UNIVERSITY OF WARWICK

Summer Examinations 2015/16

Collective Decisions

Time Allowed: 2 Hours

Answer **ONE** question from Section A and **ONE** question from Section B. Answer Section A questions in one booklet and Section B questions in a separate booklet.

Approved pocket calculators are allowed.

Read carefully the instructions on the answer book provided and make sure that the particulars required are entered on each answer book. If you answer more questions than are required and do not indicate which answers should be ignored, we will mark the requisite number of answers in the order in which they appear in the answer book(s): answers beyond that number will not be considered.

Section A: Answer ONE Question

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1. (a) Recall the Approval Voting rule: each voter can vote for ("approve") as many candidates as she likes. The winner is the candidate who gets most votes. Prove that a voter who votes for all of the candidates will not have any influence on the election outcome. **(15 marks)**

(b) In criminal cases, like a murder case with capital punishment, jury verdicts have to be unanimous. Some theorists have worried that this requirement of unanimity may in fact lead to convicting an innocent person if jurors are behaving strategically. Explain the logic behind this argument, and discuss how this drastic conclusion could be mitigated. **(25 marks)**

(Continued overleaf)

2. Consider a variation of the Hotelling-Downs spatial model where two candidates compete in an election by offering policies in the unit interval $[0, 1]$. Voters' bliss points are uniformly distributed on $[0, 1]$, and their utilities are quadratic: $U(b, x) = -(b-x)^2$ if a voter has bliss point b and policy x is implemented. Assume that instead of maximizing the probability of winning, each candidate maximizes her vote share.

(a) Write down each candidate's payoff from a pair of policies (x_1, x_2) **(5 marks)**

(b) Describe the best response correspondences for each of the candidates **(20 marks)**

(c) Find a Nash equilibrium of this game. Is it unique? **(15 marks)**

Section B: Answer ONE Question

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3. Suppose the policy space is the unit square $X = [0, 1]^2$. A point in X is denoted $x = (x_1, x_2)$. There are three voters $I = \{1, 2, 3\}$. The preferences of the voters are described by utility functions on X as follows:

$$u_1(x_1, x_2) = -2x_1 - x_2$$
$$u_2(x_1, x_2) = x_1 + 2x_2$$
$$u_3(x_1, x_2) = x_1 - x_2$$

(a) What is the ideal point of voter i, $i = 1, 2, 3$. **(5 marks)**

(b) Explain why Plott's theorem that identifies conditions for existence of the majority rule core does not directly apply here. **(25 marks)**

(c) Is there global cycling? That is, for any $x, y \in X$ does there exist a majority rule path from x to y? Prove your answer directly. **(30 marks)**

4. There are $n > 1$ agents in the economy. Agents only differ in productivity: agent i has productivity w_i. If she supplies x_i unites of labour, she earns a pre-tax income $y_i = x_i w_i$ and bears an effort cost of $\frac{1}{2}x_i^2$, which represents the tradeoff between labour and leisure. In addition, each agent pays a fraction t of her income in taxes. Tax revenues are then redistributed in equal shares. Thus payoff U_i of agent i is given by

$$U_i(w_i, x_i, t) = (1-t)w_i x_i - \frac{1}{2}x_i^2 + \frac{t}{n}\sum_{j=1}^{n} w_j x_j$$

Given a tax rate t, agent i chooses labour supply $x_i^*(w_i, t)$ that maximizes her utility taking labor supply of other agents as given.

(a) What is the optimal labour supply of individual i? How does it depend on changes in the tax rate t? **(15 marks)**

(b) What is the equilibrium payoff $U_i^*(w_i, t)$ of agent i when tax rate t is implemented and all agents supply their labour optimally? **(10 marks)**

(c) Prove that agents' equilibrium preferences over tax rates are single-peaked for any productivity w_i. **(20 marks)**

(d) Suppose agents choose the tax rate (same for all) by majority rule voting. What is the majority rule equilibrium tax rate? **(15 marks)**

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5. Suppose there are three individuals and three alternatives, with the set of alternatives $\mathcal{A} = \{a, b, c\}$. Consider a restricted domain X that consists only of the profiles where each individual can have one of the following four strict preference orders:

$$X = \{a \succ b \succ c, b \succ a \succ c, b \succ c \succ a, c \succ b \succ a\}.$$

Thus there are a total of only 64 possible joint preference profiles.

(a) Formulate and explain the meaning of three of Arrow's axioms: transitivity, unanimity (weak Pareto), and IIA. **(10 marks)**

(b) Find a non-dictatorial Social Welfare Function that satisfies these axioms on X (i.e., prove that for your chosen SWF each axiom holds on X). **(40 marks)**

(c) Explain how your result in (b) does not contradict Arrow's impossibility theorem. **(10 marks)**

(End)