UNIVERSITY OF WARWICK

Summer Examinations 2015/16

Econometrics 2: Time Series

Time Allowed: 1.5 Hours

Answer ONE Question from Section A and ONE Question from Section B. All questions carry equal marks. Answer Section A questions in one booklet and Section B questions in a separate booklet.

Read carefully the instructions on the answer book provided and make sure that the particulars required are entered on each answer book. If you answer more questions than are required and do not indicate which answers should be ignored, we will mark the requisite number of answers in the order in which they appear in the answer book(s): answers beyond that number will not be considered.

Section A: Answer ONE Question

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1. Consider the stationary ARMA(1,1) $y_t = c + \phi y_{t-1} + \epsilon_t + \theta\epsilon_{t-1}$

(a) Find a general expression for $y_{t+k|t} := E[y_{t+k}|y_t]$. **(15 marks)**

(b) Find an expression for $\omega_k^2 = E[(y_{t+k} - y_{t+k|t})^2]$, the variance of the conditional expectation. **(10 marks)**

(c) Faced with the forecast error loss function $L(e) = b[\exp(ae) - ae - 1]$, where $e = y_{t+k} - \widehat{y_{t+k}}$, describe how you would adjust your forecast compared to the predictor $y_{t+k|t}$ considered above. Assume $a, b > 0$. Describe intuitively the properties of the new optimal predictor, $\widehat{y_{t+k}}$, and how your results would change in the presence of conditional heteroskedasticity. *Hint*: if $z \sim N(\mu, \sigma^2)$ then $E[\exp(z)] = \exp(\mu + \sigma^2/2)$. **(25 marks)**

2. (a) Find the roots of the lag polynomial and determine the order of integration of the following series.

$$x_t = 0.98x_{t-1} - 0.02x_{t-2} + \epsilon_{xt}$$
$$y_t = 2y_{t-1} - y_{t-2} + \epsilon_{yt}$$

(10 marks)

(b) For each series write down the stationary transformation if it is integrated or the MA(∞) representation if it is stationary. (If you prefer, you may use generic symbols ϕ_1, ϕ_2, rather than specific coefficient values for MA representations) **(10 marks)**

(c) Let

$$w_t = \mu_w + X_t$$
$$x_t = \mu_x + \beta X_t + \tilde{x}_t$$
$$y_t = \mu_y + \beta Y_t + \tilde{y}_t$$

where $\tilde{x}_t \sim I(0)$, $\tilde{y}_t \sim I(0)$, $X_t \sim I(1)$, $Y_t \sim I(1)$ and the μ_i are non-stochastic. What is the order of integration of $z_t = x_t + y_t$? Find a stationary variable s_t which is a linear combination of w_t, x_t, y_t. Express your variable in terms of the cointegrating vector between w_t, x_t, y_t. Is your variable s_t uniquely defined? **(10 marks)**

(d) Explain how you would test for cointegration between two variables. **(10 marks)**

(e) If there are two independent cointegrating vectors between two series z_{1t}, z_{2t}, what do we know about z_{1t}, z_{2t}? Give a mathematical explanation of your argument. **(10 marks)**

Section B: Answer ONE Question

3. Consider the VAR(2) in n variables

$$y_t = c + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \epsilon_t$$

with $E[\epsilon_t] = 0_{n.1}$, $E[\epsilon_t \epsilon'_{t-1}] = 0_{n,n}$, $E[\epsilon_t \epsilon'_t] = \Sigma$

(a) Derive the effect of $\epsilon_{i,t}$ on $y_{j,t+k}$. **(10 marks)**

(b) Given that Σ is not a diagonal matrix, describe the identification problem in interpreting these quantities, and suggest a scheme for providing an interpretable set of effects. **(10 marks)**

(Question 3 continued overleaf)

(c) Let n=4. Derive VECM representation of the VAR(2), for a cointegrating rank of 2. Note the size and rank of all your matrices and vectors. Show why an identification problem arises and discuss how you would solve it. **(20 marks)**

(d) Describe how you would test for Granger causality in the VAR and comment on the relation between Granger causality and cointegration. **(10 marks)**

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4. (a) If an error series $\{\epsilon_t\}$ is a GARCH(1,1) process, derive the process for the squared errors. **(10 marks)**

(b) Find the ARCH representation of the GARCH(1,1) process and comment on what this tells us about the relative merits of estimating GARCH(1,1) versus high-order ARCH processes. **(10 marks)**

(c) Show how you would compute the j-period ahead forecasts for a GARCH(2,1) process recursively and find explicit expressions for the first 3 terms in the series. Comment on the long-run properties of these forecasts. **(20 marks)**

(d) What aspects of financial data do ARCH and GARCH models not capture and how would you test for the presence of such effects? **(10 marks)**

(End)