weights dominate $\frac{1}{N}$ ?
(c) Describe how forecast combination can be seen as a form of shrinkage for instance in the case of kitchen sink predictive regressions.
(d) Explain what is meant by LASSO and Adaptive Elastic Net, their properties and what is the Oracle property.
3. CAPM and Factors
(a) Consider a portfolio of a set of risky assets and the risk free rate, derive an equation linking the return on this portfolio and the quantity of risk on this portfolio, interpret this equation. Show how this relation leads directly to the CAPM model by considering a portfolio with a proportion $x$ invested in an asset $i$ and $(1-x)$ invested in the tangency portfolio
(b) What insight does this give you as to why the CAPM model may fail empirically?
(c) Explain how poor conditioning and hence instability in the asset variance -covariance matrix may be reduced by identifying noisy factors through an eigenvalue decomposition of the variance- covariance matrix and how the portfolio weights may then be expressed in terms of factor loadings and a reduced set of (eigen) factors
(d) Describe the Partial Least Squares (PLS) approach to factor construction and compare this with the factors constructed in (c) above. What critical insight in the Kelly and Pruitt paper is exploited to produce significant evidence of factor returns?
4. Performance Measurement;
(a) Explain what is meant by:
i. The Calmar Ratio
ii. The Sortino Ratio
iii. Max Drawdown
iv. The Information Ratio
(b) Explain what is meant by over-fitting and how k -fold Cross Validation may overcome this problem.
(c) Explain what is meant by problem of multiple hypothesis testing in performance measurement, the Familywise Error Rate (FWER) and False Discovery Rate (FDR) and how they are related
(d) Explain how the simple adjustment to the Sharpe Ratio known as "The Haircut Sharpe Ratio"
i. If we have a $p$-value of 0.05 for a single," independent", $t$ - test ( one sided for testing strategy profitability), derive the corrected $p$-value which recognises that 10 previous independent tests have been carried out prior to this final test. Comment on the result.
ii. Suppose you have a strategy, developed with 10 years of monthly data, that delivers an annualised Sharpe Ratio of 0.92 with a corresponding $t$-statistic of 2.91 . The $p$-value is $0.4 \%(0.0004)$ - would you invest in this strategy?
iii. Suppose now you discover this strategy was developed through a process that involved 200 other strategies or tests, adjust the $p$ value using the Bonferroni method, would you still invest in this strategy?

## 5. Risk Management

(a) Provide mathematical definitions of Value at Risk, Expected Shortfall and Expectiles and explain how they are related and their properties as measures of Risk.
(b) What are the principle advantages and disadvantages of Historical Simulation when used as a method of computing Value at Risk and how might you overcome its deficiencies?
(c) Explain the mathematical relationship between expectiles and expected shortfall and how this can lead to CARE models
(d) Suppose that each of two investments has a $4 \%$ chance of loss of $\$ 10$ million, a $2 \%$ chance of a loss of $\$ 1$ million and a $94 \%$ chance of a profit of $\$ 1$ million. They are independent of each other.
i. What is the VaR for one of the investments when the confidence interval is $95 \%$
ii. What is the expected shortfall when the confidence level is $95 \%$
iii. What is the VaR for a portfolio consisting of the two investments when the confidence interval is $95 \%$
iv. What is the expected shortfall for a portfolio consisting of the two investments when the confidence level is $95 \%$
v. Show that, in this example, VaR does not satisfy the subadditivity condition whereas expected shortfall does?

ECM10
MPhil in Economic Research

Wednesday 9 May 2018 9:00am - 12:00pm

R200
ADVANCED MACROECONOMICS I

Candidates are required to answer three out of four questions. Each question will carry an equal weight of $33.33 \%$ of the total marks for this paper.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Consider a production economy with a single perishable consumption good in each period. There is a continuum of measure one of identical firms and households. Each household owns a firm, is endowed with one unit of time in each period and has the following utility function:

$$
U=E_{0}\left(\sum_{t=0}^{\infty} \beta^{t} \frac{\left(c_{t}-\frac{l_{t}^{1+\theta}}{1+\theta}\right)^{1-\rho}}{1-\rho}\right), \beta \in(0,1), \theta \geq 0, \rho \geq 0
$$

where $l_{t}$ and $c_{t}$ denote labour supply and consumption respectively, at time $t$. The production technology of a typical firm in period $t$ is given by

$$
y_{t}=z_{t} n_{t}^{1-\alpha}, \quad \alpha \in(0,1)
$$

where $y_{t}$ denotes a firm's output at date $t$ and $n_{t}$ is labour input at date $t$. Variable $z_{t} \in(0, \infty)$ is an identically distributed shock drawn from a distribution with mean $\bar{z}$ and variance $\sigma_{z}^{2}$.
(a) Define a competitive equilibrium for this economy, by using either the sequence or the period 0 approach.
(b) For given prices, solve the problem of a typical household. How does the labour supply change with the wage rate? Explain.
(c) For given prices, solve the problem of a typical firm. Find the labour demand and output of a typical firm. Explain.
(d) Using the market clearing condition for labour and the consumption good, find the equilibrium wage rate, consumption, labour and output in each time period $t$. How does aggregate output change with the shock $z$ ?
(e) Write down the indirect utility of a typical household, as a function of the shock $z$. If $\rho=0$, does a typical household prefer to live in the world described above, or in another in which $z_{t}=\bar{z}$ for all $t$ ? (Hint: Use Jensen's inequality, which implies that $f(E(x)) \leq E(f(x))$ when $f(\cdot)$ is a convex function.)
2. Consider an economy populated by a continuum of measure one of identical households, each with preferences given by

$$
\mathbb{E}_{0}\left[\sum_{t=0}^{\infty} \beta^{t} \frac{c_{t}^{1-\sigma}}{1-\sigma} \theta_{t}\right],
$$

where $c_{t}$ is consumption at period $t, \sigma \geq 0,0<\beta<1$ is the subjective discount factor and $\theta_{t} \in\left\{\theta_{L}, \theta_{H}\right\}$ is a preference shock. There is a single asset tree that yields perishable dividends $d_{t}$ (fruits) in each period. Households are initially endowed with some number of shares of the tree, $s_{0}$.

Assume that $d_{t}$ is a random variable taking values from the set $\mathbf{D}=\left\{d_{L}, d_{H}\right\}$ with $0<d_{L}<d_{H}<\infty$ and is distributed according to a first-order Markov chain with a time invariant transition probability matrix. The random variable $\theta_{t}$ takes values from the set $\boldsymbol{\Theta}=\left\{\theta_{L}, \theta_{H}\right\}$, with $0<\theta_{L}<\theta_{H}<\infty$ and is distributed according to a first-order Markov chain with a time invariant transition probability matrix. Households can trade the share of the tree. The price of the tree when the current realization of $\left(\theta_{t}, d_{t}\right)$ is $(\theta, d)$ is given by the function $p: \mathbf{D} \times \boldsymbol{\Theta} \rightarrow R_{+}$. There are no other assets in this economy.
(a) State the problem of the representative household in recursive form.
(b) Define a recursive competitive equilibrium. Be precise.
(c) Assuming that the value function is differentiable and ruling out a bubble equilibrium, derive the optimal equilibrium price of a share of the tree. Explain.
(d) Let $d$ be constant and let $\theta$ be independent and identically distributed with mean 1 . Is the price of a share of the tree higher when $\theta$ is high or low? Explain.
(e) Now, let $\theta$ be constant. Does 'good news' about future output (dividends) always produce an increase in stock prices? Explain.
3. Consider a variation of the basic RBC model, where capital stock is predetermined in a given period, and owned by the households, but households decide the optimal capital utilisation. Let $\tilde{K}_{t}=v_{t} K_{t}$ be capital services offered by households to the firms, and $v_{t}$ be the utilisation of capital. Depreciation of capital is a function of utilisation, $\delta\left(v_{t}\right)$, with the following functional form:

$$
\delta\left(v_{t}\right)=\delta_{0}+\phi_{1}\left(v_{t}-1\right)+\frac{\phi_{2}}{2}\left(v_{t}-1\right)^{2},
$$

with $\phi_{1}, \phi_{2}>0$ and $0 \leq \delta_{0} \leq 1$. In steady state, capital is assumed to operate at full capacity, i.e. $v=1$. Households maximise expected discounted lifetime utility

$$
\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t}\left(\ln C_{t}+\chi \ln L_{t}\right)
$$

over consumption $C_{t}$, leisure $L_{t}$, capital $K_{t+1}$ and utilisation $v_{t}$, subject to the budget constraint

$$
C_{t}+K_{t+1}=R_{t}\left(v_{t} K_{t}\right)+W_{t} N_{t}+\left(1-\delta\left(v_{t}\right)\right) K_{t}
$$

where $N_{t}=1-L_{t}$ denotes hours worked, $R_{t}, W_{t}$ denote the price of capital services and the wage rate respectively. The parameter $0<\beta<1$ denotes the discount factor. The representative firm produces output $Y_{t}$, rents capital services $\tilde{K}_{t}=v_{t} K_{t}$ at rate $R_{t}$, hires labour at a rate $W_{t}$ and maximises profits

$$
Y_{t}-W_{t} N_{t}-R_{t} \tilde{K}_{t}
$$

in every period, subject to a Cobb-Douglas production function $Y_{t}=Z_{t} \tilde{K}_{t}^{\alpha} N_{t}^{1-\alpha}$, with $0<\alpha<1$, and a productivity shock, which evolves according to $\log Z_{t}=\rho \log Z_{t-1}+\varepsilon_{t}$, with $\varepsilon_{t} \sim N\left(0, \sigma_{\varepsilon}^{2}\right)$ and $0<\rho<1$.
(a) Explain intuitively how the inclusion of variable capital utilisation and depreciation affects the amplification mechanism of the RBC model.
(b) Define the competitive equilibrium and derive the equilibrium conditions that describe the dynamics of the variables of the model. Provide a brief interpretation of the conditions.
(c) Use the equilibrium conditions from (b) to describe
i. how equilibrium capital utilisation $v_{t}$ reacts to changes in the price of capital services $R_{t}$, and
ii. the impulse response of output and the price of capital services to a positive productivity shock.
How do your answers in (i) and (ii) change for different values of $\phi_{2}$ ? Explain.
4. Consider a New Keynesian model with nominal price and wage rigidities, with no capital accumulation. Intermediate good firms have a production function which is linear in labour and subject to a productivity shock. The model is reduced to and described by the following set of log-linear equilibrium equations:

$$
\begin{align*}
\hat{y}_{t} & =\mathbb{E}_{t} \hat{y}_{t+1}-\frac{1}{\sigma}\left(i_{t}-\mathbb{E}_{t}\left(\pi_{t+1}\right)\right)  \tag{1}\\
\pi_{t} & =\beta \mathbb{E}_{t}\left(\pi_{t+1}\right)+\frac{\left(1-\beta \psi_{p}\right)\left(1-\psi_{p}\right)}{\psi_{p}} \hat{\varphi}_{t}  \tag{2}\\
\pi_{t}^{w} & =\beta \mathbb{E}_{t}\left(\pi_{t+1}^{w}\right)-\frac{\left(1-\beta \psi_{w}\right)\left(1-\psi_{w}\right)}{\left(1+\eta \theta_{w}\right) \psi_{w}}\left(\hat{\omega}_{t}-\hat{\omega}_{t}^{f}\right),  \tag{3}\\
\hat{\varphi}_{t} & =\hat{\omega}_{t}-\hat{z}_{t}  \tag{4}\\
\hat{\omega}_{t} & =\hat{\omega}_{t-1}+\pi_{t}^{w}-\pi_{t}  \tag{5}\\
\hat{z} & =\rho \hat{z}_{t-1}+\varepsilon_{t}, \varepsilon_{t} \sim N\left(0, \sigma_{\varepsilon}^{2}\right) \tag{6}
\end{align*}
$$

All variables are in log-deviations from a steady state at which both price and wage inflation (gross) rates are equal to 1 . The variables $\pi_{t}, \pi_{t}^{w}$ and $i_{t}$ are the inflation rate, wage inflation rate and nominal interest rate respectively. Moreover, $\hat{z}_{t}, \hat{y}_{t}, \hat{\omega}_{t}, \hat{\varphi}_{t}$ are the log-deviations from their steady states of the productivity shock, demanded output, real wage, and the real marginal cost of production of a firm respectively. Finally, $\hat{\omega}_{t}^{f}$ is the equilibrium log-linear real wage in the absence of any nominal frictions (the frictionless wage rate). The parameters $\sigma>0, \eta \geq 0$ and $0<\beta<1$ are the inverse of the elasticity of intertemporal substitution, the inverse of the Frisch elasticity of labour supply and the discount factor respectively. Moreover, $0 \leq \psi_{p} \leq 1$ and $0 \leq \psi_{w} \leq 1$ are probabilities with which a firm and a household cannot change its price or wage respectively in a given period. Finally, the parameter $\theta_{w} \geq 0$ denotes the degree of substitutability among different varieties of labour.
(a) Provide a brief interpretation for each of the equations.
(b) Explain intuitively why under flexible prices and flexible wages we get $\hat{\omega}_{t}^{f}=\hat{z}_{t}$.
(c) Use (b) to rewrite expression (2) in terms of deviations of the real wage from its frictionless counterpart and comment on the relationship between (2) and (3).
(d) Use the model equations and your answers to (a)-(c) to comment on the ability of this model to reproduce the cyclicality properties of real wages observed in the data. Also, how does the model perform in this respect when prices and wages are flexible?

## END OF PAPER

ECM10<br>MPhil in Economic Research

Wednesday 16 May $2018 \quad$ 2:00pm - 4:00pm

R201
ADVANCED MACROECONOMICS II

Candidates are required to answer two out of three questions
Each question carries equal weight

Write your candidate number (not your name) on the cover of each booklet.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS

 EXAMINATIONCalculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1 (Incomplete Market Models)

Consider an economy populated by a continuum of infinitely-lived households indexed by $i$ on $[0,1]$. There is a measure unity of households, while time is discrete and indexed by $t \in\{0,1, \ldots, \infty\}$. Preferences for individual $i$ are given by:

$$
\mathcal{U}\left(c_{0}^{i}, c_{1}^{i}, \ldots\right)=\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}^{i}\right)
$$

where $\beta \in(0,1)$ is the discount factor, $c_{t} \geq 0$ denotes time $t$ consumption, $u^{\prime}>0$ and $u^{\prime \prime}<0$. Each household $i$ in period $t$ receives a stochastic idiosyncratic endowment of the consumption good $\epsilon_{t}^{i}=\left\{\epsilon_{h}, \epsilon_{l}\right\}$ which follows a Markov chain where $\pi_{h h}^{\epsilon}=\operatorname{Pr}\left[\epsilon_{t}^{i}=\epsilon_{h} \mid \epsilon_{t-1}^{i}=\epsilon_{h}\right], \pi_{l l}^{\epsilon}=\operatorname{Pr}\left[\epsilon_{t}^{i}=\epsilon_{l} \mid \epsilon_{t-1}^{i}=\epsilon_{l}\right]$, $\pi_{h l}^{\epsilon}=1-\pi_{h h}^{\epsilon}$, and $\pi_{l h}^{\epsilon}=1-\pi_{l l}^{\epsilon}$. Households can trade a risk-free asset $a$ in zero-net supply with return, $R_{t}$, and they can accumulate debt up to the exogenous borrowing limit $-\bar{b}$.
(a) Define a stationary recursive competitive equilibrium for this economy.
(b) Suppose now that $\bar{b}=0$, and that $u(c)=c^{1-\gamma} /(1-\gamma)$. What is so special about this case? Determine the equilibrium risk-free return as a function of the model's parameters and prove that the return is lower than $1 / \beta$. Explain how the risk-free rate changes with the ratio $\epsilon_{h} / \epsilon_{l}$, with the persistence parameter $\pi_{h h}$, and with risk aversion $\gamma$.

## Question 2 (Search and Matching Models)

Consider an extension of the basic Mortensen-Pissarides model described in class that allows for a search intensity decision for unemployed workers. The details of the model are as follows: There is a unit mass of ex ante homogeneous workers, each with preferences given by

$$
\sum_{t=0}^{\infty} \beta^{t}\left[c_{t}-a n_{t}-g\left(s_{t}\right)\right]
$$

where $c_{t}$ is consumption in period $t, n_{t} \in\{0,1\}$ is time spent working in period $t$, and $s_{t} \in[0,1]$ is search intensity in period $t$. The function $g$ is strictly increasing, strictly convex, twice continuously differentiable, and satisfies

$$
\lim _{s \rightarrow 0} g^{\prime}(s)=0, \lim _{s \rightarrow 1} g^{\prime}(s)=+\infty
$$

There is a unit mass of ex ante homogeneous entrepreneurs, each with preferences given by

$$
\sum_{t=0}^{\infty} \beta^{t}\left[c_{t}-k v_{t}\right]
$$

where $c_{t}$ is consumption in period $t$ and $v_{t}$ is vacancies posted in period $t$. A matched worker-entrepreneur pair produces output $y$, and with probability $\lambda$ a productivity match becomes permantently unproductive. There is a constant returns to scale matching function, written as $m\left(s_{t} u_{t}, v_{t}\right)$, where $u_{t}$ is the mass of unemployed workers in period $t$. This function gives the total number of matches that are formed if $u_{t}$ unemployed workers each search with intensity $s_{t}$ and $v_{t}$ vacancies are posted. The probability that an individual vacancy is matched with an unemployed worker is given by

$$
q_{t}=\frac{m\left(s_{t} u_{t}, v_{t}\right)}{v_{t}}
$$

and the probability that a worker who searches with intensity $s_{t}$ is matched with a vacancy is given by

$$
p_{t}=s_{t} \frac{m\left(s_{t} u_{t}, v_{t}\right)}{s_{t} u_{t}}=\frac{m\left(s_{t} u_{t}, v_{t}\right)}{u_{t}}
$$

From the perspective of a given individual worker, if $s_{t}$ is the average search intensity of all the other workers and there are $u_{t}$ unemployed workers, one
can think of $m\left(s_{t} u_{t}, v_{t}\right) / s_{t} u_{t}$ as the matching probability per unit of search intensity. This individual chooses their own search intensity $s$ with the understanding that their probability of being matched with a vacancy is given by $s \cdot m\left(s_{t} u_{t}, v_{t}\right) / s_{t} u_{t}$, i.e., it is linear in their own search intensity.

Timing is the same as in the basic model studied in class. At the beginning of each period matches from the previous period find out if they become unproductive. Vacancies are posted and previously unmatched workers and those whose matches become unproductive search. Successful matches that result from search in period $t$ become productive in period $t+1$.
(a) Consider a Social Planner who maximizes an equal weighted integral of individual utilities, i.e., who maximizes the expected present discounted value of output net of vacancy posting costs, labor disutility, and search intensity costs. Let $V(e)$ be the value function for the Social Planner when $e$ is the fraction of workers who are employed. Formulate the Social Planner's problem recursively.
(b) Show that the value function for the Social Planner is of the form $V(e)=\alpha_{0}+\alpha_{1} e$ for some constants $\alpha_{0}$ and $\alpha_{1}$.
(c) Argue that the optimal choice of search intensity is independent of the state $e$ and that the optimal choice of vacancies is such that $s^{*}(1-e) / v^{*}$ is independent of the state $e$.
(d) Show that if we increase the value of $y$ that the optimal decision rule for the Social Planner will entail greater search effort.
(e) Consider a decentralized equilibrium in which wages are determined via generalized Nash bargaining as discussed in class. Specifically, the worker's threat point is the value of being unemployed, and the entrepreneur's threat point is the value of an unfilled vacancy, which is zero in equilibrium. Assume that the worker's share of the surplus is given by $\theta$. As in class, consider an equilibrium in which the wage rate is constant. Letting $V_{e}$ and $V_{u}$ denote the values for an employed and unemployed worker respectively, define a recursive equilibrium for this economy.

## Question 3 (Overlapping Generation Models)

At each date $t \geq 1$, an economy consists of overlapping generations of a constant number $N$ of two-period-lived agents. At $t=1$, there are also some initial old people. Young agents born at $t \geq 1$ have preferences over consumption streams of a single good that are ordered by $u\left(c_{t}^{t}\right)+u\left(c_{t+1}^{t}\right)$, where $u(c)=\ln (c)$ and where $c_{t}^{i}$ is the consumption at time $t$ of an agent born at $i$. Each young agent born at $t \geq 1$ has identical preferences and endowment pattern $\left(w_{1}, w_{2}\right)$, where $w_{1}$ is the endowment when young and $w_{2}$ is the endowment when old. The consumption good is non-storable. Assume $0<w_{2}<w_{1}$. In addition, there are some initial old agents at time 1 who are endowed with $w_{2}$ of the time 1 consumption good, and who order consumption streams by $c_{1}^{0}$. The initial old (i.e., the old at $t=1$ ) are also endowed with $M$ units of an unbacked fiat currency called dollars and a stock $E$ of another unbacked fiat currency called euros. Both stocks of currency are constant over time. These two currencies are the only storable goods in the economy.
(a) Find the saving function of the young agent.
(b) Define an equilibrium with one or more valued fiat currencies.
(c) Define a stationary equilibrium with one or more valued fiat currencies.
(d) Compute a stationary equilibrium with one or more valued fiat currency.
(e) How many equilibria with valued fiat currencies are there? Compute all of them.
(f) Compute the limiting values as $t \longrightarrow+\infty$ of the rates of return on both currencies in each of the non-stationary equilibria with valued fiat currency. Justify your calculations.
(g) Does an equilibrium with valued fiat currency necessarily have an equilibrium allocation that is Pareto optimal? Explain as thoroughly as possible.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Tuesday 29 May 2018 1:30pm - 3:30pm

S140
BEHAVIOURAL ECONOMICS

Candidates are required to answer all four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. (a) Give a careful description of a public-goods game.
(b) What difficulties might you encounter when trying to implement this game in a laboratory experiment?
(c) What economic lessons have been learned from experimental implementations of such games?
2. (a) Give a careful description of the Experience-Weighted Attraction model of Camerer and Ho (1999).
(b) What are the four key parameters of this model? What is their interpretation?
(c) How well does the model fit the experimental data?
3. (a) Give a careful description of the experiment of Padoa-Schioppa and Assad (2006).
(b) What do the neurons in their experiment code? Illustrate your answer with schematic figures.
(c) What do their findings imply about the nature of decision making?
4. (a) What is meant by the endowment effect?
(b) Describe a field experiment that was conducted in order to test for the existence of this effect.
(c) What difficulties are there with the interpretation of this experiment?
(d) How (at least in principle) would you redesign the experiment in order to circumvent these difficulties?

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Monday 4 June 2018 9:00am-11:00am

S150
ECONOMICS OF NETWORKS

Candidates are required to answer three out of four questions.
Each question carries equal weight

Write your candidate number (not your name) on the cover of each booklet.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1: Consider a game in which $n$ players are located on nodes of an undirected network $g$. Players simultaneously choose actions $x_{i} \in \mathcal{R}_{+}$. Let $N_{i}(g)$ be the set of players with whom player $i$ has a link in network $g$. The payoffs of player $i$ faced with a strategy profile $x$ are given by

$$
\begin{equation*}
\Pi_{i}(x)=f\left(x_{i}+\sum_{j \in N_{i}(g)} x_{j}\right)-c x_{i} . \tag{1}
\end{equation*}
$$

where $f(0)=0, f^{\prime}()>$.0 and $f^{\prime \prime}()<$.0 and $c>0$. Suppose there exists a number $\hat{x}>0$ such that $f^{\prime}(\hat{x})=c$.

1. Show that in every non-empty network there exists an equilibrium with specialization: some players choose $\hat{x}$ and others choose 0 .
2. Show that there are only two equilibria in the star network, one in which only the center contributes and the other in which only the spokes contribute.
3. Now suppose that initially the network is empty. Players can form links with other players at cost $k>0$, in addition to investing in action $x_{i}$. Consider the equilibrium of this richer game with action choice and linking. What are the circumstances under which the equilibrium is characterized by specialization?

Question 2: Consider a $n$ player network formation game. Suppose that two players $i$ and $j$ can form a link if they both agree, and pay a cost $c>0$. The network created by this bilateral linking is denoted by $\mathbf{g}$. The payoffs to player $i$ under network $\mathbf{g}$ are

$$
\begin{equation*}
\Pi_{i}(\mathbf{g})=1+\sum_{j \neq i} \delta^{d(i, j ; \mathbf{g})}-\eta_{i}^{d}(\mathbf{g}) c \tag{2}
\end{equation*}
$$

where $d(i, j ; \mathbf{g})$ is the (geodesic) distance between $i$ and $j$ in network $\mathbf{g}$ and $\delta \in(0,1)$ is the decay factor.

1. Define a pairwise stable network.
2. Provide the range of parameter values, $c$ and $\delta$, for which the empty, the complete and star network are pairwise stable.
3. A network is efficient if it maximizes players' payoffs across all networks. Provide a characterization of efficient networks.

