EXAMINATION FOR THE DIPLOMA IN ECONOMICS

Monday 4 June 2012 9.00 - 12.00

Paper 1
MICROECONOMICS

This paper is divided into two sections. Answer **four** questions in total, with at least one question from each section.

Write *your candidate number* ***not*** *your name on the cover sheet of* ***each*** *booklet*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
20 Page booklet x 2	Approved Calculator allowed
Rough work pads	
Tags	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

A1 (a) Use the Ricardian model of international trade to evaluate the following statements:

(i) "Trade is beneficial only if your country is strong enough to stand up to foreign competition."

(ii) "Trade with countries that pay low wages hurts high wage countries and is unfair."

(b) Discuss the implications of low-skilled worker immigration into the UK for UK wages, trade and output within the framework of the Heckscher-Ohlin model.

A2 Consider an exchange economy with two individuals, A and B, and two consumption goods, x and y. A has utility function $u_A(x_A, y_A, x_B) = \ln(x_A + x_B) + y_A$ and B has utility function $u_A(x_B, y_B) = \sqrt{x_B} + y_B$, where x_i and y_i are the amounts of good x and y consumed by individual $i = A, B$. Endowments are $(e^x_A, e^y_A) = (1, 6)$ and $(e^x_B, e^y_B) = (0, 2)$. Set the price of y equal to 1 and denote by p the price of good x.

(a) Interpret the utility functions of the two individuals and derive their demand functions.

(b) Solve for the competitive equilibrium price p and the equilibrium allocations.

(c) Argue that any Pareto efficient allocation in this economy has a unique distribution of the consumption good x between consumers A and B. What are the Pareto efficient allocations of good y? Represent all Pareto efficient allocations in the Edgeworth Box diagram.

(d) Is the competitive equilibrium allocation derived in (b) Pareto efficient or not? Explain why. If it is not efficient what could society do to obtain a Pareto efficient competitive equilibrium (a discussion suffices here)?

A3 Consider a society with two consumers A and B, and two goods, x and y. Denoting by x_i, y_i the amounts of goods x and y consumed by individual i, assume that utility functions are given by

$$\begin{aligned} u_A(x_A, y_A) &= \min(x_A, 2y_A), \\ u_B(x_B, y_B) &= \min(2x_B, y_B) \end{aligned}$$

and demand functions are given by

$$\begin{aligned} x_A &= \frac{2m_A}{2p_x + p_y}, \quad y_A = \frac{m_A}{2p_x + p_y} \\ x_B &= \frac{m_B}{p_x + 2p_y}, \quad y_B = \frac{2m_B}{p_x + 2p_y}. \end{aligned}$$

Individual A's income is $m_A = 18$, and individual B's income is $m_B = 54$. The government wants to evaluate the effect of a price change on society from an initial price vector $p = (p_x, p_y) = (\frac{3}{4}, 3)$ to a new price $p' = (p'_x, p'_y) = (4, 1)$.

(a) (i) What is an indirect utility function? What are the indirect utilities $v_i(p, m_i)$ and $v_i(p', m_i)$ for individuals $i = A, B$ at the two prices p and p'? Are A and B better or worse off with p' compared to p.

(ii) What are the Compensating Variations $CV_i(p, p')$ of the above price change for individuals $i = A, B$?

(b) Assume that the government uses the following Samuelson-Bergson social welfare function defined on individuals' indirect utility functions to measure society's welfare at price p:

$$W(p) := \frac{1}{2}v_A(p, m_A) + \frac{1}{2}v_B(p, m_B).$$

Which price does society prefer under this measure? What are the assumptions about measurability and comparability of individual utility functions that underlie this welfare function?

(c) Assume instead that the government evaluates society's welfare as follows. It computes the sum of the Compensating Variations of the two prices. If the sum is positive, it considers price p' to be better for society than price p, and vice versa if the sum is negative. Using this measure, which price does society prefer? Interpret this welfare measure and comment on its underlying assumptions about measurability and comparability of individual utility functions.

(d) Will the rankings under the two measures in (b) and (c) change if individual A's utility remains unchanged but individual B's utility function is instead given by $\hat{u}_B = 4u_B$?

A4 Robinson Crusoe lives alone on an island. He has a total number of H hours each day available, which he spends either sleeping or fishing. He cannot catch more than one fish a day but to catch that fish he needs to spend at least L hours fishing. Robinson likes to sleep and to eat fish and his utility over both goods is described by the utility function $u(s,f) = sf$, where s is the number of hours Robinson sleeps and f is the amount of fish he eats. Denote by p the price of fish and by w the price of sleep.

(a) Represent this island economy graphically and carefully describe what conditions must hold in a competitive equilibrium.

(b) Under what condition does an equilibrium exist in this economy? If it exists what is the relative equilibrium price $\frac{p}{w}$ and what is the equilibrium allocation?

(c) Assume now that Robinson instead regards fish and sleep as perfect substitutes and that his utility is $u(s,f) = \alpha s + \beta f$, where $\frac{\alpha}{\beta} = \frac{1}{L}$. Show graphically that there are two competitive equilibria. What will the allocations and relative prices be in these equilibria?

(d) Give conditions that ensure existence and uniqueness of a competitive equilibrium.

SECTION B

B1 A firm is considering building a Fusion power plant that will provide power to a region. They make a rough calculation using CAPM. In the future there are two states of the world. High growth H, or low growth L; each happens with probability $\frac{1}{2}$. Profit Π depends on the state. In state H, the world is energy hungry, the demand function for electricity is high, and the firm will sell electricity, making profit Π_H. In state L, the demand for electricity is low, and the firm will make a profit Π_L. The average global market portfolio will return M, which is $M_H = 7$ in state H, and $M_L = 1$ in state L. This is assumed to be independent of the firm's investment decision. There is a risk free asset which will return $r = 2$.

(a) (i) What is the expected return of the market portfolio, $\mathbb{E}[M]$, and what is its variance $\mathrm{Var}[M]$?

(ii) What is the rate of return R of the fusion plant's stock as a function of Π and v, where v is the present value of the company stock? What is expected return $\mathbb{E}[R]$?

(iii) With $\mathrm{Cov}(R, M) = \frac{3}{2v}(\Pi_H - \Pi_L)$, what is the market β for the fusion plant's stock?

(iv) What is the CAPM equation relating R and M?

(b) Using values $\Pi_H = £6 \times 10^{11}$ (£600 billion), $\Pi_L = £0$, value the fusion investment (i.e., find v). Is it worth an investment of £60 billion?

(c) If the market portfolio was more volatile, would that increase or decrease the value of the investment?

B2 A new firm manufactures widgets. The firm can be of type B or D. If it is of type D, the widgets it manufactures are durable, while if it is of type B, they are breakable. Each widget sold, independent of its type, results in a profit of £1. The firm's manager knows the type of the firm and can engage in advertising at a level $\alpha \geq 0$ at cost $c(\alpha) = \frac{1}{2}\alpha^4$. This informs α consumers of the existence of the product.

First, suppose the firm's type is known to consumers. If the firm is of type B, no consumers buy. If the firm is of type D, α consumers buy, and they tell their friends, resulting in α more purchases.

(a) What are the profits of the firm if it is type B, as a function of α? What are the profits if it is type D? What is the optimal choice of α for each type, and the resulting profits?

Now suppose that consumers do not know the type of the firm initially. They observe the amount of advertising α and use it as information about the type of the firm and what product to expect. If they observe advertising α, they think the product is durable with probability $p(\alpha)$. This results in $\alpha p(\alpha)$ initial purchases. If it turns out that the widget is breakable, then no recommendations are made and there are no further purchases. Otherwise, another α consumers buy upon recommendations of their friends.

(b) Show that the strategies found in (a) are no longer an equilibrium.

(c) In a separating equilibrium, a firm of type B advertises at level α_B, and a firm of type D advertises at level $\alpha_D \neq \alpha_B$. What are the conditions for α_B and α_D to form an equilibrium? What is the best separating equilibrium for a firm with process D? How do strategies compare to part (a)?

(d) Intuitively, why can advertising be a signal of product quality?

B3 A consumer has the option of investing in a pharmaceutical company's shares. With probability p the company makes a breakthrough and the return is $R = 2$. With probability $1 - p$ it fails, with return $R = 0$. The consumer's initial wealth is w. He puts any money not invested into the company under his mattress, making return $r = 1$. The consumer has von Neumann-Morgenstern utility function $u(x) = \log x$.

(a) What is the threshold $k \in [0, 1]$, such that for $p > k$ he invests a positive amount in the company? How much does he optimally invest?

In what follows, assume $w = \frac{1}{2}$ and $p > k$.

(b) Let $f(p)$ be the expected utility resulting from the optimal investment choice. Show that $f(p)$ is convex.

(c) Assume that before the consumer invests, he can receive some gossip about the company. The gossip can be good (G) or bad (B). Each type of gossip is heard with probability $\frac{1}{2}$. If the gossip is good, the firm will have probability of success $p + \epsilon$; if bad, $p - \epsilon$. What is the consumer's strategy now (assume $p - \epsilon > k$)?

(d) Show that the value of access to the gossip is

$$h(\epsilon) = \frac{1}{2} f(p + \epsilon) + \frac{1}{2} f(p - \epsilon) - f(p).$$

Is $h(\epsilon)$ positive? Show that $h'(0) = 0$ and interpret this result.

B4 A risk neutral corporation contracts with a builder for a new block of offices. The quality of the work depends on the builder's investment. If he invests a monetary amount I, the quality of the building will be $Q = \sqrt{I} + \epsilon$, where ϵ is an (independent) noise term with distribution $N(0, \sigma^2)$. Assume that the builder, when facing a normally distributed risk X, has utility $U(X) = \mathbb{E}[X] - \frac{1}{2}\rho \mathrm{Var}(X)$, where $\rho > 0$. The corporation offers a linear incentive scheme to encourage investment by the builder, so that the payment for the offices is $P = \alpha + \beta Q$. Therefore, the builder's final monetary gain is $\alpha + \beta Q - I$.

(a) Given α and β, what is the builder's utility if he invests I? What is the optimal I? What are resulting profits for the builder? When is it better for him not to accept the contract?

(b) The corporation's von Neumann-Morgenstern utility function is $Q - P$. What is the utility of the corporation resulting from a choice of (α, β)?

(c) The corporation chooses (α, β) to maximize its utility, while giving the builder a reservation level of utility $\bar{u}$, so that the builder accepts the contract. Find the optimal performance related component β. Comment on the dependence on ρ and σ^2.

(d) Assume now that the corporation is risk averse and the builder risk neutral. What would be the optimal β for the corporation to propose?

END OF PAPER

UNIVERSITY OF
CAMBRIDGE

ECD1
DIPLOMA IN ECONOMICS

Monday 3 June 2013 9.00 - 12.00pm

Paper 1

MICROECONOMICS

You should answer **five** questions in total, with at least one question from each Section.

Each Section carries equal weight.

Write your **candidate number** not your name on the cover of each booklet.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

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SECTION A: General Equilibrium and Trade

1. (a) State Rybczynski's theorem and provide an intuitive proof.

   (b) Country A and Country B each have two factors of production, capital and labour, with which they produce two goods, x and y. Technology is the same in the two countries, characterized by constant returns to scale and strictly increasing production functions with decreasing marginal products. Good x is labour intensive and good y is capital intensive. Country A is capital abundant, Country B is labour abundant. Consumers' preferences are identical. The two countries trade with each other. Analyse the effect of imposing a 35 hours working week in Country A.

2. Consider an exchange economy with two consumers, A and B, and two consumption goods, x and y. Consumer A has utility function $u^A(x_A, y_A) = \ln x_A + y_A$, and consumer B has utility function $u^B(x_B, y_B) = x_B^{2/3} y_B^{1/3}$, where x_i and y_i are the amounts of good x and y consumed by individual $i = A, B$. Endowments are $(e_x^A, e_y^A) = (7, 2)$ and $(e_x^B, e_y^B) = (1, 4)$. Denote prices by p_x and p_y respectively.

   (a) Derive the consumers' demand functions and find expressions for the excess demand functions of goods x and y.

   (b) Solve for the competitive equilibrium prices p_x and p_y and the equilibrium allocations.

   (c) Now assume that consumer A's utility function is $u^A(x_A, y_A) = (x_A+1)^2 + y_A$ instead, while consumer B's utility function is unchanged.

      i. Derive an expression for consumer A's marginal rate of substitution. Does A have convex preferences?

      ii. Do you expect a competitive equilibrium to exist in this economy? Explain your answer.

SECTION B: Welfare

3. Consider an economy consisting of two individuals, A and B, who consume one private and one public good. The utility function of individual h, $h = A, B$, is $u^h(x_h, G) = \frac{1}{2}\ln x_h + \frac{1}{2}\ln G$, where x_h is the amount of the private good individual h consumes and G is the amount of the public good provided. Each individual is initially endowed with e_h units of the private good and none of the public good. There is a production technology that transforms p units of the private good into one unit of the public good.

   (a) Assume that competitive markets operate in which individuals A and B can trade the private and the public good at their respective prices, i.e., 1 for the private good and p for the public good. Denote by g_h the amount of the public good individual h decides to buy, so that $g_A + g_B = G$ is the total amount of the public good available in the economy. A consumer makes his decision on how much to buy of both goods based on his prediction of how much the other consumer buys of the public good. Set up consumer A's utility maximisation problem and derive the first-order conditions for an interior solution. Using those, find the optimality condition that must hold in a competitive equilibrium linking A's marginal rate of substitution to the goods' prices. Find a similar expression for individual B (use your answer for individual A). Interpret those conditions.

   (b) Assume now that a social planner decides on how much of the private good each individual consumes and how much of the public good will be made available. He wishes to maximise a utilitarian social welfare function of the following form
$$u^A(x_A, G) + u^B(x_B, G).$$
Set up his maximisation problem and find the first-order conditions for an interior solution for this problem. By combining the first-order conditions find the Samuleson Rule. Compare it to the solutions you found in (a). Are they the same? If not, why not?

4. Three individuals Ann, Bob and Colin would like to determine a group preference ordering over two possible joint projects X and Y, that is, they would like to find a function that maps every possible profile of individual preference orderings over X and Y into a group preference ordering.

   (a) Will **A**, **B** and **C** be able to define an Arrow social choice rule satisfying the four properties required by Arrow? Carefully explain your answer with reference to Arrow's Impossibility Theorem.

   (b) Denote the possible (strict) preference profiles over X and Y as in the following table; for example, in profile I everybody prefers X over Y. Assume that the social choice function $F(\cdot)$ selects the group preference ordering given by the bottom line:

	I	II	III	IV	V	VI	VII	VIII
A	$X \succ_A Y$	$X \succ_A Y$	$X \succ_A Y$	$Y \succ_A X$	$X \succ_A Y$	$Y \succ_A X$	$Y \succ_A X$	$Y \succ_A X$
B	$X \succ_B Y$	$X \succ_B Y$	$Y \succ_B X$	$X \succ_B Y$	$Y \succ_B X$	$X \succ_B Y$	$Y \succ_B X$	$Y \succ_B X$
C	$X \succ_C Y$	$Y \succ_C X$	$X \succ_C Y$	$X \succ_C Y$	$Y \succ_C X$	$Y \succ_C X$	$X \succ_C Y$	$Y \succ_C X$
$F(\cdot)$	$X \succ Y$	$X \succ Y$	$Y \succ X$	$X \succ Y$	$Y \succ X$	$X \succ Y$	$Y \succ X$	?

   Is it possible to define $F(\text{VIII})$, such that all four properties are fulfilled?

   (c) Assume now that there is an additional, third alternative Z, and consider the following two possible preference profiles:

	I	II
A	$X \succ_A Y \succ_A Z$	$Y \succ_A X \succ_A Z$
B	$X \succ_B Y \succ_B Z$	$Y \succ_B Z \succ_B X$
C	$X \succ_C Z \succ_C Y$	$X \succ_C Z \succ_C Y$

   Assume that $F(\text{I}) = X \succ Z \succ Y$. What does this imply for $F(\text{II})$ if the Independence of Irrelevant Alternatives property is fulfilled?

SECTION C: Game Theory

5. Consider the following game, in which two players simultaneously choose an action, i.e., player 1 chooses X, Y or Z, and player 2 chooses A, B, C, or D. Payoffs to player 1 and 2 are given by the first, respectively second entry in each cell.

	A	B	C	D
X	0,1	2,1	3,3	4,1
Y	0,0	4,2	1,1	8,0
Z	3,0	3,1	0,1	7,2

   (a) What is a *strictly dominated strategy* and what is meant by *iterated deletion of strictly dominated strategies?* Apply iterated deletion of strictly dominated strategies in the above game.

