UNIVERSITY OF WARWICK

Summer Examinations 2015/16

Topics in Economic Theory

Time Allowed: 1.5 Hours

Answer FIVE questions. Choose 2 among the questions $\{1,2,3\}$ and 3 among the questions $\{4,5,6,7\}$. All questions carry equal marks.

Approved pocket calculators are allowed.

Read carefully the instructions on the answer book provided and make sure that the particulars required are entered on each answer book. If you answer more questions than are required and do not indicate which answers should be ignored, we will mark the requisite number of answers in the order in which they appear in the answer book(s): answers beyond that number will not be considered.

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1. Preference and Choice

C({x,y,z,q})= {q} C({x,y})={x} C({y,z})={y} C({x,z})={z}

(a) Does the choice rule above satisfy WARP? Please explain. **(8 marks)**

(b) Can this choice rule be rationalized by a preference relation $\succeq$? Please explain and in case yes, write it down. In case no, write down a new choice rule which can be rationalized by a preference relation. **(12 marks)**

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2. Preference and Choice

- Outcomes={earn 1 million, earn 0, sick for a week}
- There are 3 lotteries with the respective probabilities for each of the three events above: $L_A = \{0.4, 0.4, 0.2\}$, $L_B = \{0.2, 0.8, 0\}$, $L_C = \{0.3, 0.6, 0.1\}$.
- The preferences are: $A \succ B$, $B \succ C$.

(a) Does the above preferences over lotteries satisfy the independence axiom? Please explain. **(10 marks)**

(Question 2 continued overleaf)

(b) Please write down a preference which satisfies the independence axiom over these lotteries. **(10 marks)**

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3. Preference and Choice

Consider the binary relation "divisibility" on integers, defined by $a|b \Leftrightarrow$ a divides b on the set Z of integers. Which properties does this relation satisfy (on positive integers)? Please explain each.

(a) Reflexivity/Irreflexivity

(b) Symmetry

(c) Asymmetry

(d) Antisymmetry

(e) Transitivity

(f) Negative Transitivity

(g) Completeness **(20 marks)**

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4. Adverse Selection

Consider a monopolist who is selling a good to a single buyer who can be of a high or low valuation type. The buyer's valuation per unit good is given by $\theta_i \in \{3, 5\}$ with respective probabilities 0.6 and 0.4. The cost function of the monopolist is $c(q) = q^2$. Outside options are zero.

(a) First, assume the firm can observe the type of buyer and find the optimal price proposed and quantities consumed depending on the type of buyer. What is this type of pricing called? **(5 marks)**

(b) Second, solve the firm's problem assuming that it cannot observe the individual buyer type and has to propose a single contract. (clue: there are 2 possibilities depending on whom the seller targets). **(5 marks)**

(c) Now assume the monopolist cannot observe the buyer type but can propose a menu of contracts. Write down the monopolist's objective and the constraints she has to satisfy. Show that the IR_h is slack by making use of IR_l and IC_h. Solve for the optimal menu of contracts. Compare the principal's profit with the part (a) and (b) and comment. **(10 marks)**

(Continued overleaf)

5. Adverse Selection

Assume a firm wants to contract with a supplier. The supplier's cost of production is his private information. The supplier can be of 3 different types having unit costs θ_i 2,4,6 for $i = l, m, h$, indicating low, medium and high cost types, with respective probabilities $(0.3, 0.3, 0.4)$. Assume that the firm offers a menu of contracts, (w_i, q_i) denoting respectively the payment to the supplier and quantity produced. The suppliers' utility function is $u(w, q) = w - q^2\theta_i$. The firm's utility is $v(w, q) = 20q - w$. Outside options are zero.

(a) How many different contracts will the menu consist of (Explain this by using the revelation principle.) Find out who is the "best" and "worst" type in this contract. **(5 marks)**

(b) Then, write down the incentive constraints (we need to take into account only 1 for each type as the Spence Mirrlees condition holds). Show that both types apart from the lowest type will get positive information rents. Write down the participation constraints. Show that the IR of the worst type will bind. Then, show that due to the IC constraints (in other words the ability to mimic), the two better types' IR constraints will be slack. (clue: first show that of the medium type and then the best type.) Solve for the quantities produced and comment. **(15 marks)**

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6. Moral Hazard

Suppose that in a principal agent relation the agent's effort can result in either of two possible outcomes which have value $\underline{x} = 2$ and $\overline{x} = 5$ for the principal. The agent chooses to put in effort level of $e = 0$ or $e = 1$ where $c(e) = e$. The possibilities of outcomes realizing is as follows: $\pi(\overline{x}|e = 1) = 0.8$ and $\pi(\overline{x}|e = 0) = 0.2$. The principal and the agent are risk neutral given the utilities respectively $U(x, w) = x - w$ and $V(w, e) = w - e$ where w is the payment contingent on the outcome. The agent has limited liability (wages cannot be negative).

(a) First assume the agent's effort is observable. Calculate the optimal level of effort and wage offer. **(10 marks)**

(b) Second assume the principal cannot observe the agent's effort, in other words there is a moral hazard problem. Will the principal prefer to induce effort or not? (clue: you have to solve for both cases and compare the principal's profit). Find the optimal incentive compatible contract. Finally, how much would the principal be willing to pay in order to monitor the agent's effort. (clue: by monitoring, the principal can achieve the outcome in (a)!) **(10 marks)**

7. Moral Hazard

Consider a driver who who owns a car. The driver can exert effort (e=1) or not (e=0) where the cost of effort is $c(1) = 1$. The probability of having an accident is $\frac{1}{2}$ if $e = 1$ whereas it is 1 if $e = 0$. The driver's initial wealth is $w = 50$ and in case of an accident he loses 20 and is left with $w = 30$. The driver's utility function is $u(w, e) = \sqrt{w} - c(e)$.

(a) Will the driver choose to put in effort or not in the absence of insurance? **(5 marks)**

(b) Consider a risk neutral insurance company. The contract offered specifies the final wealth of the agent (w_1, w_0) respectively in case of no accident and accident. What would be the optimal contract offered which induces the agent to participate and exert effort? (Here w_1 denotes the final wealth of agent after the transfers and in case no accident happens, given the agent bought the insurance.) **(15 marks)**

(End)