UNIVERSITY OF WARWICK

Summer Examinations 2015/2016

Microeconomics 2

Time Allowed: 3 hours.

Answer **ALL FOUR** questions in **Section A** (18 marks each), and **ONE** question from **Section B** (28 marks). Answer Section A questions in one booklet and Section B questions in a separate booklet.

Approved pocket calculators are allowed.

Read carefully the instructions on the answer book provided and make sure that the particulars required are entered on each answer book. If you answer more questions than are required and do not indicate which answers should be ignored, we will mark the requisite number of answers in the order in which they appear in the answer book(s): answers beyond that number will not be considered.

Section A: Answer ALL FOUR Questions

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1. Suppose a firm's production function has the Cobb-Douglas form

$$q = z_1^{\alpha 1} z_2^{\alpha 2}$$

where z_1 and z_2 are inputs, q is output and α_1, α_2 are positive parameters.

(a) Draw the isoquants. Do they touch the axes? Calculate the elasticity of substitution in this case? **(6 marks)**

(b) Using the Lagrangean method find the cost-minimising values of the inputs and the cost function. **(6 marks)**

(c) Under what circumstances will the production function exhibit (a) decreasing (b) constant (c) increasing returns to scale? Explain this using first the production function and then the cost function. **(6 marks)**

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2. Consider an island economy with two individuals Friday and Robinson who have Cobb-Douglas preferences. Friday likes bananas more than coconuts, and Robinson likes coconuts more than bananas. Their utility functions are given by:

$$u^f\left(x_b^f,\, x_c^f\right) = \alpha \ln x_b^f + (1-\alpha)\ln x_c^f$$

and

$$u^r(\, x_b^r,\, x_c^r,) = \alpha \ln x_c^r + (1-\alpha)\ln x_b^r$$

where x_b^f and x_c^f are Friday's consumption of bananas and coconuts, and x_b^r and x_c^r Robinson's consumption of bananas and coconuts and α is a positive parameter, α∈(0.5, 1).

The total amounts of bananas and coconuts in the island are:

$$e_b = 2;\ e_c = 2c \geq 2$$

Robinson and Friday have the same endowment of bananas and coconuts:

(a) We normalize the price of bananas to equal one or $p_b = 1$. Compute the demand functions for coconuts and bananas of Robinson and Friday as a function of the price for coconuts p_c. **(6 marks)**

(b) Find the equilibrium relative price for this island economy and determine the net trading quantities for the following cases:

i. Where c = 1, **(6 marks)**

ii. Where c > 1 and describe the properties of the solution as c increases in value. **(6 marks)**

3. A monopoly firm can produce a good Q at cost c(Q)=0. There are two types of consumer of the good, denoted L and H respectively. Assume the inverse demand functions for the two types of consumer are:

A_i-Q for i=L,H

where $A_H > A_L > 0$. There is a mass of consumers normalized to equal 1, of which $0<\alpha<1$ are type H consumers and 1-α are type L consumers.

(a) Assume the monopoly can observe the type of any consumer and is able to prevent post-purchase arbitrage. The monopoly offers two different packages denoted (T_i,Q_i) for i=L or H, where T_i is the payment made by the consumer and Q_i is volume offered by the monopoly for the payment T_i. Describe the optimal market packages offered by the monopolist. You may use a diagram to illustrate your answer. (**5 marks)**

(Question 3 continued overleaf)

(b) If the monopolist offers two packages designed for the two types L and H, call these (T_L^*,Q_L^*) and (T_H^*,Q_H^*), it must be the case that four constraints relating to consumer behavior are satisfied as part of the monopolist profit maximization programme. Write down and name each of these constraints **(4 marks).**

(c) Show why it must be the case that H type consumers enjoy a positive surplus. **(4 marks)**

(d) Suppose instead the monopoly cannot observe the type of any individual consumer but is aware that α of the consumers are H types and $1-\alpha$ of the consumers are L types. If the firm offers only one package to the market (T^*,Q^*), show why the following inequality is satisfied $(1-\alpha)/\alpha<(A_H-A_L)/A_L$. **(5 marks)**

4. Consider two individuals A and B considering how to divide a dollar between themselves. Suppose the individuals agree to the following rules:

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- First A proposes a division to B
- Second B can accept or reject the proposal
- If B accepts, the division occurs as A proposed
- If B rejects, each player obtains zero

(a) Assuming there is no discounting of payoffs, write down the extensive form of the game. **(4 marks)**