   (b) Give the definition of a Nash Equilibrium. Solve for ALL Nash Equilibria.

   (c) Assume that players move sequentially instead of simultaneously. That is, first player 1 chooses and action, and after having observed player 1's choice player 2 chooses an action. What is the subgame perfect Nash equilibrium in this game?

6. Max and Moritz both like cake and consider two procedures to share a cake.

   (a) They simultaneously call out a demand for (part of) the cake. If they ask for different amounts, the higher demand is satisfied, and the other person gets the remaining cake. If they ask for the same amount, they both obtain what they demanded if the demands add up to less than 1. If total demand is more than 1 the cake is split equally. What are Max and Moritz' best response functions (you can restrict attention to individual demands that are smaller or equal to 1)? What will Max and Moritz demand in the Nash equilibrium? How much of the cake will each of them get?

   (b) Max is first allowed to demand an amount $x_1 \in [0, 1]$ of the cake, which Moritz can accept or reject. If Moritz accepts, Max receives his demand x_1 and Moritz gets the remaining cake. If Moritz rejects, a part $(1 - \delta)$ of the cake is thrown away, where $\delta < 1$. Moritz can then make a demand $x_2 \in [0, \delta]$, which Max can accept or reject. If Max accepts, Moritz receives his demand x_2 and Max gets what is left of the cake. If Max rejects, they both receive nothing. How will they divide the cake in a subgame perfect Nash equilibrium? Would your answer differ if they played a Nash equilibrium?

   (c) Which procedure, the one in (a) or in (b), does Max prefer if they play a subgame perfect Nash equilibrium in (b)?

SECTION D: Uncertainty

7. (a) Mr Riskaverse is an expected utility maximizer: He evaluates lotteries $\{(p_1, p_2, p_3) : p_1 + p_2 + p_3 = 1\}$ on $\{\$1, \$2, \$3\}$, using the following expected utility function

$$U(p_1, p_2, p_3) = \sum_{i=1}^{3} p_i u(\$i),$$

where $u(\$1) = 1$, $u(\$2) = 10$ and $u(\$3) = 11$.

i. Represent Mr Riskaverse's preferences on a Machina Triangle.

ii. Show that Mr Riskaverse's preferences satisfy the Independence Axiom.

(b) Mrs Insured has a CARA vNM utility function $-e^{-m}$. According to her local bankruptcy laws, if she gets into debt, then all debts are paid for by the government, and she ends up with wealth 0. Her vNM utility function is then effectively

$$u(m) = \begin{cases} -e^{-m} & m \geq 0 \\ -1 & m < 0 \end{cases}$$

Suppose that Mrs Insured has wealth $w > 0$. She considers gambling an amount of money $g \geq 0$; with probability 1/2 she wins the amount g, with probability 1/2 she loses it.

i. Explain why, if she can choose to gamble any amount $g \leq w$, she will optimally choose $g = 0$.

ii. Explain why, if she can choose to gamble any amount $g > w$, she will optimally choose $g = \infty$.

iii. Is this type of bankruptcy law sensible?

8. In an Arrow-Debreu economy, there are a number of identical agents, with vNM utility functions $u(x) = \ln(x)$. (In such an economy, no trading actually occurs, but there are still prices.) Two independent events, 'Djokovic wins US Open' and 'World economy recovers', occur with probabilities $\pi_D = 60\%$ and $\pi_R = 50\%$ respectively. Overall, there are four possible states, ω_{ij}, $i = D, \bar{D}$ and $j = R, \bar{R}$, where D stands for 'Djokovic wins', $\bar{D}$ stands for 'Djokovic does not win', R stands for 'World economy recovers' and $\bar{R}$ stands for 'World economy does not recover'. Endowments in the four states are:

state	Endowment of each agent
ω_{DR}	5
$\omega_{D\bar{R}}$	4
$\omega_{\bar{D}R}$	5
$\omega_{\bar{D}\bar{R}}$	4

(a) What are the Arrow Debreu securities in this economy? Using MRS conditions, calculate the prices q_{DR}, $q_{D\bar{R}}$, $q_{\bar{D}R}$, $q_{\bar{D}\bar{R}}$ of those securities, using the normalization $q_{DR} + q_{D\bar{R}} + q_{\bar{D}R} + q_{\bar{D}\bar{R}} = 1$. Give your answers to 3 decimal places.

(b) What is the price q_D of an asset that pays 1 if Djokovic wins the US Open? What is the price q_R of an asset that pays 1 if the world economy recovers? Comment.

9. A manager needs a new website for his business. He does not know how difficult this task is: it can take either $T = 1$ months or $T = 3$ months, both with equal probability. The manager would like to ensure that the project is done, but minimise the expected payment. He goes to see an IT contractor, who knows the length of time it will take to finish the project. The contractor's utility function is $u(T, m) = -10T + m$, where m is the remuneration. His reservation utility is 0.

   (a) The gullible manager believes everything he is told. The contractor however is truthful only when it pays. The manager asks the contractor how long the project will take, and offers a contract that gives the contractor at least his reservation utility. How much will the manager offer if he is told $T = 1$ or $T = 3$? Will the contractor reveal how difficult the task is? How much will the manager pay?

   (b) Before meeting the contractor, the manager is informed that some people could be lying to him. His first thought is to offer a pair of contracts m_1 and m_3 so that the contractor will choose m_1 if $T = 1$ and m_3 if $T = 3$. Why does this not reduce the expected payment?

   (c) His next idea is to require the contractor to come into the office for a minimum amount of time τ. He knows that the contractor has disutility of 10 for every amount of time spent working on the project but only a disutility of 5 for an amount of time spent idle in the office. So, the contractor's utility is

$$u(T, \tau, m) = \begin{cases} -10T + M & T \geq \tau \\ -10T - 5(\tau - T) + m & T < \tau. \end{cases}$$

   The manager offers contracts $(\tau_1 = 1, m_1)$ and $(\tau_3 = 3, m_3)$ in the hope that if $T = i$ the contractor chooses (τ_i, m_i) for $i = 1, 3$.

   i. Why might this approach help deal with the manager's problem? What are the optimal contracts $(\tau_1 = 1, m_1)$ and $(\tau_3 = 3, m_3)$?

   ii. In the adverse selection model covered in lectures, it is optimal to distort the contracts of one of the types. Explain why this is and why it is not optimal here to distort one of the τ_is?

10. The owner of a firm is looking for a manager and drawing up terms of remuneration. He was not satisfied with the previous manager's performance. The owner is used to giving fixed salaries, but would like to follow some of his competitors and introduce performance-related pay. He thinks that the firm's profits should be a good measure of the manager's performance, but is not sure how much of the pay should be performance-related. Give the owner some advice.

END OF PAPER

ECD1
Advanced Diploma in Economics

Monday 2 June 2014 9:00am to 12:00pm

Paper 1

Microeconomics

*This paper is in five sections. Candidates are required to answer **five** questions in total, with one question from each section.*

Each section carries equal weight.

Credit will be given for clear presentation of relevant statistics.

*Write your number **not** your name on the cover sheet of **each** booklet.*

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20 Page Answer Booklet x 1
Rough Work Pad

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Calculator – students are permitted to bring an approved calculator

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Section A

1. Consider the following two models of international trade. Answer both parts of the question.

   (a) Assume that in a Ricardian world there are 3 countries, which produce Apples and Oranges, using labour as their only input. What can we say about the pattern of production in this world? Draw the production possibility frontier and discuss what the relative price of Apples to Oranges could be.

   (b) Consider a world with two countries, Home and Foreign. They both produce two goods using capital and labour as inputs: a labour intensive good X_1 and a capital intensive good X_2. Assume that the production technologies are constant returns to scale and that there is no labour mobility between countries. Assume further that Home is labour abundant and small, whereas Foreign is capital abundant and large. Discuss the effect of an import subsidy in the Home country on wages, output prices and trade. How would your answer change if Home was a large rather than a small country?

2. Robinson Crusoe lives alone on an island. He has T number of hours each day that he can either devote to work producing coconuts or consume as leisure. His utility function over those two commodities is given by $u(l, c) = \alpha \ln l + c$, where l is leisure, c is coconuts and $\alpha > 0$. His production function of coconuts is $f(L) = \ln(L + 1)$, where L is the amount of hours he works per day. Set the price of coconuts equal to 1, and denote the wage rate by w.

   (a) i. Set up the producer's problem for Robinson and find the demand for labour and the supply of coconuts as a function of the wage rate w. (Careful: your answer will depend on whether $w \leq 1$ or $w > 1$). Calculate the profit function.

   ii. Set up the consumer's problem for Robinson and find the demand for leisure and coconuts as a function of the wage rate.

   (b) i. Look for a competitive equilibrium with $w \leq 1$ and show that there is such an equilibrium if $\alpha \leq T$. What is the equilibrium wage and allocation in this equilibrium?

   ii. If $\alpha > T$, what would you expect the equilibrium wage and allocation to be?

Section B

3. Consider two different price vectors $p = (p_x, p_y)$ and $p' = (p'_x, p'_y)$, with $p_y = p'_y$, and a consumer with income m. We are interested in comparing the consumer's welfare at these two price vectors.

   (a) Define the notions of (i) (change in) consumer surplus, (ii) compensating variation and (iii) equivalent variation, and contrast the three.

   (b) Compare and rank the notions of consumer surplus, compensating variation and equivalent variation in terms of integrals of the Marshallian and Hicksian demand function. Draw a graph.

   (c) Assume that the consumer's indirect utility function is given by $v(p, m) = \frac{m^2}{4p_x p_y}$. Using results from duality theory, find the Marshallian demand functions of this consumer. Calculate the change in consumer surplus for a price movement from $p = (2, 3)$ to $p' = (1, 3)$ when the consumer has income $m = 10$.

4. Albert and Betty live in a two goods exchange economy. Albert has utility function $u_A(x_A, y_A) = x_A^{1/4} y_A^{3/4}$ and an endowment of three units of good x and two units of good y, while Betty has utility function $u_B(x_B, y_B) = \min(x_B, 2y_B)$ and an endowment of two units of good x and one unit of good y. Prices are p_x and p_y respectively.

   (a) What are the competitive equilibrium prices and the equilibrium allocation? Is the allocation Pareto efficient? Explain why.

   (b) Assume that the government wants to implement the following Pareto efficient allocation: $(x_A, y_A) = (1, 1)$ and $(x_B, y_B) = (4, 2)$ via a lump-sum transfer of good y from Albert to Betty. How much of y does Albert have to give to Betty, so that this allocation emerges as the competitive equilibrium of the economy?

   (c) Assume now that lump-sum taxes are not feasible. Instead, the government imposes a per unit tax on Albert's consumption of good y and transfers the money received to Betty. Explain whether it is possible to implement the allocation in (b) as the competitive equilibrium of the economy with such redistributive taxes.

(TURN OVER)

Section C

5. The market demand for soft drinks is given by the inverse demand function $P(Q) = 20 - 2Q$, where Q is total litres of soft drinks supplied in the market.

   (a) **A**pricots and **B**lossoms, two soft drink companies with identical marginal cost equal to 4, simultaneously decide on how many litres of soft drinks to supply to the market. How much does each firm supply and what is the price and the firms' profits?

   (b) Assume now that there is a third competitor, **C**ranberries, with the same cost structure as A and B. How much will each firm supply if firms simultaneously choose quantities as described in (a). What are the price and the firms' profits? Comment on what would happen to quantities, price and profits if more firms were in the market.

   (c) Assume now that A is the leading supplier in this market. Therefore, A chooses its quantity first, and B and C, having observed A's choice, simultaneously choose their respective quantities second.

      i. What are B's and C's optimal quantities as a function of the quantities supplied by their competitors?

      ii. Using the fact the B and C are strategically in a symmetric position, derive the optimal amount they should supply solely as a function of A's quantity.

      iii. Using your answer in ii. derive the optimal amount A should supply.

      iv. What are total quantity, the price and profits in the market? Compare your answer to your answer in (b).

6. Answer all parts of the question.

   (a) Tests of cognitive ability are used as proxies for innate ability. The military frequently uses cognitive tests when allocating jobs and tasks. In contrast private firms rarely do. Explain why the military and private firms differ in their reliance on cognitive tests.

   (b) Recent decades have witness both an increased average level of education as well as an increased reliance on cognitive tests when hiring amongst private firms. Ideological shifts aside, what could explain these concurrent trends?

Section D

7. Answer all parts of the question.

   (a) The Allais paradox involves choices between the following lotteries, where, for example, $(p_1, x_1; p_2, x_2)$ denotes a lottery that pays prizes x_1 and x_2 with probabilities p_1 and p_2 respectively:

   A: (0.9, \$0; 0.1, \$5million)

   B: (1, \$1million)

   C: (0.01, \$0; 0.89, \$1million; 0.1, \$5million)

   D: (0.89, \$0; 0.11, \$1million)

      i. What hypothetical decisions are subjects asked to make, and what choices are commonly observed?
      ii. How do these choices contradict expected utility maximisation?
      iii. Give an interpretation of the preferences expressed by subjects.
      iv. Do you think these preferences are reasonable?

   (b) The Ellsberg paradox makes use of a monetary prize equal to \$100, and two urns: Urn X, containing 50 red balls and 50 black balls, and Urn Y, containing red and black balls in unspecified numbers.

      i. What hypothetical choices are subjects asked to make, and what choices are commonly observed?
      ii. What aspect of the standard theory of decision making under uncertainty do these choices contradict?
      iii. Give an interpretation of the preferences expressed by subjects.
      iv. In what settings do you think we would we be likely to find preferences of this type?

(TURN OVER)

8. In 2050 the temperature of the world will have risen by T degrees, where $\mathrm{E}(T) = 5$ and $\mathrm{Var}(T) = 10$. A large rise will cause flooding in much of the world, but also a boom in Greenland as ice melts. The return of the market portfolio in the period to 2050 is described by $M = 100 - 10T + \epsilon_1$, i.e. investments in this portfolio are multiplied by M over the period to 2050, where $\mathrm{E}(\epsilon_1) = 0$ and $\mathrm{Var}(\epsilon_1) = 1000$. The value of Greenland Construction Company (GCC) in 2050 will be $\$\Pi$ billion, with $\Pi = T + 1 + \epsilon_2$, where $\mathrm{E}(\epsilon_2) = 0$ and $\mathrm{Var}(\epsilon_2) = 1$. The random variables T, ϵ_1 and ϵ_2 are independent. There is a risk-free asset with return $r = 10$. The return of GCC is $R = \Pi/v$, where the current value is $\$v$ billion.

   (a) Calculate the following:

      i. Expected return $\mathrm{E}[M]$ and variance $\mathrm{Var}[M]$ of the market portfolio,

      ii. $\mathrm{E}[\Pi]$ and $\mathrm{Cov}[\Pi, M]$,

      iii. $\mathrm{E}[R]$ and $\mathrm{Cov}[R, M]$ as functions of v.

   (b) What is the CAPM formula relating R and M? Under the assumptions of CAPM, what is the current value of GCC?

   (c) Is v greater or less than $\mathrm{E}[\Pi]/r$ and why?

   (d) If the variance of ϵ_2 were increased, how would that affect v? You should assume that GCC is small relative to the world market, and that $\mathrm{Var}[M]$ is not (significantly) affected.

Section E

9. Larry dislikes exercise. His preferences are given by $\mathrm{E}[\sqrt{w}]-e$, where w is wealth and $e \in \{0,1\}$ is the amount of exercise he takes. Larry initially has wealth 16 but may face an illness which results in medical costs of $l = 16$. If not, there are no costs $l = 0$. The probability of illness is $p_I = 3/4$ if $e = 0$, and $p_I = 1/4$ if $e = 1$. These probabilities are known to all parties.

   (a) Suppose there is no insurance available. Will Larry exercise? What is his resulting utility $\bar{u}$?

   (b) Suppose there is a monopolist insurer with no transaction costs. The insurer maximises expected profits, subject to giving Larry at least the utility $\bar{u}$ found in part (a). Suppose the insurer does not observe e but only whether the medical costs were incurred. He offers a contract specifying payments of π_H if Larry remains healthy and π_I if he falls ill, where $\pi_k > 0$, $k = H, I$, indicates a payment paid by Larry, and $\pi_k < 0$ indicates a payout by the insurer. So, Larry's final wealth is $w_H = 16 - \pi_H$ when healthy and $w_I = -\pi_I$ when ill.

      i. What are the conditions on w_H and w_I for Larry to accept the contract and choose $e = 1$? Why would we expect $w_H > w_I$ in this contract?

      ii. Calculate the optimal contract which makes Larry want to accept and choose $e = 1$? (Hint: first calculate $\sqrt{w_H}$ and $\sqrt{w_I}$.) What are expected profits for this contract?

      iii. Is this better than the optimal contract which causes Larry to choose $e = 0$? Explain your answer.

