EC3010

UNIVERSITY OF WARWICK
Summer Examinations 2015/16

## Mathematical Economics 2: Dynamics, Uncertainty \& Asymmetric Information

Time Allowed: 1.5 hours
Answer TWO questions. All questions carry equal marks.
Approved hand calculators may be used.
Read carefully the instructions on the answer book provided and make sure that the particulars required are entered on each answer book. If you answer more questions than are required and do not indicate which answers should be ignored, we will mark the requisite number of answers in the order in which they appear in the answer book(s): answers beyond that number will not be considered.

1. (a) Amanda $(A)$ and $\operatorname{Bob}(B)$ are involved in a joint project. Their payoffs from the project depend on whether each cooperate $(C)$ or defect $(D)$ and the productivity of the project, which Bob knows while Amanda believes it could be low $\left(B_{L}\right)$ or high $\left(B_{H}\right)$ with equal probability. Payoffs are given in the table below.

Bob
Amanda

|  | $B_{L}$ |  |
| :---: | :---: | :---: |
|  | C | D |
| $C$ | 2, 2 | $-2,0$ |
| D | 0, -2 | 0, 0 |


| $B_{H}$ |  |  |
| :---: | ---: | ---: |
| $C$ | $D$ |  |
| 4, | 4 | -1, |
| $1,-1$ | 0, | 0 |

(i) Suppose Amanda plays $C$ with probability $p \in[0,1]$ and $D$ with probability $1-p$. Find the best-response correspondence of both types of Bob (i.e., the type $B_{L}$ that knows the project's productivity is low and the type $B_{H}$ that knows productivity is high). ( 10 marks)
(ii) Find all Bayesian equilibria of the game. (15 marks)
(b) Consider the game in the figure below. Nature moves first and chooses either Top $\left(1_{T}\right)$ or Bottom ( $1_{B}$ ) with equal probability. Player 1 observes the choice by Nature and selects either Left $(L)$ or Right $(R)$. Player 2 observes the choice of player 1 ( $L$ or $R$ ), but not Nature's choice (i.e., player 2 does not know whether player 1 is type $1_{T}$ or type $1_{B}$ ). Player 2 chooses between $\operatorname{Up}(U)$ and Down $(D)$. The first number associated with a terminal node is the payoff of player 2, the second number is the payoff of player 1 .


Find a completely mixed Perfect Bayesian equilibrium (PBE); that is a PBE in which both type $1_{T}$ and type $1_{B}$ of player 1 play each of $L$ and $R$ with positive probability. ( 25 marks)
2. A risk-neutral owner (principal) needs to hire a manager (agent) to exert effort on a project. The agent may exert high $(e=H)$ or low $(e=L)$ effort. Low effort costs zero $(c(L)=0)$, while high effort cost $10(c(H)=10)$; the unit of measure is thousand pounds. If the agent exerts high effort, then the project yields a revenue of 700 with probability 0.75 and a revenue of 100 with probability 0.25 . If the agent exerts low effort revenue is 700 with probability 0.25 and 100 with probability 0.75 . The agent is risk averse; when her wage is $w$ and she exerts effort $e$, her utility is $\sqrt{w}-c(e)$. The outside option payoff of the agent is 10 . If indifferent between the outside option and the contract offered by the principal, we assume the agent accepts the contract offered. The goal of the principal is to maximise his expected profit (revenue minus wage payments).
(a) Suppose the principal observes the revenue realisation and the effort exerted by the agent and can offer a contract with the agent's wage that depends on the effort and revenue realisation.
(i) What is the contract the principal offers if he wants the agent to exert low effort? (3 marks)
(ii) What is the contract the principal offers if he wants the agent to exert high effort? (3 marks)
(iii) What is the optimal wage contract for the principal? Does the principal prefer the wage contract under high or low effort? What is the principal's profit?
(3 marks)
(b) Now suppose the principal does not observe effort and can only offer a contract with wage that depends on the revenue realisation.
(i) What is the contract the principal offers if he wants the agent to exert low effort? (5 marks)
(ii) What is the contract the principal offers if he wants the agent to exert high effort? In answering the question, describe the incentive and individual rationality constraint of the agent. Explain why both constraints have to hold with an equality. Write down the optimisation problem of the principal. (16 marks)
(iii) What is the optimal wage contract for the principal? Does the principal prefer the wage contract under high or low effort? What is the principal's profit?
(10 marks)
(c) Due to a positive shock to the economy, the project's revenue in the good state has increased. Now, if the agent exert high effort revenue is 900 with probability 0.75 and 100 with probability 0.25 . If the agent exerts low effort revenue is 900 with probability 0.25 and 100 with probability 0.75 . Suppose the principal does not observe effort.
(i) How does the shock to the economy affect the optimal wage with low and high effort? What is the preferred wage contract for the principal? (10 marks)
3. Consider a seller who has private information about the quality $q$ of her new product. The seller's product is either of high $(q=2)$ or low $(q=1)$ quality. The seller knows her product's quality, or type, but buyers do not; prior to advertising by the seller, they all attach equal probability to the product being high or low quality. Before launching the new product in the market, the seller decides how much advertising effort to exert. Advertising does not directly reveal any information about the product, but advertising effort is observable by buyers and thus can serve as a signal. There is a unit mass of identical buyers; they all form the same posterior expectation about the product's quality after observing the seller's advertising effort. Their utility is zero if they do not purchase and it is the difference between the price and the expected quality if they purchase the product. Thus, all buyers make the same purchasing decision; they buy the product if the price is not higher than the product's posterior expected quality. Knowing this and using the buyers' posterior expectation in equilibrium, the seller will charge a price $p$ for the product equal to the buyers posterior expected quality. After exerting advertising effort $e$, the payoff of the seller is $p-\frac{e^{2}}{q}$ if she sells the product (to all buyers) at price $p$; her payoff is $-\frac{e^{e}}{q}$ if she does not sell.
(i) Compute all the separating Perfect Bayesian equilibrium (PBE) outcomes of the game. For each equilibrium outcome, specify a set of posterior beliefs that, together with the action taken by each seller's type (i.e., the low and high quality seller), constitute a PBE. ( 15 marks)
(ii) Can you compare the separating PBE's from the point of view of maximising the sum of buyers and seller's utility? ( 5 marks)
(iii) Compute all the pooling PBE outcomes of the game. For each equilibrium outcome, specify a set of posterior beliefs that, together with the action taken by each seller's type, constitute a PBE. ( 15 marks)
(iv) Can you compare the pooling PBE's from the point of view of maximising the sum of buyers and seller's utility? (5 marks)
(v) Now suppose the seller cannot advertise the new product. What price will she offer to buyers in equilibrium? Which of the pooling and separating PBE's with advertising are better for the seller (i.e., give higher payoff) than the equilibrium when advertising is impossible? ( $\mathbf{1 0}$ marks)