Question 3: There are $n \geq 3$ firms located in a collaboration network, facing a linear demand, given by $Q=1-p$. The initial marginal cost of production in a firm is $\bar{c}$ and assume that $n \bar{c}<1$. Each firm $i$ chooses a level of research effort given by $s_{i}$. Collaboration between firms involves sharing of research efforts that lower costs of production. The marginal costs of production of a firm $i$, in an (undirected) network $g$, facing a profile of efforts $s$, are given by:

$$
\begin{equation*}
c_{i}(s \mid g)=\bar{c}-\left(s_{i}+\sum_{j \in N_{i}(g)} s_{j}\right), \tag{3}
\end{equation*}
$$

where $N_{i}(g)$ refers to the neighbors. Let $\eta_{i}(g)=\left|N_{i}(g)\right|$. Research effort is costly: $Z\left(s_{i}\right)=\alpha s_{i}^{2} / 2$, where $\alpha>0$. Given costs $c=\left\{c_{1}, c_{2}, \ldots c_{n}\right\}$, firms choose quantities $\left(\left\{q_{i}\right\}_{i \in N}\right)$, with $Q=\sum_{i \in N} q_{i}$.

1. Given costs $c=\left(c_{1}, c_{2}, . ., c_{n}\right)$, compute the Cournot equilibrium quantities and profits.
2. Taken as given this market equilibrium, compute the payoffs of firm $i$, located in network $g$ given the research efforts $s$.
3. Define positive and negative externalities and strategic complements and substitutes. Show that this game exhibits a positive externality across neighbors and negative externality across non-neighbors actions. And show that actions of neighbors are strategic complements, while the actions of non-neighbors are strategic substitutes.
4. Now let $k$ be the degree of a firm in a regular network. Show that equilibrium research effort is decreasing in $k$.

Question 4: There is a single SOURCE and a single DESTINATION and a collection of $n$ intermediary traders located on nodes of an undirected connected network in between the source and destination. The value of exchange is 1 . The network of traders and the valuations are common knowledge. Traders post prices simultaneously; they have zero costs. If the Buyer and Seller have a (direct) link they exchange the good and split the surplus. If they don't have a direct link then they compute the cost of using a path between them. They pick the lowest cost path if it is less than 1 (randomizing across paths if there are multiple lowest cost paths). Source and destination divide the residual surplus, after paying the cost of the path.

1. Formalize this economic exchange as a game of simultaneous moves.
2. An equilibrium outcome is said to be efficient if trade takes place with probability 1. Prove that for any network there exists an efficient pure strategy Nash equilibrium.
3. Show that in any equilibrium with trade, the distribution of surplus is extremal: intermediate traders extract either all or none of the surplus. CAMBRIDGE

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Thursday 24 May $2018 \quad 1: 30 \mathrm{pm}-3: 30 \mathrm{pm}$

S170
INDUSTRIAL ORGANISATION

Candidates are required to answer 3 compulsory questions

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Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

Consider an industry with $N \geq 2$ firms producing a homogeneous product and competing in quantities. Firms have constant marginal costs $c>0$ and face the inverse demand function $p=a-b Q$, where $Q \equiv \sum_{i=1}^{N} q_{i}$ and $q_{i}$ is quantity produced by firm $i$. There are no fixed costs of production.
(1) Solve for the Cournot equilibrium and write down expressions for equilibrium quantities, price and profits.
(2) Write down expressions for equilibrium consumer surplus $C S(N)$ and social welfare

$$
W(N)=C S(N)+\sum_{i=1}^{N} \pi_{i}(N)
$$

(3) Describe what happens to equilibrium firm output, industry output, profit, price, consumer surplus and social welfare as the number of firms $N$ increases. Briefly discuss your findings.

## Question 2

Consider an industry with $N$ firms who produce a homogeneous product and compete in quantities. They face an inverse demand function given by $p=a-b Q$ where $Q \equiv \sum_{i=1}^{N} q_{i}$ and $q_{i}$ is quantity produced by firm $i$. Firms have heterogeneous costs, with firm $i=1, \ldots, N$ having constant marginal cost $c_{i}>0$. There are no fixed costs of production.
(1) Derive the aggregate equilibrium production in the industry and derive an expression for the equilibrium price.
(2) Suppose there are two types of firms, namely $H \geq 1$ high cost firms with marginal cost $c_{H}$ and $L \geq 1$ low cost firms with low marginal cost $c_{L}<c_{H}$ and where $H+L=N$. Under this scenario, derive the aggregate equilibrium production in the industry and derive an expression for the equilibrium price.
(3) Consider a merger between two firms $i$ and $j$ with marginal costs $c_{i}$ and $c_{j}$ respectively. Each firm has a cost of either $c_{H}$ or $c_{L}$. A merged firm can produce using the production technology of any of the merging firms. Calculate the post-merger aggregate and individual equilibrium production in the industry and derive an expression for the equilibrium price, distinguishing between the cases of (i) a merger between two high cost firms, (ii) a merger between two low cost firms and (iii) a merger between a high cost and a low cost firm. Briefly discuss your findings.

## Question 3

Consider a linear city on which consumers are uniformly distributed with density one. Each consumer has unit demand and can choose between two firms $i$ and $j$, located at points $l_{i}$ and $l_{j}$ respectively, where $0 \leq l_{i} \leq l_{j}$. A consumer that travels distance $s$ pays a quadratic transport cost $t s^{2}$ with $t>0$. The value of consumption of any good is $v>0$. The two firms have constant marginal costs $c>0$ and set prices $p_{i}$ and $p_{j}$, respectively. The game is played in two stages as follows: At the first stage, the firms simultaneously and non-cooperatively choose locations $\left(l_{i}, l_{j}\right)$. At the second stage, they simultaneously and non-cooperatively set prices $\left(p_{i}, p_{j}\right)$.
(1) Consider the subgame at stage two where the locations $\left(l_{i}, l_{j}\right)$ are given. Solve for the Bertrand Nash equilibrium $\left(p_{i}^{*}\left(l_{i}, l_{j}\right), p_{j}^{*}\left(l_{i}, l_{j}\right)\right)$ of this subgame, assuming that there is full market coverage.
(2) Given the equilibrium values $\left(p_{i}^{*}\left(l_{i}, l_{j}\right), p_{j}^{*}\left(l_{i}, l_{j}\right)\right)$, write down expressions for $\pi_{i}^{*}\left(l_{i}, l_{j}\right)$ and $\pi_{j}^{*}\left(l_{i}, l_{j}\right)$.
(3) Discuss the firms' incentive to differentiate their products by locating far away from their rival.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

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Tuesday 5 June 2018 9:00am-11:00am
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S180
LABOUR: SEARCH, MATCHING AND AGGLOMERATION

Candidates are required to answer three out of four questions. Each question carries equal weight.

This written exam carries $75 \%$ of the marks for S180

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

[^3]
## Question 1

Consider a labour market with search frictions. Let $\lambda, \delta, \rho$ be the job offer arrival rate, the job destruction rate, and the interest rate respectively; the job offer arrival rate for employed and unemployed job seekers is the same. Every firm has one job (either filled or vacant) and all jobs and workers are equally productive; productivity is normalized to unity. New entry of firms comes at a one-time cost $K$. The value of leisure for an unemployed job seeker is $B$. After a job seeker and firm with a vacancy meet, the firm offers a take-it-or-leave-it wage (hence, it only includes a retention premium, not a hiring premium). Workers accept any wage offer that is greater or equal to their current wage. Labour supply is normalized to unity. Let $u$ and $v$ be the number of unemployed and vacancies respectively, let $F(W)$ be the distribution of wages among job offers, and let $\lambda_{f}$ be the rate at which firms receive applications from job seekers (either employed or unemployed).
a) What is the value of $\lambda_{f}$ ? Explain your result.
b) Let $V^{V}$ be the value of a vacancy. Write down the Bellman equations for the asset value of an employed and an unemployed jobseeker, $V^{U}$ and $V^{E}(W)$, and of a filled job $V^{J}(W)$.
c) What is the value of vacancy in this economy? Explain your answer.
d) Derive the equilibrium wage offer distribution (tip: firms are free to choose their wage offer. What does this imply for the value of $V^{J}(W)$ for different values of $W$ ?). Explain your answer.
e) What is the lowest wage any firm will offer? Explain your answer.
f) What is the asset value of a filled job? Use this expression to simplify the expression for the wage distribution derived under d ). Given an interpretation of this relation.

## Question 2

Consider a city as analyzed by Lucas and Rossi-Hansberg. The structure of this city is as follows: moving from the midpoint to the edge of the city, subsequent zones have the following use: (1) commercial, (2) residential, (3) commercial, (4) mixed, (5) commercial, (6) residential. Let $\kappa$ be the commuting cost as a percentage of remaining income per unit of distance.
a) Draw three pictures, for the direction of commuting, for wages, and for land rents, all as a function of the distance to the center of the city. Explain your result. What is the interpretation of the wage in a location?
b) The city council considers the construction of an underground system that links the center of the city to any point at a distance $r$ from the center (so, all points at the circle with radius $r$ have a station that connects them to the center). There are no stops at intermediate locations, so commuters can only use the system to travel from $r$ to the center and back. The construction cost of the system is proportional to $r$, that is, proportional to the circumference of the circle at location $r$ and is independent of the number of travellers. Let $S_{1}$ be the outer limit of zone (1), and let $S_{6}$ be the edge of the city (the outer limit of zone (6)). For travellers the cost of using the underground system for a trip is $S_{1} \kappa \frac{r}{S_{6}}$. Hence, if the city council would choose to build the ring of stations at the edge of the city, $r=S_{6}$, this cost is equal to the regular commuting cost from the center to the edge of zone (1). Commuters can travel to and from the underground stations at a cost $\kappa$ per unit of distance. The city council discusses at what location $r$ to build the stations. What would you advise the council? Motivate your answer.
c) What would you roughly expect to happen to the size of the zones (1)-(6) due to the introduction of the subway system? Explain your answer.
d) Redraw the pictures you have drawn under a). and explain the difference.
e) How would you advice the city council to fund the fixed cost of the system? Via some form of taxation (if so, what is the proper tax base?), or via tickets sold to users of the system? Explain your answer.
f) Would you advice the city council to build the system? On what rough metric would you base your evaluation of the costs and benefits of building the system?

## Question 3

a) Describe the stylized facts regarding the effect of minimum wages on employment and the wage distribution.
b) Describe in 5-10 sentences how David Lee analyzes the impact of minimum wages on the wage distribution. Mention one point of critique.
c) Describe in 5-10 sentences how Dube, Lester and Riech analyze the effect of minimum wages on employment. Mention one point of critique.
d) Explain to what extent a Walrasian assignment model of heterogeneous workers and jobs with comparative advantage can explain the stylized facts you mentioned under a). Analyze the relation between the assignment of workers to jobs and the wage function by means of a graphical representation of both functions. Explain how a minimum wage affects both this assignment and relative wages.
e) Suppose that there is an upward sloped labour supply function: higher wages for a particular worker type lead to an increase in labour supply of that type. If an increase in the minimum wage leads to spill over effects to the wages paid to worker types earning wages above the minimum, this upward supply curve leads to an increase in labour supply for these worker types. Might this lead to an increase in employment for these skill levels? And might an increase in the minimum wage therefore lead to an overall increase in employment? Explain your answer.

## Question 4

a) Explain Zipf's law for city size. Provide an expression for the class of distributions that yields Zipf's law. What parameter value yields Zipf's law?
b) What is the expected share of the largest city in total employment in a country if this law would hold literally?
c) How can you test Zipf's law empirically? What is the statistical caveat of this method?
d) What is Ades and Glaeser's exception to this law? What is their explanation for this exception?
e) Explain a stochastic process that generates Zipf's law.
f) Should cities diversify their portfolio of activities (e.g. a mix of tradeable industries)? What are the arguments pro and contra? Is diversification consistent with Zipf's law?
g) Explain the argument of Desmet and Rossi-Hansberg on agglomeration of industries in relation to the introduction of new GPTs. How does this relate to Zipf's law?

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

## S301 <br> APPLIED ECONOMETRICS

This paper is in two sections:
Section A: Candidates are required to answer two compulsory question.
Section B: Candidates are required to answer two out of four questions.

Each section carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator New Cambridge Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A Answer BOTH questions from this Section

1. Suppose you are interested in estimating the effect of imprisonment on reoffending. Let $y_{1}$ be the probability of reoffending if a suspected criminal is imprisoned, and $y_{0}$ be the probability of reoffending if the suspected criminal is not imprisoned. Let $D=1$ if the suspected criminal is imprisoned.
(a) Explain, in words, the economic meaning of $E\left(y_{1} \mid D=1\right), E\left(y_{1} \mid D=\right.$ $0)$. Which one of these is not observed in your data?
(b) Suppose that we are interested in estimating the average treatment effect on the treated (ATT): $\alpha=E\left(y_{1} \mid D=1\right)-E\left(y_{0} \mid D=1\right)$. One of your friends suggest that you regress the probability of reoffending on $D$. Show the bias in using the coefficient on D from this regression as an estimator for the ATT. What is the direction of the bias you would expect?
(c) For suspected criminals in a non-confession case, imprisonment decisions will be determined by a judge. Judges differ systematically in their stringency. Suppose that you observe the type of judge assigned to each defendant in your data. Judges are of one of the following two types: some are lenient and some are strict, and the assignment of judges to the applicant is random (determined by a lottery). Discuss how the additional variable in the data may help you reduce the bias you showed in part (b). Be specific about the assumptions you make and how you plan to use this additional information in a regression framework.
(d) Suppose that the estimated effects of imprisonment using your strategy in part (c) are statistically different from the OLS estimates in part (b). Provide at least three explanations that could explain the difference in estimates. If the differences are not statistically significant, should we conclude that OLS estimate is unbiased?
2. In a study on investment by eleven US firms over the period 1935 to 1954 the following relationship is suggested

$$
I_{i t}=\alpha_{i}+\beta_{1} F_{i t}+\beta_{2} C_{i t}+u_{i t} \quad i=1, \ldots, 11 \quad t=1935, \ldots, 1954
$$

where $I_{i t}$ is gross investment, $F_{i t}$ is the market value of the firm at the end of the previous year and $C_{i t}$ is the value of stock of machines and equipment at the end of the previous year. The results below are obtained using the following estimation methods (i) pooled OLS (ii) the fixed effects (FE) estimator (iii) the random effects estimator (RE) and (iv) the between estimator (BE)

| Estimation Method | $\hat{\beta}_{1}$ | $\hat{\beta}_{2}$ |
| :---: | :---: | :---: |
| Pooled OLS | 0.114 | 0.227 |
|  | $(0.006)$ | $(0.024)$ |
| FE | 0.110 | 0.310 |
|  | $(0.011)$ | $(0.016)$ |
| RE | 0.109 | 0.307 |
|  | $(0.009)$ | $(0.016)$ |
| BE | 0.134 | 0.029 |
|  | $(0.027)$ | $(0.175)$ |
| Hausman test FE vs RE | 3.97 |  |
|  | $[0.137]$ |  |

Note: Standard errors in round brackets, and $p$-values in square brackets
(a) Explain what assumptions would be required for each of these methods to provide a good estimator (your answer should include a discussion of what you mean by "good").
(b) What does the pattern of results suggest to you about the likely values of $\beta_{1}$ and $\beta_{2}$ ?
(c) Comment on the advantages and disadvantages of using panel data for this sort of exercise compared to using time series or cross-sectional approaches.

SECTION B. Answer TWO questions from this Section

1. (a) Define $D_{i}=1$ if an individual $i$ attends college and $D_{i}=0$ if an individual $i$ does not attend college. Assume that the effect of college attendance on earnings is homogeneous across the population. Describe the econometric problem of "ability bias" in the estimation of the impact of college attendance on earnings with a simple econometric model. Show how instrumental variables (IV) can be used to address the problem.
(b) Your friend proposes to use the availability of college in a local area as an instrumental variable for college attendance. Formally, let $Z_{i}=0$ when there is no college in the area of residence, and $Z_{i}=1$ if a college exists. Specify the assumption(s) under which the instrument is valid.
(c) Suppose that the assumption(s) you described in part (b) is satisfied in the data you collected. In addition assume that the effect of college attendance on earnings is heterogeneous across the population and there are no defiers. Under the potential outcome framework, derive that the IV estimate is equivalent to the Local Average Treatment Effect.
(d) Suppose that you are interested in estimating the return from attending a local college separately from the return from attending an outside college. Let us define the treatment variable $T_{i}$ in the following way: $T_{i}=0$ if the person $i$ does not attend any college, $T_{i}=1$ if the person goes to college outside his/her area of residence and $T_{i}=2$ if the person goes to college in the same area of residence. Assume that you have the same binary instrument defined in part (b). Are we able to identify the causal effect of attending a local college relative to not attending a college? Explain. (Hint: you might want to start by defining the compliers.)
2. Economists are interested in understanding the determinants of intergenerational correlations in earnings. They provide answers to this question by estimating the effect of individual parental attributes on the outcomes of their children.
(a) Discuss the potential problems if one just regresses an outcome of the children on one particular parental attribute.
(b) Suppose you are interested in estimating the effect of parental education on children's outcomes. One method used to identify causation involves finding variation in parents' education that is arguably unrelated to other parental characteristics and using this variation to identify the effect of parental education on the outcomes of children.
Black, Devereux, and Salvanes (2005) apply this approach using register data from Norway. During the 1960s, there was a change in the compulsory schooling laws affecting primary and middle schools. Pre-reform, the Norwegian education system required children to attend school through the seventh grade; after the reform, this was extended to the ninth grade, adding two years of required schooling. Additionally, implementation of the reform occurred in different municipalities at different times, starting in 1960 and continuing through 1972, allowing for regional as well as temporal variation.
Suppose you were given a data set with parental education levels, children's outcome (education and earnings at age 30), children's characteristics (year of birth, municipality of residence) and details of the law change (first birth cohort affected by the reform). Discuss why the availability of school reform may help us to answer the question. Explain how you would use the data to estimate the effect of parental education on children's earnings. You need to specify the estimating equations and any assumptions you make.
(c) You are interested in estimating the effect of parental education on children's education. In your data, besides observing the education level of mothers and their children, you also observe that some of the mothers are siblings. Discuss how you would use the sibling information to estimate the intergenerational transmission of education. Write down the estimating equation and be clear about any assumptions you make. Would your results be biased if the first-born child of a family always gets more family resources?
3. A researcher wishes to understand the relationship between alcohol consumption (alcons - litres per year per capita) and life expectancy of females (lefemale65 is the life expectancy of female currently aged 65). An unbalanced panel of annual data between 1960 and 2015 for 40 OECD countries is available. Across the countries the minimum time series observations is 8 and maximum is 56 .

To begin with he estimates the following equation for the UK alone

$$
\begin{equation*}
\text { lefemale } 65^{t}=\alpha+\beta \text { alcons } s_{t}+u_{t} \tag{1}
\end{equation*}
$$

and obtains the following estimates with 30 observations (standard errors in parentheses)

| Parameter | Estimate |
| :---: | :---: |
| $\alpha$ | $8.06(0.97)$ |
| $\beta$ | $1.03(0.10)$ |
| $R^{2}$ | 0.80 |
| \#observations | 30 |

Examination of the data shows that life expectancy has risen dramatically everywhere over the sample period. Concerned that there are other variables that may have driven this increase he includes a time trend (defined as year-1980)

$$
\begin{equation*}
{\text { lefemale } 65_{t}}=\alpha+\beta \text { alcons }{ }_{t}+\gamma \text { Trend }_{t}+u_{t} \tag{2}
\end{equation*}
$$

and obtains

| Parameter | Estimate |
| :---: | :---: |
| $\alpha$ | $16.7(0.81)$ |
| $\beta$ | $-0.002(0.09)$ |
| $\gamma$ | $0.12(0.009)$ |
| $R^{2}$ | 0.97 |
| \#observations | 30 |

(a) How would you interpret the results above in the light of the researchers concerns? To what extent does including a linear trend solve any issues that arise?
(b) Concerned that the linear trend is too restrictive he tries to allow for general trending behaviour by adding 30 time dummies instead of the linear time trend. What effect might this have on the estimation?

He decides an alternative approach is to specify a panel using all 40 countries and includes both individual and time fixed effects

$$
\text { lefemale } 65_{i t}=\alpha_{i}+f_{t}+\text { Balcons }_{i t}+u_{i t}
$$

This gives the following estimates

| Parameter | Estimate |
| :---: | :---: |
| $\beta$ | $-0.12(0.01)$ |
| $R^{2}$ | 0.42 |
| \#observations | 1737 |

$F$-statistic all time dummies zero $F(55,1641)=245$
$F$-statistic all $\alpha_{i}$ the same $F(39,1641)=319$
(c) What interpretation would you give to the coefficients $\alpha_{i}$ and $f_{t}$ ?
(d) To what extent does the panel fixed effects formulation help in dealing with the trending behaviour relative to that in equation (2)? What extra assumptions are required for the panel approach to be valid?
(e) A colleague suggests that alcons is very likely measured with error. How might classical measurement error affect your interpretation of these results?
4. Suppose data is generated by the following process

$$
y_{i t}=\alpha_{i}+\lambda y_{i t-1}+\varepsilon_{i t} \quad i=1, \ldots, N \quad t=1, \ldots, T
$$

where $y_{i 0}$ are fixed constants, $\varepsilon_{i t}$ are IID $N\left(0, \sigma^{2}\right)$ and $0 \leq \lambda<1$.

Discuss what the properties of estimates of $\lambda$ might be when the estimation technique is:
(a) Pooled ordinary least squares
(b) LSDV estimation
(c) Anderson Hsiao Instrumental Variables on the differenced equations using $\Delta y_{i t-2}$ as instruments.

Your discussions may include suggestions on how to improve estimation.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Thursday 7 June 2018 9.00am-11.00am

S500
DEVELOPMENT ECONOMICS

Answer two out of three questions. Each question carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1(a) Individuals born in origin L, characterized by social connectedness $p$, decide whether to move to destination $M$ and start a business or stay in the traditional occupation in origin $L$ at the beginning of their lives on the basis of the payoffs at the two locations. Each individual is endowed with ability $\omega$ and $\log \omega \sim U[0,1]$ in each birth cohort. The payoff at the origin is $v \omega^{\sigma}$ and the payoff at the destination for the cohort born in period $t$ is $e^{A(p, t)} K_{t}^{\alpha} \omega^{1-\alpha}$, where $A(p, t)$ measures the contribution of the migrant network to the individual's firm's productivity; $A_{p}>0, A_{t}>0, A_{p t}>0$, and $K_{t}$ is its endogenously determined capital stock. $\sigma \in(0,1), \alpha \in(0,1)$, and each firm faces an interest rate $r$. Show that the initial capital selected by the marginal migrant from cohort $t$ is decreasing in $p$ but that the subsequent growth in his capital stock, say at $t^{\prime}>t ; \log K_{t^{\prime}}-\log K_{t^{\prime}-1}$, is increasing in $p$. Provide a concise explanation for this result.
(b) Suppose that origin-based networks are absent and, instead, that there is exogenous productivity growth at the destination. $A(p, t)$ is thus replaced by $B(t)$ in the production function, with $B_{t}>0$. Under what conditions on $v$ will the results on initial capital still be obtained? What information can be used to distinguish this alternative model from the model with networks?

2 Business plan competitions in which potential entrepreneurs pitch their ideas and the most capable contestants receive a cash grant to start up their businesses are becoming increasingly popular in developing countries. What criteria should be considered when selecting the winners in an economy where community networks are active?

3 Marriage in India is endogamous; i.e. individuals marry within their caste, whereas marriage in Africa is exogamous; i.e. individuals marry outside their birth clan. Provide an explanation for these differences, paying attention to the economic tradeoffs. Comment, in addition, on the consequences of these historical differences for the functioning of business networks today.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Wednesday 06 June $2018 \quad$ 1:30pm-3:30pm

## S620 <br> INSTITUTIONS AND ECONOMIC GROWTH IN HISTORICAL PERSPECTIVE

Candidates are required to answer two out of four questions

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. (a) Why does contract enforcement matter for economic growth and what are the main problems in providing it?
(b) Present one theory showing how a 'private-order' institution might have ensured contract-enforcement in an economy before c. 1800.
(c) How well does the theory you have presented stand up to confrontation with the empirical findings?
2. Discuss the usefulness of economic efficiency compared to other factors in explaining the existence and survival of one of the following historical institutions:
(a) serfdom
(b) merchant guilds
(c) craft guilds
3. Property rights are widely regarded as fundamental institutional determinants of economic growth.
(a) What are the key characteristics of property rights that determine their impact on econonomic growth?
(b) What are the most important features of the historical development of property rights in England between c. 1200 and c. 1800?
(c) Does history show that property rights are always good for economic growth?
4. Does historical experience provide any useful policy implications for assessing the role of institutions in economic growth?

ECM9<br>MPhil in Economics

Thursday 9 May $2019 \quad 2: 00 \mathrm{pm}-3: 30 \mathrm{pm}$

E100
MICROECONOMICS I

Candidates are required to answer three compulsory questions

Write your candidate number (not your name) on the cover of each booklet.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

An online newspaper is considering introducing dynamic advertising alongside each article. They realise that advertisers value different articles differently (depending on the topic of the article, the popularity of the article, and the advertiser itself) but they are unable to observe these valuations directly.