10. Consider the following model of reciprocation. There are two stages. In the first stage, A(gatha) has the opportunity of doing a favour of size $x \geq 0$ for B(eatrice). In the second stage B, having received the favour, can reciprocate by doing a favour of size $y \geq 0$ for A. A can be a selfish type (S) or a generous type (G). If A is selfish her utility is $\sqrt{y} - x$. If A is generous, A's utility is $\sqrt{y} - x + 0.1\,(\sqrt{x} - y)$. If B believes A is selfish, B's utility is $\sqrt{x} - y$. If B believes A is generous, B's utility is $\sqrt{x} - y + 0.1\,\left(\sqrt{y} - x\right)$.

    (a) Interpret the preferences of A and B.

    (b) Suppose B knows A's type. What choices of x and y would you expect?

    (c) Suppose B does not know A's type but attaches positive probabilities to A being S or G. What are the conditions for a separating equilibrium? (You should give the equations that need to be satisfied but you do not need to solve for the range of separating equilibria).

    (d) Describe the signalling motive that A has.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECAD1
Advanced Diploma in Economics

Monday 1 June 2015   9:00am to 12:00pm

Paper 1

Microeconomics

*This paper is in five sections. Candidates are required to answer **five** questions in total, with one question from each section.*

Each section carries equal weight.

Credit will be given for clear presentation of relevant statistics.

*Write your number **not** your name on the cover sheet of **each** booklet.*

STATIONERY REQUIREMENTS
20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator – students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1. Jacques and Gilles live up on a hill. They would like to plant new trees on the hill, where there are currently no trees. They have identical utility functions:

$$U = 4\log(X) + 2\log(T)$$

where X is a (private) consumption good, with price = £1, and T is the total number of trees on the hill. The cost of planting a tree is also £1. Jacques and Gilles also have identical incomes: $Y_J = Y_G = £100$. Assume that Jacques and Gilles are the only agents in this society.

   (a) Suppose that trees on the hill are planted privately, where T_J is the number of trees planted by Jacques and T_G is the number of trees planted by Gilles. The two tree lovers act independently to maximize their individual utility, while anticipating the planting actions of each other. What is the private equilibrium number of trees that would be planted on the hill?

   (b) What is the Pareto efficient number of trees that should be planted on the hill? Explain the similarities or differences between the Pareto efficient number of trees and the private equilibrium obtained in part (a).

2. Consider a small open economy, Home, which produces and consumes three goods, labelled good 1, good 2, and good 3. The goods differ in the factor intensity of their production, and each good can be traded with the rest of the world. Suppose that, as a result of political turmoil overseas, a large amount of (physical) capital is repatriated back to Home.

   (a) How would you expect this 'capital shock' to affect the composition of output in Home? Explain.

   (b) Suppose that initially, before the increase in capital stock, good 1 was being exported, and goods 2 and 3 were being imported. Is it possible that the capital shock might lead to good 2 being exported? Explain.

   (c) Suppose now that Home is a 'large open economy', at least in the market for good 3. Describe the impact on welfare, at Home and in the rest of the world, of an import tariff imposed by Home on good 3.

SECTION B

3. Christie spends a significant share of her income on football boots, and the rest on a composite good, whose price is normalized to 1. She is currently employed at the market wage of w. Her uncle Joe urges her to join his shoe factory, where she receives the same as the market wage w, but where she will be entitled to the employee discount on the purchase of football boots. With the discount, she will pay p' for each pair of boots, which is lower than the market price p. She also receives an offer from a competing shoe company called "Rival", who offer Christie a fixed amount E to endorse their products. Uncle Joe will not hire Christie if she accepts Rival's offer. Still, Rival's offer is attractive enough to make Christie indifferent between accepting it and taking her uncle Joe's job. Christie accepts the endorsement deal. Unfortunately, before she gets a chance to cash the endorsement cheque, Rival goes bankrupt. So the endorsement deal is off, and uncle Joe has already refused to hire Christie. However, uncle Joe is willing to give Christie the job again (with the employee discount on football boots), but only if Christie agrees to pay him a bribe B to get the job.

   (a) Suppose that both goods are normal goods. In that case, is the highest bribe B that Christie is willing to pay greater than or less than the endorsement fee E? Explain. (You may find it helpful to draw a diagram).

   (b) Now suppose that Christie's preferences can be represented by a quasi-linear utility function $u(s) + g$, where s is football boots, and g is the composite good. Again, compare E with B. Explain your answer.

4. Consider an exchange economy populated by three individuals, Kurly, Larry, and Moe, and two goods, guavas and nuts. The preferences of the three individuals can be characterized as follows. Kurly always demands equal quantities of both goods. Larry spends two-thirds of his income on guavas, and the rest on nuts. Moe never consumes nuts. Let nuts be the numeraire good, so its price can be normalized to one, and use p to denote the price of guavas.

   (a) What utility functions would describe the behaviour of each of these individuals? Draw indifference curves for each of the three.

   (b) Suppose that the endowments of Kurly, Larry, and Moe are (5,0), (3,6) and (0,4) respectively, where each pair of numbers denotes the number of units of guavas and nuts that the individual is endowed with. Compute the competitive equilibrium. What is the equilibrium price ratio, and what are the equilibrium consumption levels for each individual?

SECTION C

5. Mr. Average has constant relative risk aversion with coefficient $\rho > 0$, implying that his vNM utility function $u(x) = \frac{x^{1-\rho}}{1-\rho}$.

   (a) Suppose a representative portfolio of equities has gross return over 1 year X, which is .9 with probability 1/2 and 1.26 with probability 1/2, while a risk-free bond has gross return $r = 1.01$. Mr. Average chooses to invest 30% of his savings in equities and the rest in bonds. Calculate ρ.

   (b) Suppose Mr. Average is given a hypothetical choice between having wealth W and having wealth given by the lottery $\left(\frac{1}{2}, 0.9W; \frac{1}{2}, 2W\right)$. Which would he choose?

   (c) How do the calculations above illustrate the *equity premium puzzle*? Suggest a possible explanation of the puzzle.

6. Country X would like to sell GDP bonds to country Y. One GDP bond pays $\Pi = 1€$ in 1 year's time is the GDP growth of country X is greater than 1%, and $\Pi = 0$ in one year's time if GDP growth is less than 1%. There is a market portfolio with gross return M which can be either high, $M = 1.3$, or low, $M = 0.9$. The joint probabilities of high and low market returns and high and low growth are given below:

$\mathbb{P}$	growth in country X > 1%	growth in country X < 1%
$M = 1.3$	0.4	0.1
$M = 0.9$	0.1	0.4

   There is a risk free asset with gross return $r = 1$ (i.e. no interest is paid over the year).

   (a) Calculate $\mathbb{E}[M]$, $\mathrm{Var}[M]$, $\mathbb{E}[\Pi]$ and $\mathrm{Cov}[M, \Pi]$.

   (b) Assuming the CAPM equation holds, find the current market value of a single GDP bond.

   (c) An inverse GDP bond pays 1€ in 1 year's time only when the GDP growth of country X is less than 1%. Which has a greater market value, a GDP bond or an inverse GDP bond? Explain.

SECTION D

7. A town hires a company to carry out a construction project. The weather may be good or bad with equal probability. The company has a choice $r \in \{1, 2\}$ of resources to devote to the project; this choice of r is not observed by the town. The project may face delays. The joint probabilities of good weather and delays depend on r and are as follows:

$\mathbb{P}$ when $r = 1$	good weather	bad weather
on time	45%	25%
delayed	5%	25%

and

$\mathbb{P}$ when $r = 2$	good weather	bad weather
on time	35%	10%
delayed	15%	40%

(a) Suppose the town is risk neutral, receiving utility $10 - T$ if the project is completed on time, and $5 - T$ if it is completed late. The company is risk averse, receiving utility $u(T) - r$, where $u'' < 0$, and has known reservation utility $\bar{u}$. The town offers a contract in which the company's payment T may depend on whether the project was completed on time and may also depend on the weather.

i. Describe the optimal contract, assuming that this involves the company choosing $r = 2$. How will T depend, if at all, on whether there are delays and on the weather? Explain how risk is distributed in this contract. (You do not need to set up any optimization problems or calculate the optimal contracts.)

ii. Does the town's inability to observe r reduce the utility it can achieve, compared with when it can observe r? Explain.

(b) How would your answer to part (ii) change if the town is risk averse and the company is risk neutral?

8. Cambridge Cycle Company (CCC) has managed to monopolize the market for bikes in Cambridge. Cambridge residents are composed of two groups, Students and Professionals. Students form $\frac{3}{4}$ of the population. Each resident demands at most 1 bike. The monetary value that each student attaches to a bike is q, where q is the quality of the bike. The monetary value that each professional attaches to a bike is $2q$. CCC is profit-maximizing, and producing a bike of quality q costs q^2.

(a) Initially CCC offers a special deal (q_s, F_s) to students, i.e., a bike of quality q_s at price F_s while professionals are offered (q_p, F_p). Find the optimal choices (q_s^C, F_s^C) and (q_p^C, F_p^C).

(b) Professionals Against Discrimination successfully lobbies for a law against student deals. Now neither type can be prevented from taking the deal intended for the other type. What qualities and prices (q_s, F_s) and (q_p, F_p) will CCC want to set now? (You should state any results that you need to use here but do not need to prove them.)

(c) CCC anticipates that the proportion of Cambridge residents who are professionals will rise. How would you expect this to affect the qualities and prices of bikes on offer at CCC?

SECTION E

9. (a) Why might economies of scale in production be a reason for trade between countries? Discuss.

   (b) It is often argued that, when markets are interlinked, then the monopoly or monopsony power that an agent has in one market can worsen asymmetries in power between market participants. Discuss.

10. An employee (player 1) works for his boss (player 2). The employee can either shirk (S) or work (W). Working imposes a disutility cost g on the employee but produces output of value v for the boss. The boss can either inspect (I) or not inspect (NI). An inspection costs the boss h but perfectly reveals whether the employee has shirked. The boss pays the employee a wage of w (which cannot be dependent on output) unless she has caught him shirking, in which case he is paid 0. Both players choose their strategies simultaneously, and the payoffs are as summarised below. Assume that $w > g$, and that $g > h > 0$.

	I	NI
S	$0, -h$	$w, -w$
W	$w - g, v - w - h$	$w - g, v - w$

(a) Find the pure strategy Nash equilibria, if any, in this game. Explain your answer briefly.

(b) Find any mixed strategy Nash equilibria, using x to denote the probability that the employee shirks, and y to denote the probability that the boss inspects. If you are unable to compute the equilibrium probabilities, then provide an intuitive discussion

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECAD1
ADVANCED DIPLOMA IN ECONOMICS

Monday 30 May 2016 9:00am-12:00pm

Paper 1

MICROECONOMICS

Candidates are required to answer a total of **five** questions, with at least **one** question from each section

Each section carries equal weight.

Write your **candidate number** (not your name) on the cover of **each** booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

1. Xavier, Yolanda and Zeke, plan a holiday together and need to decide how much money they are going to spend. X(avier) would like to spend as much money as possible; his utility from the trip is $U^X(m) = m$, where m is the total sum of money spent on the trip. Y(olanda) would like to spend exactly 4000 pounds; her utility is $U^Y(m) = -(m - 4000)^2$. Z(eke) likes either cheap backpacking trips or expensive resort vacations; his utility is $U^Z(m) = (m - 5000)^2$. They are considering three possible budgets for their trip: $t = 3000$, $t' = 4500$ and $t'' = 6000$.

   (a) Assume that they use majority voting to decide successively over two alternative budgets, i.e. they first vote over the pair (t, t'), then over (t', t'') and then over (t, t''). Will this procedure allow them to determine a "best" budget? Explain why or why not.

   (b) A fourth friend, Zzazz, offers: "Take me on the trip. I have exactly the same preferences as one of you. If you let me vote as well, we'll be able to sort this out!" Is he correct?

   (c) Another friend suggests: "Instead of taking Zzazz, just change the budget t' to a different amount!" What new amount t' should they choose and why?

2. Nigel and David discuss the possibility of "Brexit" and its consequences.

   (a) Nigel is in favor of the UK leaving the European Union. He argues: "The UK's access to European markets and therefore exports as well as imports are not going to be affected because of the global World Trade Organization rules. On the other hand, there will be a massive reduction in foreign workers employed in the UK. This implies that wages will rise and UK workers will be much better off." Assuming Nigel's assumptions are correct, can you use the Heckscher-Ohlin model of international trade to support Nigel's claim? Discuss all implications of such a model.

   (b) David answers as follows: "I am against such a move. Even if your assumptions about the consequences of Brexit are true, foreign workers are mostly unskilled workers, and a reduction in the UK's unskilled labour force will put the UK at a competitive disadvantage. Imports and exports will increase and the terms of trade will deteriorate." Can you change the model you built for your answer in (a) to accommodate David's view of the UK economy? Does this allow you to support his claim? Discuss all its implications.

SECTION B

3. Answer both parts of this question.

   (a) Equilibrium Existence

   i. Using graphs, demonstrate and discuss the sufficient conditions for the existence of a competitive equilibrium.

   ii. Give an example of an economy in which these conditions are not satisfied but a competitive equilibrium nevertheless exists. A graph with explanations will suffice.

   (b) Excess Demand Functions

   i. Explain why excess demand functions are necessarily homogenous of degree zero.

   ii. Consider the following functions. Could they be excess demand functions of some economy? Explain why or why not.

$$E(p) = \begin{cases} E_x(p) = \frac{p_x}{p_y} + 3p_x - p_y \\ E_y(p) = 3p_y p_x \end{cases}$$

$$F(p) = \begin{cases} F_x(p) = \frac{p_z^2}{p_x^2} + 5 \\ F_y(p) = 7\frac{p_x p_z}{p_y^2} \\ F_z(p) = -6 \end{cases}$$

4. Robinson Crusoe lives alone on an island. He has a total number of $H > 1$ hours available each day, which he spends either lounging on a deck chair or collecting melons. If he spends $L < H$ hours searching for melons he will find an amount $y = 2\sqrt{L}$. His utility from lounging and eating is described by a Cobb-Douglas utility function, but in addition, searching for melons gives Robinson a nasty headache. Denoting by x the amount of melons Robinson eats, by l the number of hours he lounges and by y the amount of melons he collects, his overall utility is described by:

$$u(x, y, l) = xl - y,$$

   Let the price of melons be p and the price of labour be w.

   (a) Interpret Robinson's utility function.

   (b) Solve for the competitive equilibrium price ratio and the competitive equilibrium allocations.

   (c) Suppose that Robinson decides he would like to spend just the right amount of time collecting melons and the rest lounging on his deck chair to maximize his welfare. Will he be better off by doing this rather than by relying on the market? Solve for the optimal amount of time spent on either activity and comment on the (possible) difference to your answer in (b).

SECTION C

5. A coin has probability p of landing heads. Chantal has a utility function $u(x) = \ln x$ and she is offered a gamble in which she will be paid $x = £2^n$ if the first head occurs on the nth flip.

   (a) Derive the coefficient of absolute risk aversion for Chantal.

   (b) What is the expected value of the gamble when $p = 0.5$?

   (c) For general p, express the expected utility of the gamble to Chantal as a sum.

   (d) Evaluate the sum obtained in (c), i.e. find a closed form formula in p without the summation sign. *Hint*: you can use the following summation formula:

   $$\sum_{n=0}^{\infty}(1-p)^n = \frac{1}{p}$$

   (e) What is Chantal's certainty equivalent for the gamble?

6. Mr. Charles is a striker in football team J who has scored an average of 23 goals per season in his first 3 years playing for the club. His current contract stipulates a fixed annual salary of $18,000,000$ for the next 3 years. He is approached by football club L who offers him the following contract:

   You will receive a fixed compensation of 17,500,000 per year for the next three years. In addition, you will receive a bonus of 100,000 for each goal scored in official matches. An additional clause is that in case the team were not to manage to qualify for an European football competition, then 1,000,000 will be deducted from your salary in the subsequent year (in which the team is not playing in Europe).

   Discuss the factors that will influence Mr. Charles' decision to accept or decline the offer, and how the new contract is likely to affect his performance in case he were to accept it. Can football club L change the terms of its offered contract - in a way that (approximately) does not change the expected payment - to improve the chance of Mr. Charles' acceptance? Write down an improved contract offer and discuss the changes that would make it more likely for Mr. Charles to accept it compared to the above offer.