(b) What is the set of Nash equilibria for this game? **(4 Marks)**

(c) Show what happens to the set of Nash equilibria when we apply Sequential Rationality. **(4 marks)**

(d) Provide a brief interpretation about the nature of the equilibrium set in (c), in particular what can we say about those Nash equilibria that do not survive sequential rationality? **(3 marks)**

(e) Suppose that B can offer a payment to A before the rules are finalized, which if accepted means that B proposes the division and A chooses to accept or reject the proposal. Describe the subgame perfect equilibrium for this modification and discuss in relation to the equilibrium payoffs. **(3 marks)**

Section B: Answer ONE Question

5. Suppose a consumer has a two-period utility function of the form $\alpha log\ x_1 + (1-\alpha) log\ x_2$ where x_i is the amount of consumption in period *i*. The consumer's resources consist just of inherited endowment A in period 1, which is partly spent on consumption in period 1 and the remainder saved in an asset paying a rate of interest r.

(a) Draw the lifetime budget constraint and the indifference curves and interpret the parameter α in this case. **(6 marks)**

(b) Obtain the optimal allocation of $(x_1; x_2)$. **(6 marks)**

(c) Explain how consumption varies with A, r and α. **(6 marks)**

(d) Comment on your results and examine the "income" and "substitution" effects of the interest rate on consumption. **(10 marks)**

6. With no fishing, Lake Bass has a stock of 100 fish. The benefit earned by a fisherman who fishes at this lake depends on the probability of catching a fish, which decreases as the stock of fish is depleted. As an approximation of this, suppose that the benefit earned by any one fisherman from sitting down and fishing at the lake is $B = 0.25(100 - a)$, where *a* is the number of other fishermen at the lake. The cost borne by any one fisherman sitting down to fish is C = 1, the opportunity cost of his/her time.

(a) If a person decides to fish when B>C, compute '*a'* such that another person decides to fish? **(6 marks)**

(b) What is the reduction in benefits to the first fisherman, if a second fisherman comes to the lake? What is the reduction in the total benefits of the first two fishermen, if a third fisherman comes to the lake? Use this information to compute the externality generated by one additional fisherman coming to the lake to fish. **(6 marks)**

(c) Use your answer in (b) to calculate the social optimal number of fishermen who should fish at the lake. Compare your answer to the answer in (a). Why do they differ? **(6 marks)**

(d) If the lake is a common resource to which access is free, what is the number of fishermen who will fish at the lake? If the lake is now managed by a profit-maximising private company that is allowed to limit the number of fishermen using the lake and charge an entry fee, what would be the level of this fee? What is the number of fishermen who will be allowed to fish at the lake and how does it compare with the social optimum? **(10 marks)**

7. Consider the Varian model of sales in which price dispersion features in equilibrium.

(a) Outline the assumptions in the model. Describe and show the method used to identify a Mixed Strategy Nash Equilibrium (MSNE). **(14 marks)**

(b) Show diagrammatically for some price p lying between the consumers' reservation price *r* and p*, where p* is the lower bound on prices in the MSNE, the profit of 'success' and the profit of 'failure'. What can we say about expected profits in the MSNE? **(4 marks)**

(c) If firms have identical linear costs C(q)=*c*q+*f* where q is a firm' output and *c*, *f*>0, show why the lowest price p* in a MSNE equals *r* when the proportion of informed consumers tends to zero. **(10 marks)**

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8. (a) Consider the following simultaneous move two-player game in which the payoff to the row player is always the first value in each cell. Assuming common knowledge of rationality, identify the unique solution in the game. Briefly explain your reasoning. **(6 marks)**

	L	M	R
U	2,2	1,1	4,0
D	1,2	4,1	3,5

(b) If $\mathbf{s^*}=\{S_1^*, S_2^*\}$ is a unique pure strategy solution in a two player game found by application of iterative elimination of strictly dominated strategies, show why it is also a Nash equilibrium. **(8 marks)**

(c) In the two-stage Kreps-Scheinkman capacity constrained duopoly model, efficient rationing is assumed and for certain parameter values a pure strategy subgame perfect equilibrium is shown to exist in which the firms choose Cournot capacities. Briefly describe the structure of the model and the need for assuming some form of rationing. If instead proportional rationing were assumed, speculate as to how this may affect firm behavior and the nature of the subgame perfect equilibrium. **(14 marks)**

(End)