The newspaper decides to allow two advertisements on each page, and recognises that the advertisement closest to the start of the article will generate twice as many referrals as the advertisement lower on the page. Potential advertisers are invited to submit a price they will pay for each online referral generated. At any specific time the advertising location closest to the top of the page is awarded to the advertiser submitting the highest bid at that moment, while the second location is allocated to the advertiser offering the second highest bid, or not at all if there is only one bid. Firms will be allowed to change their bid whenever they choose throughout the day.
(a) Set up this problem as a Position Auction (consider just one page in isolation), and determine the prices you would expect to see bid in equilibrium as the number of potential advertisers rises from 2 to 5 .
(b) How will the equilibrium bid prices respond to the differing popularity of the articles?
(c) How would your answers to (a) and (b) change if the newspaper instead allowed three advertising locations on each page?
(d) Under what situations would it be worthwhile for the newspaper to increase the number of advertising locations from two to three?
(e) Should the newspaper introduce a minimum allowable bid (consider the case with two locations)? And if so, how might they determine the minimum allowable bid?

## Question 2

Consider a society inhabited by a continuum of citizens and normalize the size of the population to 1 . Suppose that the preferences of agent $i$ over a public good $y$ and a private good $c$ are

$$
\begin{equation*}
w_{i}=c_{i}+\ln y . \tag{1}
\end{equation*}
$$

Individuals are endowed with different levels of wealth, denoted $e_{i}$. The wealth distribution is uniform and defined on the interval $[\underline{e}, \bar{e}]$, with $\bar{e}>\underline{e}>1$. Public goods are produced from private goods according to the technology

$$
\begin{equation*}
y=a Q, \tag{2}
\end{equation*}
$$

where $Q$ is the total input of private goods and $a>0$ is the productivity of public production. The production of public goods is financed via a tax, $t$, on the wealth of each individual and so, individual $i^{\prime} s$ budget constraint is

$$
\begin{equation*}
c_{i} \leq e_{i}(1-t) \tag{3}
\end{equation*}
$$

(a) What is individual $i$ 's preferred choice of the public good and the tax rate? Are the conditions for the Median Voter Theorem satisfied? Explain.
(b) Under majority rule and universal franchise, what is the selected policy?
(c) Explain what happens to the supply of public goods and to the tax rate if
(i) The productivity of government production $a$ increases.
(ii) Average wealth increases.
(d) Suppose the franchise is restricted in such a way as to exclude some poor individuals. Assume that only individuals with wealth above $\hat{e} \in(\underline{e}, \bar{e})$ are allowed to vote. How does that affect the supply of public goods and the tax rate? Can an extension of the franchise or an increase in wealth explain an increase in the size of the public sector? Explain.

## Question 3

Consider an economy with two firms $i \in\{1,2\}$, whose Marginal Abatement Cost (MAC) curves are given by the following exponential functions: $\mathrm{MAC}_{1}=e^{\frac{10}{3} a_{1}}$ and $\mathrm{MAC}_{2}=$ $e^{\frac{10}{7} a_{2}}$, where $a_{i}$ is the abatement of firm $i$. The Social Cost of Carbon is given by $\operatorname{SCC}(a)=$ $100-e^{a}$, where $a$ is total abatement, and the initial (unpriced) quantity of emissions is 5 tonnes.
(a) Find the economy-wide MAC.
(b) Find the the optimal carbon price and quantity of emissions under both an emissions tax and an emissions trading scheme.
(c) Explain why both these policies are efficient.
(d) Suppose there is a third firm, with $\mathrm{MAC}_{3}=30$. Find the optimal emissions price and quantity of abatement by each firm.
(e) State and explain two reasons why, in the real world, carbon pricing alone may not be an efficient policy response to climate change.

ECM9/ECM11
MPhil in Economics
MPhil in Finance and Economics

Tuesday 14 May 2019 9:00am to 11:00am

## E101 <br> APPLIED MICROECONOMICS

This paper is in two sections. Section A: Candidates are required to answer four out of six questions. Section B: Candidates are required to answer one compulsory question.

Each Section carries equal weight; questions within Section A carry equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Section A

1. Define the three types of labour supply elasticities discussed in the lecture. Is there a case in which the three coincide? (Explain your answer)
2. For each of the following claims, state clearly whether you find the statement to be true, false, or uncertain and explain why. (Credit is given for the argument, rather than the correct classification of the claims.)
(a) Denote with $E V$ the equivalent variation, with $C V$ the compensating variation, and with $\Delta C S$ the change in consumer surplus. Then, $C V \leq \Delta C S \leq E V$.
(b) The deadweight loss arises solely due to substitution effects.
(c) In the presence of irremovable distortions, it is still optimal to try to satisfy as many Pareto conditions as possible.
3. Consider an economy in which there are three goods, $x_{1}, x_{2}$ and $x_{3}$ and one fixed factor of production, labour, $\ell_{0}$. Good 1 is produced in the (competitive) private sector with constant returns to scale using labour as the only input ( $x_{1}=\ell_{0}$ ) where the marginal product of labour is assumed to be one. The other goods are produced in the public sector. The private good $x_{1}$ can either be consumed or be used as an input into the production of $x_{2}$ and $x_{3}$. Suppose that the utility of the representative consumer is quasi-linear in the private good and separable in the two goods produced by the state owned firm. The utility function is given by $U=x_{1}+v\left(x_{2}\right)+w\left(x_{3}\right)$.

Suppose, in contrast with the example we saw, that the government wants to use the state own firm that produces $x_{2}$ and $x_{3}$, to earn a profit (which will be used to subsidize, say, other government activities). How would that affect the Ramsey pricing rule? Explain your answer.
4. Suppose consumers live for two periods, and they draw utility from a single good in the economy. Their preferences are given by $u(c)=\log (c)$ in each period. Let $c_{1}$ denote consumption in period $1, c_{2}$ consumption in period 2 , and $\beta<1$ the discount factor (i.e. the factor at which future consumption is discounted compared to current consumption). Consumers receive a deterministic income $y_{1}$ in the first period. Consumers can freely borrow (e.g., run a credit card balance) and lend (e.g., deposit in a saving account) in financial markets at the prevailing interest rate $r$. The government introduces a tax $\tau$ on capital gains, reducing the effective interest rate faced by the consumer to $r(1-\tau)$.
Can you predict how a given consumer's consumption will change?
5. Do all households in an economy experience the same inflation rate? Explain your answer.
6. Suppose that you have "reliable" estimates of the compensated elasticity of labour supply of 0.2647 for single women with one child under 2 years old and that the revenue from income taxes from these individuals in 2016-2017 was approximately $£ 182 \mathrm{bn}$. Suppose there is a proposed income tax increase that, due to its progressivity, can be approximated with an ad valorem $\operatorname{tax} \tau=0.15$ on the wage of individuals (so that the new wages $w^{t_{1}}$ would be approximated with $\left.w^{t_{1}}(1-\tau)\right)$. Can you calculate the DWL associated with this tax generated by the group of single women with one child under the age of 2 ? If so, provide an expression for it.

## Section B

A researcher estimates the following system of demands by OLS,

$$
\omega_{j h}=\alpha_{j 0}+\alpha_{j, a d} N_{a d u l t s, h}+\alpha_{j, k i d} N_{k i d s, h}+\sum_{k=1}^{7} \gamma_{j k} \log \left(p_{k, h}\right)+\beta_{j} \log \left(\frac{x_{h}}{P_{h}}\right)+u_{j h}
$$

where $\omega_{j h}$ is the budget share on good $j$ for household $h$, the price index $P_{h}$ is approximated with the Stone price index for household $h,{ }^{1}$ and we assume that $u_{j h}$ is measurement error in the dependent variable that is uncorrelated with prices and disposable income.

There are seven goods in this demand system: 1) food at home, 2) food in restaurants, 3) transport, 4) services, 5) recreation, 6) alcohol and tobacco (considered together as a single good), and 7) clothing. The coefficient estimates from each OLS regression are given in the columns in Table 1.

Table 1: Estimates from the Demand System

|  | food at <br> home <br> $\omega_{1}$ | food at <br> restaurants <br> $\omega_{2}$ | Transport <br> $\omega_{3}$ | Services <br> $\omega_{4}$ | Recreation <br> $\omega_{5}$ | Vice <br> $\omega_{6}$ | Clothing <br> $\omega_{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $N_{\text {adults,h }}$ | 0.079 | -0.040 | 0.022 | -0.014 | -0.026 | -0.019 | -0.001 |
| $N_{\text {kids, } h}$ | 0.053 | -0.020 | -0.018 | 0.019 | -0.003 | -0.015 | -0.016 |
| $\log \left(p_{1}\right)$ | -0.086 | 0.075 | -0.120 | 0.064 | 0.032 | -0.039 | 0.073 |
| $\log \left(p_{2}\right)$ | -0.014 | -0.021 | 0.150 | -0.065 | 0.085 | -0.069 | -0.066 |
| $\log \left(p_{3}\right)$ | 0.010 | 0.009 | 0.006 | -0.016 | -0.009 | -0.004 | 0.004 |
| $\log \left(p_{4}\right)$ | 0.029 | 0.002 | 0.017 | -0.028 | -0.089 | 0.089 | -0.020 |
| $\log \left(p_{5}\right)$ | 0.066 | -0.064 | -0.013 | 0.087 | 0.014 | -0.079 | -0.010 |
| $\log \left(p_{6}\right)$ | -0.008 | 0.007 | -0.029 | -0.019 | -0.035 | 0.069 | 0.016 |
| $\log \left(p_{7}\right)$ | -0.093 | -0.003 | 0.006 | 0.070 | 0.066 | 0.000 | -0.046 |
| $\log \left(x_{h} / P_{h}\right)$ | -0.160 | 0.050 | 0.022 | -0.011 | 0.046 | 0.017 | 0.037 |
| $\operatorname{Constant}$ | 1.624 | -0.306 | -0.043 | 0.283 | -0.284 | -0.030 | -0.244 |

a) What conditions need to hold for additivity and homogeneity to be satisfied? Does Slutsky symmetry seem to hold in the data?
b) Using the Stone Index approximation of the price deflator in the system (as the true price deflator), derive an expression for the compensated price elasticity $\varepsilon_{j k}^{c}$. (Hint: Derive an expression for the uncompensated price elasticity and the income elasticity and use the Slutsky equation.)

[^4]c) Suppose that the compensated cross-price elasticities are zero for this one exercise (i.e. suppose that somehow, regardless of the estimated coefficients, there are no interdependencies in consumption across goods). Suppose also that the budget shares are, on average, given by

| Good | Average Share |
| :---: | :---: |
| food home | 0.39 |
| food rest | 0.03 |
| transport | 0.25 |
| services | 0.07 |
| recreation | 0.05 |
| vice | 0.13 |
| clothing | 0.08 |

and that consumers in the economy are more or less homogeneous in consumption. If you were the planner of this economy, which good would you tax the most in order to raise revenue while minimising the DWL? (Hint: Use the expression you derived above for $j=k$.)
d) Now suppose that there are two types of households and their average shares are as follows.

| Shares | Poor | Rich |
| :---: | :---: | :---: |
| food home | 0.39 | 0.12 |
| food rest | 0.03 | 0.1 |
| transport | 0.25 | 0.08 |
| services | 0.07 | 0.2 |
| recreation | 0.05 | 0.32 |
| vice | 0.13 | 0.08 |
| clothing | 0.08 | 0.1 |

Without doing any more calculations, would your answer to the question above change if you took into account equity considerations? Explain.
e) Suppose the VAT applies to all goods in the system and it increases by $10 \%$, changing the prices from $p^{t_{0}}=\left(p_{1}^{t_{0}}, p_{2}^{t_{0}}, \ldots, p_{7}^{t_{0}}\right)$ to $p^{t_{1}}=\left(p_{1}^{t_{1}}, p_{2}^{t_{1}}, \ldots, p_{7}^{t_{1}}\right)$. Assuming that (nominal) disposable income stays the same before and after the price change, explain how you would measure the compensating variation in order to assess the welfare costs of this change in prices?

## END OF PAPER

ECM9/ECM11<br>MPhil in Economics<br>MPhil in Finance \& Economics

Wednesday 8 May $2019 \quad 2: 00 \mathrm{pm}$ to $4: 00 \mathrm{pm}$

E200
MACROECONOMICS

The paper is in two sections: You should answer all four questions in Section A and one question from Section B. Each Section carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A

A.1. Consider the following standard $q$-theory model assuming that investment costs are $(1-\theta) I_{t}$ where $\theta$ is an exogenous tax credit that decreases the cost of investing. The following dynamics for the capital stock $K_{t}$ and the shadow price of capital $q_{t}$ emerges:

$$
\begin{aligned}
q_{t}+\theta & =1+\chi \Delta K_{t+1} \\
\Delta q_{t+1} & =r q_{t}-A F^{\prime}\left(K_{t}+\frac{1}{\chi}\left(q_{t}-1\right)\right)
\end{aligned}
$$

where $r$ denotes the real interest rate, with $1+r>0$; the production function is given by $Y_{t}=A_{t} F\left(K_{t}\right)$ with $F^{\prime}(\cdot)>0$ and $F^{\prime \prime}(\cdot)<0$, where $A$ is a positive productivity parameter; and $\chi$ is a positive parameter related to the investment adjustment costs. The initial capital stock $K_{0}>0$ is given and the transversality condition is $\lim _{T \rightarrow \infty}\left(\frac{1}{1+r}\right)^{T} q_{T} K_{T+1}=0$. Assume that at period $t$ the government unexpectedly decreases permanently the tax credit to $\theta^{\prime}<\theta$. Carefully explain how $K_{t+1}$ and $q_{t}$ are affected over time using a phase diagram and provide an intuitive explanation for the effect on investment.
A. 2 Consider a representative consumer who lives for only two periods denoted by $t=1,2$. The consumer is born in period 1 without any financial assets and leaves no bequests or debt at the end of period 2. The consumer's exogenous labor income is $w_{t}$ in each period and $r \in(0,1)$ is the exogenous real interest rate at which the consumer can borrow or lend in period 1. Her preferences over consumption, $c_{t}$, can be represented by the utility function:

$$
U\left(c_{1}, c_{2}\right)=u\left(c_{1}\right)+\beta u\left(c_{2}\right), \quad 0<\beta<1
$$

where $\beta$ is the discount factor and the momentary utility function is given by:

$$
u(c)=\left\{\begin{array}{cc}
\frac{c^{1-\theta}-1}{1-\theta}, & \theta \neq 1 \\
\log (c) & \theta=1
\end{array}\right.
$$

with $\theta>0$.
Derive optimal consumption in period 1 as a function of $w_{1}, w_{2}, \beta, r$ and $\theta$. Now, consider the effect of an increase in the real interest rate $1+r$ on optimal first-period consumption, $c_{1}$. What is the overall effect of this increase if labor income in the second period is zero, $w_{2}=0$ ? Explain.
A.3. Suppose the economy of Britannia is described by the following flexibleprice monetary model, which is based on purchasing power parity, money market equilibrium in Britannia (and abroad) and uncovered interest parity, respectively:

$$
\begin{aligned}
p_{t} & =s_{t}+p_{t}^{*} \\
m_{t}^{(*)}-p_{t}^{(*)} & =y_{t}^{(*)}-i_{t}^{(*)} \\
i_{t} & =i_{t}^{*}+\mathrm{E}_{t}\left[s_{t+1}\right]-s_{t}
\end{aligned}
$$

where $p_{t}$ denotes the log aggregate price level, $s_{t}$ the log nominal exchange rate, defined as the domestic price of foreign currency, $m_{t}$ the log money supply, $y_{t} \log$ aggregate output, and $i_{t}$ the nominal interest rate. Foreign variables are indicated by an asterisk, and subscripts indicate the time period.
Solve for the nominal exchange rate $s_{t}$ (assuming no bubbles). In addition, explain how Britannia's nominal exchange rate in period $t$ is affected if it is suddenly expected in period $t$ that Britannia's output from period $t+2$ onwards will be $4 \%$ lower than previously anticipated due to developments related to Britannia's abandonment of frictionless trade with Europia.
A. 4 Suppose the government of Britannia sets the income tax rate $\tau_{t}$ in each period to minimize the following loss function:

$$
L_{t}=\sum_{s=0}^{\infty} \frac{1}{(1+r)^{s}} \frac{1}{2} \tau_{t+s}^{2} Y_{t+s}
$$

where $Y_{t+s}$ denotes Britannia's real national income in period $t+s$, and $r$ is the constant real interest rate. The government's intertemporal budget constraint is given by

$$
(1+r) B_{t}+\sum_{s=0}^{\infty} \frac{G_{t+s}}{(1+r)^{s}}=\sum_{s=0}^{\infty} \frac{\tau_{t+s} Y_{t+s}}{(1+r)^{s}}
$$

where $B_{t}$ denotes government debt at the start of period $t$, and $G_{s}$ government purchases in period $s$. Assume that initially, $B_{t}>0, Y_{s}=\bar{Y}$ and $G_{s}=\bar{G}$ for $s=t, t+1, \ldots$
Suppose that due to developments related to Britannia's abandonment of frictionless trade with Europia, it is suddenly expected in period $t$ that Britannia's output from period $t+2$ onwards will be lower by an amount $\beta$, such that $Y_{s}=\bar{Y}-\beta$ for $s=t+2, t+3, \ldots$. Explain how this would affect the income tax rate $\tau_{t}$ over time.

## SECTION B

## B1. Real Business Cycle

Consider the standard Real Business Cycle (RBC) model. The economy is closed and the horizon infinite. There is a continuum of measure one of identical, profit maximising firms. The production technology of the representative firm is represented by the following function:

$$
Y_{t}=K_{t}^{\alpha}\left(A_{t} L_{t}\right)^{1-\alpha},
$$

where subscript $t$ denotes the time period, $Y_{t}$ is output, $L_{t}$ is labour used in production, $K_{t}$ is capital. $A_{t}>0$ is an exogenous, stochastic productivity factor that follows: $A_{t}=(1-\rho) \bar{A}+\rho A_{t-1}+\varepsilon_{t}$ where $\varepsilon_{t} \sim N\left(0, \sigma^{2}\right)$ and $0 \leq \rho \leq 1 . \alpha \in(0,1)$. There is perfect competition in the product and labour market.
Investment in the economy is given by: $I_{t}=K_{t+1}-(1-\delta) K_{t}$ where $\delta \in(0,1)$ is the depreciation rate.
There is a continuum of measure one of identical individuals. A representative individual maximises the following utility function:

$$
U=E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\ln \left(c_{t}\right)+\chi \ln \left(1-L_{t}\right)\right]
$$

where $c_{t}$ denotes consumption, $L_{t}$ labour supply and $\beta \in(0,1)$ and $\chi>0$ are constant parameters. The time endowment is equal to one in each period.
Individuals supply labour to the firms at a real wage $w_{t}$ per unit of labour. They can also borrow or lend in each period and the credit markets are in equilibrium in every period, which implies that investment is equal to aggregate savings. The budget constraint of the individuals is

$$
c_{t}+K_{t+1}=\left(1+r_{t}^{K}-\delta\right) K_{t}+w_{t} L_{t},
$$

where $r_{t}^{K}$ is the return on capital.
(a) Formulate the optimization problem of the representative firm and take the first order conditions associated with the problem. Give an economic interpretation.
(b) Formulate the optimization problem of the representative individual and take the first order conditions associated with the problem. Give an economic interpretation.

Now assume instead that

$$
U=E_{0} \sum_{t=0}^{\infty} \beta^{t}\left[\ln \left(c_{t}-b C_{t-1}\right)+\chi \ln \left(1-L_{t}\right)\right]
$$

where $C$ corresponds to average consumption in the economy, which the individual takes as given, with $b \geq 0$.
(c) What is the intuition behind this utility function? Derive the Euler equation for this problem and comment on your results.
(d) Figure 1 presents the impulse response functions, as percentage deviations from steady state values, of various macroeconomic variables to a one percent positive productivity shock under three assumptions on $b: b=0, b=0.7$, and $b=0.9$. The model is otherwise the same workhorse RBC model as described before. Identify which of the three sets of impulse response functions ((i) solid, (ii) dashed, (iii) dotted line) correspond to each value of $b$. Explain intuitively how the impulse responses depend on the value of $b$.

Figure 1






## B. 2 Shapiro-Stiglitz model with income taxes

Suppose the economy of Britannia is described by the following ShapiroStiglitz model. The representative firm faces perfect competition and sets the level of employment $L_{t}$ and the real wage $w_{t}$ to maximize real profits

$$
\Pi_{t}=\left(e_{t} L_{t}\right)^{\alpha}-w_{t} L_{t}
$$

where $e_{t}$ denotes the employee's effort level, the subscript $t$ indicates the time period, and the technology parameter satisfies $0<\alpha<1$.
The representative worker maximizes the expected value of lifetime utility

$$
U=\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^{t}} v_{t}
$$

where $\rho$ is the worker's intertemporal discount rate ( $\rho>0$ ), and instantaneous utility is given by

$$
v_{t}= \begin{cases}(1-\tau) w_{t}-e_{t} & \text { if employed } \\ (1-\tau) b & \text { if unemployed }\end{cases}
$$

where $\tau$ is the income tax rate $(0<\tau<1)$ and $b$ is the unemployment benefit $(b>0)$. The employee's effort level equals

$$
e_{t}= \begin{cases}\bar{e}>0 & \text { if not shirking } \\ 0 & \text { if shirking }\end{cases}
$$

Assume that in each period, $s$ is the probability that an employee loses the job for exogenous reasons, $q$ is the probability that an employee is fired at the end of the period due to shirking, and $f=f(u)$ is the probability that an unemployed worker finds a job, where $u$ is the unemployment rate, which satisfies $u=s /(s+f)$ in equilibrium.
(a) Write down the expected value of lifetime utility for the worker if (i) employed and not shirking, (ii) employed but shirking, and (iii) unemployed. Explain briefly.
(b) Derive the no-shirking condition. Provide an intuitive explanation for the relation between the wage $w$ and the unemployment rate $u$.
(c) Show graphically and explain intuitively the effect on the equilibrium wage $w$ and employment $L$ if Britannia's government would like to improve public finances by
i. reducing the unemployment benefit $b$.
iii. raising the income tax rate $\tau$.

Carefully explain how the post-tax wage $(1-\tau) w$ is affected in each case.

ECM9/ECM11
MPhil in Economics
MPhil in Finance and Economics

Thursday 16 May 2019 10:00am to 11:30am

E201
APPLIED MACROECONOMICS I

Candidates are required to answer two out of three questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Question 1 Consider the optimization problem

$$
\begin{gathered}
\max _{\left\{c_{t}, b_{t+1}, x_{t+1}\right\}} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right) \\
\text { subject to } \quad p_{t} c_{t}+b_{t+1}+x_{t+1}=w_{t-1}+b_{t}\left(1+i_{t}\right)+x_{t}-T_{t} \\
x_{t+1} \geq 0, \quad t=0,1, \ldots
\end{gathered}
$$

where $p_{t}$ denotes the price level; $i_{t}$ the nominal interest rate; $c_{t}$ real consumption; $b_{t}$ nominal bonds; $w_{t-1}$ nominal non-financial income received in period $t ; T_{t}$ lump-sum taxes; $x_{t}$ excess cash holdings; and $t=0,1,2 \ldots$ The function $u(\cdot)$ represent the flow utility received from real consumption, and $\beta$ is a discount factor.

The government's budget constraint is given by

$$
p_{t} g_{t}+\left(1+i_{t}\right) d_{t}=d_{t+1}+T_{t}+m_{t}-m_{t-1}
$$

where $g_{t}$ denotes (real) government consumption; $d_{t}$ nominal government debt; and $m_{t}$ the money stock.
(a) Derive the intertemporal optimality conditions for both $b_{t+1}$ and $x_{t+1}$, and provide an intuitive explanation. Assume that $p_{-1} y_{-1}+x_{0}=m_{-1}$, use bond and goods market clearing conditions together with $w_{t-1}=p_{t-1} y_{t-1}$ to derive the equation of exchange.
(b) Suppose that potential output in both period $t$ and $t+1$ are given by $y_{t}=y_{t+1}=y$, and that the marginal utility is of the type $u^{\prime}(c)=c^{-\gamma}$. There is no government spending, and the money supply is kept constant $m_{t}=m_{t+1}=m$. Suppose furthermore that there is a sudden change in the discount factor from $\beta$ to $\beta^{\prime}$. Which value of $\beta^{\prime}$ brings the economy to a liquidity trap? Denote this value $\hat{\beta}$. If prices in period $t, p_{t}$, are sticky and such that $p_{t}=\bar{p}=m / y$, is a recession induced by $\beta^{\prime}>\hat{\beta}$ depending on the parameter $\gamma$ ? Explain.
(c) Under the assumptions outlined in part (b) above, suppose that the discount factor increases such that $\beta^{\prime}>\hat{\beta}$, and remains elevated in the subsequent period with probability $q$. With the complementary probability, $(1-q)$, it reverts back to normal, and so on. Prices remain sticky throughout the duration of the liquidity trap. The intertemporal optimality condition is therefore

$$
u^{\prime}(\hat{y})=\beta^{\prime}\left[q u^{\prime}(\hat{y})+(1-q) u^{\prime}(y)\right] .
$$

How is $\hat{y}$ related to $q$ ? Provide an intuitive explanation.

