UNIVERSITY OF WARWICK

Summer Examinations 2015/16

Mathematical Economics 1B

Time Allowed: 1.5 Hours

Answer TWO questions only. All questions carry equal weight. Answer each question in a separate answer booklet.

Read carefully the instructions on the answer book provided and make sure that the particulars required are entered on each answer book. If you answer more questions than are required and do not indicate which answers should be ignored, we will mark the requisite number of answers in the order in which they appear in the answer book(s): answers beyond that number will not be considered.

1. Consider a two-consumer $(i = 1, 2)$ exchange economy where consumption sets are given by $X_i = R_+^2$, $i = 1, 2$ and preferences can be represented by the following utility functions:

$$u_1(x_{11}, x_{12}) = x_{11}$$
$$u_2(x_{21}, x_{22}) = x_{21}x_{22}$$

Now assume that the economy has the aggregate endowment vector $\omega = (2, 2)$ where the initial endowments of the consumers are $\omega_1 = (1, 1)$, $\omega_2 = (1, 1)$.

   (a) Prove that $\mathbf{p} \gg 0$ in any Walrasian equilibrium for this economy. **(10 marks)**

   (b) Find a Walrasian equilibrium for this economy. **(15 marks)**

Now consider a general economy specified by $\left(\{(X_i, \succsim_i)\}_{i=1}^I, \{Y_j\}_{j=1}^J, \omega\right)$ with an allocation denoted $(\mathbf{x}, \mathbf{y})$, wealth levels $(w_1, ..., w_I)$ and prices $\mathbf{p} = (p_1, ..., p_L)$.

   (c) Define what is meant by a price-equilibrium with transfers and state any difference between this and a Walrasian equilibrium. **(10 marks)**

   (d) State then prove the First Welfare Theorem of Economics. **(15 marks)**

2. Consider a two-consumer ($i = 1, 2$) exchange economy where the aggregate endowment is given by $\omega = (1, 1)$. Consumption sets are given by $X_i = R_+^2$, $i = 1, 2$ and preferences can be represented by the following utility functions:

$$u_1(x_{11}, x_{12}) = \sqrt{x_{11}} + \sqrt{x_{12}}$$
$$u_2(x_{21}, x_{22}) = x_{21}$$

(a) Define what is meant by locally non-satiated preferences. **(5 marks)**

(b) Are consumer 2's preferences locally non-satiated? Provide a proof to accompany your answer. **(10 marks)**

Let $x = [(0, 1), (1, 0)]$. Denote prices $\mathbf{p} = (p_1, p_2)$ and assume for the rest of this question that $\mathbf{p} \in R_+^2$.

(c) Prove that x is a Pareto optimal allocation. **(10 marks)**

(d) Does there exist a price vector $\mathbf{p} \neq (0, 0)$ and wealth levels $w_1, w_2 \geq 0$, such that $(\mathbf{p}, x)$ is a price-equilibrium with transfers? If yes, find one. If no, explain why. **(10 marks)**

(e) Does there exist a price vector $\mathbf{p} \neq (0, 0)$ and wealth levels $w_1, w_2 \geq 0$, such that $(\mathbf{p}, x)$ is a price quasi-equilibrium with transfers? If yes, find one. If no, explain why. **(10 marks)**

(f) In a few sentences, provide a comparison of your answers to parts (d) and (e) making reference to a welfare theorem. **(5 marks)**

....

3. Consider a two-good economy (good 1 and good 2) where there are two firms (firm 1 and firm 2). Firm j produces good j using capital (K_j) and labour (L_j) as inputs of production, $j = 1, 2$. There is a total amount of capital and labour in the economy given by $\bar{K}, \bar{L} = 1$ respectively. Assume the prices for the two consumption goods and factor inputs $[(p_1, p_2), (w, r)] \gg 0$ throughout the question.

(a) Define what is meant by a technically efficient factor allocation in this economy. **(5 marks)**

The firms are characterised by the following production functions:

$$f_1(L_1, K_1) = \sqrt{L_1 K_1} \qquad \text{for } L_1, K_1 \geq 0$$
$$f_2(L_2, K_2) = L_2 + K_2 \qquad \text{for } L_2, K_2 \geq 0$$

(b) Find the set of all technically efficient factor allocations and draw it in an appropriate diagram. In the diagram, select one such efficient allocation and show the isoquants of each firm that pass through it. **(20 marks)**

Suppose there are also two individuals in the economy, Mr.1 and Mr.2. Mr.1 owns firm 1 and has 1 unit of labour. Mr.2 owns firm 2 and has 1 unit of capital. Consumption sets are $X_i = \mathbb{R}^2_{++}$, $i = 1, 2$ and preferences are given by:

$$u_i(x_{i1}, x_{i2}) = \log(x_{i1}) + \log(x_{i2})$$

(c) Define a Walrasian equilibrium for this economy. **(10 marks)**

(d) Find a Walrasian equilibrium for this economy. **(15 marks)**