

# **UNIVERSITY COLLEGE LONDON**

## **EXAMINATION FOR INTERNAL STUDENTS**

**MODULE CODE : ECON3003**

**ASSESSMENT : ECON3003A**  
**PATTERN**

**MODULE NAME : Econometrics for Macroeconomics and Finance**

**DATE : 05 May 2016**

**TIME : 10:00 am**

**TIME ALLOWED : 2 hours**

This paper is suitable for candidates who attended classes for this module in the following academic year(s):

**2014/15 and 2015/16**

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SUMMER TERM EXAMINATIONS 2016  
ECON3003: Econometrics for Macroeconomics and  
Finance

TIME ALLOWANCE: 2 HOURS

**Answer SIX questions: All FOUR from part A and TWO questions from part B. Questions in Part A carry 60% of the total mark and questions in Part B carry 40% of the total mark.**

Important Notes: In cases where a student answers more questions than requested by the examination rubric, the policy of the Economics Department is that the student's first set of answers up to the required number will be the ones that count (not the best answers). All remaining answers will be ignored.

At the end of the paper, critical values for the  $N(0, 1)$ ,  $F$  distribution and Dickey-Fuller unit root test are tabulated.

Part A  
Answer all **four** questions.  
The marks of each question are provided in brackets.

A.1 [15 PTS]. You have collected data of a time series  $Y_t$  and wish to model its time series dependence as described by its autocorrelation function,  $\rho_Y(k) = \text{Corr}(Y_t, Y_{t-k})$ .

(i) Suppose that the time series solves the following autoregressive (AR) model of order 2,

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t,$$

where  $\varepsilon_t$  is i.i.d. with  $E[\varepsilon_t] = 0$  and  $E[\varepsilon_t^2] = \sigma_\varepsilon^2$ . State a sufficient condition for the process to be stationary and mixing. Why is stationarity and mixing an important property in modelling and estimation of time series?

(ii) Suppose that the AR process  $Y_t$  as given in (i) indeed is stationary and mixing. Show that  $\rho_Y(k)$  satisfies

$$\rho_Y(k) = \phi_1 \rho_Y(k-1) + \phi_2 \rho_Y(k-2), \quad k \geq 2.$$

(iii) Suppose instead that  $Y_t$  solves the following moving average (MA) model of order 2,

$$Y_t = \mu + \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2},$$

where  $\varepsilon_t$  is as in (i). Under which conditions is this MA process stationary? Suppose that the MA process indeed is stationary; show that  $\rho_Y(k)$  in this case satisfies

$$\rho_Y(1) = \frac{\alpha_1 + \alpha_1 \alpha_2}{1 + \alpha_1^2 + \alpha_2^2}, \quad \rho_Y(2) = \frac{\alpha_2}{1 + \alpha_1^2 + \alpha_2^2},$$

and  $\rho_Y(k) = 0, k > 2$ .

A.2 [15 PTS]. You wish to analyze whether it's possible to affect people's decision to have children through taxes. Using annual US data of the fertility rate (#children born per 1,000 women of childbearing age),  $FR$ , and the personal tax exemption for having a child (in US Dollars),  $EX$ , from 1913 to 2015, you obtain the following regression fit,

$$FR_t = 73.68 + 0.075EX_t - 0.0051EX_{t-1} + 0.039EX_{t-2} + \hat{\varepsilon}_t, \quad \bar{R}^2 = 0.41 \quad (1)$$

(3.21) (0.124)      (0.1550)      (0.121)

where  $\hat{\varepsilon}_t$  is the residual, and heteroskedastic-robust standard errors are reported in brackets.

(i) Under which condition can we interpret the above estimated regression coefficients as dynamic causal effects? Is this condition likely to be satisfied in the regression model (1)? Explain.

(ii) Plot the estimated dynamic multipliers as a function of time for the first six years together with their 95% confidence bands. Plot the corresponding estimated cumulative dynamic multipliers as a function of time. Interpret the two figures.

(iii) In order for the reported standard errors in (1) to be correct (i.e., consistent), what does the error term  $\varepsilon_t$  need to satisfy? Is this assumption likely to be satisfied here? How would you adjust the standard errors in case the assumption is violated?