Question 2 Consider the optimization problem

$$
\begin{gathered}
\max _{\left\{c_{t}, b_{t+1}, \ell_{t}\right\}} \sum_{t=0}^{\infty} \beta^{t}\left[u\left(c_{t}\right)+v\left(1-\ell_{t}\right)\right], \\
\text { subject to } \quad p_{t} c_{t}+b_{t+1}=w_{t-1} \ell_{t-1}+b_{t}\left(1+i_{t}\right)-T_{t}
\end{gathered}
$$

where $p_{t}$ denotes the price level; $i_{t}$ the nominal interest rate; $c_{t}$ real consumption; $b_{t}$ nominal bonds; $\ell_{t}$ labour supply income received in period $t ; w_{t}$ the nominal wage in period $t ; T_{t}$ lump-sum taxes; and $t=0,1,2 \ldots$ The function $u(\cdot)$ represent the flow utility received from real consumption, $v(\cdot)$ the flow utility received from leisure, and $\beta \in(0,1)$ is a discount factor. Define inflation as $\pi_{t}=\left(p_{t} / p_{t-1}-1\right)$.

The government's budget constraint is given by

$$
p_{t} g_{t}+\left(1+i_{t}\right) d_{t}=d_{t+1}+T_{t}+m_{t}-m_{t-1}
$$

where $g_{t}$ denotes (real) government consumption; $d_{t}$ nominal government debt; and $m_{t}$ the money stock.
(a) Derive the intratemporal optimality condition for $\ell_{t}$ in terms of the real wage, $\bar{w}_{t}=$ $w_{t} / p_{t}$, and inflation, $\pi$. Under common circumstances the corresponding optimality condition would not involve inflation. Intuitively explain why it does contain inflation under the current circumstances.
(b) Derive the equivalent optimality condition for the social planner. Under the assumption that the inflation rate as well as quantities are constant. What is the optimal inflation rate? Provide an intuitive explanation.
(c) Now suppose that consumption evolves according to $c_{t+1}=(1+g) c_{t}$, for $t=0,1, \ldots$, and that $u^{\prime}(c)=c^{-\gamma}$. How is the optimal inflation rate related to the growth rate of the economy? Explain why this relationship emerges.

Question 3 A government's budget constraint in real terms is given by

$$
g_{t}-\tau_{t}+\left(1+r_{t}\right) d_{t}=d_{t+1}+\frac{m_{t}-m_{t-1}}{p_{t}}, \quad \forall t
$$

where $g_{t}$ denotes government spending; $\tau_{t}$ lump-sum taxes; $d_{t}$ government debt; $r_{t}$ the associated real interest rate; $m_{t}$ the money stock; and $t=0,1,2 \ldots$ The no-Ponzi condition is given by

$$
\lim _{s \rightarrow \infty} \frac{d_{t+1+s}}{\Pi_{j=0}^{s}\left(1+r_{t+j}\right)}=0, \quad \forall t
$$

Assume throughout this question that excess cash holdings are zero; output is constant and equal to one; and the real interest rate is constant and equal to $r$.
(a) Use the equation of exchange and show that government debt, $d_{t}$, is equal to the net present value of two components: fiscal surpluses and seignorage (with the latter expressed in terms of inflation). Provide an intuitive explanation. [Use the approximation $\pi_{t} \approx\left(p_{t}-p_{t-1}\right) / p_{t}$ for simplicity].
(b) Define government surpluses as $n_{t}=\tau_{t}-g_{t}$, and assume that the government sets the path of fiscal surpluses according to the rule

$$
n_{t+s+1}=(1-\rho) \bar{n}+\rho n_{t+s},
$$

for $\rho \in[0,1)$ and with $n_{t-1}$ given. Conditional on a certain value of $d_{t}$, how is the net present value of seignorage in period $t$ affected by $n_{t-1}, \bar{n}$ ? Why is this referred to as "some unpleasant monetarist arithmetic"?
(c) Suppose that the central bank sets inflation at a constant value, $\bar{\pi}$, such that the government debt can be honored under a given path for fiscal surpluses. Denote the present value of the resulting seignorage as $\bar{X}_{t}$, with $\bar{X}_{t}>0$. Express the constant value of inflation as a function of $\bar{X}$ and $r$.

Now suppose that in period $t$, inflation suddenly declines to zero for one period only. What must the constant value of inflation be in periods $t+1$ onwards to ensure that the public debt remains sustainable, and how does this value of inflation relate to $\bar{\pi}$ above?

## END OF PAPER

ECM9/ECM11/MGM3
MPhil in Economics
MPhil in Finance and Economics
MPhil Finance

Friday 10 May 2019 2:00pm to 5:00pm

## E300 <br> ECONOMETRIC METHODS

Answer all six questions from Section A and two questions from Section B. Part A is $60 \%$ of total marks.

All questions in Section A carry equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly. Credit will be given for clear presentation of relevant statistics.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator Dickey-Fuller \& Engle Granger Table
New Cambridge Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

A1 Consider a regression model

$$
y_{i}=\alpha+\beta x_{i}+\gamma z_{i}+\varepsilon_{i}
$$

with $E\left(\varepsilon_{i} \mid x_{i}, z_{i}\right)=0$ and i.i.d. triples $\left(y_{i}, x_{i}, z_{i}\right) i=1, \ldots .100$. The following is the STATA outcome of the OLS regression of $y_{i}$ on the constant, $x_{i}$, and $z_{i}$

- reg $\mathrm{y} \times \mathrm{z}$


Let us denote the residuals from the OLS regression of $y_{i}$ on the constant and $z_{i}$ as yres ${ }_{i}$, and let us denote the residuals from the OLS regression of $x_{i}$ on the constant and $z_{i}$ as ares ${ }_{i}$. The following is the STATA outcome of the OLS regression of yres $_{i}$ on tres $_{i}$ (constant is excluded). Note that some of the table's entries were intentionally removed.
. reg yres xres, nocon

(a) (5 points) In the above table, what does STATA report as the OLS estimate of the coefficient on sres? Explain your answer.
(b) (5 points) In the above table, what does STATA report as the standard error of the coefficient on xres? Explain your answer.

A2 Consider a simple univariate regression model

$$
y_{i}=\alpha+\beta x_{i}+\varepsilon_{i}
$$

with i.i.d. pairs $\left(y_{i}, x_{i}\right) i=1, \ldots, 100, E\left(\varepsilon_{i} \mid x_{i}\right)=0$, and $\operatorname{Var}\left(\varepsilon_{i} \mid x_{i}\right)<\infty$.
(a) (5 points) A student argues that, as long as the model is correct, since pairs ( $y_{i}, x_{i}$ ) $i=1, \ldots, 100$ are i.i.d., the pairs $\left(\varepsilon_{i}, x_{i}\right) i=1, \ldots, 100$ must also be i.i.d. Is this correct? Explain your answer.
(b) (5 points) The student further argues that, as long as $\left(\varepsilon_{i}, x_{i}\right) i=1, \ldots, 100$ are i.i.d., $\operatorname{Var}\left(\varepsilon_{i} \mid x_{i}\right)$ must be the same for all $i$, and hence, there should be no heteroskedasticity. Is this argument correct? Explain your answer.

A3 Consider linear regression model

$$
y_{i}=\beta x_{i}+u_{i},
$$

where $\left(y_{i}, x_{i}\right)$ with $i=1, \ldots, n$ are i.i.d., $x_{i}$ and $u_{i}$ are independent, $\mathrm{E}\left(x_{i}\right)=\mathrm{E}\left(u_{i}\right)=0, \mathrm{E}\left(x_{i}^{2}\right)=$ $\mathrm{E}\left(u_{i}^{2}\right)=1$.
(a) (4 points) Define the concept of the convergence in distribution.
(b) (6 points) Find the limit in distribution of $\frac{1}{\sqrt{n}} \sum_{i=1}^{n}\left(y_{i}-\bar{x}\right)$ as $n \rightarrow \infty$, where $\bar{x}=\frac{1}{n} \sum_{i=1}^{n} x_{i}$.

A4 Consider the following time series regression

$$
y_{t}=\alpha+\beta x_{t}+\varepsilon_{t}, t=1, \ldots, T
$$

where $x_{t}$ is a random walk, $\beta \neq 0$, and $\varepsilon_{t}, t=1, \ldots, T$, are i.i.d. $N(0,1)$ random variables independent from $x_{t}, t=1, \ldots, T$.
(a) (4 points) Are $y_{t}$ and $x_{t}$ cointegrated? Explain your answer.
(b) (6 points) Suppose that a student would like to test a hypothesis that $\beta=1$. She claims that, despite the fact that $x_{t}$ is nonstationary, the hypothesis can be tested using the standard $t$ test. Is this claim correct? Explain your answer.

A5 Suppose $\left(y_{i}, x_{1 i}, x_{2 i}\right), i=1, \ldots, n$, is an i.i.d. sequence with

$$
y_{i}=x_{1 i}^{\prime} \beta_{1}+x_{2 i} \beta_{2}+\varepsilon_{i}
$$

where $x_{1 i}$ is a $k \times 1$ vector and $x_{2 i}$ is a scalar for $i=1, \ldots, n$. Let $\beta=\left(\beta_{1}^{\prime}, \beta_{2}\right)^{\prime}$. Assume $E\left(\varepsilon_{i} \mid x_{1 i}, x_{2 i}\right)=0$ for $i=1, \ldots, n$. So the OLS regression of $y_{i}$ on $x_{1 i}$ and $x_{2 i}$ is consistent for $\beta$. For the following questions state any additional assumptions needed.
(a) (5 points) Suppose that $x_{2 i} \neq 0$ for all $i$. Let $w_{i}=\left(x_{1 i}^{\prime}, x_{2 i}^{-1}\right)^{\prime}$. Suppose we estimate $\beta$ using $2 S L S$ with instrument $w_{i}$. Give an explicit expression for $\hat{\beta}_{2 S L S}$. In doing so, clearly define all the necessary matrices.
(b) (5 points) Is $\hat{\beta}_{2 S L S}$ consistent? Prove your answer.

A6 Let $y$ be a random variable with the following distribution.

$$
y= \begin{cases}1 & \text { with probability } 3 \theta \\ 0 & \text { with probability } 1-3 \theta\end{cases}
$$

Suppose that you observe a random sample $y_{1}, \ldots, y_{n}$ from the above distribution. Let $n_{1}$ and $n_{0}$ be the number of the observations equal to 1 and 0 respectively.
(a) (5 points) Find the maximum likelihood estimator of $\theta$ in terms of $n_{1}$ and $n_{0}$.
(b) (5 points) Find the Cramér-Rao lower bound as a function of $n$ and $\theta$.

## SECTION B

B1 Consider the following Logit model

$$
\begin{aligned}
y_{i} & =\left\{\begin{array}{cc}
1 & \text { if } y_{i}^{*}>0 \\
0 & \text { otherwise }
\end{array}\right. \\
y_{i}^{*} & =x_{i} \beta+\varepsilon_{i}
\end{aligned}
$$

where $\left(y_{i}^{*}, x_{i}\right)$ are i.i.d. and $\varepsilon_{i} \mid x_{i}$ has logistic distribution with the cumulative distribution function

$$
F(z)=\frac{e^{z}}{1+e^{z}}
$$

(a) (3 points) Why such a model may be preferred to the linear probability model?
(b) ( 7 points) You are given data

| $i$ | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- |
| $y_{i}$ | 1 | 0 | 1 |
| $x_{i}$ | -1 | 1 | 1 |

Compute the maximum likelihood estimate of $\beta$.
(c) (3 points) What is your estimate of the marginal effect of $x$ on the conditional probability $\operatorname{Pr}(y=1 \mid x)$ at $x=1 ?$
(d) (7 points) Now suppose that $\varepsilon_{i} \mid x_{i}$ does not have the standard logistic distribution. Instead, it is $3 \varepsilon_{i} \mid x_{i}$ that does. That is, the conditional distribution of $\varepsilon_{i} \mid x_{i}$ has the cumulative distribution function

$$
G(z)=\frac{e^{3 z}}{1+e^{3 z}}
$$

The data are still the same as given in (b). How does your answer to (b) change? Explain your answer.

B2 A researcher studies the relationship between the unemployment rates in France and Spain. They collect data on quarterly unemployment rates for these two countries from 1986Q2 to 2018 Q 4.
(a) (5 points) First, the researcher runs the OLS regression of the French unemployment rate (variable $U f r$ ) on the Spanish unemployment rate (variable $U s p$ ). They obtain the following results with the standard errors in parentheses

$$
\widehat{U f r}_{t}=\underset{(0.34)}{7.33}+\underset{(0.019)}{0.156 U s p_{t},} R^{2}=0.34
$$

The researcher suspects that the relationship is spurious. Explain, how they can test this suspicion.
(b) (5 points) The time series plots for $U f r$ and $U s p$ are as follows


A colleague suggests that before testing for spurious regression, the researcher should check whether $U f r$ and $U s p$ have unit roots. They provide the following OLS results (standard errors in parentheses)

$$
\begin{aligned}
\widehat{U f r}_{t} & =\underset{(0.11)}{0.20}+\underset{(0.011)}{0.979 U f r_{t-1}}+\underset{(0.067)}{0.65} \Delta U f r_{t-1} \\
\widehat{U s p}_{t} & =\underset{(0.11)}{0.23}+\underset{(0.007)}{0.986 U s p_{t-1}}+\underset{(0.051)}{0.82} \Delta U s p_{t-1}
\end{aligned}
$$

where $\Delta U f r_{t}=U f r_{t}-U f r_{t-1}$ and $\Delta U s p_{t}=U s p_{t}-U s p_{t-1}$. Test the hypotheses that $U f r$ and $U s p$ have unit roots.
(c) (5 points) What is the rational for including the lags $\Delta U f r_{t-1}$ and $\Delta U s p_{t-1}$ to the regressions in (b)? The researcher would like to even use a more flexible specification of the two regressions by including a time trend as well. However, the colleague advices them against including the trend. What may be the colleague's argument?
(d) (5 points) Assuming that $U f r$ and $U s p$ are cointegrated, interpret the following OLS result

$$
\begin{aligned}
\widehat{\Delta U f r_{t}}= & \underset{(0.10)}{0.10}+\underset{(0.08)}{0.49} \Delta U f r_{t-1}+\underset{(0.04)}{0.10} \Delta U s p_{t}+\underset{(0.04)}{0.01} \Delta U s p_{t-1} \\
& -\underset{(0.013)}{0.014}\left(U f r_{t-1}-0.156 U s p_{t-1}\right) .
\end{aligned}
$$

B3 Consider a random sample $\left\{\left(y_{i}, x_{i}\right), i=1, \ldots, n\right\}$, where the conditional distribution of $y_{i}$ given $x_{i}$ is exponential with density $\lambda_{i} e^{-\lambda_{i} y}$ and $\lambda_{i}=x_{i} / \mu$. Suppose that the marginal distribution of $x_{i}$ is uniform on the segment [1,2].
(a) (5 points) Find the maximum likelihood estimator of $\mu$ as a function of $\left(y_{i}, x_{i}\right), i=1, \ldots, n$.
(b) (5 points) Show that the maximum likelihood estimator from (a) is conditionally unbiased.
(c) (5 points) Find the conditional Cramér-Rao lower bound on the conditional variance of a conditionally unbiased estimator of $\mu$.
(d) (5 points) Show that the maximum likelihood estimator obtained in (a) achieves the conditional Cramér-Rao lower bound.

B4 Discuss the following two topics.
(a) (10 points) Likelihood ratio, Lagrange multiplier, and Wald tests.
(b) (10 points) HAC standard errors.

## END OF PAPER

ECM11
MPhil in Finance \& Economics

Monday 13 May $2019 \quad 2.00 \mathrm{pm}$ to 4.00 pm

F100
FINANCE I

Candidates are required to answer all four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS

## EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Suppose that $c_{1}, c_{2}, c_{3}, c_{4}$, and $c_{5}$ are the prices of European call options with strike prices $X_{1}, X_{2}, X_{3}, X_{4}$, and $X_{5}$, respectively, where $X_{1}<X_{2}<X_{3}<X_{4}<X_{5}$ and $X_{2}-X_{1}=X_{3}-X_{2}=X_{4}-X_{3}=X_{5}-X_{4}$. All options have the same maturity. Show that

$$
c_{2}+c_{4} \leq c_{3}+\frac{c_{1}+c_{5}}{2}
$$

2. A stock price is currently 40 . Over this coming year it is expected to go up by $50 \%$ or down by $50 \%$. Over the following year it is expected to go up by $70 \%$ or down by $30 \%$. The risk-free interest rate is $10 \%$ per annum over these two years.
(a) Using the binomial model, what is the value of a two years European option to sell with a strike price of 40 ?
(b) Would there be a difference in value if the option was American?
(c) Calculate the variance of returns of the underlying stock in each of the coming years. Would using the Black-Scholes pricing formula be wrong?
3. A 6 percent six-year bond has a yield-to-maturity of $12 \%$. A $10 \%$ six-year bond has a yield-to-maturity of $8 \%$. Assume coupons are paid once a year, the next coupon being in one year.
(a) Calculate the prices of these two bonds.
(b) Calculate the sixth year's spot rate, $r_{6}$.
4. Consider a firm whose activities have value, $V_{t}$, which follows a Geometric Brownian Motion:

$$
d V_{t}=\mu V_{t} d t+\sigma V_{t} d z_{t}
$$

where $z_{t}$ is a standard Brownian motion. Assume that a riskless asset exists that pays a constant rate of interest $r$. Let $\psi\left(T_{\underline{V}} \mid V_{t}\right)$ denote the probability density function of the first passage time $T_{\underline{V}}$ to a lower boundary $\underline{V}$, conditional on the current state being $V_{t}$ (where $\left.V_{t}>\underline{V}\right)$. The Laplace transform of $\psi\left(T_{\underline{V}} \mid V_{t}\right)$ is here equal to

$$
\int_{t}^{\infty} e^{-r\left(T_{\underline{V}}-t\right)} d \psi\left(T_{\underline{V}} \mid V_{t}\right)=\left(\frac{V_{t}}{\underline{V}}\right)^{\lambda}
$$

where $\lambda=-2 r / \sigma^{2}$.
The firm has outstanding only equity and a perpetual debt contract which promises an infinitely long stream of constant coupon payments, $c$, each unit of time, as well as the residual value of the firm in case of bankruptcy. In the absence of taxes and bankruptcy costs, if the debt remains outstanding without time limit unless bankruptcy is triggered by the value of the firm's assets reaching the value $\underline{V}$, then the value of the debt and equity are

$$
\begin{aligned}
D\left(V_{t}\right) & =\frac{c}{r}+\left[\underline{V}-\frac{c}{r}\right]\left(\frac{V_{t}}{\underline{V}}\right)^{\lambda}, \\
S\left(V_{t}\right) & =V_{t}-\frac{c}{r}+\left\{0-\left[\underline{V}-\frac{c}{r}\right]\right\}\left(\frac{V_{t}}{\underline{V}}\right)^{\lambda} .
\end{aligned}
$$

(a) Consider that (i) debt service payments are tax deductible with $\tau$ being the tax advantage of debt and (ii) bankruptcy entails a proportional cost, $\alpha V_{t}$.
Derive the value of the debt and equity.
(b) Consider that (i) debt service payments are tax deductible with $\tau$ being the tax advantage of debt, (ii) bankruptcy entails a proportional cost, $\alpha V_{t}$, and (iii) shareholders freely choose when to abandon servicing the debt, in a non-cooperative fashion, and that in this event, debt-holders respond triggering bankruptcy. Derive the share-holders non-cooperative optimal closure rule.

## END OF PAPER

ECM11
MPhil in Finance \& Economics

Wednesday 15 May $2019 \quad 2: 00 \mathrm{pm}$ to $4: 00 \mathrm{pm}$

F200
FINANCE II

Candidates are required to answer all four questions..

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS

## EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Consider a firm that, at date $t$, has $n$ outstanding shares and $m$ European warrants. Each warrant can be converted at the maturity date $T$ into one newly created share, upon payment of an exercise price $K$. Denote $S_{t}$ and $W_{t}$ the values at date $t$ of a share and a warrant, respectively. The value of the firm is then $V_{t}=n S_{t}+m W_{t}$.
(a) Establish the values of the $m$ warrants at date $t$, in terms of the Black-Scholes formula. Denote $C\left(S_{t} \mid T-t, K\right)$ the Black-Scholes formula for the value at date $t$ of a European call option on a stock whose current price is $S_{t}$, with maturity date $T$ and exercise price $K$.
(b) Establish the values of the $n$ shares.
2. The price of a 1 -year zero-coupon bond is $\$ 934.6$. The price of a 2 -year $4 \%$ coupon bond is $\$ 949.2$. The price of a 3 -year $8 \%$ coupon bond is $\$ 1036.4$. Assume coupons are paid once a year, the next coupon being in one year.
(a) What are the spot rates, $r_{1}, r_{2}, r_{3}$, and the one-year forward rates, $f_{1,2}, f_{2,3}$, embedded in these bond prices?
(b) A 3 -year $4 \%$ coupon bond is selling at $\$ 950$. Is there a profit opportunity? If so, how would you take advantage of it?
3. Suppose the Hi-fin and Low-emi Companies have expected returns and standard deviations shown below, with a correlation of $\rho_{H L}=0.3$.

|  | Expected return | Standard Deviation |
| :--- | :---: | :---: |
| Hi-fin | $\bar{r}_{H}=6 \%$ | $\sigma_{H}=0.18$ |
| Low-emi | $\bar{r}_{L}=10 \%$ | $\sigma_{L}=0.2$ |

(a) Calculate (i) the expected return and (ii) the standard deviation of a portfolio that is equally invested in Hi-fin's and Low-emi's stock.
(b) If only the correlation between Hi-fin's and Low-emi's stock were to increase,
(i) Would the expected return of the portfolio rise or fall?
(ii) Would the standard deviation of the portfolio rise or fall?
(c) Calculate (i) the expected return and (ii) the standard deviation of a portfolio that consists of a long position of $\$ 6,000$ in Hi-fin and a short position of $\$ 2000$ in Low-emi's.
4. Consider a model with two dates $(t=1,2)$, and no discounting. At $t=1$, an entrepreneur is running a firm with assets in place which generate $X \in\left\{X^{L}, X^{H}\right\}$ at $t=2$, with $\Delta_{X} \equiv X^{H}-X^{L}>0$ and $\theta \equiv \operatorname{Pr}\left[X=X^{H}\right]$.
Existing investors hold claims on $X$ with promised repayments at $t=2$ equal to $R^{L}$ if $X=X^{L}$ and $R^{H} \equiv R^{L}+\Delta_{R}$ if $X=X^{H}$. More precisely, they hold debt with face value $K$ so that $R^{L}=\min \left\{K ; X^{L}\right\}$ and $R^{H}=\min \left\{K ; X^{H}\right\}$.
At $t=1$, the entrepreneur considers undertaking a project: Investing $F$ increases $\theta$ to $\left(\theta+\Delta_{\theta}\right)$ and this investment has a positive value, i.e., $\Delta_{\theta} \Delta_{X}-F>0$.
(a) Show that with risky debt, the investment policy is distorted towards underinvestment.
(b) Show that debt forgiveness can be Pareto improving, establishing the face value of the debt, $\hat{R}$, the debtholders are willing to accept, conditionally on the investment being made.

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Tuesday 7 May 2019 2:00pm to $4: 00 \mathrm{pm}$

F300
CORPORATE FINANCE

Candidates are required to answer all four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

(Banks, negative interest rates, and QE) Negative interest rates have become commonplace in recent times yet they are unprecedented and controversial. Commercial banks' shortterm liabilities consist mostly of deposits and financial intermediaries are unwilling to pass on negative rates to their depositors fearing withdrawals. Imagine that you wanted to investigate how QE and negative policy rates jointly affect bank lending, and whether the efficacy of either policy depends on the presence of the other. Guided by theory, detail an empirical identification strategy and describe all major identification threats. [Avoid lengthy discussions around the topic; concise and accurate answers will be rewarded.]

## Question 2

(Costly state verification) Consider an economy composed of entrepreneurs and outside investors. Both types are risk neutral and can always invest their wealth in outside capital markets and earn an expected gross return $R$. Each entrepreneur has wealth, $w \sim U[0,2]$, so wealth is distributed uniformly between zero and two among the entrepreneurs. Each entrepreneur also has the option to undertake a project that requires an indivisible investment of 1 and has an i.i.d. return of $x \sim U[0,2 \bar{x}]$.

Outside investors have lots of wealth but no access to projects. They are willing to lend money to entrepreneurs if their expected return from lending money is not exceeded by the return to their outside option. However, outside investors cannot verify the returns from the project unless they pay a fixed cost, $c$.

Assume that the contract between the investor and entrepreneur takes the form of a standard debt contract: the entrepreneur pays a return $D$ to the outside investor when possible. Yet if the entrepreneur cannot afford to pay, the outside investor pays the verification cost and takes all the profits.

There is no bargaining in this model. Investors are perfectly competitive, so the entrepreneurs will never offer more than necessary to get the financing, $1-w$, that they need to do the project. Assume that the entrepreneur is willing to undertake the project, and analyze the problem from the point of view of the outside investor.
(a) Find the investor's expected gain if she invests in the project. What is this expected gain a function of? What are the expected verification costs of the investor? Provide intuition for the expression you derive.
(b) Graph this expected return as a function of $D$ and discuss how the equilibrium value $D^{\star}$ will be chosen.
(c) What is the sign of the derivative, $\partial D^{\star} / \partial w$ ? Interpret.
(d) Under what circumstances will there be no lending? List all potential causes.