SECTION D

7. Students apply for internships with an employer E. A student can be one of three types t: low quality, $t = 0$, medium quality, $t = 6$, or high quality, $t = 12$. E does not know an individual student's type but knows that there is an equal proportion of each type in the applicant pool. Students send CVs to obtain an interview, and they can put anything they want on the CV, that is, the CV is an "announcement" $a \in \{0, 6, 12\}$ of their type. E after reading the CV gives a recommendation r for an interview, with $r \in \{0, 1, 2, ..., 11, 12\}$.

   E's and a student S's payoffs are:

$$u_E(r,t) = \begin{cases} r - t & \text{if } t \geq r \\ t - r & \text{if } t < r \end{cases} \qquad \text{and} \qquad u_S(r,t) = \begin{cases} r - (t+4) & \text{if } t + 4 \geq r \\ (t+4) - r & \text{if } t + 4 < r \end{cases}$$

   (a) Consider the following strategy profile. Each student sends a CV that announces her truthful type, i.e. $a = t$. E believes the CV and assigns a recommendation equal to the announcement, i.e. $r = a$. Compute the payoff of each type of student and the average payoffs for E. Explain carefully whether or not this is an equilibrium.

   (b) Consider the following strategy profile. All students of all types send a CV that announces that they are of high quality, i.e. $a = 12$. E does not believe the CV and assigns all students a recommendation $r = 6$. Compute the payoff of each type of student and the average payoffs for E. Explain carefully whether or not this is an equilibrium.

   (c) Consider the following strategy profile:

   - Students of type 0 send a CV that announces $a = 6$.
   - Students of type 6 and of type 12 both send a CV that announces $a = 12$.

   E identifies students who announce $a = 6$ to be $t = 0$, and he assigns them $r = 0$. E identifies students who announce $a = 12$ to be equally likely to be type 6 or type 12, and he assigns them $r = 9$. Compute the payoff of each type of student and the average payoffs for E. Explain carefully whether or not this is an equilibrium. (E only receives CVs with $a = 6$ or $a = 12$, but you can assume that in case E were to receive a "strange" CV with a different a he assigns $r = 0$ to that CV).

8. Consider the following experiment. In each treatment, participants have to pick one of two events, A or B, to gamble on:

- Treatment 1:
  - A drawing a red marble from a bag containing 50% red and 50% white marbles
  - B drawing a red marble 7 times in a row, with replacement, from a bag containing 90% red and 10% white marbles
- Treatment 2:
  - A drawing a red marble from a bag containing 50% red and 50% white marbles
  - B drawing a red marble at least once out of 7 attempts, with replacement, from a bag containing 10% red and 90% white marbles

(a) Compute the probability of event B in treatment 1 and in treatment 2.

(b) Describe what the majority of experimental participants are likely to choose for the gambles in treatments 1 and 2. Comment on your findings.

(c) What bias might explain the inconsistent choices observed in the experiments? Carefully describe how the bias explains the choices in both treatments.

(d) Describe how the inconsistent choices observed in this experiment have implications for insurance.

SECTION E

9. (a) Whether welfare programs discourage work is a long-standing issue of academic research and public debate. Suppose that the British government decides to give every individual with income Y below the poverty line, Y^*, a transfer of size $B = Y^* - t \times Y$, where $t = 1$. Assume non-labour income is zero and the individual consumes all labour income. Show the budget constraint for the individual. Discuss whether this transfer program has any work disincentives and why. Does the impact on labour supply depend on the level of the poverty line, Y^*, and if so, how?

   (b) How do models that incorporate increasing returns to scale in production, or monopolistically competitive product markets, offer an explanation for intra-industry trade? Be sure to describe what is meant by 'intra-industry' trade, and to contrast these explanations with those offered by Ricardian and Heckscher-Ohlin models.

10. Two individuals, $i = 1, 2$, simultaneously decide on how much to contribute to a public good. They have identical wealth $w_1 = w_2 = w$ and can contribute an amount $0 \leq c_i \leq w$; the remainder of the wealth $w - c_i$ is spent on private goods. The amount of the public good is equal to the sum of the contributions, and each person cares about the amount of the public good, her consumption of private goods and a third term $(w - c_i)(c_1 + c_2)$ that reflects an interaction between the amount of the public good and her private consumption. An individual i's utility is therefore

$$u_i(c_1, c_2) = c_1 + c_2 + w - c_i + (w - c_i)(c_1 + c_2), \; i = 1, 2.$$

   (a)

      i. Calculate and draw the best response functions.

      ii. Solve for the Nash Equilibrium and show that both players are worse off than if they both contribute one half of their wealth to the public good.

   (b)

      i. Extend the analysis to the case of n people, i.e. solve for the best response functions and the Nash Equilibrium when n people can contribute to the public good. As the number of people increases, how does the total amount contributed in a Nash equilibrium change?

      ii. Compare the players' equilibrium payoffs with their payoffs when each contributes half her wealth to the public good, as n increases without bound.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECAD1
ADVANCED DIPLOMA IN ECONOMICS

Monday 5 June 2017 09:00am-12:00pm

Paper 1

MICROECONOMICS

Candidates are required to answer one question from each section. There is a total of five sections.

Each section carries equal weight.

Write your **candidate number** (not your name) on the cover of **each** booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated .

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Section A

1. Suppose there are three parties, $j = A, B, C$, seeking election. Party j is located at point x_j, where $0 \leq x_j \leq 1$, 0 represents being liberal and 1 represents being conservative. These locations are such that $x_A < x_B < x_C$. There is an odd number of voters, n. Let each voter i have a bliss point l_i in terms of ideology, where $0 \leq l_i \leq 1$. Voter i's payoffs from electing party j is $1 - (l_i - x_j)^2$.

   (a) Define a Condorcet winner for this setting.

   (b) Suppose the voters simultaneously cast a vote for one of the three alternatives and the election is decided by electing the party with the most votes. In the event of a tie a party with most votes is elected, with each party with most votes being elected with equal probability. Suppose each person votes for their most preferred party. Find an example in which B is a Condorcet winner, but A wins the election.

   (c) Suppose we model voting as a simultaneous move game. The players are the n voters, the action set of each voter i is the set of available alternative parties $\{A, B, C\}$, the election is decided as described in the previous part and, as before, i's payoffs from electing party j is $1 - (l_i - x_j)^2$. Are the voting decisions in your example to the previous part of the question a Nash equilibrium?

   (d) Argue that there is always a Condorcet winner in the environment considered in this question.

2. There are five bank robbers, A, B, C, D and E. They have just stolen 10 identical gold bars and need to decide how to split them among themselves. The gold bars are indivisible (there is no time to melt them without risking being caught). They agree to the following game. The game consists of a sequence of rounds. In each round a bank robber proposes a split of the gold (an integer x_i for each bank robber in the game such that $\sum_i x_i \leq 10$). The bank robbers in the game then vote on whether to accept the proposed division or not. If **strictly more than half** of those in the game vote in favor of the division, then the game ends and the proposed split of gold bars is implemented. **If strictly more than half** of those in the game vote in favour of the division, then the game ends and the proposed split of gold bars is implemented. If half or fewer of the bank robbers in the game vote in favor of the division, then the bank robber making the proposal is killed, an outcome worse than receiving no gold bars. The game then moves onto the next round. In the first round bank robber A makes the proposal. If the second round is reached bank robber B makes the proposal. If the third round is reached bank robber C makes the proposal, then D if the fourth round is reached and finally E. Bank robbers prefer more gold bars to less. You may assume that **indifferent bank robbers never vote for the proposal unless they are offered all the gold bars, in which case they vote in favour of the proposal.**

   (a) Write down an example of a strategy for Bank robber B.

   (b) Find the payoffs of all the bank robbers in the subgame perfect Nash equilibrium.

   (c) Suppose bank robber A understands game theory, calculates the subgame perfect Nash equilibrium, and then remarks: "There is no way I'm making that offer, game theory is stupid." Discuss in detail (this part of the question is a substantial portion of the whole question).

Section B

3. Two opposing nations are disputing a nearby territory. Each nation must decide on an action to take. They can keep peace if both nations choose the action "Peace" (P). However, they can choose to fight. Specifically, each nation has two "weapons": sanctions (S) or arms (A). This circumstance leads to the following stage game :

		Nation 2		
		Peace	Sanctions	Arms
	Peace	(10, 10)	(2, 8)	(−5, 13)
Nation 1	Sanctions	(8, 2)	(5, 5)	(0, 0)
	Arms	(13, −5)	(0, 0)	(1, 1)

(a) Suppose this game is played once. Find all the *pure* strategy Nash Equilibria.

(b) Suppose this game is played twice and players have time preferences represented by the discount factor δ, where $0 \leq \delta \leq 1$. Write down an example of a strategy for Nation 1.

(c) Suppose $\delta = 1$. Show that there is a *pure* strategy Subgame Perfect Nash Equilibrium where both nations choose the action "Peace" in the first period.

(d) Suppose that the game is played infinitely often. Consider the grim trigger strategy in which player i alternates between playing Peace and Arms, so that Peace is played in odd periods and Arms is played in even periods, as long as player $j \neq i$ has never deviated from the same sequence of Peace in odd periods and Arms in even periods, and otherwise, i plays Sanctions forever. Find the smallest δ^*, where $0 < \delta^* < 1$, such that for all $\delta \geq \delta^*$, the strategy pair in which each nation uses this strategy is a subgame perfect equilibrium. [*Hint*: If $0 < r < 1$, then $\sum_{t=0}^{\infty} r^t = \frac{1}{1-r}$, and $r^{2t} = (r^2)^t$].

4. Suppose that two workers indexed $i = 1, 2$ compete for bonus of value B. Each worker produces output $y_i(e_i) = a_i + e_i$, where $0 \leq a_i \leq 1$ denotes the workers' abilities (which are common knowledge) and where $e_i \geq 0$ denotes workers' effort choices. Let $a_1 > a_2$. Worker i's cost of effort is e_i. Worker 1 receives the bonus if $y_1 - y_2 > \phi$. If $y_1 - y_2 = \phi$ the workers share the bonus evenly, and if $y_1 - y_2 < \phi$ worker 2 receives the bonus (where ϕ may be positive or negative). Workers' payoffs are the value of the bonus they receive less their cost of effort. The game proceeds by workers first simultaneously choosing their effort levels before the bonus is allocated as described above.

   (a) Show that there is no pure strategy Nash equilibrium in which both $e_1 > 0$ and $e_2 > 0$.

   (b) Suppose both players play mixed strategies. Let player 1 mix as described by the cumulative distribution function F_1 and player 2 mix as described by the cumulative distribution function F_2. It will be convenient to work with $\tau \equiv a_1 - a_2 - \phi$. Assume that $B > \tau > 0$.

   Use the iterated deletion of strictly dominated strategies to argue that worker 1 only ever plays $0 \leq e_1 \leq B - \tau$ and worker 2 only ever plays $e_2 = 0$ or $\tau \leq e_2 \leq B$.

   (c) We will look for a mixed strategy Nash equilibrium in which the workers choose zero effort with positive probability and otherwise mix over effort levels according to the (strictly increasing) cumulative distribution functions F_1 and F_2.

   Show that for these mixed strategies to constitute a mixed strategy Nash equilibrium the following equations must hold at every $0 < e_1 \leq B - \tau$ and every $\tau \leq e_2 \leq B$:

$$
\begin{aligned}
F_2(e_1 + \tau) &= \frac{\pi_1 + e_1}{B} \\
F_1(e_2 - \tau) &= \frac{\pi_2 + e_2}{B},
\end{aligned}
$$

where π_1 and π_2 are constants.

Section C

5. Consider the following variation of the Holt-Laury (HL) risk-elicitation method. As in HL, a participant has to choose either Option A or B for each of the 10 decisions. For each decision, Option A gives outcome A1 if the coin toss yields Heads (i.e. with probability 0.5) and outcome A2 if the coin toss yields Tails (i.e. with probability 0.5). Similarly, Option B gives outcome B1 if the coin toss yields Heads and outcome A2 if the coin toss yields Tails.

	Option A		**Option B**	
Decision number	Heads Outcome A1	Tails Outcome A2	Heads Outcome B1	Tails Outcome B2
1	0.05	4.95	2.65	2.75
2	1.10	3.90	2.65	2.75
3	2.40	2.60	2.65	2.75
4	2.40	2.60	2.00	3.40
5	2.40	2.60	1.90	3.50
6	2.40	2.60	1.75	3.65
7	2.40	2.60	1.60	3.80
8	2.40	2.60	1.45	3.95
9	2.40	2.60	1.05	4.35
10	2.40	2.60	0.20	5.20

(a) List the choices of a risk neutral individual for each of the decisions. Explain your answers.

(b) Consider Decision 3. Can you rank Option A and Option B in terms of first order stochastic dominance? Can you rank Option A and Option B in terms of a mean preserving spread? Please explain your answers and interpret what they mean in terms of the returns and/or riskiness of the two gambles.

(c) Consider Option B for Decision 6 and 7. Can you rank them in terms of first order stochastic dominance? Can you rank them in terms of a mean preserving spread? Explain your answers and interpret what they mean in terms of the returns and/or riskiness of the two gambles.

(d) Assume an expected utility maximizer with utility function $u(x) = x^{0.5}$ takes the test above. What would she pick for each of the 10 decisions?

6. A (risk-neutral) start-up P wants to recruit a (risk-averse) coder A to improve its algorithm for ad-targeting. An improvement would bring P a benefit of $\pi = \overline{\pi} > 4$. No improvement brings P a benefit of $\pi = 0$. A can either exert high effort e_H at a cost of $c > 0$ or low effort e_L at zero cost. If A exerts e_H then the probability of an improvement is $p_H = 0.5$, otherwise the probability of an improvement is $p_L = 0.25$. Let A's outside option be $\overline{u} = 0$ and let w indicate the wage P pays to A. The utility of P is equal to π minus the wage w, and the utility of A is equal to $\ln w$ minus the cost of exerting effort. The timing of the game is as follows: (i) P offers a contract to A; (ii) A accepts the contract or rejects it (and gets the outside option); (iii) if A has accepted the contract, he chooses low or high effort.

   (a) Assume that P can see A's effort choice. Solve for the optimal contract. Find a condition that ensures that P prefers A to exert high effort.

   Hereafter assume that P cannot observe A's effort choice.

   (b) State P's maximization problem formally.

   (c) Derive the cheapest contract that implements the low effort e_L.

   (d) Derive the cheapest contract that implements the high effort e_H. What is the expected cost to P? How does it compare with your answer in (a)? Comment.

Section D

7. "What you learn is irrelevant, the sole benefit of a Cambridge education is to earn a degree with Cambridge written on it." Comment.

8. Kahneman, Knetsch and Thaler (1990, KKT hereafter) describe the results of a series of experiments that show the existence of the *endowment effect*.

   (a) Give a definition of the endowment effect.

   (b) Describe the design of the experiments and how the endowment effect can explain the results.

   (c) Give at least 4 alternatives to the endowment effect to explain the results in the experiments. For each alternative, explain whether and how the experiments in KKT are able to exclude it.

   (d) "If the endowment effect is real then indifference curves are not parallel." Comment.

Section E

9. Consider an exchange economy with two consumers, A and B, and two consumption goods, x and y. A has utility function $u^A(x_A, y_A) = x_A + y_A$, and B has utility function $u^B(x_B, y_B) = \sqrt{x_B y_B}$, where x_i and y_i are the amounts of good x and y consumed by individual $i = A, B$. Endowments are $(e_x^A, e_y^A) = (0, 5)$ and $(e_x^B, e_y^B) = (5, 0)$. Denote prices by p_x and p_y respectively.

   (a) Solve for the consumer's demand functions and the competitive equilibrium (price and allocation).

   (b) Solve for the pareto efficient allocations in this economy. If B's utility function was the same as A's, what would the pareto efficient allocations be?

   (c) "A competitive equilibrium allocation must be Pareto-efficient because, if it were not, two agents would negotiate a mutually beneficial trade". Comment.

10. Larry and King are flatmates. Larry likes to cook but does not read books. King likes to read books and also likes Larry's cooking but cannot cook himself. Both like money. Their preferences over these different goods and activities are represented by the following utility functions:

$$\begin{aligned} u^L(c_L, m_L) &= \ln c_L + m_L \\ u^K(b_K, c_L, m_K) &= \ln b_K c_L + m_K, \end{aligned}$$

   where c_L is the amount Larry cooks, b_K is the amount of books King reads and m_L (m_K) is the amount of money that Larry (King) has left over to spend. Each has an allowance of M_i, $i = L, K$, and can buy books or cooking ingredients at prices p_b and p_c respectively. The price of money is normalized to 1. Assume that $M_K > p_c > p_b$ and $M_L > 1$.