A.3 [15 PTS]. You worry about structural breaks in the regression model (1) and so decide to include three time dummies,  $WW1$ , which is one during World War 1, 1914-18,  $WW2$ , which is one during World War 2, 1941-1945, and  $PILL$ , which is one from 1963, the year when the birth control pill became available, and onwards. This yields the following fit,

$$FR_t = 95.68 + 0.073EX_t - 0.0058EX_{t-1} + 0.034EX_{t-2} \quad (2)$$

(3.28) (0.126) (0.1557) (0.126)

$$-4.32WW1_t - 18.13WW2_t - 31.30PILL_t + \hat{\varepsilon}_t,$$

(2.67) (7.89) (3.98)

with  $\bar{R}^2 = 0.46$ .

(i) Interpret the estimated coefficients for  $WW1$ ,  $WW2$  and  $PILL$ .

(ii) State the null hypothesis that there was no structural break in fertility during World War 1, and write up the test statistic you would use. Report the value of the test statistic and its corresponding  $p$ -value. Carry out the same analysis for World War 2 and the period after the birth control pill was introduced. Conclude.

(iii) Suppose that the impact of  $EX$  on  $FR$  changed after the introduction of the birth control pill. How would you adjust the above regression model (if at all) to take this into account? Explain.

A.4 [15 PTS].

(i) You are interested in forecasting the US fertility rate for 2016 and decide to use the above regression model (2). Do you have to be concerned about endogeneity of  $EX$  for this purpose? Explain.

(ii) Suppose that your last three observations are  $EX_{2013} = \$238$ ,  $EX_{2014} = \$230$  and  $EX_{2015} = \$228$ . Provide a point forecast of the fertility rate in 2016.

(iii) Suppose that you wish to forecast fertility for 2017. Can you use (2) for this? Explain. How would you modify and/or extend the model to obtain such a forecast?

## PART B

Answer **2** questions out of 3 questions.

The marks of each question are provided in brackets.

**B.1 [20 PTS].** You wish to examine whether a given company's stock returns are predictable or not. To this end, you have collected annual data of the asset return,  $R_t$ , together with an additional possible predictor, the company's earnings in year  $t$ ,  $E_t$ .

(i) State formally the hypothesis that the company's return,  $R_t$ , cannot be predicted by past returns of the company stock,  $R_{t-1}, R_{t-2}, \dots$ , together with past earnings,  $E_{t-1}, E_{t-2}, \dots$

(ii) You decide to implement the so-called variance ratio test for testing predictability. Write up the precise form of the variance ratio test statistic, state the null that it is testing, and explain how you would carry out the test. (you are here not required to provide the precise expression of the standard errors)

(iii) Suppose that the variance-ratio test fails to reject the null. Does that mean that the hypothesis in (i) is satisfied? Explain.

(iv) A friend recommends that instead of using the variance-ratio test, you should use the following regression model,

$$R_t = \mu + \sum_{i=1}^K \phi_i R_{t-i} + \sum_{i=1}^L \beta_{E,i} E_{t-i} + \varepsilon_t,$$

for suitably chosen values of  $K, L \geq 1$ , to test the hypothesis in (i). How would you test for return predictability using this regression? Suppose your test based on the above regression model fails to reject the null. Does that mean the hypothesis in (i) holds? Explain.

**B.2 [20 PTS].** Consider the CAPM for a given asset,

$$E[\bar{R}_t | \mathcal{F}_{t-1}] = \beta_t \bar{R}_{m,t},$$

where  $\bar{R}_t$  and  $\bar{R}_{m,t}$  are the excess returns of the asset and the market portfolio, respectively, at time  $t$  and  $\mathcal{F}_{t-1}$  denotes the information available to the investors at time  $t - 1$ .

(i) Write up the definition of  $\beta_t$  in terms of the conditional covariance between  $\bar{R}_t$  and  $\bar{R}_{m,t}$  and the conditional variance of  $\bar{R}_{m,t}$ . Provide an interpretation of  $\beta_t$  as a measure of risk relative to the market portfolio.

(ii) To learn about  $\beta_t$  you compute the OLS estimator  $\hat{\beta}$  obtained from the following regression model,

$$\bar{R}_t = \hat{\alpha} + \hat{\beta} \bar{R}_{SP,t} + \hat{\varepsilon}_t,$$

where  $\bar{R}_{SP,t}$  is the S&P 500 stock index, and  $\hat{\varepsilon}_t$  is the residual. Under which conditions is  $\hat{\beta}$  a consistent estimator of  $\beta_t$ ?