## Question 3

(Diamond-Dybvig) Assume there is a continuum 1 of individuals that are each endowed with one unit of currency. There are three time periods, $t=0,1,2$. At $t=0$, individuals have two options with regards to how they can invest their money. They can either stuff it in their mattress, where it gets a return equal to 1 , or they can invest it in a long-term project that yields a return $R=4$ in period $t=2$. Individuals have the option of withdrawing their money from the long-term project early at $t=1$. In this case, there is a penalty and they only receive a return $\ell=1 / 4$, rather than the return $R=4$ at $t=2$.

At time $t=2$, a fraction $\pi=1 / 2$ of the individuals receive a liquidity shock. These individuals are "impatient" and only value consumption in period $t=1$. The fraction $1-\pi$ individuals that do not receive a liquidity shock are "patient" and only value consumption in period $t=2$. At time $t=0$, each individual has an equal chance of being hit by the liquidity shock. Assume that individuals have the following ex-ante utility:

$$
\begin{aligned}
& \mathcal{U}=\pi u\left(c_{1}\right)+(1-\pi) u\left(c_{2}\right) \\
& \text { where: } u(c)=-\frac{1}{c}
\end{aligned}
$$

where $c_{1}$ and $c_{2}$ are the consumption periods $t=1$ and $t=2$, respectively.
(a) Assume there are no markets available to individuals, so that individuals must simply invest on their own. Given that the individual has invested an amount $I$ at time $t=0$, what will be the optimal levels of consumption, $c_{1}$ and $c_{2}$, if:
(i) The individual receives a liquidity shock (i.e., is impatient)?
(ii) The individual does not receive a liquidity shock (i.e., is patient)?
(b) What is the optimal level of investment, $I^{\star}$ ? Given $I^{\star}$, what is the ex-ante expected utility of an individual? Explain intuitively why both patient and impatient individuals regret their their initial investment decision ex-post at $t=1$ after their type is realized.
(c) Now suppose an ex-post financial market exists where individuals can trade bonds at time $t=1$. Each bond costs $p$ units of goods at time $t=1$, and the bond pays 1 unit of goods at time $t=2$. Assume all individuals invest an initial amount $I=1 / 2$.
(i) What is the aggregate demand and supply of bonds at $t=1$ ?
(ii) What is the equilibrium price $p$ ?
(iii) How much do impatient individuals consume in each period?
(iv) How much does a patient individual consume in each period?

## Question 4

(International corporate finance) What are the effects of monetary policy induced nominal devaluations on firms over time? Describe key micro-transmission mechanisms with empirical support. How would you test their relevance in the data? Be specific about empirical research designs and methods to alleviate identification threats. [Avoid lengthy discussions around the topic; concise and accurate answers will be rewarded.]

## UNIVERSITY OF

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

F400
ASSET PRICING

Candidates are required to answer all four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Suppose that an investor has utility function $u$ given by the formula

$$
u(w)=(2 w+b)^{\frac{1}{2}} .
$$

(a) For what range of wealth is her utility function defined?
(b) What is her absolute risk aversion? How does it vary with her wealth? How would this be expected to affect her investment in risky assets?
(c) What is her relative risk aversion? How does it vary with her wealth?
2. Suppose that there are two assets: a safe asset and a risky asset. The initial price of both assets is 1 . In each period, there is a shock $s \in\{d, u\}$. If $s=d$, then the price of the risky asset falls by $\frac{1}{3}$. If $s=u$, then the price of the risky asset rises by $\frac{1}{2}$. The price of the safe asset rises by $\frac{1}{6}$ whatever the realisation of $s$.
(a) In this context, what is meant by:
(i) a forward contract with strike price $K$ and maturity $T$;
(ii) a call option with strike price $K$ and maturity $T$;
(iii) a put option with strike price $K$ and maturity $T$ ?
(b) Now suppose that an investor wishes to go long 2 units of a call option with strike price $\frac{1}{2}$ and maturity 2 , short 3 units of a call option with strike price 1 and maturity 2 and long 1 unit of a call option with strike price 2 and maturity 2. Assuming that there is no arbitrage, how much will it cost her to form this portfolio?
(c) Why might the investor want to form the portfolio described in part (b)?
3. (a) Describe a model of the dynamics of forward rates. Be careful to explain all the elements of your model.
(b) What form would bond prices take in this model? Give a brief explanation of the logic behind your answer.
(c) Derive the stochastic differential equation governing the evolution of these bond prices.
4. There are two assets: a safe asset and a risky asset. The price of the safe asset evolves according to the s.d.e.

$$
d B=r B d t
$$

with initial condition $B(0)=B_{0}$, and the price of the risky asset evolves according to the s.d.e.

$$
d S=\mu S d t+\sigma S d Z,
$$

with initial condition $S(0)=S_{0}$, where $Z$ is a standard Wiener process.
(a) Explain carefully what is meant by an up-and-in contract in this context.
(b) Explain carefully what is meant by an up-and-out contract in this context.
(c) Derive a formula for the price of an up-and-out contract.

Hint: suppose that we are given an arithmetic Wiener process $X$ starting at $\alpha$ with drift $m$ and diffusion $s$, i.e. we have

$$
d X=m d t+s d \widehat{Z}
$$

with initial condition $X(0)=\alpha$, where $\widehat{Z}$ is a standard Wiener process. Suppose further that the process $X^{\beta}$ is obtained by stopping the process $X$ when it first reaches the point $\beta$. Then the random variable $X_{t}^{\beta}$ (i.e. the position of $X^{\beta}$ at time $t$ ) has an atom (or mass point) at $\beta$ and a density

$$
\begin{aligned}
g(x)= & \phi(x ; m T+\alpha, s \sqrt{T}) \\
& -\exp \left(-\frac{2 m(\alpha-\beta)}{s^{2}}\right) \phi(x ; m T+2 \beta-\alpha, s \sqrt{T})
\end{aligned}
$$

on the rest of its support. Here $\phi$ is the p.d.f. of the standard normal distribution, and the support of $X_{t}^{\beta}$ is $[\beta, \infty)$ if $\beta<\alpha$ and $(-\infty, \beta]$ if $\beta>\alpha$.

## END OF PAPER

## UNIVERSITY OF

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Friday 24 May $2019 \quad$ 1:30pm to $3: 30 \mathrm{pm}$

F510
INTERNATIONAL FINANCE

Candidates are required to answer four out of five questions.

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Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

(a) Define the Covered Interest Parity (CIP) condition and briefly discuss why it may not hold if countries adopt capital controls.
(b) List and briefly discuss possible causes that may prevent the CIP conditions from holding also when investors can freely trade assets across borders. In your answer, please use the theoretical framework discussed in class.
(c) Show that counterparty risk in the forward market cannot be an explanation of CIP deviations.

## Question 2

Consider the Uncovered Interest Parity (UIP) condition

$$
q_{t}=-\left[r_{t}-r_{t}^{*}\right]+E_{t} q_{t+1}-E_{t} \lambda_{t+1}
$$

where $q_{t}$ is the real exchange rate, $r_{t}$ and $r_{t}^{*}$ are, respectively, the home and foreign real interest rate and $\lambda_{t+1}$ is the excess return.
(a) What is the UIP puzzle? Write down the Fama regression and briefly explain the puzzle, carefully distinguishing between the "textbook" (or classical) and the "new UIP puzzle".
(b) Assume that $E_{t} \lambda_{t+1}$ is a risk premium. According to Fama (1984), which properties should $E_{t} \lambda_{t+1}$ have for risk to explain the "textbook UIP puzzle"?
(c) Engel (2016) argues that, if the Fama solution to the textbook UIP puzzle is right at each point in time, another puzzle arises. Explain briefly.

## Question 3

Debt forgiveness: Consider a small open economy with a debt stock with face value 100. In the coming year, the country will be able to repay its debt in full only with probability .5. Otherwise, the country can only pay 20.
An International Fund is deciding whether to help the country to buy back its debt, up to a stock with face value of 40 . As an alternative, it would help the country to organize a swap of new safe debt (paying in all circumstances) for old risky debt.
Assume that all investors are risk neutral investors and do not discount the future. Assume $R^{\text {World }}=1$.
(a) Detail and discuss the benefits of each alternative for the debtor country and the creditors relative to the status quo with no debt relief.
(b) Suppose per effect of reducing the outstanding debt stock, the probability of full repayment of the bonds in the hands of the investors rises from .5 to . 8 . Would both creditors and debtors be better off relative to the status quo? Explain.

## Question 4

Saving glut and precautionary saving. Consider a small open economy with a representative agent with preferences

$$
\begin{equation*}
\ln C_{0}+E_{t} C_{1}=\ln C_{0}+1 / 2 \ln C_{1}^{H}+1 / 2 \ln C_{1}^{L} \tag{1}
\end{equation*}
$$

and endowment $Y_{0}=4$, and $Y_{1}$ either 6 in the good (H) state or 4 in the bad (L) state. Each state has probability $1 / 2$. Assume for simplicity that the world interest rate is zero and international investors are risk neutral and do not discount the future.
(a) Define and calculate the interest rate under financial autarky. Is the country going to be a net lender $\left(B_{0}>0\right)$ or a net borrower $\left(B_{0}<0\right)$ ? Explain.
(b) Would your conclusion change if the projections for future output $Y_{1}$ were revised to be either 8 in the good (H) state or 2 in the bad (L) state. Explain carefully why a rise in macroeconomic uncertainty may affect the optimal saving plan of the residents in the country.
(c) How would your answers change if the country could issue state contingent debt, i.e. debt with interest payment contingent on the realization of output?
(d) Discuss the conditions that makes it possible for countries to write international debt contracts as under (a)/(b) or (c).

## Question 5

Debt sustainability: Consider a small open economy that only operates for two periods only, $t=1,2$. In each period its output can be High or Low with probability $1 / 2$.

Investors believe that the maximum sustainable primary surplus, $\mathcal{P} \mathcal{S}_{t}^{M a x}$ the country can generate in each period is 20 if output is High, 10 if output is Low. For simplicity assume that $R^{\text {World }}=1$, investors are risk neutral and do not discount the future.
(a) Denote the largest stock of debt that the economy can sustain with $\mathcal{B}^{M a x}$. Assume that debt consists of one-period discount bonds only, and that, in case of default, bonds only pay a fraction $X<1$ of their face value. Explain how investors set the price $Q$ at which they are willing to buy the (discount) bonds issued by the country, as a function of $\mathcal{B}^{M a x}$, and the expected losses that investors are going to suffer in case of default.
(b) Write down the equation showing the determinants of $\mathcal{B}^{\text {Max }}$ and explain its terms.
(c) Set $\mathrm{X}=0$ (in case of default bonds pay nothing). Using your answers to (a) and (b), calculate the maximum sustainable debt $\mathcal{B}^{\text {Max }}$ in the final period 2, then in period 1. Are they the same? Explain.

Now you are asked to assess the sustainability of the country debt, knowing that, in period 1, (a) the country runs a primary deficit of 2 and (b) debt (issued in the past) coming to maturity is 4 .
(d) Assuming that $\mathrm{X}=0$, at which equilibrium price $Q_{2}$ can the country borrows and satisfy its financing need in period 1 ? Explain carefully.

Investors know that an "International Fund" stands ready to lend to the country $\mathcal{B}^{I F}$ at the riskless price plus a small $(10 \%)$ penalty $Q_{1}^{I F}=1.1$.
(e) How would the presence of the IF affect the equilibrium? Explain carefully the consequences on bond issuance and prices.
(f) How would the size of the IF lending capacity if, in case of default, investors recover a rate $X=10 \%$ of their bond investment?

## END OF PAPER

 CAMBRIDGEECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Thursday 23 May 2019 1:30pm to $3: 30 \mathrm{pm}$

F520
BEHAVIOURAL FINANCE

Candidates are required to answer two out of three questions

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## QUESTION 1

a. Discuss what financial economists mean when they say an individual is rational, and explain how behavioural finance relaxes this assumption.
b. Explain what we mean when we say the stock market is informationally efficient, and discuss the three conditions under which informational efficiency arises.
c. Investors believe that the fundamental value of company ABC in the next year (year 1 ) is normally distributed random variable, with a mean of $£ 100 \mathrm{M}$ and variance of 2500 . Investors, after observing that ABC changed its CEO, construct a new signal, S, which says that the fundamental value of $A B C$ in year 1 will be $£ 80 \mathrm{M}$. This signal is also normally distributed, with a variance of 1400 . Given the riskiness in ABC's cash-flows, investors require an expected return of $11 \%$ to be appropriately compensated for investing in this company. Use Bayes Rule to calculate investor's posterior expectation for ABC's cash flow in year 1, as well as the price of $A B C$ in the current year (year 0 ). Suppose that an arbitrager, who is fully rational, understands that the signal that the market created ( $S$ ) under-estimates the fundamental value of $A B C$ by $£ 16 \mathrm{M}$. Explain what trade would this arbitrager make, and calculate what abnormal return (in \%) he can expect to earn in year 1. Assume that the variance of the signal $S$ is correctly calculated by the market. Moreover, assume that the trades of the arbitrager in year 0 do not have any effect on the price of ABC in year 0 , and that the expected return of $11 \%$ reflects the appropriate rate of return that the investors and the arbitrager should expect to earn to be compensated for investing in ABC.

## QUESTION 2

a. Describe the phenomenon of overconfidence, and discuss how it can lead to biased learning.
b. Is there any evidence that trading volume in financial markets is excessive? Discuss this issue, describing in detail the findings of academic studies to support your arguments. In your answer, explain in detail the tests and results found by these studies.
c. Discuss the intuition behind the asset pricing model with overconfident investors of Daniel, Hirshleifer and Subrahmanyam (2001) ("Overconfidence, Arbitrage, and Equilibrium Asset Pricing", Journal of Finance). Consider the case of orthogonal dividends only. Provide necessary analysis to support the intuition.

## QUESTION 3

a. Suppose that a two-period economy is populated by identical risk-neutral investors. In this economy all stock prices equal the expected value of company earnings in quarter 1. Investors calculate that the earnings of company FCB in quarter 1 could take three possible values: In state 1 they could be $£ 1 \mathrm{M}$, in state $2 £ 10 \mathrm{M}$ and in state $3 £ 15 \mathrm{M}$. Investors, however, face ambiguity about the probabilities of these states, and believe that three different probability distributions are possible: (A: $\pi_{1}=0.65, \pi_{2}=0.26$ and $\pi_{3}=0.09$ ); (B: $\pi_{1}=0.58, \pi_{2}=0.23$ and $\pi_{3}=0.19$ ); (C: $\pi_{1}=0.43, \pi_{2}=0.17$ and $\pi_{3}=0.40$ ). Assume that investors are ambiguity averse and use the max-min expected utility model (MMEU) to evaluate ambiguous assets. What price would FCB have in this market in the current quarter (quarter $0)$ ? Without doing any calculations, discuss how your previous answer would change if investors in this economy are ambiguity neutral.
b. Suppose now that investors calculate expectations using the Choquet integral, where the capacities for each state are $v(\{1\})=v(\{2\})=v(\{3\})=0.2$, and $v(\{1,2\})=v(\{1,3\})=v(\{2,3\})=0.4$ and $v(\{1,2,3\})=1$. What would the price of FCB be in quarter 0 be in this case?
c. Describe the Ellsberg paradox, and highlight how such choices contradict rational decision making. Can such choices provide an explanation for the equity premium puzzle? Discuss this issue, drawing on evidence from academic studies to support your arguments. In your answer, explain in detail the tests and results found by these studies.

END OF PAPER

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UNIVERSITY OF
    CAMBRIDGE
ECM10/ECM11
MPhil in Economic Research
MPhil in Finance and Economics
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Thursday 9 May $2019 \quad 2.00 \mathrm{pm}-4.00 \mathrm{pm}$

R100
MICROECONOMICS

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## Question 1

1. Consider an endowment economy with two people, two goods and endowment vector $e$. Use subscripts to index goods $l=1,2$ and superscripts to index people $i=1,2$. Normalize $p_{1}=1$. Both people have strictly positive endowments of both goods. Suppose the preferences of the two people can be represented by the following utility functions:

$$
\begin{aligned}
u^{1}\left(x_{1}^{1}, x_{2}^{1}\right) & =\log \left(x_{1}^{1}\right)+\alpha \log \left(x_{2}^{1}\right) \\
u^{2}\left(x_{1}^{2}, x_{2}^{2}\right) & =\log \left(x_{1}^{2}\right)+\beta \log \left(x_{2}^{2}\right),
\end{aligned}
$$

where $\alpha>\beta>0$.
(a) Do person 1 and person 2 have preferencesthat are locally non-satiated?
(b) Derive expressions for the Marshallian demands.
(c) Write down the aggregate excess demand functions for goods 1 and 2.
(d) Show that a necessary and sufficient condition for a Walrasian equilibrium is that $z_{1}\left(p_{2}\right)=0$ (note that by the normalization $p_{1}=1$ the excess demand function can be defined as a function of $p_{2}$ instead of the price vector $p$ ).
(e) Find $\lim _{p_{2} \rightarrow 0} z_{1}\left(p_{2}\right)$ and $\lim _{p_{2} \rightarrow \infty} z_{1}\left(p_{2}\right)$. Hence show that there exists a Walrasian equilibrium.
(f) Show that the Walrasian equilibrium is unique.

## Question 2

2. Amy is a rational decision maker who has preferences that admit an expected utility representation. Amy's preferences over lotteries are denoted by $\succeq$. Suppose the prize space (in dollars) is:

$$
\mathcal{X}=\{25,20,10,5,2\} .
$$

Amy prefers having strictly more money to less money. Consider the following lotteries:

$$
\begin{array}{ll}
l_{1}=(0,0,1,0,0), & l_{2}=\left(0, \frac{1}{2}, 0, \frac{1}{2}, 0\right), \\
l_{4}=\left(0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0\right) \\
l_{4}=\left(0, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right), & l_{5}=\left(\frac{1}{4}, 0,0, \frac{3}{4}, 0\right),
\end{array} \quad l_{6}=\left(0,0, \frac{4}{5}, 0, \frac{1}{5}\right) .
$$

Amy is indifferent between lottery $l_{1}$ and lottery $l_{2}$.
(a) Which of the following is true (fully justify your answer):
i. $l_{3} \succ l_{4}$.
ii. $l_{4} \succ l_{3}$.
iii. $l_{3} \sim l_{4}$.
iv. There is too little information to conclude any of i.-iii. are true.
(b) Which of the following is true (fully justify your answer):
i. $l_{1} \succ l_{5}$.
ii. $l_{5} \succ l_{1}$.
iii. $l_{1} \sim l_{5}$.
iv. There is too little information to conclude any of i.-iii. are true.
(c) Which of the following is true (fully justify your answer):
i. $l_{1} \succ l_{4}$.
ii. $l_{4} \succ l_{1}$.
iii. $l_{1} \sim l_{4}$.
iv. There is too little information to conclude any of i.-iii. are true.
(d) Which of the following is true (fully justify your answer):
i. $l_{4} \succ l_{6}$.
ii. $l_{6} \succ l_{4}$.
iii. $l_{4} \sim l_{6}$.
iv. There is too little information to conclude any of i.-iii. are true.
(e) Consider the lottery $(0, \alpha, 0,0,1-\alpha)$. Does there exist an $\alpha \in(0,1)$ such that $l_{1} \sim$ ( $0, \alpha, 0,0,1-\alpha$ ) (fully justify your answer)?

## Question 3

3. (a) Two countries $A$ and $B$ are bargaining over a deal. There is a finite set of possible agreements they can reach denoted by $X$, and they each have rational preferences over these outcomes described by $\succeq_{A}$ and $\succeq_{B}$ respectively. The countries only agree that one agreement is weakly better than another when they both weakly prefer it. Represent these joint preferences by the preference relation $\succeq_{J}$ such that $x \succeq_{J} y$ if and only if $x \succeq_{A} y$ and $x \succeq_{B} y$.
i. Is $\succeq_{J}$ transitive? Prove or disprove.
ii. Is $\succeq_{J}$ reflexive? Prove or disprove.
iii. Is $\succeq_{J}$ complete? Prove or disprove.
iv. A. Define the choice set for the preferences $\succeq_{J}$, denoting it by $C\left(X, \succeq_{J}\right)$.
B. Could it ever be empty? Prove or disprove.
(b) Now suppose that transfers between countries $A$ and $B$ are possible. Thus the two countries now have rational preferences over $X \times \mathbb{R}$ described by $\succeq_{A T}$ and $\succeq_{B T}$ respectively. Letting $t \in \mathbb{R}$, assume that for $i=A, B$

- For all $x \in X,(x, t) \succeq_{i T}\left(x, t^{\prime}\right)$ if and only if $t \geq t^{\prime}$.
- For all $x, x^{\prime} \in X$ there exists $t \in \mathbb{R}$ such that $(x, t) \sim_{i T}\left(x^{\prime}, 0\right)$.
- If $(x, t) \succeq_{i T}\left(x^{\prime}, t^{\prime}\right)$ then for all $t^{\prime \prime} \in \mathbb{R},\left(x, t+t^{\prime \prime}\right) \succeq_{i T}\left(x^{\prime}, t^{\prime}+t^{\prime \prime}\right)$.

Define a new preference relation $\succeq_{T}$ such that $x \succeq_{T} y$ if and only if there exists a $t \in \mathbb{R}$ such that $(x, t) \succeq_{A T}(y, 0)$ and $(x,-t) \succeq_{B T}(y, 0)$.
i. Is $\succeq_{T}$ transitive? Prove or disprove.
ii. Is $\succeq_{T}$ reflexive? Prove or disprove.
iii. Is $\succeq_{T}$ complete? Prove or disprove.
iv. A. Define the choice set for the preferences $\succeq_{T}$, denoting it by $C\left(X, \succeq_{T}\right)$.
B. Could it ever be empty? Prove or disprove.
C. Discuss and interpret $C\left(X, \succeq_{T}\right)$.

Candidates are required to answer all four questions.