   (a) Assume Larry, given his allowance and preferences, chooses how much to spend on cooking and how much money to keep. King, given *his* allowance and preferences, decides on how many books to buy and how much money to keep. What will their optimal choices (as functions of prices) be?

   (b) Their friend Magda tells them that they both could benefit if they jointly decided on how to spend their combined allowances.

      i. Is she correct and why?

      ii. If they follow Magda's advice, how much will Larry cook and King read? Compare your answer to your answer in (a).

      iii. Will Magda's solution result in a unique allocation of the left-over money between Larry and King? Why or why not?

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECAD1
ADVANCED DIPLOMA IN ECONOMICS

Monday 4 June 2018 9:00am - 12:00pm

Paper 1

MICROECONOMICS

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Section A

1. Suppose there are n countries, indexed by i, and each country decides how much to pollute. Pollution by country i generates some benefits for i by increasing i's output. Denote the amount of pollution generated by country i by q_i, and let $Q = \sum_i q_i$ be total pollution. Suppose there is a threshold $\bar{Q} > 0$, such that if $Q > \bar{Q}$, the world irreversibly changes. If instead $Q \leq \bar{Q}$ then the world remains as it is. Let the expected utility of country i be

$$q_i - \lambda_i \text{prob}(Q > \bar{Q}),$$

where $\lambda_1 > \lambda_2 > \cdots > \lambda_n > 1$. You may assume that the countries are expected utility maximizers.

The timing of the game is as follows. First nature draws $\bar{Q} \geq 0$ from a cumulative distribution function F, where F is a strictly increasing and concave function with $F(0) = 0$. This draw is *not* observed by any country, and then the countries simultaneously choose $q_i \geq 0$.

(a) What might λ_i represent?

(b) Define a Nash equilibrium in this context.

(c) Under what conditions is $q_i = 0$ a best response for i to others choosing quantities $Q_{-i} = \sum_{k \neq i} q_k$?

(d) Find a pure strategy Nash equilibrium.

2. A smuggler is trying to bring illegal goods into a country. There are three possible entry points: By land (l), by sea (s) and by air (a). The police have total resources of 1 and must allocate these resources to the entry points according to a vector (r_l, r_a, r_s), such that $r_l \geq 0$, $r_a \geq 0$, $r_s \geq 0$, and $r_l + r_a + r_s = 1$. Simultaneously, the smuggler chooses a route and the police choose where to allocate their resources. The smuggler may randomise his choice. Let (p_l, p_a, p_s) be the probability vector with which the smuggler chooses *land*, *air* and *sea* respectively. Suppose the chance of the smuggler being caught is $p_l r_l^2 + p_a r_a^2 + p_s r_s^2$. The police want to maximize the chance the smuggler is caught, while the smuggler wishes to minimize this probability.

   (a) Write down the game corresponding to this situation.

   (b) Show that there is no Nash equilibrium in which the smuggler does not play a totally mixed strategy (i.e., does not choose each entry point with strictly positive probability).

   (c) Show that there is no Nash equilibrium in which the police allocate no resources to one of the entry points.

   (d) Find the Nash equilibria.

   (e) Now suppose that the police move first, committing resources to the three entry points and then, after observing this allocation, the smuggler chooses which route to take. Use backward induction to find a Nash equilibrium of this game.

   (f) Now suppose that the smuggler moves first, choosing an entry point, and after observing this choice, the police choose how to allocate their resources. Use backward induction to find a Nash equilibrium of this game.

(TURN OVER)

Section B

3. Consider the following normal form game:

		Player 2				
		A	B	C	D	E
Player 1	a	(5, 7)	(4, 11)	(6, 4)	(8, 0)	(3, 0)
	b	(14, 0)	(5, 0)	(7, 2)	(9, 0)	(4, 5)
	c	(12, 5)	(15, 5)	(8, 6)	(7, 6)	(2, 7)
	d	(18, 1)	(5, 4)	(4, 2)	(3, 5)	(6, 0)
	e	(0, 1)	(0, 2)	(4, 2)	(0, 3)	(7, 3)

(a) Write down mathematically what it means for a pure strategy to be strictly dominated by another pure strategy.

(b) Show that playing A is strictly dominated by playing a mixture of the other actions available to player 2.

(c) Show also that player 2 cannot play actions B or C with positive probability in any Nash equilibrium (including a mixed strategy Nash equilibrium), and that player 1 cannot play actions a or c with positive probability in any Nash equilibrium (including a mixed strategy Nash equilibrium).

(d) Assume that player 1 plays strategies b, d and e with probabilities p, q and $1 - p - q$ respectively, and player 2 plays D and E with probabilities r and $1 - r$ respectively. Find all the Nash equilibria (pure and mixed) of the game.

4. Suppose there are three political candidates who differ in how conservative/liberal they are. This is represented by their locations on the unit interval. Candidate A's location is $l_A = 0.2$, candidate B's location is $l_B = 0.6$, and candidate C's location is $l_C = 0.7$. There is a unit mass of voters, who each also have a location on the unit interval. Suppose these voters are distributed over the unit interval according to a continuous and atomless probability density function f. Let voter i's utility from candidate j being elected be $U_i(j) = -(l_i - l_j)^2$, where l_i is i's location.

   (a) Suppose everyone votes for their most preferred candidate. Who would vote for which candidate?

   For the rest of the question you may assume that people are located uniformly over the unit interval.

   (b) What proportion of the vote does each candidate receive?

   (c) Who would receive the most votes if candidate B dropped out?

   (d) Show that there is a Condorcet winner among A, B and C, and identify who it is.

   (e) When voters' preferences are as described in the question, what voting system would you recommend be used and why?

Section C

5. Gneezy and Rustichini (*QJE*, 2000) conducted a field experiment in which high school students collecting charitable contributions were divided into three compensation schemes:

   - No compensation
   - 1% of the total amount collected
   - 10% of the total amount collected

   Compensation came from the researchers and it was not subtracted from the amount collected for charity.

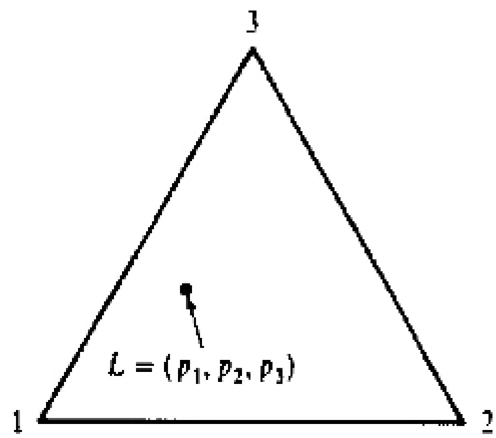
   (a) What are the advantages and disadvantages of a field experiment compared to a laboratory experiment?

   (b) Describe the results of the experiment and how they compare with the assumptions and theoretical predictions of the standard Principal-Agent model with moral hazard.

   (c) Give a formal definition of anchoring. How can anchoring explain the results of the experiment?

   (d) Discuss in detail at least one further explanation other than anchoring that Gneezy and Rustichini give for their findings.

6. Consider the following 2-dimensional simplex to represent lotteries in probability space.



(a) Draw the points representing the following lotteries: $L_1 = \{0, 1, 0\}$, $L_2 = \{0.5, 0, 0.5\}$ and $L_3 = \{1/3, 1/3, 1/3\}$.

(b) Consider lottery $L = \{p_1, p_2, p_3\}$. What type of triangle is the 2-dimensional simplex above if we want to have that $p_i \in [0, 1]$ and $p_1 + p_2 + p_3 = 1$?

Consider the following lotteries on the set of outcomes $C = \{1, 2, 3, 4, 5\}$:

- $L_A = \{0, \frac{3}{10}, \frac{2}{5}, \frac{3}{10}, 0\}$
- $L_B = \{0, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}, 0\}$
- $L_C = \{0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$
- $L_D = \{\frac{1}{10}, \frac{1}{5}, \frac{2}{5}, \frac{1}{5}, \frac{1}{10}\}$

(c) Rank them in terms of first order stochastic dominance (FOSD) and show graphically each FOSD relation.

(d) Rank them in terms of second order stochastic dominance (SOSD) and show graphically each SOSD relation.

(e) Using your results in (c) and (d), what are the most and least preferred lotteries for a risk averse expected utility maximizer?

Section D

7. 'When drafting a contract, an employer should always include any available measure to evaluate the performance of the employee.' Discuss.

8. Gigi decides to retire after a successful year. He goes to the bank to choose how to invest his £$42m$ wealth (you can assume this is Gigi's total wealth). The Investment Advisor shows Gigi the following 4 options with payoffs that depend on either a good or a bad outcome:

	Bad	*Good*
No investment	42	42
Investment 1	25	59
Investment 2	36	49
Investment 3	20	70

For each investment option, there is a 50% chance of a good outcome and a 50% chance of a bad outcome. The Advisor tells Gigi that if he does not make a choice then the *No Investment* option is automatically selected. Notice that to simplify calculations the outcomes are expressed in millions of pounds.

(a) Assume that Gigi is a risk neutral expected utility maximizer. What is his choice?

(b) Assume that Gigi is an expected utility maximizer with utility function $u(x) = \sqrt{x}/10$. What is his choice?

(c) Give a definition of *status quo* bias. How would status quo bias affect Gigi's choice? Describe in detail an experiment that supports your answer.

(d) Now assume that the Advisor gives Gigi one further option: *Investment 4* that has a bad outcome of 25 with probability $\frac{1}{2}$ and a good outcome of 80 with probability $\frac{1}{2}$. How does this change your answers in (a), (b) and (c)? Discuss.

Section E

9. There are two consumers, Al and Bill, and two goods, the quantities of which are denoted by x and y. Al and Bill each own 100 units of the y-good; Al owns 12 units of the x-good and Bill owns 3 units. Their preferences are described by the utility functions

$$u_A(x_A, y_A) = y_A + 60x_A - 2x_A^2 \qquad \text{and} \qquad u_B(x_B, y_B) = y_B + 30x_B - x_B^2.$$

   (a) Al proposes that he will trade one unit of the x-good to Bill in exchange for some units of the y-good. Al and Bill turn to you, their economic consultant, to tell them how many units of the y-good Bill should give to Al in order for this trade to make them both strictly better off. What is your advice? Using marginal rates of substitution, explain your answer.

   (b) Draw the Edgeworth box diagram, including each person's indifference curve through the initial endowment point. Use different scales on the x- and y-axes or your diagram will be very tall and thin. Derive and then draw the Contract Curve.

   (c) Determine the general equilibrium prices and allocations given the initial endowment point.

(TURN OVER)

10. Al, Bill and Cal live together in a flat and consider buying a TV to be shared between them. They agree that they will all have equal access to it and watch the same programs, i.e. the TV is a public good. The TV costs £150. Al's value for the TV (measured in £) is 60; Bill's value is either 20 or 90, depending on his social commitments; and Cal's value is either 30 or 75, depending on the weather. Bill's social commitments are independent of the weather. Everybody knows how much Al values the TV, but Bill's and Cal's value is their private information. They only know each others' *possible* valuations. The three friends discuss whether or not they should buy the TV, and if so, how much each should pay.

(a) Bill suggests that he and Cal should each announce their value, i.e. v_B and v_C, and that they should buy the TV if and only if their total value $V = v_A + v_B + v_C$ ($v_A = 60$) is greater than or equal to the cost. A person with (announced) value v_i, $i = A, B, C$, should pay

$$\frac{v_i}{V} \times 150.$$

Assume that B and C tell the truth, is this scheme efficient? Will the scheme work, i.e. do B and C want to tell the truth?

(b) Bill, an economists, suggests that they should use a Groves-Clarke mechanism to decide on the allocation and how much each should pay. Explain this mechanism. For each possible realization of the flatmates' valuations, compute how much each should contribute to the TV.

(c) Explain why Cal does not want to lie about his valuation in the Groves-Clarke mechanism when his value is 30.

END OF PAPER

EXAMINATION FOR THE DIPLOMA IN ECONOMICS

Tuesday 5 June 2012 9.00 to 12.00

Paper 2
MACROECONOMICS

The examinations is divided into five sections, Section A to Section E.
Answer **five** questions. You must choose questions from at least **four** of the sections
Write your number ***not*** *your name on the cover sheet of* ***each*** *booklet*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
20 Page booklet x 2	Approved calculators allowed
Metric Graph Paper	
Rough work pads	

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A *Intertemporal economics*

A1 Suppose a consumer optimally chooses consumption c_t and leisure ℓ_t in period t, given the real wage w_t.

Consider the following dynamic optimization problem:

$$\max_{c_1,c_2,\ell_1,\ell_2} \{u(c_1) + v(\ell_1) + \beta [u(c_2) + v(\ell_2)]\}$$
$$\text{s.t.} \quad (1-\ell_1)w_1 + \frac{(1-\ell_2)w_2}{1+r} = c_1 + \frac{c_2}{1+r}$$

where $u'(.) > 0$, $u''(.) < 0$, $v'(.) > 0$ and $v''(.) < 0$, β is the subjective intertemporal discount factor $(0 < \beta < 1)$ and r is the real interest rate.

(a) Derive the first-order conditions relating the marginal utility of consumption to the marginal utility of leisure.

(b) Derive the Euler equation for consumption, and the Euler equation for labour supply.

(c) Suppose that $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, but for simplicity assume that $v(\ell_t) = \ln(\ell_t)$, $\beta = 1$ and $r = 0$. Solve for leisure in period one, ℓ_1, as a function of the relative wage w_1/w_2. How does labour supply in period one, $1 - \ell_1$, relate to the relative wage w_1/w_2? In what way is this relationship affected by the value of σ?

A2 A pervasive argument in the debate concerning the effectiveness of fiscal policy is of the type: "A dollar spent by the government is a dollar taxed, in the present or in the future. So, government spending cannot stimulate the economy, let alone private demand." Discuss to what extent this argument is faulty, and to what extent it is correct.

SECTION B *Unemployment; economic growth*

B1 Consider the following continuous-time Solow growth model with human capital accumulation and no exogenous growth on productivity.

Technology: The production technology is represented by a constant returns to scale Cobb-Douglas function:

$$F(K, L, Z) = K^{\alpha}(huL)^{1-\alpha}, \quad \alpha \in (0, 1),$$

where K represents the capital stock, which depreciates at rate δ; L is raw labour which grows at rate n; h is the per capita human capital; and u corresponds to time spent at work.

Human capital: Following Lucas (1988), assume that human capital evolves according to:

$$\dot{h} = B(1-u)h^{\beta} - \delta_h h, \ B > 0$$

where $(1-u)$ is time spent accumulating skill. Assume that $\beta \in (0, 1)$.

Households: Households save a constant fraction of income, $s \in (0, 1)$. The economy is closed which implies that investment equals savings.

(a) If $\beta \in (0, 1)$, derive expressions for $\dot{k}$, where $k = \frac{K}{L}$. Using $\dot{k}$ and $\dot{h}$, derive equations that describe the set of combinations of $\dot{k} = 0$ and $\dot{h} = 0$. Sketch these in the (h, k) space. What is the growth rate of output per capita in the long-run? Explain.

(b) Suppose that the efficiency of the capital accumulation equation increases, i.e. there is an increase in B. Show in a graph the transition dynamics of output per capita to the new long-run equilibrium.

B2 The United States and Europe have had very different experiences in terms of unemployment during the past 40 years. Briefly describe three theories of hysteresis, and discuss how the idea of hysteresis may explain the diverging patterns observed in the labour markets.

SECTION C *Monetary economics*

C1 It has been argued that the adoption of 'quantitative easing' by the Bank of England has not been effective since the funds provided by the Bank through its Gilt purchases appear to be all held as additional reserve balances at the Bank. In addition, despite the massive increase in the monetary base, there has been a decline in the growth rate of M4. Explain why this could be the case and discuss whether it means that quantitative easing has been ineffective.

C2 On 9 February 2012 the Bank of England announced that it would increase Gilt purchases by £50bn in an expansion of its 'quantitative easing' program.

(a) Discuss how plans by the Bank to buy up to £50bn in gilts are likely to affect the money supply, bank lending, long term interest rates and economic activity in the UK.

(b) After the announcement by the Bank the pound appreciated by 0.3%. Explain whether this could be consistent with the asset market model of the exchange rate.

