UNIVERSITY OF WARWICK

Summer Examinations 2015/16

Mathematical Economics 1A

Time Allowed: 1.5 hours

Answer TWO questions ONLY. All questions carry equal weight. Answer each question in a separate answer booklet.

Approved hand calculators may be used.

Read carefully the instructions on the answer book provided and make sure that the particulars required are entered on each answer book. If you answer more questions than are required and do not indicate which answers should be ignored, we will mark the requisite number of answers in the order in which they appear in the answer book(s): answers beyond that number will not be considered.

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1. Suppose n firms compete in a market in a Cournot fashion. The inverse demand is given as $p = a - bQ$, where $Q = \sum_{j=1}^{n} q_j$ is total output in the market.

(i) Suppose the marginal cost of each firm is identical and equal to the constant c. Write up the profit maximisation problem of firm i. **(5 marks)**

(ii) Solve for the best-reply function of firm i, that is the function which expresses its output as a function of the outputs of all other firms in the market, $q_{\neg i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} q_j$. **(5 marks)**

(iii) Solve for output of firm i, the profit of firm i and the market price. **(5 marks)**

(iv) Suppose now firms incur a fixed cost of entry into the market equal to k. There are n potential firms in the market. Suppose a firm will first decide whether or not to enter the market. Firm i will enter if it can make non-negative profits, that is if $\pi_i \geq 0$. If the firm decides to enter, it will set quantities strategically with the other firms who decided to enter. Solve for the equilibrium number of firms. **(10 marks)**

(v) Assume now there is no free entry, and that there are only two firms in the market; a Stackelberg leader, and a follower. The leader sets the level of output first, and marginal cost of the two firms is still given by the constant c. Derive outputs of the two firms in equilibrium. **(10 marks)**

(Question 1 continued overleaf)

(vi) Derive profits of, respectively, the Stackelberg leader and the follower as well as the market price. **(5 marks)**

(vii) Now assume that the Stackelberg leader faces uncertainty regarding the cost of the follower, but the follower has complete information regarding the cost of the leader. More specifically, suppose the follower can be efficient with marginal cost equal to c_E or inefficient with marginal cost c_I. Either type of follower occurs with equal probability. Derive the output of all firms in equilibrium. **(10 marks)**

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2. Suppose there are two players: a Worker (W) and a Firm (F). W has private information about her level of ability. F knows that with probability ρ, W is high ability type (H), and with probability $(1-\rho)$, W is low ability type. After W observes her own type (a draw by nature), she decides whether to get a degree (play E) or not (play N). F observes W's education, but not her type. F then decides whether to employ W in a managerial job, or a clerical job (C). The pay-offs are as follows: if F employs a worker of type H in a managerial job, it obtains a payoff equal to 10; if F employs a worker of type L in a managerial job, it obtains payoff equal to 0; if F employs a worker in a clerical job, it obtains a payoff equal to regardless of the type of W. A worker of either type obtains a payoff of 10 in a managerial job, and 4 in a clerical job. The cost of education differs, however. For an H-type worker the cost of eduction is 4, whereas for an L-type worker it is 7.

(i) Draw this game in extensive form. **(5 marks)**

(ii) Assuming that the two types of worker are equally likely, is there a separating equilibrium in which the H-type chooses to do a degree, and the L-type doesn't? Carefully explain your answer. **(10 marks)**

(iii) Is there a pooling equilibrium in which both workers play N? Carefully explain your answer. **(10 marks)**

(iv) Is there a pooling equilibrium in which both workers play E? Carefully explain your answer. **(10 marks)**

(v) Suppose now the cost of doing a degree for an $L-type$ is 5. Is there still a separating equilibrium in which the H-type chooses E and the L-type chooses N? Carefully explain your answer. **(10 marks)**

(vi) Is there still a pooling equilibrium in which both types of worker chooses N? Carefully explain your answer. **(5 marks)**

3. Consider the following version of Chicken. Two drivers are on a collision course. The drivers may Swerve (play S) or be Tough (play T). If they both play S, they each get a payoff of 5; if they both play T, they each get -10; if one swerves and the other plays tough, the one who swerved gets 0, while the one who played tough gets 10.

(i) Suppose the game is played simultaneously. Represent the game in normal and extensive forms. Find all pure-strategy Nash equilibria of the game, and state the equilibrium payoffs. **(4 marks)**

(ii) Suppose one player gets to move first. Represent this new game in normal and extensive forms. Find all pure-strategy Nash equilibria of the game. Which of these Nash equilibria are subgame perfect? **(4 marks)**

(iii) Assume once again that the game is played simultaneously. Find the mixed-strategy Nash equilibrium, illustrate it in a digram and calculate the equilibrium payoffs. **(8 marks)**

(iv) Players can now be of two types – competitive (C) or ultra competitive (UC). For a competitive player, the pay-offs are as in the previous sub-questions. For an ultra competitive player, however, the pay-off of playing T when the other player plays S is 15, and the pay-off to playing S when the other player plays T is -5. In plain English, the ultra competitive player gets a higher pay-off from winning and a lower pay-off from losing, relative to the competitive player. The ultra competitive player gets the same pay-off from a tie as the competitive player. Construct a table of pay-offs for this extended game. **(4 marks)**

(v) Suppose the two types are equally likely. Show that the pair of strategies:

$$[(C \to S, UC \to T), (C \to S, UC \to T)],$$

is a Bayes-Nash equilibrium. **(10 marks)**

(vi) Assume now that the probability that a player is competitive (C-type) is equal to $\frac{1}{10}$. Is

$$[(C \to S, UC \to T), (C \to S, UC \to T)],$$

still a Bayes-Nash equilibrium? **(10 marks)**

(vii) We now assume that everyone knows that Player 1 is of the competitive type (C-type). Player 2, however, can be either competitive (C-type) or ultra competitive (UC-type). Player 2 knows her own type, but Player 1 does not know her type. The probability that Player 2 is a C-type is μ. Find all pure-strategy Bayes-Nash equilibria. **(10 marks)**

(End)