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You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. (a) Consider the following two player normal form game

|  | $A_{2}$ | $B_{2}$ | $C_{2}$ | $D_{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| $A_{1}$ | 0,7 | 6,1 | 7,0 | $-1,1$ |
| $B_{1}$ | 4,2 | 3,3 | 1,2 | 0,1 |
| $C_{1}$ | 7,0 | 2,4 | 0,7 | $-1,1$ |
| $D_{1}$ | 0,0 | $0,-2$ | 0,0 | $10,-1$ |

i. Find the set of pure strategy profiles that survive 1-level iterated deletion of strictly dominated strategies.
ii. Compute the set of pure strategies (for both players) that survive (an indefinite) iterated deletion of strictly dominated strategies. State the rationality/knowledge assumptions corresponding to each round.
iii. Find the set of (pure and mixed strategy) Nash equilibria of this game.
(b) Consider the following simultaneous three player game

|  | $A_{2}$ | $B_{2}$ |  | $A_{2}$ | $B_{2}$ |  | $A_{2}$ | $B_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A_{1}$ | $0,0,3$ | $0,0,0$ | $A_{1}$ | $2,2,2$ | $0,0,0$ | $A_{1}$ | $0,0,0$ | $0,0,0$ |
| $B_{1}$ | $1,0,0$ | $0,0,0$ | $B_{1}$ | $0,0,0$ | $2,2,2$ | $B_{1}$ | $0,1,0$ | $0,0,3$ |
|  |  |  |  |  |  |  |  |  |
|  | $A_{3}$ |  |  | $B_{3}$ |  | $C_{3}$ |  |  |

where player 1 chooses one of the two rows, player 2 chooses one of the two columns and player 3 chooses one of the three matrices $A_{3}, B_{3}$ or $C_{3}$.
i. Show that there is a correlated equilibrium in which player 3 chooses $B_{3}$ and players 1 and 2 play $\left(A_{1}, A_{2}\right)$ and $\left(B_{1}, A_{2}\right)$ with equal probabilities.
ii. Explain the sense in which player 3 prefers not to have the information that players 1 and 2 use to coordinate their actions.
2. Consider a sealed-bid auction of a single indivisible object. There are $n \geq 2$ bidders. The value of the object to each bidder $i$ is $v_{i}=\theta_{i}+\gamma$ where $\theta_{i}$ is a random variable that is independently drawn from a uniform distribution over $[0,1]$ and $\gamma \geq 0$ is some constant. Before the auction, each bidder $i$ privately observes $\theta_{i}$. The auction rules are such that each player simultaneously submits a bid in a sealed envelope. The envelopes are then opened, and the bidder who has the highest bid receives the object and pays the auctioneer the amount prescribed by the specific auction format; the other bidders receive zero utility and pay according to the specific auction format. If different bidders submit the highest bid, each obtains the object with equal probability. All aspects of the structure of this game are common knowledge with the exception that each bidder $i$ does not know $\theta_{j}$ for all $j \neq i$.
(a) Suppose that the auctioneer runs a first price auction where the highest bidder pays her bid.
i. Derive a symmetric (pure strategy) Bayesian Nash equilibrium in which each bidder $i$ uses a linear bidding strategy of the form $b\left(\theta_{i}\right)=\alpha \theta_{i}+\beta$ for all $\theta_{i} \in[0,1]$ with some constants $\alpha>0$ and $\beta$.
ii. In this equilibrium, what is the expected payoff of bidder $i$ if his type is $\theta_{i}$ ?
iii. Study each bidder's equilibrium bidding strategy as $n$ increases.
(b) Suppose that the auctioneer runs a first price auction where the highest bidder pays her bid. Assume $n=2$. Suppose now that each bidder $i$ 's valuation for the object is $v_{i}=\delta \theta_{i}+\lambda \theta_{j}+\gamma$ for some constants $\delta>\lambda>0$ and $\gamma \geq 0$. Find a symmetric (pure strategy) Bayesian Nash equilibrium with linear bid functions.
(c) The auctioneer runs an average price auction where the highest bidder obtains the object and pays the average of all bids; the other bidders receive zero utility and pay nothing. Assume that $n=2$ and that $v_{i}=\theta_{i}+\gamma$ with $\gamma \geq 0$.
i. Derive a symmetric (pure strategy) Bayesian Nash equilibrium.
ii. Compare each bidder's equilibrium bidding strategy in this auction with the first price auction.
iii. Calculate the expected payoff to bidder $i$ in this equilibrium.
iv. In this equilibrium, calculate the ex ante expected revenue of the seller.
3. (a) Consider a three period finite horizon bargaining game with alternating offers where there are two players, 1 and 2 , and an arbitrator (A). Players 1 and 2 bargain to determine the split of a surplus, normalised to one. The bargaining proceeds as follows: the game begins in period 1; player 1 submits an offer of a split $\left(y_{1}, y_{2}\right) \in \Delta$ to the arbitrator where $\Delta=\{(a, 1-a) \mid a \in[0,1]\}$. The offer is observable to player 2 , who decides whether to accept or reject. If she accepts, the proposed split is immediately implemented, the players' payoffs are ( $y_{1}, y_{2}$ ), and the game ends. If she rejects, nothing happens until the next period. In period 2 , player 2 submits an offer $\left(z_{1}, z_{2}\right) \in \Delta$ to the arbitrator, which is observable, and then player 1 may accept or reject. If he accepts the offer, the split $\left(z_{1}, z_{2}\right)$ is implemented and the game ends. Otherwise, the arbitrator makes the decision at period 3, choosing one of the submitted offers. Assume common discount factor $\delta<1$. If offer $\left(x_{1}, x_{2}\right)$ is implemented in period $t$, then the (discounted) payoffs to players 1 and 2 are $\delta^{t-1} x_{1}$ and $\delta^{t-1} x_{2}$, respectively. The arbitrator's utility function at period $t=3$ is a social welfare function $u_{A}\left(x_{1}, x_{2}\right)=x_{1} x_{2}$ for any $\left(x_{1}, x_{2}\right) \in \Delta$.
i. Show that there is a subgame perfect equilibrium where player 1 offers $1 /(1+$ $\delta)$ to player 2 in the first period, and the offer is accepted.
ii. Suppose now that there are more rounds of alternating-offer negotiation prior to arbitration. Specifically, the two players are to go to arbitration exogenously at some given date $T \geq 4$ and the arbitrator chooses between the last two offers in the negotiation. What is the subgame perfect equilibrium outcome in this case?
(b) Consider a bargaining situation in which if two players cooperate, they can obtain a surplus, normalised to one. All players are risk neutral and share a discount factor $\delta<1$, but players may differ in their attitudes towards other players' material payoffs. Specifically, the utility function of player $i$ is given by

$$
u_{i}\left(x_{i}, x_{j}\right)=x_{i}+\alpha_{i} \frac{x_{j}}{2}
$$

where $j \neq i, x_{i}$ is the material payoff of player $i, x_{i} \geq 0$ for all $i, x_{i}+x_{j} \leq 1$, and $-1<\alpha_{i}<1$ for all $i$. Assume all features of the game are common knowledge. Suppose no player receives any money in case of disagreement.
i. Interpret the parameter $\alpha_{i}$.
ii. Find the Nash bargaining solution to this problem.
4. (a) Consider the following three player entry game

i. Compute the subgame perfect equilibria of this game.
ii. Find a perfect Bayesian equilibrium that is not a subgame perfect equilibrium.
(b) Consider a market where there is an incumbent (firm I) and a challenger (firm $\mathrm{E})$. The challenger is strong $\theta^{s}$ with probability $\frac{1}{2}$ and weak $\theta^{w}$ with probability $\frac{1}{2}$, it knows its type, but the incumbent does not. The challenger may either build capacity $(C)$ or remain with the current structure $(Q)$. The incumbent observes the challenger's capacity level, but not its type, and chooses whether to fight $(F)$ or acquiesce $(A)$. The extensive form and the payoffs are given by the following figure. The challenger's payoff is listed first, the incumbent's second.

i. Describe the (pure) strategy set for firm E.
ii. Find a separating perfect Bayesian equilibrium.
iii. Find a pooling perfect Bayesian equilibrium.

## END OF PAPER

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1. Consider the following stochastic neoclassical growth model. There is a continuum of identical firms, of measure one, each with access to the following constant returns to scale production function $Y_{t}=z_{t} K_{t}^{\alpha} L_{t}^{1-\alpha}, 0<\alpha<1$, where $K_{t}$ and $L_{t}$ denote aggregate capital stock and labour respectively, in period $t$. The technology shock $z_{t}$ follows a finite Markov chain with transition matrix $\pi\left(z_{t+1} \mid z_{t}\right)$ and associated invariant distribution $\Pi$. The initial aggregate capital stock is given by $K_{0}$. The aggregate resource constraint in the economy is given by

$$
C_{t}+K_{t+1}=z_{t} K_{t}^{\alpha} L_{t}^{1-\alpha}+(1-\delta) K_{t}
$$

where $C_{t}$ is aggregate consumption in period $t$. There are two type of households, $i=1,2$, of equal measure, normalised to 1 . A type $i$ household gets utility in every period from consumption $c_{t}^{i} \geq 0$ and leisure $0 \leq h_{t}^{i} \leq 1$ and

$$
U^{i}=\mathbb{E}_{0}\left(\sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}^{i}, h_{t}^{i}\right)\right), 0<\beta<1 .
$$

The instantaneous utility function $u(\cdot, \cdot)$ is strictly concave, differentiable and increasing in both arguments. Each household of type $i$ is endowed with one unit of time that can be used for either leisure or labour $l_{t}^{i}$ so that $l_{t}^{i}+h_{t}^{i}=1$. A type $i$ household has constant labour productivity $\theta_{i}$, therefore its effective labour supply is $\theta_{i} l_{t}^{i}$. We assume that $\theta_{1}+\theta_{2}=1$. Finally, each household of type $i$ owns initial capital stock $k_{0}^{i}=\theta_{i} K_{0}$, and thus $k_{0}^{1}+k_{0}^{2}=K_{0}$.
(a) Suppose the social planner assigns Pareto weight $\mu_{i}$ to households of type $i=1,2$. State the social planner's problem recursively. Assuming that the value function is differentiable, derive the optimal conditions of this problem and provide some intuition for each condition.
(b) Now assume that households can save only with capital and state the problem of household type $i$ in recursive form. Assuming that the value function is differentiable, derive the optimal conditions of this problem and provide some intuition for each condition.
(c) State also the problem of the representative firm and derive the optimal conditions for this problem. Define a Recursive Competitive Equilibrium. Be precise.
Next, we introduce a one-period risk free bond into this economy and assume that

$$
u\left(c_{t}^{i}, h_{t}^{i}\right)=\frac{\left(c_{t}^{i}\right)^{1-\sigma}-1}{1-\sigma}+\frac{\left(h_{t}^{i}\right)^{1-\nu}-1}{1-\nu} .
$$

(d) Rewrite the problem of type $i$ household in recursive form. Impose any new assumption needed so that the recursive problem is well defined.
(e) Find the price of this risk free bond, as a function of consumption allocations, without solving for the equilibrium consumption allocations. Explain. Would there be trade of this bond in equilibrium? Justify your answer.
2. Consider an economy populated by a continuum of measure one of identical households, each with preferences given by

$$
U=\sum_{t=0}^{\infty} \beta^{t} \frac{c_{t}^{1-\sigma}}{1-\sigma},
$$

where $c_{t}$ is consumption at period $t, \sigma \geq 0$ and $0<\beta<1$ is the subjective discount factor. There are two sectors in this economy. The final goods sector consists of a continuum of identical firms of measure one, that combine labour, $L_{t}$, and a continuum of intermediate goods, where the $i-t h$ good is denoted by $x_{t}(i)$, to produce a homogeneous final output good $Y_{t}$

$$
Y_{t}=\left(\int_{0}^{A} x_{t}(i)^{\alpha} d i\right) L_{t}^{1-\alpha}, \quad 0<\alpha<1,
$$

where $A$ is an exogenous constant. The homogenous final good can be used for consumption or investment, such that

$$
K_{t+1}=(1-\delta) K_{t}+I_{t}, \quad 0<\delta<1
$$

where $K_{t}$ and $I_{t}$ are capital stock and investment at period $t$ respectively. Households own the initial capital stock $K_{0}$ and rent it out to firms. Households are also endowed with one unit of productive time in each period $t$. Intermediate goods are produced using capital according to the following production function:

$$
x_{t}(i)=k_{t}(i)
$$

where $k_{t}(i)$ is the capital stock used in the production of intermediate good $x_{t}(i)$. There are competitive markets for the final good, for labour and for capital. Intermediate goods are produced and sold by monopolists in monopolistic competition. Any profits are distributed lump-sum to households.
(a) State the problem for each of the following agents and derive the optimal conditions associated with each optimisation problem. Provide intuition for each condition.
i. the representative household,
ii. the representative firm in the final goods sector, and iii. a monopolist in the intermediate goods sector.
(b) State the market clearing conditions and write the equilibrium system of difference equations in consumption and capital and two terminal conditions, after eliminating any other endogenous variables. Find the steady state level of capital and explain how it is affected by an increase in $A$.
(c) Is the market equilibrium Pareto optimal? Explain why or why not.
3. Consider the following RBC model. Households own capital and also supply labour to firms. The representative household maximises expected discounted lifetime utility

$$
\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t}\left(\ln C_{t}-\chi \frac{N_{t}^{1+\nu}}{1+\nu}\right)
$$

with $\chi>0, \nu \geq 0$, over consumption $C_{t}$, hours worked $N_{t}$, capital $K_{t+1}$, subject to the budget constraint

$$
C_{t}+K_{t+1}=R_{t} K_{t}+W_{t} N_{t}+(1-\delta) K_{t},
$$

and initial capital stock $K_{0} . R_{t}, W_{t}$ denote the rental rate of capital and the wage rate respectively, in period $t$. The parameter $0<\beta<1$ denotes the discount factor and $0<\delta<1$ denotes the capital depreciation parameter. Firms operate under perfect competition. The representative firm produces output $Y_{t}$, rents capital $K_{t}$ at rate $R_{t}$, hires labour at a rate $W_{t}$ and maximises profits $Y_{t}-W_{t} N_{t}-R_{t} K_{t}$ in every period, subject to the following production function

$$
Y_{t} \equiv F\left(K_{t}, N_{t}\right)=Z_{t}\left[\alpha K_{t}^{\frac{\theta-1}{\theta}}+(1-\alpha) N_{t}^{\frac{\theta 1}{\theta}}\right]^{\frac{\theta}{\theta-1}}
$$

with $0<\alpha<1, \theta>0$ and a productivity shock, which evolves according to $\log Z_{t}=\rho \log Z_{t-1}+\varepsilon_{t}$, with $\varepsilon_{t} \sim N\left(0, \sigma_{\varepsilon}^{2}\right)$ and $0<\rho<1$.
(a) Provide an intuitive interpretation for the parameter $\theta$. Show that this production function exhibits constant returns to scale and that as $\theta \rightarrow 1$, it converges to a Cobb-Douglas production function.
(b) Derive the competitive equilibrium conditions that describe the dynamics of the model. Provide a brief interpretation of the conditions.
(c) Using the conditions derived in (b) work out an expression for the labour share in total income, defined by $W_{t} N_{t} / Y_{t}$, as a function of the capitallabour ratio $\kappa_{t} \equiv K_{t} / N_{t}$. Explain under what conditions the labour share in total income is always constant.
(d) Use the derived expressions in (b) and (c), and your economic intuition, to discuss whether the labour share in income is procyclical, acyclical or countercyclical. Explain carefully.
4. Consider a version of the Dynamic New Keynesian model described by the following system of equations:

$$
\begin{aligned}
\varkappa_{t} & =\mathbb{E}_{t}\left(\varkappa_{t+1}\right)-\frac{1}{\sigma}\left(i_{t}-\mathbb{E}_{t}\left(\pi_{t+1}\right)-z_{t}\right), \\
\pi_{t} & =\beta \mathbb{E}_{t} \pi_{t+1}+\kappa \varkappa_{t}, \\
i_{t} & =\gamma i_{t-1}+(1-\gamma)\left(\rho_{\pi} \pi_{t}+\rho_{y} \varkappa_{t}\right)+\varepsilon_{t} .
\end{aligned}
$$

where $\varkappa_{t}$ output gap, $i_{t}$ is nominal interest rate, $\pi_{t}$ is the inflation rate, and $z_{t}$ and $v_{t}$ are exogenous shocks. We assume $\sigma>0,0<\beta<1, \kappa>0,0<\gamma<1$ and $\rho_{\pi}, \rho_{y}>0$.
(a) Describe what each of the equations represents and give a brief interpretation.
(b) Rewrite the system in matrix form and derive the matrix $J$ that we need in order to analyse the model's determinacy. What are the BlanchardKahn conditions that ensure that the model has a unique equilibrium?
(c) Consider the limiting case in which $\gamma=0$ and show that in that case, if

$$
\begin{equation*}
\rho_{y}(1-\beta)+\kappa\left(\rho_{\pi}-1\right)>0 \tag{1}
\end{equation*}
$$

then the model always has unique equilibrium.
(d) Explain heuristically (without proof) whether you think condition (1) is in general sufficient for ensuring uniqueness of equilibrium, for any $0<\gamma<1$. Why or why not?

For question 4 you may find the following theorem useful:
Theorem: Let $B$ be a $2 \times 2$ matrix with characteristic equation

$$
f(\lambda)=\lambda^{2}-\operatorname{tr}(B) \lambda+\operatorname{det}(B)
$$

Then, the following are true:
(i) If $f(-1)>0, f(1)>0$ and $f(0)<1$ then all eigenvalues are stable, i.e. $\left|\lambda_{1,2}\right|<1$.
(ii) If $f(-1)>0, f(1)>0$ and $f(0)>1$ then all eigenvalues are (complex and) unstable, i.e. $\left|\lambda_{1,2}\right|>1$.
(iii) If $f(-1)<0$ and $f(1)<0$ then all eigenvalues are (real and) unstable, i.e. $\left|\lambda_{1,2}\right|>1$.
(iv) If $[f(-1)>0$ and $f(1)<0]$, or $[f(-1)<0$ and $f(1)>0]$, then one eigenvalue is stable and one is unstable.

## END OF PAPER

ECM10<br>MPhil in Economic Research

Wednesday 15 May 2019 2:00pm to 4:00pm

R201
ADVANCED MACROECONOMICS II

Candidates are required to answer two out of three questions
Each question carries equal weight

Write your candidate number (not your name) on the cover of each booklet.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS

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You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

(Hopenhayn) This question has you explore the properties of decision rules in the Hopenhayn model with particular types of adjustment costs. Assume that the establishment level technology now takes the form:

$$
y_{i t}=z_{i t} f\left(k_{i t}, h_{i t}, h_{i t-1}\right)
$$

(a) Define a recursive competitive equilibrium for this economy.
(b) Assume that adjustment costs on labor are linear, so that $f$ takes the form:

$$
f\left(k_{i t}, h_{i t}, h_{i t-1}\right)=g\left(k_{i t}, h_{i t}\right)-d \cdot\left|h_{i t}-h_{i t-1}\right|
$$

where $d>0$ and $|\cdot|$ denotes the absolute value. Assume the economy is in steady state. Argue that the optimal decision rule for hours takes the following form. For each realization of $z_{t}$ that does not induce exit, there is an interval $\left(\underline{h}\left(z_{t}\right), \bar{h}\left(z_{t}\right)\right)$ such that if $h_{t-1} \in\left(\underline{h}\left(z_{t}\right), \bar{h}\left(z_{t}\right)\right)$ then it is optimal to set $h_{t}=h_{t-1}$. But if $h_{t-1}$ is outside this interval then the optimal choice of $h_{t}$ depends on $z_{t}$ and is independent of $h_{t-1}$. The interesting property here is that even though adjustment costs are continuous, the optimal decision rule is discontinuous in $z$. In proving this assume that the value function is differentiable.
(c) If we had instead assumed quadratic adjustment costs, i.e.:

$$
f\left(k_{i t}, h_{i t}, h_{i t-1}\right)=g\left(k_{i t}, h_{i t}\right)-d \cdot\left(h_{i t}-h_{i t-1}\right)^{2}
$$

how would the optimal decision rule differ from that in part (b)? Provide intuition.

## Question 2

(Lucas-Prescott island economy) This question has you consider a variation of the LucasPrescott island economy that we studied in class. In particular, in class we assumed that if a household left their island in period $t$ that they could choose which island they would go to for the next period. This assumption is often referred to as directed search to reflect that fact that the individual can choose where to direct their search effort. An alternative assumption, often referred to as undirected or random search, is that when an individual chooses to leave an island, they will be randomly assigned to an island as of the beginning of the next period and that the probability of ending up on any particular island is the same. This implies that if there is a unit mass of islands and a total mass $m_{t}$ of households decide to leave the islands where they began period $t$, then each island will receive a mass of $m_{t}$ additional workers as of the beginning of period $t+1$. As in class assume that the production function on each island is given by:

$$
y_{i t}=z_{i t} f\left(e_{i t}\right)
$$

where $z_{i t}$ is the value of the technology shock on island $i$ in period $t$, and $e_{i t}$ is total labor input on island $i$ in period $t$. The production function $f$ satisfies decreasing returns to scale and the stochastic process for the technology shocks is described by the transition function $Q\left(z_{i t}, z_{i t+1}\right)$. Technology shocks on each island all follow the same stochastic process, but the realizations of technology are iid across islands. Each household values only consumption, has linear period utility and discounts the future at rate $\beta$. Ownership of firms is equally distributed among all households and households spend all of their income each period on consumption.
(a) Define a steady state recursive competitive equilibrium for this economy under the assumption of random search.
(b) Suppose that in a steady state equilibrium the expected continuation utility for a household leaving its island is given by $\hat{v}_{u}$. Additionally, assume that $\hat{m}$ is the total mass of households that leave their islands each period (i.e., the total mass of unemployed households each period). Derive a Bellman equation that will give the expected present discounted utility that a household can achieve if it begins a period on an island that has $n$ households and current technology shock $z$ given the two steady state values for $\hat{v}_{u}$ and $\hat{m}$.
(c) Using your answer in part (b), describe a procedure that you might use to find the steady state values $\hat{v}_{u}$ and $\hat{m}$.

## Question 3

(Financial frictions) Consider the following economy that features idiosyncratic managerial ability and financing constraints. There is a unit mass of ex ante identical individuals. Each individual has preferences given by:

$$
\sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right)
$$

where $c_{t}$ is consumption in period $t$. Each individual is endowed with one unit of time in each period but does not value leisure. Each individual is also endowed with stochastic managerial ability, denoted by $z_{i t}$, that evolves according to the transition function $Q\left(z_{i t}, z_{i t+1}\right)$. Every period, each individual must choose between either supplying their labor to the market or working as a manager. A manager with ability $z_{i t}$ operates a technology given by:

$$
y_{i t}=z_{i t} k^{\phi} h^{\eta}
$$

where $\phi>0, \eta>0$ and $\phi+\eta<1$, so that the function exhibits decreasing returns to scale. The stochastic process on idiosyncratic productivity is the same for each individual, but the realizations are iid across individuals. Output can be used either as investment or consumption.

The labor market is competitive and if the individual chooses to operate their own technology they can hire as much labor as they desire at the market wage rate. There is also a competitive market for renting capital, but limited contract enforcement restricts the amount of capital that a manager can hire. We assume that there is limited enforcement in the context of contracts for capital rental. Specifically, it is assumed that the manager must pay its labor costs out of revenues, but that it can renege on its payments to capital, and keep all of the (undepreciated) capital that it rented. If the firm does this, the only recourse is for creditors to seize the assets and revenues of the firm, but we assume that they can only seize a fraction $\theta$. To be more explicit, consider a manager with ability $z$ who uses inputs of capital and labor of $k$ and $h$, respectively, when the rental rate and wage rate are given by $r$ and $w$, respectively. The value of revenues and assets for the firm after payments to labor are given by:

$$
z k^{\phi} h^{\eta}-w h+(1-\delta) k
$$

If the manager does not renege on its capital rental contract it receives:

$$
z k^{\phi} h^{\eta}-w h-r k
$$

If it reneges and the creditors seize a fraction $\theta$, it will instead receive:

$$
(1-\theta)\left[z k^{\phi} h^{\eta}-w h+(1-\delta) k\right]
$$

The parameter $\theta$ indexes the degree of contract enforcement: if $\theta=1$, then there is no scope for an individual to renege on their contract with capital, whereas if $\theta=0$, then no activity in the capital rental market can be sustained. We assume that if an individual reneges on their capital contract that there is no future punishment, i.e., they are eligible to enter the market again in the following period with no negative consequences. Assume that a firm makes its decision about how much labor to hire after it makes its choice about how much capital to rent.
(a) Suppose an individual enters the period with a capital stock of $a$ and has a current realization of managerial ability $z$, that the capital rental rate is $r$, and the wage rate is $w$. Derive an expression for the maximum amount of capital that the firm could employ that is consistent with the firm not reneging on its capital contract. Assume that $z$ and $a$ are public information.
(b) Define a recursive competitive equilibrium for this economy assuming that the only capital contracts that are offered are those that will be honored (i.e., that the firm will not renege).
(c) Consider a steady state equilibrium for this economy. Let $m(z, a)$ denote the decision of whether an individual with managerial ability $z$ who has assets equal to $a$ works as a manager. What can you say about this function in the economy with perfect enforcement, i.e., $\theta=1$ ? How do you conjecture this would change if $\theta<1$ ?

END OF PAPER

ECM10
MPhil in Economic Research

Friday 10 May $2019 \quad$ 2:00pm to $5: 00 \mathrm{pm}$

R300
ADVANCED ECONOMETRICS

Candidates are required to answer three out of four questions
Each Question carries equal weight

Write your candidate number (not your name) on the cover of each booklet.

Write legibly. Credit will be given for clear presentation of relevant statistics.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
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## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator New Cambridge Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Suppose that aggregate demand for cigarettes in the U.S. is given by

$$
\log q_{i}=\beta_{0}+\beta_{1} \log p_{i}+\beta_{2} \log y_{i}+\varepsilon_{i}
$$

for $q_{i}$ the annual consumption of cigarettes in state $i$ measured in number of packs per capita, $p_{i}$ is the price of a pack in dollars, and $y_{i}$ is income in dollars per capita.
(i) Using data on 48 states, a least-squares regression gives the regression line

$$
\widehat{\log q_{i}}=\underset{(.61)}{3.86}-\underset{(.25)}{1.41} \log p_{i}+\underset{(.23)}{.34} \log y_{i}
$$

(appropriate standard errors are given in parenthesis) with $R^{2}=.433$ and $S S R=1.578$. Give an accurate interpretation to the estimated least-squares slopes on $\log p_{i}$ and $\log y_{i}$.
(ii) Evaluate the following statement. 'For predicting $q_{i}$ given levels of price and income we need to be concerned about the potential endogeneity of $p_{i}$ because, after all, we are trying to estimate a demand equation'.
(iii) Besides observations on quantity, price, and income, the data set also contains measurements on the variables 'sales tax' and 'cigarette tax'. The former is a general sales tax on commodities, the latter is a specific tax on cigarettes only. Both are in dollars per pack. Comment on the appropriateness of using 'cigarette tax' as instrument for price. Be complete and accurate in your evaluation.
(iv) Comment on the appropriateness of using 'sales tax' as instrument for price.
(v) Using both taxes as instruments, the optimal instrumental-variable estimator gives

$$
\widehat{\log q_{i}}=\underset{(.62)}{3.91}-\underset{(.24)}{1.29} \log p_{i}+\underset{(.32)}{.24} \log y_{i} .
$$

The statistical software used also reports a $J$-statistic of .336 . Which null hypothesis can we test with this statistic? Perform the test. Report in details on how you proceed and state your findings in words.
2. Suppose that

$$
y_{i}=\beta y_{i-1}+\varepsilon_{i}+\theta \varepsilon_{i-1}, \quad \varepsilon_{i} \sim \text { i.i.d. } N\left(0, \sigma^{2}\right),
$$

is a (strictly) stationary and ergodic (i.e., strong mixing with mixing coefficients that decay sufficiently fast) process. Note that, by stationarity, $|\beta|<1$.
(i) Derive the probability limit of the least-squares estimator of a regression of $y_{i}$ on $y_{i-1}$.
(ii) Construct a consistent estimator of $\beta$.
(iii) Derive the asymptotic variance of your estimator.
(iv) Suppose now that, aside from $\beta$, you also care about learning $\theta$ and $\sigma^{2}$. Derive a set of moment conditions for these three parameters. How would you use these to construct an estimator? Explain.
3. Consider the linear model with one regressor and no intercept

$$
y_{i}=x_{i} \beta+\varepsilon_{i}, \quad \varepsilon_{i} \sim N\left(0, \sigma_{i}^{2}\right),
$$

where

$$
\sigma_{i}^{2}=\sigma^{2}+x_{i} \gamma
$$

All the coefficients $\beta, \gamma, \sigma^{2}$ are unknown. We are interested in testing the null of conditional homoskedasticity, i.e., $\gamma=0$, against the alternative that $\gamma \neq 0$.
(i) Write down the log-likelihood function.
(ii) Derive the first-order conditions for $\beta, \gamma, \sigma^{2}$.
(iii) Give the maximum-likelihood estimator of $\beta$ under the null and under the alternative.
(iv) Compute the information matrix for the parameters $\gamma$ and $\sigma^{2}$. You may find it useful to recall that if a matrix $A$ is block diagonal, i.e.,

$$
A=\left(\begin{array}{cc}
A_{1} & 0 \\
0 & A_{2}
\end{array}\right)
$$

then its inverse is simply

$$
A^{-1}=\left(\begin{array}{cc}
A_{1}^{-1} & 0 \\
0 & A_{2}^{-1}
\end{array}\right)
$$

(provided, of course, that the inverses exist).
(v) Set up the Lagrange-Multiplier statistic for our null of homoskedasticity.
4. You have a random sample from the density

$$
f_{\theta}(x)=\theta x^{\theta-1}, \quad x \in(0,1) .
$$

(i) Compute the maximum-likelihood estimator (MLE) of $\theta$ from a random sample $x_{1}, \ldots, x_{n}$.
(ii) Now suppose that the density is

$$
f_{\tau}(x)=\frac{1}{\tau} x^{\frac{1}{\tau}-1} .
$$

Again compute the MLE.
(iii) How does your result under (ii) relate to (i)? Explain.
(iv) Derive the density of $y=\log x$. Use the original parametrization of the density of $x$.
(v) Use your result obtained under (iv) to compute the expectation of the MLE obtained under (ii).
(vi) Derive the efficiency bound for $\theta$ and for $\tau$. Does the MLE attain the bound in any of the two cases? Explain.