SECTION D *Open economy economics*

D1 Consider the following model of a small open economy (Home), which interacts with the rest of the world (Foreign):

$$\begin{aligned} i &= i^* + \mathbf{E}[\mathbf{e}] - \mathbf{e} + \rho \\ \mathbf{M} - \mathbf{P} &= \mathbf{Y} - \frac{1}{2} i \\ \mathbf{Y} &= (\mathbf{e} + \mathbf{P}^* - \mathbf{P}) - \theta i \end{aligned}$$

where i is the Home nominal interest rate, i^* the Foreign nominal interest rate, $\mathbf{e}$ the nominal exchange rate expressed as the Home price of Foreign currency, $\mathbf{E}[\mathbf{e}]$ the expected future exchange rate, ρ a risk premium, $\mathbf{M}$ the Home money supply, $\mathbf{P}$ the Home aggregate price level, $\mathbf{P}^*$ the Foreign aggregate price level, $\mathbf{Y}$ Home aggregate output, and all boldface variables are in logs. The coefficient θ depends on the interest rate sensitivity of investment. People have rational expectations. In the short run, the aggregate price level $\mathbf{P}$ is fixed. In the long run, the price level $\mathbf{P}$ is flexible and output is at its natural rate $\bar{\mathbf{Y}}$. Assume that $\bar{\mathbf{Y}} = 0$, $\mathbf{M} = \mathbf{0}$, $\mathbf{P}^* = 0.05$ and $i^* = 0.10$, and let $\theta = 1$.

(a) Give a brief economic interpretation of the three displayed equations above.

(b) Compute the long run equilibrium values of i, $\mathbf{P}$, $\mathbf{e}$ and $\mathbf{E}[\mathbf{e}]$ in terms of ρ. Give an intuitive explanation of the long run effect of ρ.

(c) Suppose now that there is an unexpected one-period increase in the risk premium ρ from 0 to 0.10. Compute the short run equilibrium level of the nominal interest rate i, the nominal exchange rate $\mathbf{e}$ and output $\mathbf{Y}$, and give an intuitive explanation of the short run effects.

D2 Some commentators have suggested that Greece should not be in the European Monetary Union. Discuss the criteria under which it is optimal for a country to be part of a currency union.

SECTION E *Economic policy*

E1 The relationship describing the evolution over time of the debt to income ratio can be written approximately as:

$$\Delta b = d + (r - g)b,$$

where b is the debt-to-GDP ratio, d is the primary deficit as a proportion of GDP, r is the real interest rate, and g is the growth rate of real GDP.

(a) Explain the economic intuition behind this relationship and derive it from the budget identity of the government.

(b) Suppose the initial stock of debt is positive, the country runs a primary deficit and the real interest rate is smaller than the growth rate of the economy ($r < g$). Explain and show in a diagram how the debt-income ratio evolves over time.

(c) Now, assume that the economy is initially in a steady state with $r < g$ when a financial crisis occurs in which a large bank goes bankrupt. The government nationalises the bank and bring its debts onto the public sector balance sheet. Concerns about sovereign default raise the risk premium on government bonds so that now $r > g$. Explain and show diagrammatically how this affects the debt-to-GDP ratio b over time.

E2 Discuss under what circumstances an activist monetary policy generally might be ineffective.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECD1
ECONOMICS DIPLOMA

Tuesday 4 June 2013 9.00 - 12.00pm

Paper 2

MACROECONOMICS

This paper comprises five Sections, A, B, C, D and E. Answer five questions. You must choose questions from at least four Sections.

Write your **candidate number** not your name on the cover of each booklet.

STATIONERY REQUIREMENTS
20 Page booket x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

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SECTION A: unemployment and growth

1. The employment rate, n_t, in an economy evolves according to

$$n_{t+1} = u_t f + n_t(1-\lambda)$$

where f denotes the job-finding probability, λ the separation rate, and u_t is the unemployment rate in period t. Derive the steady-state unemployment rate, u_{ss}.

Now suppose that both the job-finding probability and the separation rate changes to $\hat{f} = \gamma f$ and $\hat{\lambda} = \gamma\lambda$, respectively, for some $\gamma > 0$. Show mathematically how this change affects the steady-state unemployment rate, and briefly discuss how it may affect the welfare of an unemployed worker.

Lastly, for any $u_t \neq u_{ss}$, how does the above change in the job-finding probability and separation rate affect the transition back to the steady-state rate of unemployment? Is it faster or is it slower? Explain briefly.

2. Consider the following simplified version of Schumpeterian growth model. Discrete time and 1-period lived individuals. Linear preferences in consumption. There is a final consumption good produced with the following technology

$$Y_t = (LA_t)^{1-\alpha} x_t^{\alpha}$$

where $0 < \alpha < 1$, x_t is intermediate, L is labor and we set it to 1, A is productivity of intermediate good at time t. One unit of the intermediate good is produced with one unit of the final good. The structure of the market is the following: final good operates in perfect competition and the intermediate good operates under monopoly. Entrepreneurs producing the intermediate good invest resources to improve its quality. If innovation is successful, the intermediate good increases its productivity in the final good sector by $\gamma > 1$ as follows:

$$A_t = \gamma A_{t-1}.$$

Innovation is uncertain. To obtain probability of innovation μ the entrepreneur invests resources in terms of final good R_t. The innovation cost function is

$$\mu_t = \phi\left(\frac{R_t}{A_t^*}\right) = \lambda n_t^{\sigma}$$

where $n_t = R_t/A_t^*$ is the productivity-adjusted resources invested in innovation, and $A_t^* = \gamma A_{t-1}^*$ is productivity of new intermediate good, and $\sigma > 0$. This cost function embeds the indea that as technology advances innovation is harder.

(a) Characterize the optimal problem of the final good firm, of the intermediate good firm and derive the equilibrium profits for the intermediate good monopolist.

(b) Derive the equilibrium resources devoted to innovation n_t, and the equilibrium innovation probability μ_t. Derive the equilibrium growth rate of the economy as a function of parameters.

(c) Define $\pi = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}$, what is the effect of an increase in π on the equilibrium growth rate of this economy? What is the effect of an exogenous increase in parameter λ, and of an exogenous increase in γ? Explain the mechanism behind these effects.

SECTION B: Intertemporal Economics

3. (a) Consider the following static labour/leisure choice problem

$$\begin{aligned} &\max_{c,\ell,L}\{\ln c + \ln \ell\} \\ s.t. \quad & c = Lw \\ & 1 = \ell + L \end{aligned}$$

where c denotes consumption, ℓ leisure, L labour, and w the wage rate. Derive the optimal choice of c, ℓ and L. How does the optimal choices depend on the wage rate, w? Explain intuitively.

(b) Now consider instead the following version of the above problem

$$\begin{aligned} &\max_{c,\ell,L}\{\ln c + \ln \ell\} \\ s.t. \quad & c = L(1-\tau)w + T \\ & 1 = \ell + L \end{aligned}$$

where τ is an income tax, and T is a lump-sum *rebate*. For simplicity, assume that the government rebates all revenue from the income tax. Derive the optimal choice of c, ℓ, and L. Specify your answer in terms of the tax τ. Explain how this answer differs from that of part (a).

(c) Compute the value of τ that maximises welfare in this setting. Explain the intuition underlying this result.

4. Illustrate the effect of a (lump-sum) tax-financed permanent expansion in government spending on output and on the real interest rate in the context of an intertemporal macroeconomic model. Explain intuitively and use graphs to illustrate your answer. *(Feel free to ignore the presence of investments throughout this question).*

   Now suppose that instead of lump-sum taxes, the government must use distortionary labour income taxes to finance its expansion in spending. How does this change alter your answer above? Again explain intuitively and use graphs to illustrate.

SECTION C: Monetary Economics

5. Suppose a gilt and an index-linked gilt maturing in 2055 on the same date have a coupon rate of 4.25% and 1.25%, and a redemption yield of 3.40% and -0.06%, respectively. Compute break-even inflation (up to three decimals). Carefully explain what would be your estimate of expected inflation and the risk-free real interest rate in each case if it is known that (i) the inflation measure used for index-linked gilts is biased and overstates actual inflation by one percent point, and (ii) the index-linked gilt is less liquid and has a liquidity premium of 50 basis points.

6. Since the start of 'quantitative easing' (QE) by the Bank of England in March 2009, banks' reserves at the Bank of England have risen by about £240bn, the monetary base has more than tripled, whereas M4 is about the same. Analyze what the likely effect of QE would have been on reserves, the monetary base and M4 if banks had used it fully to expand their lending, and discuss how the actual effects could be explained.

SECTION D: International Economics

7. Suppose the UK government suddenly decides to permanently reduce government purchases. Assume that the UK is a small open economy described by the DD-AA model and initially at its long-run equilibrium. Carefully explain what would happen to UK aggregate output and the nominal exchange rate in the short run.

8. Since the recent financial crisis, the United Kingdom has faced a challenging global macroeconomic environment in which foreign nominal interest rates declined and foreign output dropped. Analyze the effect of these two changes on the UK economy and discuss how macroeconomic policy could respond to them.

SECTION E: Policy

9. Suppose that a Government is in fiscal balance on its current account and the real interest rate, r, is lower than the growth rate of output, γ_y.

   (a) Write down an expression describing the evolution of the national debt-income ratio, explaining the economic intuition.

   (b) If we start with a positive debt ratio, in the absence of any shocks to the economy how will the debt ratio evolve?

   (c) Suppose there is a financial crisis what raises the real interest rate and lowers the growth rate so that now $r > g$. How does the debt ratio evolve now, assuming that the current account is kept in balance.

10. Suppose a central bank using monetary targeting minimizes the social welfare loss function

$$L = \frac{1}{2}\alpha\left(\pi - \pi^*\right)^2 + \frac{1}{2}\left(y - y^*\right)^2$$

where π is inflation, y aggregate output, π^* the inflation target, y^* the output target and α a positive parameter. Assume that $y^* = \kappa\bar{y}$, where $\bar{y}$ is the natural rate of output and $\kappa > 1$. The economy is described by the aggregate supply relation

$$y = \bar{y} + \theta\left(\pi - \pi^e\right) + s$$

where π^e denotes private sector inflation expectations, s is a white noise aggregate supply shock with variance σ_s^2, and θ is a positive parameter. The central bank adjusts the rate of money growth m to control inflation:

$$\pi = m + v$$

where v is a white noise velocity shock with variance σ_v^2.

At the beginning of the period, the private sector forms its inflation expectations π^e using rational expectations. Subsequently, the supply shock s and velocity shock v are observed. Then, the central bank sets its monetary policy instrument m, after which inflation π and output y are realized.

   (a) Derive the rate of money growth m that the central bank sets for a given level of private sector inflation expectations π^e. Explain intuitively how m depends on π^*, y^*, s and v.

   (b) Derive the level of private sector inflation expectations π^e, and the outcome for inflation π and output y. Explain intuitively how π and y depend on π^*, y^*, s and v.

(TURN OVER)

(c) Compute the expected value and variance of inflation and output: $E[\pi]$, $E[y]$, $Var[\pi]$ and $Var[y]$. Give an economic interpretation of the results.

(d) Suppose the government decides to delegate monetary policy to a new central banker. Explain the effect on the expected value and variance of inflation and output if the new central banker (indicated by CB) has an objective function with

(i) a higher parameter α, such that $\alpha_{CB} = 2\alpha$; or

(ii) a lower parameter κ, such that $\kappa_{CB} = 1$.

In each case, discuss whether delegation to such a central banker would be desirable.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECD1
Advanced Diploma in Economics

Tuesday 3 June 9:00am to 12:00pm

Paper 2

Macroeconomics

This paper comprises three sections: A, B and C.

Candidates are required to answer ***all six*** *questions in Section A,* ***one out of three*** *questions in Section B, and* ***one out of six*** *questions in Section C.*

Section A carries 50% of the marks, with each question equally weighted. Sections B and C each carry 25% of the total marks for this paper.

Write your candidate number ***not*** *your name on the cover sheet of* ***each*** *booklet.*

STATIONERY REQUIREMENTS
20 Page Answer Booklet x 1
Rough Work Pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator – students are permitted to bring an approved calculator

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SECTION A

A.1 Consider the following stylised Phillips curve

$$\pi_t = \pi_t^e + \alpha - \beta u_t,$$

where π_t denotes the inflation rate, π_t^e the expected inflation rate and u_t the unemployment rate. The subscript t denotes the time period.

Define the natural rate of unemployment, u_n, and rewrite the above relationship in terms of u_n.

Suppose that the population can be divided into two subgroups. The first group, which comprises a fraction γ of the population, has rational expectations such that $\pi_t^e = \pi_t$. The second group, which then comprises the fraction $(1-\gamma)$, has adaptive expectations such that $\pi_t^e = \pi_{t-1}$. Derive an expression for π_t in terms of u_t and u_n. How does the slope of the Phillips curve change depending on the parameter γ? Use the logic of the Aggregate Supply curve to provide an intuitive explanation.

A.2 Firm i produces output according to

$$y_i = F(L_i e_i),$$

where y_i denotes firm i's output, L_i labour inputs and e_i is effort per worker. As usual in the efficiency-wage literature, effort depends on firm i's wage, w_i, relative to the average wage in the economy, w, according to $e_i = e(w_i, w, T)$, where T denotes the economy's *temperature*. Effort is increasing in w_i and T, but decreasing in w.

Set up the firm's optimisation problem and show that, in the optimum, the elasticity of effort with respect to w_i is equal to unity. (Functions $F(\cdot)$ and $e(\cdot)$ happen to be such that the first order conditions are both necessary and sufficient.)

In equilibrium all firms will set identical wages. Suppose that the effort function is given by $e = (w_i - \gamma \frac{w}{T})^\alpha$, with $\gamma > 0$ and $\alpha \in (0,1)$. Derive the equilibrium temperature, T^* (as in Barro [1989], "An Efficiency Wage Theory of the Weather").

A.3 Suppose investors are concerned that the US government will temporarily default on its debt payments in three months due to a failure to agree on lifting the debt ceiling. Analyse the likely effect of this on the price and yield of US Treasury bills and bonds with a maturity of one month, three months and three years, and explain how it would affect the shape of the US yield curve.

A.4 Suppose euro area banks are making unexpected early repayments of large three-year loans made by the European Central Bank during the sovereign debt crisis. Analyse the short-run effect of this on the money supply, nominal interest rate, nominal exchange rate and aggregate output in the euro area using the DD-AA model.

A.5 Consider a model in which aggregate demand is given by

$$y_t = m_t - p_t + v$$

and aggregate supply by

$$y_t - y_t^p = \beta(p_t - E_{t-1}[p_t])$$

where y_t denotes the log of real output, y^p the log of the trend of real output, m_t the log of the money stock, p_t the log of the price level, and v is the log of the velocity of money. As usual, the subscript t denotes the time period, and $E_{t-1}[\cdot]$ denotes the rational expectation of some variable given information available in period $t-1$.

The monetary policymaker responds to changes in output in the previous period according to

$$m_t = \alpha y_{t-1} + \varepsilon_t$$

where ε_t is the unsystematic, unpredictable component of money with $E_{t-1}[\varepsilon_t] = 0$. The policymaker can vary α so that the responsiveness of monetary policy to output changes.

Show that α has no impact on output, y_t. Apart from rational expectations what other assumption is crucial for this result to go through?

A.6 Consider a small open economy which is a member state of a monetary union with free movement of capital. Suppose that external lenders require the country to improve its balance of trade in the short run. Assume that the private consumption function, $C(Y-T)$, takes the form

$$C(Y-T) = C_0 + c(Y-T),$$

where C_0 is a positive constant, and $c \in (0,1)$ is the marginal propensity to consume. As usual, Y denotes income and T taxes. Assume further that the import content of total final expenditure is a constant proportion m. Let $c = 0.8$ and $m = 0.25$. Calculate the change required in the fiscal balance, holding taxation constant, to improve the trade balance by 1% of GDP in the short run. Calculate also the effect of this policy on GDP. Explain how GDP might respond in the long run. Discuss briefly the relevance of this analysis to the Eurozone.

SECTION B

B.1 Consider the Bernanke-Blinder extension to the ISLM model. Banks are assumed to hold bonds B, loans L and reserves R as assets, and have deposits D as liabilities, so that the representative bank's balance sheet is

$$B + L + R = D.$$

Reserves are equal to the legal minimum reserve requirement $R = \tau D$, where $\tau = \frac{1}{4}$. The demand for deposits is given by

$$D^d = Y - \frac{1}{2} i_B,$$

where Y is real aggregate output and i_B the bond interest rate. The demand for loans is described by

$$L^d = Y - (i_L - i_B),$$

where i_L is the loan interest rate. The supply of loans is given by

$$L^s = \frac{2}{3} (D - R).$$

Goods market equilibrium is described by

$$Y = 100 - i_L - \frac{1}{2} i_B.$$

(a) Derive the equilibrium bond interest rate, i_B, in the money market in terms of output, Y, and reserves, R; and derive the equilibrium loan interest rate, i_L in the loan market in terms of Y, R, and i_B. Give an intuitive explanation.