(iii) You are concerned that in fact  $\beta_t$  is not constant and does vary over time. You speculate that  $\beta_t$  satisfies

$$\beta_t = b_0 + b_1 Z_t, \quad (3)$$

where  $Z_t$  is growth in US consumption, which you have collected data on. How would you estimate  $\beta_t$  assuming eq. (3) is correct?

(iv) A friend of yours recommends to use a rolling-window estimator to learn about  $\beta_t$ . Explain how the rolling-window estimator is computed. Discuss the merits of the rolling-window estimator of  $\beta_t$  relative the one you obtained in (iii).

B.3 [20 PTS]. Consider the following ARCH(1) model for the return of an asset,  $R_t$ :

$$R_t = \mu + \sigma_t \varepsilon_t,$$

where  $\varepsilon_t$  is i.i.d. with  $E[\varepsilon_t] = 0$ ,  $E[\varepsilon_t^2] = 1$  and  $E[\varepsilon_t^4] < \infty$ , and

$$\sigma_t^2 = \omega + \alpha (R_{t-1} - \mu)^2. \quad (4)$$

(i) Show that the autocorrelation function of  $R_t$ ,  $\rho_R(k)$ , satisfies  $\rho_R(k) = 0$  for all  $k \geq 1$ .

(ii) Suppose that  $\alpha < 1$ . Derive an expression of the autocorrelation function of  $(R_t - \mu)^2$ ,  $\rho_{(R-\mu)^2}(k)$ .

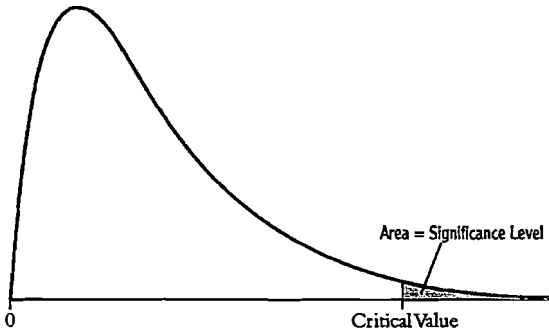
(iii) Suppose that  $\varepsilon_t \sim N(0, 1)$ . Write up the log-likelihood function for the above ARCH(1) model, which we denote  $L_T(\mu, \omega, \alpha)$ , given  $T \geq 1$  observations of  $R_t$ .

(iv) How are the maximum-likelihood estimators of  $\mu$ ,  $\omega$  and  $\alpha$  defined in terms of  $L_T(\mu, \omega, \alpha)$ ? Why are these reasonable estimators of the parameters?



Normal Deviate z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-4.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.7	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483

**TABLE 4** Critical Values for the  $F_{m,\infty}$  Distribution



Degrees of Freedom	Significance Level		
	10%	5%	1%
1	2.71	3.84	6.63
2	2.30	3.00	4.61
3	2.08	2.60	3.78
4	1.94	2.37	3.32
5	1.85	2.21	3.02
6	1.77	2.10	2.80
7	1.72	2.01	2.64
8	1.67	1.94	2.51
9	1.63	1.88	2.41
10	1.60	1.83	2.32
11	1.57	1.79	2.25
12	1.55	1.75	2.18
13	1.52	1.72	2.13
14	1.50	1.69	2.08
15	1.49	1.67	2.04
16	1.47	1.64	2.00
17	1.46	1.62	1.97
18	1.44	1.60	1.93
19	1.43	1.59	1.90
20	1.42	1.57	1.88
21	1.41	1.56	1.85
22	1.40	1.54	1.83
23	1.39	1.53	1.81
24	1.38	1.52	1.79
25	1.38	1.51	1.77
26	1.37	1.50	1.76
27	1.36	1.49	1.74
28	1.35	1.48	1.72
29	1.35	1.47	1.71
30	1.34	1.46	1.70

This table contains the 90<sup>th</sup>, 95<sup>th</sup>, and 99<sup>th</sup> percentiles of the  $F_{m,\infty}$  distribution. These serve as critical values for tests with significance levels of 10%, 5%, and 1%.



**TABLE 14.5** Large-Sample Critical Values of the Augmented Dickey-Fuller Statistic

Deterministic Regressors	10%	5%	1%
Intercept only	-2.57	-2.86	-3.43
Intercept and time trend	-3.12	-3.41	-3.96