## END OF PAPER

ECM10/MGM3<br>MPhil in Economic Research<br>MPhil Finance

Friday 17 May $2019 \quad 9.00 \mathrm{am}$ to 11.00 am

R301A
TIME SERIES

Candidates are required to answer five questions from Section A and one question from Section B.

Section A is weighted as $60 \%$ and Section B as $40 \%$.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

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You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A

1. (a) Derive the variance of the stationary $\operatorname{AR}(1)$ model

$$
y_{t}=\phi y_{t-1}+\varepsilon_{t}, \quad|\phi|<1,
$$

where $\varepsilon_{t} \sim N I D\left(0, \sigma^{2}\right)$, that is normally and independently distributed with zero mean and variance $\sigma^{2}$. (b) Suppose there are observations for $t=1,2$ and 4 , but 3 is missing. Write down the exact likelihood function and hence obtain an expression for the maximum likelihood (ML) estimator of $\sigma^{2}$ for a given of $\phi$.
2. Derive the autocorrelation function (ACF) of the stationary $\operatorname{AR}(2)$ process

$$
y_{t}=\phi_{1} y_{t-1}+\phi_{2} y_{t-2}+\varepsilon_{t}, \quad t=1, . ., T
$$

where $\varepsilon_{t}$ is serially uncorrelated with mean zero and variance $\sigma^{2}$. How would you test for a unit root?
3. Write down the sample spectral density (periodogram). What happens when the sample spectral density is computed for a white noise process?
4. The cycle plus noise model is

$$
y_{t}=\mu+\psi_{t}+\varepsilon_{t}, \quad t=1, \ldots, T
$$

where $\mu$ is a constant, the cycle is given by $\psi_{t}$ in the recursion

$$
\left[\begin{array}{c}
\psi_{t} \\
\psi_{t}^{*}
\end{array}\right]=\rho\left[\begin{array}{rr}
\cos \lambda_{c} & \sin \lambda_{c} \\
-\sin \lambda_{c} & \cos \lambda_{c}
\end{array}\right]\left[\begin{array}{c}
\psi_{t-1} \\
\psi_{t-1}^{*}
\end{array}\right]+\left[\begin{array}{c}
\kappa_{t} \\
\kappa_{t}^{*}
\end{array}\right],
$$

where $\lambda_{c}$ is frequency in radians, $\rho$ is a damping factor between zero and one, and the disturbances $\kappa_{t}$ and $\kappa_{t}^{*}$ are uncorrelated with the irregular term $\varepsilon_{t}$. What are the values of $p$ and $q$ in the $\operatorname{ARMA}(\mathrm{p}, \mathrm{q})$ reduced form? Put the model in state space form and explain briefly how this enables the likelihood function to be computed. What is the innovations form of the Kalman filter here?
5. A random variable $y_{t}, t=1, \ldots, T$, is defined as

$$
\begin{aligned}
& y_{t}=\varepsilon_{t}, \quad \text { for } t \text { even } \\
& y_{t}=\left(\varepsilon_{t-1}^{2}-1\right) / \sqrt{2}, \quad \text { for } t \text { odd }
\end{aligned}
$$

where $\varepsilon_{t} \sim \operatorname{NID}(0,1)$. Show that $y_{t}$ is white noise. Given a sample in which $T$ is even, write down the minimum mean square linear estimator (MMSLE) of $y_{T+1}$ and find its MSE. Can you construct a better predictor ? NB The kurtosis of a standard normal variate is three.
6. In an exponential distribution,

$$
f(x)=\theta^{-1} \exp (-x / \theta), \quad x \geq 0
$$

observations less than one are recorded as zero. Write down the log-likelihood function and hence formulate a dynamic conditional score (DCS) model for a changing $\theta$. Would your answer be different if the observations set to zero were recorded as missing ?
7. The Clayton copula is defined as

$$
C\left(u_{1}, u_{2}\right)=\left\{\begin{array}{lr}
\left(u_{1}^{-\theta}+u_{2}^{-\theta}-1\right)^{-1 / \theta}, & \theta>0 \\
u_{1} u_{2}, & \theta=0
\end{array}\right.
$$

where $0 \leq u_{1}, u_{2} \leq 1$. Define the coefficient of lower tail dependence (the lower tail index) and derive this coefficient for the Clayton copula. Comment briefly on the value taken by the lower tail index when $\theta=5$ and $\theta=0$.

## SECTION B

B1 (a) A time series is constructed as a rolling sum of the last four observations, that is $y_{t}=S_{4}(L) x_{t}, t=4, \ldots, T$, where $S_{4}(L)$ is the summation operator and $L$ is the lag operator. If $x_{t}$ is white noise, what is the autocorrelation function (ACF) of $y_{t}$ ?
(b) Now suppose that $y_{t}=\Delta_{4} x_{t}$, where $\Delta_{4}$ is the fourth difference operator. If $x_{t}$ is white noise, is $y_{t}$ invertible? Find the ACF and spectrum of $y_{t}$.
(c) Find the ACF of $y_{t}=\Delta_{4} x_{t}$, when $x_{t}$ is a random walk plus noise with signalnoise ratio $q$. Comment on Figure 1 which shows fourth differences of a quarterly time series on bicycle thefts in Cambridge.


Figure 1: Correlogram of fourth differences of bicycle thefts in Cambridge
B. 2 How would you estimate the underlying level of daily volatility using (i) daily returns (ii) daily range and (iii) realized volatility? Begin your answer to each part by explaining briefly how the observations are constructed.

ECM10/MGM3
MPhil in Economic Research
MPhil Finance

Friday 17 May 2019 9:00am to 11:00am

R301B
CROSS SECTION AND PANEL DATA

Candidates are required to answer three questions from Section A and one question from Section $B$.
Section A is weighted as $60 \%$ and Section B as $40 \%$.

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## STATIONERY REQUIREMENTS

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## SECTION A

A1 Decision maker $i$ faces a finite choice set $\Omega_{J}$ of dimension $J$. Writing the utility of choice $j$ for individual $i$ as

$$
U_{i j}=V_{i j}+\varepsilon_{i j},
$$

alternative $j$ is chosen if $U_{i j}-U_{i j^{\prime}}>0, \forall j^{\prime} \neq j \in \Omega_{J}$. The probability that $j$ is chosen may be written

$$
\begin{equation*}
P_{i j}=\int_{\varepsilon} \mathbf{1}\left(\varepsilon_{i j}-\varepsilon_{i j^{\prime}}>V_{i j^{\prime}}-V_{i j} \forall j^{\prime} \neq j \in \Omega_{J}\right) f\left(\varepsilon_{i}\right) d \varepsilon_{i}, \tag{1}
\end{equation*}
$$

where $\varepsilon_{i}=\left(\varepsilon_{i 1}, \ldots, \varepsilon_{i J}\right)$.
(a) (i) Given $V_{i j}=\mathbf{v}_{j}^{\prime} \boldsymbol{\omega}$, for fixed parameters $\boldsymbol{\omega}$, and $\varepsilon_{i} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon}\right)$, what is the dimension of the integral in (11)? How many parameters are estimable?
(ii) Given $V_{i j}=\mathbf{v}_{j}^{\prime} \boldsymbol{\omega}_{i}$, with $\boldsymbol{\omega}_{i} \sim N\left(\overline{\boldsymbol{\omega}}, \boldsymbol{\Sigma}_{\omega}\right)$, and $\varepsilon_{i} \sim$ Type 1 Extreme Value, how does the tractability of the probability expression compare to that of (i) above? How many parameters are estimable?
(b) For $V_{i j}=\mathbf{v}_{j}^{\prime} \boldsymbol{\omega}$ and $\varepsilon_{i} \sim$ Type 1 Extreme Value, write down the choice probability for alternative $j$.
(c) In what sense do the two models implied by (a) ii.) and (b) represent a trade-off between tractability and the representation of choice behaviour in discrete choice?

A2 Consider the following latent variable model

$$
\begin{equation*}
y_{i}^{*}=\psi_{0}+\psi_{1} x_{i}+u_{i}, \quad i=1, \ldots, N . \tag{2}
\end{equation*}
$$

where $x_{i}>0$. The error $u_{i}$ is heteroskedastic such that $u_{i} \mid x_{i} \sim \mathrm{~N}\left(0, \sigma\left(x_{i}\right)^{2}\right)$, where $\sigma\left(x_{i}\right)^{2}=x_{i}^{2}$.
We observe $y_{i}=\mathbf{1}\left(y_{i}^{*}>0\right)$, where $\mathbf{1}($.$) is the indicator function.$
(a) Show that $\operatorname{Pr}\left(y_{i}=1 \mid x_{i}\right)=\Phi\left(\psi_{0} \frac{1}{x_{i}}+\psi_{1}\right)$, where $\Phi($.$) is the standard$ normal cumulative density function.
(b) Show that if $\psi_{0}$ and $\psi_{1}$ are both positive, the marginal effect of $x_{i}$ on $\operatorname{Pr}\left(y_{i}=1 \mid x_{i}\right)$ has the opposite sign to the marginal effect of $x_{i}$ on the latent dependent variable $y_{i}^{*}$.
(c) Now suppose that the model of interest is

$$
\operatorname{Pr}\left(y_{i}=1 \mid x_{i}, c_{i}\right)=\Phi\left(\psi_{0}+x_{i} \psi_{1}+\gamma c_{i}\right)
$$

where $u_{i} \mid x_{i}, c_{i} \sim \mathrm{~N}(0,1) . c_{i} \sim \mathrm{~N}\left(0, \tau^{2}\right)$ is an omitted variable and independent of $x_{i}$.
Given this model specification evaluate the following statement.
In probit models neglected heterogeneity is a more serious problem than in linear models. That is, even if

$$
E\left(\gamma c_{i}+u_{i} \mid x_{i}\right)=E\left(\gamma c_{i}+u_{i}\right)=0,
$$

the probit coefficients are inconsistent.
A3 Potential outcomes in the following binary model are given by

$$
y(x)=\mathbf{1}(\alpha+\beta x+u \geq 0)
$$

where $x$ is a continuous variable, $y(x)$ is a zero-one variate which varies with $x$, and $u$ is an error term. $\alpha$ and $\beta$ are unknown parameters and $\mathbf{1}($. denotes the indicator function.
(a) Find an expression for the effect of a change from $x$ to $x^{\prime}$ for an individual with error $u$.
(b) Find an expression for the average effect $E\left[y\left(x^{\prime}\right)-y(x)\right]$.

A4 (a) Consider the observational rule $y_{i}=\mathbf{1}\left(y_{i}^{*}>0\right)$, where $y_{i}^{*}=\boldsymbol{x}_{i}^{\prime} \boldsymbol{\beta}+u_{i}$, and $u_{i}$ is distributed Type 1 Extreme Value. We may then write

$$
\operatorname{Pr}\left(y_{i}=1 \mid x_{i}\right)=\frac{\exp \left[\boldsymbol{x}_{i}^{\prime} \boldsymbol{\beta}\right]}{1+\exp \left[\boldsymbol{x}_{i}^{\prime} \boldsymbol{\beta}\right]}
$$

Assuming that the prior distribution is given by $\pi(\boldsymbol{\beta})=\mathcal{N}\left(\boldsymbol{\beta}_{0}, \boldsymbol{B}_{0}\right)$, show that the posterior distribution of the logit model with a normal prior is given by

$$
\begin{aligned}
\pi(\boldsymbol{\beta} \mid y) \propto & \prod_{i=1}^{N}\left(\frac{\exp \left[\boldsymbol{x}_{i}^{\prime} \boldsymbol{\beta}\right]}{1+\exp \left[\boldsymbol{x}_{i}^{\prime} \boldsymbol{\beta}\right]}\right)^{y_{i}}\left(\frac{1}{1+\exp \left[\boldsymbol{x}_{i}^{\prime} \boldsymbol{\beta}\right]}\right)^{1-y_{i}} \\
& \times \exp \left[-\frac{1}{2}\left(\boldsymbol{\beta}-\boldsymbol{\beta}_{0}\right)^{\prime} \boldsymbol{B}_{0}^{-1}\left(\boldsymbol{\beta}-\boldsymbol{\beta}_{0}\right)\right] .
\end{aligned}
$$

If we increase the dimension of the posterior to include $y^{*}$, does this simplify the distribution of the posterior?
(b) Contrast the use of data augmentation in the multinomial probit and mixed logit model.
A. 5 (a) In what sense is bootstrap aggregation (bagging) a solution to the problem of instability of single regression trees? In what sense is interpretability of single trees compromised by this method?
(b) Evaluate the statement that

Certain variable transformations, that would be redundant for a standard linear regression model and create collinearities, can be valuable in penalized linear models or other machine-learning predictors.
(c) In machine learning a dataset is often divided into training and testing data, denoted by $\mathcal{S}_{t r}$ and $\mathcal{S}_{t e}$ respectively. Evaluate the basis for further sample splitting on $\mathcal{S}_{t r}$.
(d) Evaluate the statement that

From a Bayesian point of view there is no distinction between fixed and random effects panel data models, only between hierarchical and non-hierarchical models.

## SECTION B

B1 The linear unobserved effects panel data model may be written as

$$
\begin{equation*}
y_{i t}=x_{i t}^{\prime} \beta+c_{i}+\varepsilon_{i t} \tag{3}
\end{equation*}
$$

where $x_{i t}=\left(1, x_{i t 1}, x_{i t 2}, \ldots, x_{i t K}\right)^{\prime}$ is a $(K+1) \times 1$ vector, $\varepsilon_{i t}$ denotes an idiosyncratic error, and $c_{i}$ is an unobserved effect. $\beta$ is a vector of unknown parameters.
(a) i) Write down the conditional mean of $y_{i t}$ implied by the assumption of contemporaneous exogeneity, conditional on the unobserved effect $c_{i}$.
ii) Write down the conditional mean of $y_{i t}$ implied by the assumption of strict exogeneity, conditional on the unobserved effect $c_{i}$.
(b) With reference to (3), an analyst now decides to model the relationship between $c_{i}$ and $x_{i t}$, by assuming

$$
\begin{align*}
c_{i} & =\tau+\bar{x}_{i} \xi+a_{i},  \tag{4}\\
E\left(a_{i}\right. & \left.\mid \boldsymbol{x}_{i 1}, \ldots, \boldsymbol{x}_{i T}\right)=0 \tag{5}
\end{align*}
$$

Given strict exogeneity of $\varepsilon_{i t}$ in (3), along with (4) and (5), write down the estimating equation. In what sense is this model unifying for the pooled OLS, random effects and fixed effects estimators?
(c) Consider the following unobserved effects panel data model

$$
\begin{equation*}
y_{i t}=\omega+x_{i t}^{\prime} \beta+\mu_{i}+\varepsilon_{i t}, \tag{6}
\end{equation*}
$$

where $\mu_{i}$ denotes an individual-specific effect and $\varepsilon_{i t}$ is an IID error term. $\mathbf{x}_{i t}$ denotes a $K \times 1$ vector of exogenous variables.
An analyst proposes the following estimator for $\beta$

$$
\begin{align*}
\widehat{\boldsymbol{\beta}}_{P}= & {\left[\frac{1}{T} \sum_{i=1}^{N} \boldsymbol{X}_{i}^{\prime} \mathbf{Q} \boldsymbol{X}_{i}+\psi \sum_{i=1}^{N}\left(\overline{\mathbf{x}}_{i}-\overline{\mathbf{x}}\right)\left(\overline{\mathbf{x}}_{i}-\overline{\mathbf{x}}\right)^{\prime}\right]^{-1} }  \tag{7}\\
& \times\left[\frac{1}{T} \sum_{i=1}^{N} X_{i}^{\prime} \mathbf{Q} \mathbf{y}_{i}+\psi \sum_{i=1}^{N}\left(\overline{\mathbf{x}}_{i}-\overline{\mathbf{x}}\right)\left(\bar{y}_{i}-\bar{y}\right)\right]
\end{align*}
$$

where $\boldsymbol{X}_{i}$ is a $T \times K$ matrix, $\mathbf{Q}=\mathbf{I}_{T}-\frac{1}{T} \mathbf{i i}^{\prime}$ is a $T \times T$ matrix, $\mathbf{i}$ is a $T \times 1$ unit vector, and $\mathbf{I}_{T}$ is a $T \times T$ identity matrix.
$\psi=\sigma_{\varepsilon}^{2} /\left(\sigma_{\varepsilon}^{2}+T \sigma_{\mu}^{2}\right)$, where $\sigma_{\mu}^{2}$ and $\sigma_{\varepsilon}^{2}$ denote, respectively, the variance of $\mu_{i}$ and $\varepsilon_{i t}$.
i) Carefully explain the basis for the choice of estimator $\widehat{\boldsymbol{\beta}}_{P}$.
ii) We now rewrite (6) as

$$
\begin{equation*}
y_{i t}=\omega+z_{i} \gamma+\boldsymbol{w}_{i t} \delta+\mu_{i}+\varepsilon_{i t}, \tag{8}
\end{equation*}
$$

where $z_{i}$ denotes a $1 \times L$ vector of time constant variables and $\boldsymbol{w}_{i t}$ denotes a $1 \times J$ vector of time variables that vary over $i$ and $t$. Given (8), an analyst proposes the following null hypothesis

$$
H_{0}: E\left[\left(\overline{\boldsymbol{w}}_{i}-\overline{\boldsymbol{w}}_{i}^{*}\right)^{\prime} \mu_{i}\right]=\mathbf{0}
$$

as a basis for choosing between the random and fixed effects estimator of $\delta$.
Noting that $\overline{\boldsymbol{w}}_{i}^{*}$ denotes the conditional expectation of $\overline{\boldsymbol{w}}_{i}$ on $\boldsymbol{z}_{i}$, provide a rationale for the use of this null hypothesis.
B. 2 Let the utility agent $i$ obtains from alternative $j$ in choice situation $t$ be written as

$$
U_{i j t}=x_{i j t}^{\prime} \beta_{i}+\varepsilon_{i j t} .
$$

$x_{i j t}$ is a $K \times 1$ vector of explanatory variables, and $\varepsilon_{i j t}$ is distributed IID Type 1 extreme value.
We observe $\boldsymbol{y}_{i}=\left(y_{i 1}, \ldots, y_{i T}\right)^{\prime}$ where $y_{i t}=j$ if individual $i$ chooses alternative $j$ at time $t$. Variables $\boldsymbol{x}_{i j t}$ are denoted collectively (for all time periods and alternatives) by $\boldsymbol{x}_{i}$.
(a) i) Assuming $\boldsymbol{\beta}_{i} \sim f(\boldsymbol{\beta} \mid \boldsymbol{\theta})$, where $\boldsymbol{\theta}$ denote the hyperparameters of the distribution of $\beta$, write down an expression for the conditional and unconditional probability of individual $i^{\prime}$ s sequence of choices, respectively $\mathcal{P}_{i}\left(\boldsymbol{\beta}_{i}\right)=P\left(\boldsymbol{y}_{i} \mid \boldsymbol{x}_{i}, \boldsymbol{\beta}_{i}\right)$ and $\mathcal{P}_{i}(\boldsymbol{\theta})=P\left(\boldsymbol{y}_{i} \mid \boldsymbol{x}_{i}, \boldsymbol{\theta}\right)$.
ii) Find an expression for $h\left(\boldsymbol{\beta} \mid \boldsymbol{y}_{i}, \boldsymbol{x}_{i}, \boldsymbol{\theta}\right)$, the density of $\boldsymbol{\beta}$ in the subpopulation of individuals who would choose $\mathbf{y}_{i}$ when facing $\mathbf{x}_{i}$.
Provide an intuitive interpretation of $h\left(\boldsymbol{\beta} \mid \boldsymbol{y}_{i}, \boldsymbol{x}_{i}, \boldsymbol{\theta}\right)$.
(b) We now consider a different model where the distribution of any given parameter $\beta$ is defined by a grid of $C$ points, with fixed value $\beta_{c}$ at the $c^{\text {th }}$ point.
The probability mass at point $\beta_{c}, s_{c}$, is an unknown parameter.
Find an expression for $\mathcal{P}_{i}(s)$, the probability that agent $i$ makes the sequence of choices $\boldsymbol{y}_{i}=\left\{y_{i t}\right\}$, with unknown parameters $s=\left\{s_{c}\right\}$.
(c) For the model given in (b), show that the Expectation Maximisation (EM) algorithm may be written as

$$
\begin{equation*}
\mathcal{Q}\left(\boldsymbol{s}, \boldsymbol{s}_{j}\right)=\sum_{i} \sum_{c} h_{\boldsymbol{\beta}_{c}} \log \left(s_{c}\right), \tag{9}
\end{equation*}
$$

where $\mathcal{Q}\left(s, s_{j}\right)$ denotes the expected value of the log-likelihood function of $s$, with respect to the current conditional distribution of $\boldsymbol{\beta}_{c}$ given $y$, and the current estimates of the parameters $s_{j}$. Note that $h_{\boldsymbol{\beta}_{c}}=h\left(\boldsymbol{\beta}_{c} \mid \boldsymbol{y}_{i}, \boldsymbol{s}_{j}\right)$.
(d) Table 2 presents parameter estimates based on a stated preference experiment designed to elicit consumer preferences over energy supplier.
The suppliers were differentiated based on price, the length of the contract in years, whether the supplier was local, whether the supplier was well known, whether the supplier charged time-of-day (TOD) rates, and whether the supplier charged seasonal rates.

A mixed logit model was estimated under the assumption that the parameters are independently normally distributed in the population.
i.) Interpret the results.
ii.) Demonstrate how these results can be used to determine preference heterogeneity for the length of contract.
iii.) Test whether a mixed logit model is to be preferred to a standard logit specification.

Table 2: Mixed logit model of choice among energy suppliers

|  | $\hat{\theta}$ | $\widehat{\sigma}_{\theta}$ |
| :--- | :---: | :---: |
| Price coeff. | -0.8827 | - |
| $(0.050)$ |  |  |
|  | -0.2125 | 0.3865 |
|  | $(0.026)$ | $(0.028)$ |
| Local | 2.2277 | 1.7514 |
|  | $(0.127)$ | $(.137)$ |
| Well-known | 1.5906 | 0.9621 |
|  | $(0.100)$ | $(0.098)$ |
| TOD | 2.1328 | 0.4113 |
|  | $(0.054)$ | $(0.040)$ |
| Seasonal | 2.1577 | 0.2812 |
|  | $(0.051)$ | $(0.022)$ |

Log Likelihood Logit: -4959
Log Likelihood Mixed Logit -3619

Standard errors in parenthesis
$\widehat{\theta}$ denotes the mean and $\widehat{\sigma}_{\theta}$ the standard deviation of each parameter estimated.

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research
MPhil Finance

Tuesday 28 May 2019 1:30pm to $3: 30 \mathrm{pm}$

S140
BEHAVIOURAL ECONOMICS

Candidates are required to answer all four questions.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. (a) Give a careful description of Nagel's (1995) guessing-game experiment.
(b) What can we learn from her results about how one player in a game may reason about the thinking and behaviour of another?
(c) How robust are these conclusions?
2. (a) Give a careful description of a model of the consumption and savings decisions of an individual.
(b) What predictions does the model make about the decisions of an exponential consumer? What predictions does it make about the decisions of a hyperbolic consumer? What are the qualitative differences between the two sets of predictions?
3. (a) Give a careful description of the experiment of McClure, Berns \& Montague (2003).
(b) What was the purpose of this experiment?
4. (a) What is meant by anchoring?
(b) Give a careful description of the Beggs and Graddy (2009) econometric model of the art market.
(c) What do we learn from this model about anchoring in the art market? Justify your answer carefully.