(b) Derive output, Y, in terms of R and i_B such that there is equilibrium in both the goods market and the loan market. Give an intuitive explanation.

(c) Suppose the initial level of reserves is $R = 10$ and that the central bank conducts an open market purchase of bonds for an amount of 5. Compute the initial and new equilibrium levels of output, Y, the bond interest rate, i_B, and the loan interest rate, i_L. Illustrate the effect graphically and provide an economic explanation.

(d) Explain carefully how the effect in part (c) would be different if the economy were in a liquidity trap.

B.2 Consider a price-taking representative agent who faces the following optimisation problem

$$\max_{c_1,c_2,b_2} \{u(c_1) + \beta u(c_2)\}$$
$$\text{subject to} \quad c_1 + b_2 = (1 + r_1)b_1 + y_1 - T_1$$
$$c_2 = (1 + r_2)b_2 + y_2 - T_2,$$

where c_t denotes consumption, b_t savings, y_t income, T_t lump-sum taxes, and r_t the (real) interest rate. As usual, $t = 1, 2$, denotes time. The utility function, $u(c)$, is given by $\frac{c^{1-\sigma}}{1-\sigma}$, with $\sigma > 1$.

Assume that the economy is closed with no investments, such that the GDP identity is given by $y_t = c_t + g_t$, where g_t denotes government spending. Income, y_t, is a pure endowment. The government runs a balanced budget such that $T_t = g_t$.

(a) Derive the Euler equation associated with the above optimisation problem and provide an intuitive explanation.

(b) Suppose for simplicity that $g_t = 0$, $t = 1, 2$. Derive the equilibrium interest rate, r_2, as a function of the ratio of y_1 and y_2. What happens to the interest rate, r_2, if agents receive unexpected news in period 1 that y_2 will fall? What is the intuition behind this result?

(c) Given a certain value of y_1 and y_2, suppose that g_1 suddenly rises but that g_2 remains at zero. What is the effect on output, y_1, and the interest rate, r_2? Provide the intuition for this result. Illustrate this situation graphically in the (y_1, r_2)-plane. How does this differ from the standard intertemporal model, and why?

(d) Lastly, suppose that both y_1 and y_2 increase by a constant proportion $\gamma > 1$. For simplicity $g_t = 0$, $t = 1, 2$. How does the interest rate, r_2, respond to this change? Illustrate this situation graphically in the (y_1, r_2)-plane. Explain intuitively.

B.3 Consider the following variation of the continuous time Solow growth model. There are J identical firms in this economy. The production technology is represented by

$$Y_j(t) = \bar{A}(t)F(K_j(t), L_j(t)) = \bar{A}(t)K_j(t)^{\alpha}L_j(t)^{1-\alpha}, \quad \alpha \in (0,1),$$

where $K_j(t)$ denotes the capital stock used by firm j, $L_j(t)$ denotes labour input used in production by firm j, and t denotes the time period. Each firm is small relative to the aggregate. Households save a constant fraction of income, $s \in (0,1)$. The economy is closed which implies that investment equals savings. The labor force $L(t)$ grows at a constant rate n and capital depreciates at rate δ. In this economy technology is a by-product of economic activity, such that

$$\bar{A}(t) = A\left(\frac{K(t)}{L(t)}\right)^{\mu},$$

where $A > 0$ is a constant, and $0 < \mu < (1-\alpha)$ is a parameter describing the strength of external effects in this economy.

(a) Write down the equilibrium equation that describes the evolution of capital per unit of labour. Does the economy exhibit a Balanced Growth Path with positive long-run growth per worker output?

(b) Calculate the elasticity of the output per worker with respect to the productivity factor A (i.e. $\partial \ln y / \partial \ln A$) in the long-run. Does the externality parameter μ affect this elasticity? Explain.

(c) Illustrate graphically and explain carefully the dynamics of the output per worker and consumption per worker after a permanent increase in the productivity factor A.

SECTION C

C.1 Explain the implications of the efficient market hypothesis and discuss whether they are consistent with empirical evidence.

C.2 On 23 January 2014, the Argentina peso dropped about 15% against the US dollar. Discuss what could be the causes of a currency crisis.

C.3 Explain Ricardian equivalence, and discuss what implication it might have on the efficacy of expansionary fiscal policy.

C.4 Labour productivity is ten times higher today than 100 years ago. Yet the unemployment rate is roughly the same. Discuss possible explanations.

C.5 It is believed that time-consistent discretionary monetary policy normally results in excess inflation. Argue why this is the case, why a policymaker would want to reduce the inflation bias of discretionary monetary policy, and how she could do this.

C.6 Discuss the main business cycle facts for the United Kingdom. What are the sources of business cycles? Is it desirable to eliminate business cycle fluctuations?

END OF PAPER

ECAD1
Advanced Diploma in Economics

Tuesday 2 June 2015 9:00am to 12:00pm

Paper 2

Macroeconomics

This paper comprises three sections: A, B and C.

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20 Page Answer Booklet x 1
Rough Work Pad

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SECTION A

A.1 An economy is populated by $\bar{L}_p$ physicists and $\bar{L}_j$ janitors. A representative firm operates under perfect competition and maximizes profits

$$\Pi = \max_{\{L_p, L_j\}} \{F(L_p, L_j) - w_p L_p - w_j L_j\},$$

where $F(L_p, L_j) = [\eta L_p^\gamma + (1-\eta)L_j^\gamma]^{\frac{1}{\gamma}}$ denotes the production function, and w_p and w_j denote the real wages for physicists and janitors, respectively. The parameters η and γ satisfy $\eta \in (0,1)$ and $\gamma \in (-\infty, 1)$.

Derive the firm's first order conditions. For $\eta > 0.5$, under which condition will janitors earn higher equilibrium wages than physicists? Intuitively explain why.

A.2 A representative household faces the following optimization problem,

$$\begin{aligned} &\max_{C_0, C_1, B_1} \{u(C_0) + u(C_1)\} \\ \text{subject to} \quad & C_0 + B_1 = (1+r_0)B_0 + Y_0 - T_0 \\ & C_1 = (1+r_1)B_1 + Y_1 - T_1, \end{aligned}$$

where C_t denotes consumption, B_t government bond holdings, Y_t labor income, T_t taxes, and r_t the real interest rate, where the subscript t indicates the time period, $t = 0, 1$. The utility function $u(\cdot)$ satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Derive the Euler equation, and provide a brief intuitive explanation.

The government's budget constraints are given by

$$\begin{aligned} G_0 - D_1 &= T_0 - D_0(1+r_0) \\ G_1 &= T_1 - D_1(1+r_1), \end{aligned}$$

where G_t denotes government purchases and D_t government debt, $t = 0, 1$. Both Y_t and G_t are exogenous. In equilibrium $D_t = B_t$.

Using this equilibrium condition, substitute the government's budget constraint in period 0 into the representative household's budget constraint in period 0. Do the same for period 1. Substitute these results into the Euler equation to derive the equilibrium real interest rate. Interpret your results.

A.3 Consider the following Baumol-Tobin model of money demand. A representative economic agent can choose between two assets: cash M, which is required for spending, but earns zero nominal return; and bonds, which yield the nominal interest rate i. The agent receives a nominal income PY in the form of bonds at the beginning of the period and spends a fraction γ of it uniformly during the period. P denotes the aggregate price level, Y real income, and $\gamma \in (0, 1)$. The agent incurs a real transaction cost $c > 0$ each time she sells bonds.

Derive the average level of real money holdings M/P that minimizes total costs for the agent, and the real money demand elasticities with respect to real income Y and the nominal interest rate i. How do the results depend on the fraction γ of income that is spent?

A.4 Suppose the central bank suddenly announces that it will perform a large outright open market purchase six weeks from now. Using the asset market model and assuming prices remain sticky for three months, analyze how this affects the nominal exchange rate at *(i)* the time of announcement, and *(ii)* the time of implementation.

A.5 Suppose a policymaker faces the constraint of the Phillips Curve

$$\pi_t =_{t-1} \pi_t^e + \alpha(y_t - \bar{y})$$

where y_t is output, $\bar{y}$ is potential output, so $y_t - \bar{y}$ is the output gap. π_t is the inflation rate, and ${}_{t-1}\pi_t^e$ is the rational expectation of inflation in period t formulated in period $t - 1$. The policymaker sets output, y_t, to minimize the loss function

$$L_t = (y_t - y^T)^2 + \beta(\pi_t - \pi^T)^2.$$

It is assumed that $y^T \geq \bar{y}$.

Derive equilibrium inflation and output.

Suppose that you suspect that with probability ρ, y^T is equal to y^* which strictly exceeds $\bar{y}$, while with the complementary probability $(1 - \rho)$, y^T is equal to $\bar{y}$. What is the *expected* inflation bias? Explain why.

A.6 Consider a small open economy which is a member state of a monetary union with free movement of capital. Assume that the private consumption function, $C(Y-T)$, takes the form

$$C(Y-T) = C_0 + c \times (Y-T),$$

where $C_0 > 0$ is a positive constant, and $c \in (0,1)$ is the marginal propensity to consume. As usual, Y denotes income and T taxes. Assume further that the import content of total final expenditure is a constant proportion $m \in (0,1)$, such that

$$M = m \times (C + I + G + X),$$

where M denotes imports, C private consumption, G government expenditures, and X exports.

Let $c = 1/2$ and $m = 1/2$, and assume that investment represents 25 percent of income. Suppose that investment increases by 50 percent because of a boom in speculative housebuilding. Calculate the resulting change in net exports as a proportion of initial GDP.

SECTION B

B.1 The reservation wage, w_r, in McCall's search model is characterized by the equation

$$w_r - b = \frac{\beta}{1-\beta}\left(\int_{w_r}^{\infty}(w - w_r)dF(w)\right),$$

where b denotes unemployment benefits, $\beta \in (0,1)$ the discount factor, w wages, and and $F(w)$ denotes the cumulative distribution function (CDF) for wage offers.

(a) Intuitively explain how the reservation wage depends on both b and β.

(b) Suppose that b increases. What happens to the job finding rate, and what is the implication for the unemployment rate? What are the possible effects on output/welfare?

Let $V(w)$, denote the value of having a job paying a perpetual wage w,

$$V(w) = \frac{w}{1-\beta}.$$

Let U denote the value of not having a job,

$$U = b + \beta E[\max\{V(w), U\}],$$

where E denotes the expectations operator associated with the CDF above.

(c) For a given value of U, graphically illustrate the value

$$W = \max\{V(w), U\},$$

with w on the x-axis, and with W on the y-axis. Carefully mark out the reservation wage, w_r, in your graph, and explain.

(d) Suppose w has a uniform distribution with the following CDF

$$F(w) = \frac{w - \underline{w}}{\overline{w} - \underline{w}}, \quad w \in [\underline{w}, \overline{w}]$$

where the mean, μ_w, and variance, σ_w^2, are given by

$$\mu_w = \frac{1}{2}(\overline{w} + \underline{w}), \quad \sigma_w^2 = \frac{1}{12}(\overline{w} - \underline{w})^2.$$

Suppose now that $\overline{w}$ and $\underline{w}$ changes to $\overline{w}_{new} = \overline{w} + \varepsilon$ and $\underline{w}_{new} = \underline{w} - \varepsilon$, respectively, with $\varepsilon > 0$. What is the effect on the mean and the variance of the wage offer?

How do you think this change of the wage offer distribution will affect the reservation wage? Explain intuitively why.

B.2 Consider the following model of a small open economy (Home), which interacts with the rest of the world (Foreign):

$$\begin{aligned} i &= i^* + E[\mathbf{e}] - \mathbf{e} \\ \mathbf{M} - \mathbf{P} &= \mathbf{Y} - \frac{1}{2} i \\ \mathbf{Y} &= (\mathbf{e} + \mathbf{P}^* - \mathbf{P}) - \theta i \end{aligned}$$

where i is the Home nominal interest rate, i^* the Foreign nominal interest rate, $\mathbf{e}$ the nominal exchange rate expressed as the Home price of Foreign currency, $E[\mathbf{e}]$ the expected future exchange rate, $\mathbf{M}$ the Home money supply, $\mathbf{P}$ the Home aggregate price level, $\mathbf{P}^*$ the Foreign aggregate price level, $\mathbf{Y}$ Home aggregate output, and all boldface variables are in logs. The coefficient θ depends on the interest rate sensitivity of investment. People have rational expectations. In the short run, the aggregate price level $\mathbf{P}$ is fixed. In the long run, the price level $\mathbf{P}$ is flexible and output is at its natural rate $\bar{\mathbf{Y}}$. Suppose that $\mathbf{M} = \bar{\mathbf{M}}$, $\bar{\mathbf{Y}} = 0$, $\mathbf{P}^* = 0.05$ and $i^* = 0.10$, and assume initially that $\theta = 0$.

(a) Give an economic interpretation of the three displayed equations above.

(b) Compute the long run equilibrium values of i, $\mathbf{P}$, $\mathbf{e}$ and $E[\mathbf{e}]$. Explain how they are affected by the level of $\bar{\mathbf{M}}$.

(c) Suppose now that there suddenly is a permanent increase in the money supply $\bar{\mathbf{M}}$ from 0 to 0.25. Compute the short run equilibrium level of the nominal interest rate i, the nominal exchange rate $\mathbf{e}$ and output $\mathbf{Y}$. Analyze the short-run effects, both graphically and intuitively. Is there exchange rate overshooting in this case?

(d) Assume now that $\theta = 1$ and again analyze the short-run effects of a sudden, permanent increase in the money supply $\bar{\mathbf{M}}$ from 0 to 0.25. Compare the results to part (c).

B.3 Consider the following cash in advance model

$$\max_{c_0,c_1,B_1,X_1} \{u(c_0) + \beta u(c_1)\}$$

$$\begin{aligned} \text{subject to} \quad p_0c_0 + B_1 + X_1 &= B_0(1+i_0) + W_{-1} + X_0 - T_0 \\ p_1c_1 &= B_1(1+i_1) + W_0 + X_1 - T_1 \\ X_1 &\geq 0 \\ X_0, W_0, B_0, i_0, &\text{ are given, and } W_0 + X_0 = m_{-1}. \end{aligned}$$

Here c_t denotes consumption of the output good, B_t nominal government bonds, X_t excess cash holdings, Y_t nominal income, i_t the nominal interest rate, and T_t are nominal taxes, $t = 0, 1$. The utility function, $u(\cdot)$, satisfies the normal requirements, $u'(\cdot) > 0$, and $u''(\cdot) < 0$.

(a) Derive the Euler equations related to both bond holdings, B_1, and excess cash holdings, X_1.

(b) The government's budget constraint is

$$G_t = T_t + (m_t - m_{t-1})$$

where G_t denotes nominal government expenditures, and m_t money supply. The equilibrium conditions are $B_t = 0$, and $Y_t = C_t + G_t$, $t = 0, 1$. The cash in advance structure implies that $W_0 = Y_0$.

Derive the equation of exchange in period 0 and then in period 1.

(c) Potential output in period 0 is equal to one. Find the value of real output in period 1, y', that brings the economy to the cusp of a liquidity trap. That is, $y_1 = y'$ should satisfy

$$u'(1) = \beta \frac{p_0}{p_1} u'(y'),$$

with $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma > 1$.

(d) Suppose that $y_1 < y'$ and $p_0 = m_0$, such that $y_0 < 1$. Consider implementing two different policies in period 0: (1), to increase government spending, G_0 such that $y_0 = 1$, or (2) to increase m_1 such that $y_0 = 1$. Which policy yields the largest increase in private consumption, c_0? What is the intuition for this result?

Section C

C.1 In 2008, the Consumer Price Index (CPI) fell and remained 4 percent below its pre-crisis trend. The unemployment rate rose to around 10 percent, and declined over the course of six years. Is this pattern consistent with the NAIRU theory?

C.2 During the recent financial crisis, output, consumption, investment, *and* the real interest rate declined substantially. Which types of events/shocks can reconcile these facts with the predictions of the neoclassical intertemporal model? Which events/shocks cannot?

C.3 On 22 January 2015, the European Central Bank (ECB) announced a program of large-scale asset purchases, while its main refinancing rate was virtually zero. Analyze how this is likely to affect bond yields, equity prices, bank lending and economic activity in the euro area.

C.4 In the aftermath of the financial crisis, several central banks in advanced economies have engaged in 'quantitative easing'. This has lead to the allegation of a 'currency war' that negatively affects other countries. Analyze whether 'quantitative easing' is likely to have a detrimental effect on economic activity in other countries and discuss how these countries could adjust their macroeconomic policy to counter any negative effects.

C.5 In the Bank of England act of 1998, the UK government formally delegated monetary policy to an independent Monetary Policy Committee. Why would the government want to restrict its policy options in this way?

