

UNIVERSITY OF WARWICK

Summer Examinations 2015/16

Mathematical Techniques B

Time Allowed: 1.5 Hours, plus 15 minutes reading time during which notes may be made (on the question paper) BUT NO ANSWERS MAY BE BEGUN

Answer **ALL SEVEN** questions. Answer questions 1-4 in one booklet and questions 5-7 in a separate booklet.

Approved pocket calculators are allowed.

Read carefully the instructions on the answer book provided and make sure that the particulars required are entered on each answer book.

1. Find the stationary points of the following functions:

(a)

$$f(x, y) = x^3 - x^2y + y^2 \quad (6 \text{ marks})$$

(b)

$$g(x, y) = xye^{4x^2 - 5xy + y^2} \quad (12 \text{ marks})$$

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2. Radium is a radioactive element which decays at the rate of 0.04 percent per year. Compute the number of years (in a continuous fashion, rounded to the closest integer) that passes until one-half of an initial quantity x_0 of radium has decayed. **(12 marks)**
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3. Derive the formula for the quadratic approximation to a differentiable function $f(x)$ around $x = a$ where a is a specific value. Briefly explain your approach in words. **(15 marks)**
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(Continued overleaf)

4. Given constants α and β , a difference equation of the form

$$x_n = \alpha x_{n-1} + \beta$$

for $n = 0, 1, 2, 3, \dots$ is called a first-order linear difference equation. By repeatedly applying the previous equation, derive the value for x_n as a function of α , β and the initial value x_0 only. What happens in the special case of $\alpha = 1$? **(15 marks)**

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5. A differential equation contains terms involving the derivatives of the dependent variable y with respect to its argument x . Determine whether the function

$$y = 2e^{-x} + 3e^{2x}$$

is a solution to the differential equation

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + 2y.$$

(12 marks)

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6. Consider the equation system

$$\begin{aligned}xe^y + yf(z) &= a \\ xg(x, y) + z^2 &= b\end{aligned}$$

where $f(z)$ and $g(x, y)$ are differentiable functions, and a and b are constants. Suppose that the system defines x and y as differentiable functions of z . Find expressions for dx/dz and dy/dz . **(16 marks)**

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7. If $\mathbf{A} = (a_{ij})$ is an $n \times n$ matrix, the trace $tr(\mathbf{A})$ of \mathbf{A} is defined by $tr(\mathbf{A}) = \sum_{i=1}^n a_{ii}$. Thus, $tr(\mathbf{A})$ is the sum of the diagonal elements of \mathbf{A} . Show that if \mathbf{A} and \mathbf{B} are $n \times n$ matrices, then:

(a) $tr(\mathbf{A} + \mathbf{B}) = tr(\mathbf{A}) + tr(\mathbf{B})$. **(4 marks)**

(b) $tr(c\mathbf{A}) = c tr(\mathbf{A})$ where c is a scalar. **(4 marks)**

(c) $tr(\mathbf{A}') = tr(\mathbf{A})$. **(4 marks)**

(End)