## END OF PAPER

ECM9/ECM10/ECM11/MGM3
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research
MPhil Finance

Monday 3 June 2019 1.30pm to 3.30 pm

S170
INDUSTRIAL ORGANISATION

Candidates are required to answer 3 compulsory questions

Write your candidate number (not your name) on the cover of each booklet.

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## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1.

Consider two firms $i$ and $j$ that are each potentially active in two distinct geographical markets 1 and 2. A firm $k=i, j$ produces output $q_{k n} \geq 0$ for market $n=1,2$ and can thus decide whether to produce in both markets, in only one of the markets or in no market at all. Output produced in one market cannot be transferred to another market and the production costs for a firm are independent across markets. In particular, a firm's costs when producing in a given market are given by

$$
C(q)=\frac{c q^{2}}{2}
$$

with $c>0$. The inverse demand in each market is given by

$$
p=a-b Q
$$

with $a, b>0$ where $Q=q_{i}+q_{j}$ is aggregate output in a given market. When two firms compete in a given market, they do so by simultaneously and non-cooperatively setting quantities.
(a) Consider a non-competition agreement between the firms under which firm $i$ is the sole producer in market 1 and firm $j$ is the sole producer in market 2. Derive the equilibrium production levels $\left(q_{i}^{M}, q_{j}^{M}\right)$ in this case.
(b) Suppose now that the two firms do not sign any agreement and act non-cooperatively in each market. Derive the equilibrium production levels $\left(q_{i}^{C}, q_{j}^{C}\right)$ in each market in this case.
(c) Consider an agreement under which the two firms set quantities cooperatively in order to maximise joint profits in each market. Derive the equilibrium production levels $\left(q_{i}^{*}, q_{j}^{*}\right)$ in each market in this case.
(d) Denote by $Q^{M}=q_{i}^{M}, Q^{*}=q_{i}^{*}+q_{j}^{*}$ and $Q^{C}=q_{i}^{C}+q_{j}^{C}$ the aggregate industry output in each market under the three scenarios considered above. (i) Rank the three levels of aggregate output ( $Q^{M}, Q^{*}, Q^{C}$ ) and briefly interpret the ranking. (ii) Briefly outline how the equivalent ranking would be if the firms instead had linear cost functions

$$
C(q)=c q
$$

## Question 2.

Consider an upstream industry consisting of $N \geq 2$ symmetric firms that provide an intermediate good to a downstream firm $D$. The firm $D$ is a monopolist on a market for a final product that is characterised by the demand function

$$
p=a-b q
$$

where $a, b>0$ are constants, $p$ is the price charged to final consumers and $q$ is the total quantity produced. In order to produce the final good, $D$ must buy its inputs from the upstream industry and it needs one unit of the input to produce one unit of the final output. The upstream firms produce the intermediate good at constant marginal cost $c \geq 0$. The downstream firm does not bear any cost of turning intermediate goods into final goods. Assume that $a>c$.
(a) First consider the benchmark in which the downstream firm vertically integrates with an upstream firm. Calculate the optimal price $p^{M}$, quantity $q^{M}$ and profits $\pi^{M}$ under full integration, i.e. under the assumption that $D$ and the upstream firm coordinate with a view to maximise joint profits.
(b) Now consider the case in which the upstream firms non-cooperatively compete in quantities to supply the downstream firm. Calculate the equilibrium price $p^{U}$ for the intermediate good and the price $p$ charged to final consumers.
(c) Compare the outcomes under full integration and under no integration and determine what happens in the limit as $N \rightarrow \infty$. Briefly interpret your results.

## Question 3.

Consider two firms $i$ and $j$ producing differentiated goods. The inverse demand functions for the two goods are given by

$$
\begin{aligned}
& p_{i}=a-b q_{i}-d q_{j} \\
& p_{j}=a-b q_{j}-d q_{i}
\end{aligned}
$$

where $a, b>0$. The firms produce at marginal cost $c \geq 0$ and compete in quantities.
(a) Compute the Cournot equilibrium quantities $\left(q_{i}{ }^{C}, q_{j}{ }^{C}\right)$.
(b) Now consider a merger between firms $i$ and $j$ and assume that after the merger, the firms produce at the same marginal cost and retain both product varieties. Compute the profit maximising quantities $\left(q_{i}^{*}, q_{j}^{*}\right)$.
(c) Compare the Cournot quantities $\left(q_{i}^{C}, q_{j}^{C}\right)$ and the profit maximising quantities $\left(q_{i}^{*}, q_{j}^{*}\right)$ and determine when the merged firm produces a higher output of each good than the two Cournot competitors. Interpret your result.
(d) Now suppose that the merged firm could choose to produce only one variety (and thus withdraw the other variety from the market). Determine a condition under which it is profitable for the firm to do so and interpret this condition.

END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research

Friday 31 May 20199 9:00am to 11:00am

S180
LABOUR: SEARCH, MATCHING AND AGGLOMERATION

Candidates are required to answer three out of four questions. Each question carries equal weight.

This written exam carries $75 \%$ of the marks for S180

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

Consider a labour market with search frictions. Let $\delta, \rho, \lambda$ be the job destruction rate, the interest rate, and the job offer arrival rate respectively. Every firm has one job (either filled or vacant). All workers are identical. We consider the special case that all firms are equally productive; productivity is normalized to unity. Entry of a firm comes at a one-time cost $K$. The value of leisure for an unemployed job seeker is $B$. Workers accept any wage offer that is greater or equal to their current wage. Let $F(W)$ be the distribution of wages among employed workers. Firms set wages as to maximize the value of a filled job. Let $J(W)$ be the asset value of a filled job as a function of the wage $W$ they pay and let $V$ be the value of a vacancy.
a. Derive the Bellman equation for the asset value of a filled job and give an interpretation of the terms.
b. Derive the first order condition for the optimal wage offer and give an interpretation of the terms. In this first order condition, the derivative of $J(W)$ can be ignored. Why is this the case?
c. Even in this special case where all jobs are equally productive, there will be wage differences between firms. Explain why this is the case.
d. Claim: $J(W)$ will be the same for all $W$. True of false? Explain your answer.
e. Use your answer under 4. to derive the distribution $F(W)$.
f. Consider the equation $F(W)=0$. What is the solution for $W$ ? Explain your answer.
g. Make a graph of $F(W)$.
h. How would a minimum wage higher than $B$ affect this graph.
i. Discuss the evidence of David Lee regarding minimum wages. What identification strategy does he use in his research? Explain how the model discussed above could explain this evidence.

## Question 2

Consider a city as analyzed by Lucas and Rossi-Hansberg, which has a structure with a single Central Business District CBD/commercial areas surrounded by suburbs/residential areas, where $\delta$ is the decay rate of knowledge spill overs and $\kappa$ is the commuting cost (or: the decay rate of labour supply due to commuting).
a. Discuss the evidence of Ahlfeldt et.al. on Berlin regarding $\delta$ and $\kappa$. Is this evidence consistent with this structure of the city?
b. Suppose that commuting cost $\kappa$ increase over time by some exogenous process. How will wage and land rent differentials between locations within the city change by the gradual in crease in commuting cost?
c. At some point in time, $\kappa$ has increased so much that a mixed neighborhood emerges, where people both live and work. Where do you expect this mixed neighborhood to emerge? Explain your answer.
d. Why is there no commuting at all in a mixed neighborhood in the model of Lucas and Rossi-Hansberg?
e. At the moment the mixed neighborhood emerges in the existing city, a city planner starts a new city elsewhere in the country. In order to offset the agglomeration externalities, she introduces a system of land taxes and uses the revenues of this tax system for subsidies. The new city quickly obtains a similar size as the existing city. Would you expect this new city to have a mixed neighborhood at its centre? Explain your answer.
f. Suppose that different city planners start competing for citizens to live in their cities. What is the equilibrium tax rate on land rents?

## Question 3

a. In the comparative advantage model of Teulings, the return to the marginal unit of skill of a worker is determined by the complexity of the job to which she is assigned in equilibrium. Explain the mechanism.
b. This model implies that there are two reasons for a better skilled worker to earn a higher wage. Explain.
c. Teulings and Van Rens show that the return to human capital depends on its demand and supply. Discuss their parameterization. What is the demand for human capital and what is the supply? How does this claim relate to the mechanism under 1 ?
d. Gabaix and Landier show that CEO pay depends on firm size. They explain this phenomenon using an argument related to the mechanism discussed above. What is their argument? What does comparative advantage of better skilled workers in complex jobs mean in this particular context?
e. Furthermore, Gabaix and Landier show that - over time - CEO pay depends on the size of $250^{t h}$ firm in the country. How would you explain this result?

## Question 4

a. Discuss the evidence of Gennaioli et.al. on the inter-regional differences in GDP per capita. What explains these differences? Why are these differences problematic from a standard return to human capital perspective?
b. Their evidence on the return to human capital is consistent with a strong regional sorting. Explain why this is the case. Why does not everybody move towards the region with high GDP per capita? Explain how this force encourages sorting.
c. Glaeser and Mare give an alternative explanation for the high wages in particular regions. What is their explanation? What evidence do they provide for this view?
d. Desmet and Rossi-Hansberg discuss the role of General Purpose Technologies in the agglomeration of activities at particular locations. Their model has two forces: agglomeration and de-agglomeration. Discuss both forces and apply them to the two agglomeration waves in the $20^{\text {th }}$ century.
e. How does this evidence relate to the findings of the Gennaioli et.al.?
f. Shortly discuss the historical evidence on urbanization in Europe. How does this evidence relate to the Gennaioli et.al. findings?

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

## S301 <br> APPLIED ECONOMETRICS

This paper is in two sections:
Section A: Candidates are required to answer two compulsory question.
Section B: Candidates are required to answer two out of four questions.

Each section carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator New Cambridge Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## SECTION A. ANSWER BOTH QUESTIONS FROM THIS SECTION

1. Job Corps (JC) is the largest vocationally focused education and training program for disadvantaged youths in the US. Through a nationwide network of campuses, Job Corps offers a comprehensive array of career development services to at-risk young women and men to prepare them for careers. Define $D_{i}=1$ if an individual $i$ enrolls in JC and $D_{i}=0$ if an individual $i$ does not enroll in JC. Suppose you are interested in estimating the effect of JC on labor market outcomes (e.g. earnings). Let $y_{1}$ be the potential outcome if an individual enrolls in JC, and $y_{0}$ be the potential outcome if the individual does not enrolls in JC.
(a) Explain, in words, the economic meaning of $E\left(y_{1} \mid D=1\right), E\left(y_{1} \mid D=0\right)$. Which one of these is not observed in your data?
(b) Suppose that we are interested in estimating the average treatment effect on the treated (ATT): $\alpha=E\left(y_{1} \mid D=1\right)-E\left(y_{0} \mid D=1\right)$. One of your friends suggest that you regress the observed outcome on $D$. Show the bias in using the coefficient on D from this regression as an estimator for the ATT. What is the direction of the bias you would expect?
(c) An experimental evaluation of JC was conducted in the mid-1990s. The experiment randomly assigned eligible applicants to either a treatment group or to a control group. Individuals in the treatment group are eligible to enroll in Job Corps. Individuals in the control group are excluded from Job Corps. 72 percent of the treatment group enrolled in the Job Corps. Explain how the experimental evaluation helps to obtain the causal effect of JC. Can we use the experimental evaluation to estimate the ATT? Why?
(d) Besides JC, there exist alternative training programs which individuals can also choose. Suppose we define the treatment variable $T_{i}$ in the following way: $T_{i}=0$ if the person $i$ does not attend any training, $T_{i}=1$ if the person enrolls in alternative training programs and $T_{i}=2$ if the person enrolls in JC. Are we able to use the experimental evaluation defined in part (c) of this question to identify the causal effect of attending JC relative to no training? Explain.
2. In a panel of 46 US states over the 30 years 1963-1992 demand for cigarettes $\left(C_{i t}\right)$ in state $i$ in year $t$ is hypothesised to follow the following relation

$$
\ln \left(C_{i t}\right)=\alpha+\beta \ln \left(P_{i t}\right)+\gamma \ln \left(\operatorname{Pmin}_{i t}\right)+\delta \ln \left(Y_{i t}\right)+u_{i t}
$$

where $C_{i t}=$ cigarette sales in packs per capita, $P_{i t}=$ price per pack of cigarettes, $\operatorname{Pmin}_{i t}=$ minimum price in an adjoining states per pack of cigarettes (in case people buy across state borders) and $Y_{i t}=$ per capita disposable income each measured in state $i$ in year $t$ and $u_{i t}$ is a random error component. The following results are obtained by various estimation methods (standard errors in parentheses)

|  | Dependent |  |  |  |  | Variable | $\ln \left(C_{i t}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ |  |  |  |  |
| Estimation | Pooled OLS | OLS/LSDV | GLS | Between |  |  |  |  |
| const | 4.167 | 4.601 | 4.569 | 4.366 |  |  |  |  |
|  | $(0.286)$ | $(0.440)$ | $(0.422)$ | $(1.791)$ |  |  |  |  |
| $\ln \left(P_{i t}\right)$ | -1.277 | -1.023 | -1.026 | -1.496 |  |  |  |  |
| $(0.060)$ | $(0.042)$ | $(0.041)$ | $(0.403)$ |  |  |  |  |  |
| $\ln \left(\right.$ Pmin $\left._{i t}\right)$ | 0.160 | -0.117 | -0.110 | 0.321 |  |  |  |  |
|  | $(0.059)$ | $(0.054)$ | $(0.053)$ | $(0.360)$ |  |  |  |  |
| $\ln \left(Y_{i t}\right)$ | 0.569 | 0.520 | 0.522 | 0.600 |  |  |  |  |
|  | $(0.031)$ | $(0.047)$ | $(0.045)$ | $(0.166)$ |  |  |  |  |
| Individual Effects | No | Fixed | Random | No |  |  |  |  |
| Time dummies | Yes | Yes | Yes | No |  |  |  |  |
| ResidualSumSquares | 38.719 | 7.243 | 39.503 | 1.030 |  |  |  |  |
| $\log$ likelihood | 507.57 | 1664.18 | 493.76 | 22.10 |  |  |  |  |
| No of Obs | 1380 | 1380 | 1380 | 46 |  |  |  |  |

F-test of poolability Column (2) vs (1) $F(45,1302)=125.73$
Breusch Pagan test for RE Column (3) vs (1) $\chi^{2}(1)=12738$
Hausman test Column (3) vs (2) - Null hypothesis: GLS estimates are consistent: Asymptotic test statistic: $\chi^{2}(32)=1.9$
(a) Explain what assumptions (about $u_{i t}$ ) are required for the appropriateness of each column estimation.
(b) From the information given what can you conclude about the validity of these assumptions?
(c) What would you conclude about the price elasticity of demand? What further estimation might you consider?

## SECTION B. ANSWER TWO QUESTIONS FROM THIS SECTION

1. An econometrician, Robert, collects data containing total employment (L) and average wages (W) for all the counties in the UK. Robert considers the following model of supply and demand for labor:

$$
\begin{align*}
D_{i} & =\alpha_{0}+\alpha_{1} W_{i}+\epsilon_{i},  \tag{1}\\
S_{i} & =\beta_{0}+\beta_{1} W_{i}+\mu_{i} \tag{2}
\end{align*}
$$

where $D_{i}, S_{i}$ and $W_{i}$ are labor demand, labor supply and wages in county $i, i=1, \ldots, n$. The parameter of interest is $\alpha_{1}$. Robert assumes that the labor market in each county is independent and competitive, so labor market clears ( $D_{i}=S_{i}$ ) for any county i and $\left\{\epsilon_{i}, \mu_{i}\right\}$ are i.i.d..
(a) Draw a simple labor demand and supply diagram for a given county i and derive its equilibrium wage, $W_{i}$.
(b) To estimate labor demand (equation (11), Robert regresses $L_{i}$ on $W_{i}$ and obtains the least square estimator ( $\widehat{\alpha}_{1}$ ) for $\alpha_{1}$. Is $\widehat{\alpha}_{1}$ unbiased? If so, provide a proof. If not, derive the bias of the least square estimator for $\alpha_{1}$.
(c) Suppose that, in addition to variables on employment and wages, Robert observes the number of new migrants in a county $\left(M_{i}\right)$ in his data. Assume that $E\left(\mu_{i} \mid M_{i}\right)=E\left(\epsilon_{i} \mid M_{i}\right)=0$. Can Robert use the additional variable on migrants to improve the previous estimates of $\alpha_{1}$, and if so, how? Please explain your answers carefully.
(d) Is it possible to identify $\beta_{1}$ with Robert's dataset containing information on $M_{i}, L_{i}$, and $W_{i}$ ? Why? How would you advise Robert to estimate $\beta_{1}$ ? Be clear on any assumptions you are making.
2. Imagine you have conducted a randomized social experiment to study the efficacy of a job search counselling program on unemployment durations of young job-seekers. The package includes a short training session on how to search for a job, along with an individual interview and job search plan. Starting with a pool of 4000 eligible new job seekers, you carefully randomized 2000 into the treatment group (to receive the program) and 2000 into a control group. Data was collected from all 4000 individuals for a subsequent period of one year. Unfortunately, only 1500 individuals in the treatment group actually showed up to receive the course and interview. Here are the data:

|  | Control Group | Treatment Group |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  |  | Attended Program | Did not attend | Total |
| Sample | 2000 | 1500 | 500 | 2000 |
| \% back in work <br> at one year | $50 \%$ | $80 \%$ | $40 \%$ | $70 \%$ |

(a) Is a comparison of the employment rates of program participants and non-participants likely to give a "good" estimate of the effect of the program? Explain.
(b) Calculate the intent to treat (ITT) effect of the program. How is this estimate useful for informing policy and what are the limitations?
(c) Calculate the average treatment effect for the treated. How is this estimate useful for informing policy and what are the limitations?
(d) Suppose you also collect observable background characteristics of all individuals in the control and treatment group. Would controlling for these characteristics give you a "better" estimate of the treatment effect? Explain.
3. A company wants to know the price elasticity of demand for their product and collect data on annual sales $\left(Y_{i t}\right)$ and price $\left(P_{i t}\right)$ across a panel of US states $i=1, \ldots, N$ and years $t=1, \ldots, T$. To take account of a dynamic response they specify the following fixed effects panel model

$$
y_{i t}=\alpha_{i}+\lambda y_{i t-1}+\delta p_{i t}+\gamma^{\prime} z_{i t}+\epsilon_{i t}
$$

where $y_{i t}=\ln \left(Y_{i t}\right), p_{i t}=\ln \left(P_{i t}\right), z_{i t}$ is a vector of other observed control variables (such as state income etc.), the $\alpha_{i}$ are state specific constants to take account of any time invariant state factors and $\epsilon_{i t}$ are IID error terms. They believe price is effectively exogenous in this setup so the model is to be estimated by ordinary least squares (OLS) and they propose to use $\frac{\hat{\delta}}{1-\lambda}$ as an estimate of the long run price elasticity where $\hat{\delta}$ and $\hat{\lambda}$ are the OLS estimates of $\delta$ and $\lambda$ respectively.
(a) Outline any criticisms you have for this strategy.
(b) What modifications would you suggest that might improve their estimates of this elasticity?
4. A cinema chain wants to know whether a marketing promotion encourages households to visit the cinema. They have data on whether household $i$ visits the cinema in week $t\left(y_{i t}=1\right)$ or not $\left(y_{i t}=0\right)$ and whether the household received marketing that week $\left(x_{i t}=1\right)$ or not $\left(x_{i t}=0\right)$. They decide to specify a model

$$
P\left(y_{i t}=1 \mid x_{i t}, \eta_{i}\right)=F\left(x_{i t} \beta+\eta_{i}\right) \quad i=1, \ldots, N t=1 \ldots, T
$$

where $F$ is the logistic cumulative distribution function

$$
F(r)=\frac{e^{r}}{1+e^{r}}
$$

and $\eta_{i}$ is a household fixed effect. The $y_{i t}$ are assumed independent (conditional on $x_{i t}$ and $\eta_{i}$ ).
(a) One team member recalls that there are problems estimating fixed effect binary choice models on panel data and suggests assuming $\eta_{i}=\eta$ for all $i$. What econometric problems might this cause?
(b) They then decide to conduct a trial where for a random selection of $N$ households they produce no marketing in week $1\left(x_{i 1}=0\right.$ all $\left.i\right)$ and then deliver marketing in week $2\left(x_{i 2}=1\right.$ all $\left.i\right)$ and observe $\left(y_{i 1}, y_{i 2}\right)$. They decide to use the fixed effect formulation. Explain how you would derive the log likelihood function for this model and the first order conditions for estimation of $\left(\beta, \eta_{i}\right)$
(c) After the trial they identify three types of household:
i. Those for whom $y_{i 1}+y_{i 2}=0$
ii. Those for whom $y_{i 1}+y_{i 2}=2$
iii. Those for whom $y_{i 1}+y_{i 2}=1$

Explain the effect each of these types has in the likelihood function and how they could be used to estimate $\beta$.

## END OF PAPER

ECM9/ECM10/ECM11
MPhil in Economics
MPhil in Finance \& Economics
MPhil in Economic Research

Wednesday 29 May 2019 1:30pm to $3: 30 \mathrm{pm}$

## S500 <br> DEVELOPMENT ECONOMICS

Answer two out of three questions. Each question carries equal weight.

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

## Question 1

a. Describe the main shortcomings of a standard agricultural crop insurance product and discuss the following possibility for a new insurance product: a two-tier system in which the insurance company makes insurance payouts to the village as a whole, while a village council decides on how to allocate the payouts to individual residents.
b. Describe a typical rainfall index insurance product and discuss whether and how it overcomes the shortcomings of standard insurance.
c. What do we mean by basis risk in index insurance products and why does it matter? Discuss.
d. A researcher suggested that setting up more rainfall stations can tackle basis risk and improve rainfall insurance products. Do you agree? Why or why not? Discuss.
e. Can the presence of informal risk-sharing agreements affect the demand for rainfall index insurance? If so, how? Does your answer depend on the extent of basis risk? Discuss.
f. Suppose you want to empirically test whether and how the presence of informal risk-sharing agreements affect the demand for rainfall index insurance. How would you do this and why? Discuss.

## Question 2

a. Outline the Target Input Model and describe how learning affects farmers' technology adoption decisions.
b. Explain the challenges involved in providing evidence for the extent of social learning and describe how Conley and Udry (2010) addresses these challenges in the article "Learning About a New Technology: Pineapple in Ghana", American Economic Review, Vol. 100, No. 1, pp. 35-69.
c. A new agricultural technology is introduced in two countries, A and B. Country A displays greater ethnic fractionalization than country B. Do you expect this to matter for adoption in the two regions? If so, how would you extend the Target Input Model to capture this? Discuss.
d. The government of country C would like to promote farmers' adoption of a new fertilizer. Historically, the government hired extension agents to visit villages and communicate with farmers about new agricultural practices. This generally had poor results. Can you propose any new policy that promotes adoption by leveraging the power of social learning? Discuss.

## Question 3

a. ROSCAs are one of the most common informal financial institutions across the developing world. Some researchers have linked the success of ROSCAs to the need to purchase indivisible goods. Describe their argument and discuss whether it is supported by what we observe in the data.
b. Another hypothesis for the success of ROSCAs is lack of self-control. Discuss how lack of self-control affects savings and explain why ROSCAs might be valuable if individuals lack self-control.
c. How would you test empirically the relationship between ROSCAs and lack of self-control? Discuss.
d. Which other strategies would you expect to be adopted by individuals that lack self-control? Describe at least one such strategy and propose a randomized experiment that can empirically assess its relevance.

## END OF PAPER

ECM9/ECM10
MPhil in Economics
MPhil in Economic Research
Monday 3 June 2019 1:30pm-3:30pm

## S620 <br> INSTITUTIONS AND ECONOMIC GROWTH IN HISTORICAL PERSPECTIVE

Candidates are required to answer two out of four questions

Write your candidate number (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

## STATIONERY REQUIREMENTS

20 Page booklet x 1
Rough work pads

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator - students are permitted to bring an approved calculator
You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. "For long-term economic growth, property rights institutions are more important than contracting institutions." (Acemoglu \& Johnson 2005).
(a) Explain the theoretical argument underlying this statement.
(b) To what extent is this conclusion supported by empirical evidence on the historical development of the European economy?
2. "Guilds had both positive and negative effects on the pre-modern European economy, but their net effect on economic growth was positive." Assess this statement with regard to EITHER merchant guilds OR craft guilds.
3. (a) What are the theoretical mechanisms by which the institution of the family might have affected economic growth in Europe between c. 1500 and c. 1800?
(b) To what extent do the empirical findings support these theoretical arguments?
4. Does the fact that an economic institution existed in many different societies over many centuries demonstrate that it was an efficient solution to an economic problem? Why or why not? Answer with reference to at least one economic institution in Europe before c. 1800.

[^0]:    You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

[^1]:    You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

[^2]:    ${ }^{1}$ In their paper, labour reforms are measured with a time lag, but this is not central.

[^3]:    1 of 5

[^4]:    ${ }^{1}$ The Stone Index is defined by $\log \left(P_{h}\right)=\sum_{j=1}^{7} \omega_{j h} \log \left(p_{j h}\right)$.