C.6 'The Real Business Cycle model provides a surprisingly accurate account of economic fluctuations.' Critically evaluate this claim.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECAD1
Advanced Diploma in Economics

Tuesday 31 May 2016 09:00am - 12:00pm

Paper 2
MACROECONOMICS

This paper comprises three Sections, A, B and C.
Answer **ALL FIVE** questions from Section A.
Answer **ONE** question from Section B.
Answer **ONE** question from Section C.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A.1 Consider the following intratemporal (i.e. *within period*) optimisation problem:

$$\max_{c_t,\ell_t} \left\{ \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \ell_t \right\}$$
$$\text{subject to} \quad c_t = (1-\ell_t)w_t,$$

where c_t denotes consumption, ℓ_t leisure, w_t the real wage, $L_t = (1-\ell_t)$ hours worked, and the parameter σ is positive. Compute optimal consumption c_t and hours worked L_t as a function of the real wage w_t.
Suppose that the economy is growing at a rate g every year such that $w_{t+1} = (1+g)w_t$, for $t = 0, 1, \ldots$ In a famous essay, John Maynard Keynes predicted that from 1930 to 2030 the real wage would rise sixfold, while hours worked would decline to a third of their level in 1930. Which value of g and σ in the above framework are consistent with Keynes's predictions? Explain what your answer implies about the relative strengths of income and substitution effects of the real wage on labour supply? (Feel free to use the approximation $\ln(1+x) \approx x$ for a sufficiently small value of x.)

A.2 Consider the following McCall search model, in which the reservation wage w_r is determined by:

$$w_r - b = \underbrace{\frac{\beta}{1-\beta} \sum_{w \in W} \max\{w - w_r, 0\} \Pr(w)}_{h(w_r)},$$

where b denotes unemployment benefits ($b \geq 0$) and β the worker's intertemporal discount factor ($0 < \beta < 1$). Assume that wage offers w are drawn from the set $W = \{w_1, w_2, w_3\}$, with $0 < w_1 < w_2 < w_3$. The probability $\Pr(w)$ that any $w \in W$ occurs is given by $\Pr(w_1) = p_1$, $\Pr(w_2) = p_2$, and $\Pr(w_3) = 1 - p_1 - p_2$, where p_1 and p_2 are positive and satisfy $p_1 + p_2 < 1$.
Carefully draw $w_r - b$ and $h(w_r)$ in a graph with w_r on the horizontal axis. In the standard McCall search model the probability of finding a job is strictly decreasing in the level of unemployment benefits b. Explain whether this is still true in the above framework.

A.3 Using the Bernanke-Blinder model, analyse the effects of a large outright open market purchase by the central bank when the economy is in a liquidity trap. Carefully explain how the effect on aggregate output is affected by the liquidity trap.

A.4 On 3 December 2015 the European Central Bank announced a reduction in its deposit rate by 10 basis points, and an extension of its large-scale asset purchase program of €60bn per month until at least March 2017. In response to this news, the euro experienced a significant appreciation. Explain whether this could be consistent with the asset market model of the exchange rate.

A.5 In the Romer model, it is assumed that the production function for new technology (ideas), A, is given by:

$$\dot{A} = \theta L_A^{\lambda} A^{\phi}$$

where L_A denotes the labour employed in the technology sector and $\theta > 0$. Assume ϕ and λ to be positive numbers lower than 1 ($0 < \phi, \lambda < 1$) and that the share of labour devoted to research, s_A, is constant, i.e. $L_A = s_A L$, where L is total labour force and $0 < s_A < 1$. Moreover assume that the labour force grows at a constant rate n.

(a) On the balanced growth path, calculate the growth rate of output per capita (which is equal to the growth rate of A) as a function of ϕ, λ, and the growth rate of the labour force, n.

(b) Explain what the parameter ϕ is supposed to capture, then explain the relationship you found in part (a) between this parameter and the long-run growth rate of output per capita.

SECTION B

B.1 Consider a price-taking representative agent who faces the following optimisation problem

$$\max_{\{c_1,c_2,b_2,\ell_1,\ell_2\}} \{\ln c_1 + \ln \ell_1 + \beta (\ln c_2 + \ln \ell_2)\},$$
$$\text{subject to} \quad c_1 + b_2 = w_1(1-\ell_1) - T_1,$$
$$c_2 = w_2(1-\ell_2) + b_2(1+r_2) - T_2,$$

where c_t denotes consumption, ℓ_t leisure, b_t savings, T_t lump-sum taxes, w_t the real wage, r_t the real interest rate, and β the intertemporal discount factor $(0 < \beta < 1)$. The subscript $t = 1, 2$ denotes the time period.

(a) Derive the agent's intertemporal budget constraint. Give an economic interpretation of the result.

(b) Find the optimality conditions for c_1, c_2, b_2, ℓ_1 and ℓ_2. Derive the intertemporal Euler equation for consumption, and the intratemporal (i.e. *within period*) Euler equation characterizing the optimal trade-off between consumption and leisure. Give an intuitive explanation of these Euler equations.

Assume that the economy is closed and that there is no investment. So the GDP identity is given by $y_t = c_t + g_t$, where g_t denotes government purchases, which are exogenous, and y_t denotes aggregate output, which is equal to $y_t = w_t(1-\ell_t)$. Suppose that $w_t = A_t$, where A_t represents productivity in period $t = 1, 2$. In addition, assume that the government runs a balanced budget such that $T_t = g_t$.

(c) Solve for the equilibrium level of output y_t in terms of A_t and g_t for $t = 1, 2$. Derive the fiscal multiplier $\partial y_1 / \partial g_1$ for a permanent increase in g_1. Provide an intuitive explanation of the results.

(d) Now suppose that $A_1 = A_2 = 1$ and $g_2 = \rho g_1$, where $0 \leq \rho \leq 1$. What are the fiscal multipliers $\partial y_t / \partial g_t$ for $t = 1, 2$? Compare the result to part (c). Derive the equilibrium real interest rate r_2 and explain how it is affected by a rise in g_1 when $\rho = 0$ and when $\rho = 1$. Explain the intuition underlying these results.

B.2 Suppose a central bank using monetary targeting minimizes the social welfare loss function

$$L = \frac{1}{2}(\pi - \pi^*)^2 + \frac{1}{2}\lambda (y - y^*)^2$$

where π is inflation, y the output gap, π^* the inflation target, y^* the output gap target and λ a positive parameter. Assume that $\pi^* > 0$ and $y^* > 0$. The central bank sets the rate of money growth m, which affects inflation:

$$\pi = m + v$$

where v is a white noise velocity shock with variance σ_v^2. The aggregate supply relation is described by

$$y = \theta (\pi - \pi^e) + s$$

where π^e denotes private sector inflation expectations, s is a white noise aggregate supply shock with variance σ_s^2, and θ is a positive parameter. Assume that v and s are independent.

At the beginning of the period, the private sector forms its inflation expectations π^e using rational expectations. Subsequently, the supply shock s and velocity shock v are observed. Then, the central bank adjusts its monetary policy instrument m, after which inflation π and the output gap y are realized.

(a) Derive the rate of money growth m that the central bank sets for a given level of private sector inflation expectations π^e. Explain intuitively how m depends on π^*, y^*, π^e, s and v.

(b) Derive the level of private sector inflation expectations π^e, and the outcome for inflation π and the output gap y. Compute the expected value and variance of inflation and the output gap: $\mathrm{E}[\pi]$, $\mathrm{E}[y]$, $\mathrm{Var}[\pi]$ and $\mathrm{Var}[y]$. Give a brief economic interpretation of the results.

(c) Suppose the government is dissatisfied with macroeconomic performance and considering whether to require the central bank to announce and commit to a rate of money growth m_C before the start of the period. Derive the level of m_C that minimizes expected social welfare losses L and the resulting macroeconomic outcomes π_C, y_C, $\mathrm{E}[\pi_C]$, $\mathrm{E}[y_C]$, $\mathrm{Var}[\pi_C]$ and $\mathrm{Var}[y_C]$. Would commitment improve macroeconomic outcomes?

(d) Alternatively, the government could delegate monetary policy to a new central banker (indicated by CB) who minimizes L but with a different λ or y^*, such that either (i) $\lambda_{CB} = 0$, or (ii) $y^*_{CB} = 0$. Explain whether these delegation options would improve macroeconomic outcomes compared to part (b). In addition, analyse which of the three alternatives would be best: commitment as in part (c), delegation (i) or delegation (ii).

B.3 Consider a small open economy with perfect capital mobility described by the Mundell-Fleming model. Focus on the short run and assume that prices are fixed.

(a) Assume that exchange rates are flexible. Show graphically the effect of a fiscal and of a monetary expansion. Discuss the economic intuition of the results and compare them with the effects of these policies in closed economy.

(b) Now consider a fixed exchange rate regime and repeat the same exercise as in part (a).

(c) In 2009 fears developed about Greece's ability to honor its debt obligations, which were triggered by revelations that the Greek government had misreported the data on government deficits. This caused a crisis of confidence and bond yield spreads widened compared to the other Eurozone countries. Soon after, Greece entered a period of harsh and prolonged recession driven by capital ouflows and soaring costs of public debt. Use the Mundell-Fleming model to show how a crisis of confidence in a country's debt can trigger a deep recession.

SECTION C

C.1 For an extensive time series covering post-war data in the United States, the fiscal multiplier is commonly estimated to be about 0.8. Yet, during the financial crisis of 2008-2010 the IMF estimated the fiscal multiplier to be around 1.5. Assume that these estimates are accurate for their respective time periods. Presuming any rise in government spending is debt financed, discuss to which extent the presence of both non-Ricardian (i.e. liquidity constrained) and Ricardian households can explain this notable difference.

C.2 The table below illustrates the evolution of some macroeconomic variables for the US economy at the indicated time periods. In particular, it shows the unemployment rate u_t, the employment-to-population ratio n_t, and the percent deviations from trend for the consumer price index (CPI) $\tilde{p}_t$ and for real GDP $\tilde{y}_t$, where the trend paths for the CPI and real GDP have been computed based on their average growth rates during the decade before the financial crisis.

Variable	Period t		
	2007Q1	2009Q4	2015Q4
u_t	4.5%	9.8%	5.0%
n_t	63.3%	58.5%	59.4%
$\tilde{p}_t$	0%	-4.6%	-11.5%
$\tilde{y}_t$	0%	-8.2%	-11.3%

Discuss to which extent these data are consistent with the NAIRU, and which other theories (if any) may help to provide a more complete understanding.

C.3 On 1 March 2016, the Japanese government issued a ten-year government bond at a negative yield. At the same time, five-year government bonds in Germany had reached yields of around -0.4%. Explain carefully what it means to issue or buy a bond with a negative yield, and discuss what factors have contributed to the unprecedented phenomenon of negative government bond yields in much of Europe and Japan.

C.4 Suppose that exit from the European Union by the United Kingdom ('Brexit') would have a negative effect on UK net exports. In addition, assume that the uncertainty created by the prospect of Brexit makes investment in the UK less attractive. Analyse the effects of these 'Brexit shocks' on the nominal exchange rate, the nominal interest rate and aggregate output in the UK in the short run.

C.5 Imperfect competition is a necessary but not a sufficient condition to generate money non-neutrality. Discuss.

END OF PAPER

ECAD1
Advanced Diploma in Economics

Tuesday 6 June 2017 9:00am - 12:00pm

Paper 2
MACROECONOMICS

This paper comprises three Sections, A, B and C.
Answer **ALL FIVE** questions from Section A.
Answer **ONE** question from Section B.
Answer **ONE** question from Section C.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the Invigilator, who will notify the Examiner. A general announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A1 The yield on two-year German government bonds reached close to -1% in February, while the interest rate on the standing deposit facility of the European Central Bank (ECB) was -0.40%.
Explain carefully what it means to buy a bond with a negative yield. Why would European banks and investors be willing to buy a two-year bond at such a negative yield below the ECB deposit rate?

A2 After the referendum on 23 June 2016, the British pound experienced a large depreciation while the UK had a big current account deficit. Derive the Marshall-Lerner condition for the case in which initially $EX = \kappa IM$, where EX denotes exports, IM imports, and κ is a positive parameter, assuming no net factor income or unilateral current transfers from abroad. Explain whether the presence of an initial current account deficit makes it more or less likely that the exchange rate depreciation improves the current account for a given level of aggregate output.

A3 In McCall's search model it is commonly assumed that a job lasts for perpetuity. As a consequence, the reservation wage, w_r, is implied by the equation

$$U = \frac{1}{1-\beta} w_r,$$

where U denotes the value of unemployment, and β is the discount factor. Now assume that a job does not last for perpetuity, but terminates in each period with probability δ. The value of having a job paying wage w – denoted V – is therefore

$$V = w + \beta[(1-\delta)V + \delta U].$$

(a) Derive the value of U in terms of the reservation wage and the relevant parameters.

(b) Holding w_r constant, how is U related to δ? Explain.

(c) Lastly, assume that unemployment follows the law of motion

$$u_{t+1} = (1-f)u_t + \delta(1-u_t),$$

where f denotes the job-finding rate. Provide an intuitive explanation of the above law of motion, and derive the steady state unemployment rate, u_{ss}. If $F(w)$ denotes the cumulative distribution function for job offers, how do you think f is related to w_r?

A4 Consider the following static optimisation problem

$$\max_{c,\ell} \{\ln(c) + \ln(\ell)\}$$

$$\text{subject to} \quad c = (1-\ell) - T,$$

where c denotes consumption, ℓ leisure, and T lump-sum taxes.

(a) Derive optimal labour supply $L = 1 - \ell$ as a function of T. Explain intuitively how L is related to T.

(b) Suppose that output, Y, is given by L. In addition, the government runs a balanced budget such that $G = T$. The GDP identity is $Y = C + G$. What is the fiscal multiplier?

(c) Suppose now that taxes are distortionary such that the budget constraint is

$$c = (1 - \ell)(1 - \tau),$$

where τ is the tax rate. The government still runs a balanced budget such that $G = \tau L$. What is the fiscal multiplier under these conditions?

(d) How does your answer differ from the previous case in which taxes were lump-sum? Provide an intuitive explanation.

A5 Country A at time period t has a real Gross Domestic Product of 1000 bitcoins, a Price level of 1 and a nominal national Debt of 600 bitcoins. It has a real growth rate of 2.5% and a real interest rate of 3%. Government expenditure is 200 bitcoins and the government raises taxes of 200 bitcoins.

(a) What is the government's budget constraint and what are the debt dynamics describing the evolution of national debt?

(b) Draw the phase diagram for the evolution of debt and comment upon its features.

(c) Suppose the government wishes to stabilise the debt-income ratio. What must it do?

(d) Suppose the real interest rate falls to 2.5% and the real growth rate of the economy rises to 3%. What effect does this have on your answer?

SECTION B

B1 Consider the following Bernanke-Blinder model. Banks are assumed to hold bonds B, loans L and reserves R as assets, and have deposits D as liabilities, so that the representative bank's balance sheet is:

$$B + L + R = D$$

Reserves are equal to the legal minimum reserve requirement $R = \tau D$, where $\tau = \frac{1}{2}$. The demand for deposits is given by

$$D^d = \frac{1}{2}Y - \frac{1}{2}i_B$$

where Y is real aggregate output and i_B the bond interest rate. The demand for loans is described by

$$L^d = Y - i_L + i_B$$

where i_L is the loan interest rate. The supply of loans is given by

$$L^s = \frac{1}{2}(D - R)$$

Goods market equilibrium is described by

$$Y = \alpha - i_L - i_B$$

where α is an aggregate demand parameter.

(a) Derive the equilibrium bond interest rate i_B in the money market in terms of output Y and reserves R; and derive the equilibrium loan interest rate i_L in the loan market in terms of Y, R and i_B. Explain intuitively how i_B and i_L are affected by Y and R.

(b) Derive output Y in terms of α, R and i_B such that there is equilibrium in both the goods market and the loan market. Explain intuitively how this equilibrium output is affected by i_B and R.

(c) Assume that initially $\alpha = 340$ and $R = 40$. Suppose that there is a negative aggregate demand shock that reduces α by 20. Compute the initial and new equilibrium levels of output Y and the bond interest rate i_B. Give an intuitive explanation for the effects.

(d) Suppose that the central bank would like to completely offset the effect on output of the negative aggregate demand shock in part (c). Compute the change in reserves R that would be needed to achieve this, and the resulting bond interest rate i_B. Show the effects of these changes in α and R on Y and i_B in *one* diagram and give an economic interpretation of the results.

B2 Consider a population of measure (size) one. The measure of unemployed, U_t, employed, N_t, and out-of-the-labour-force, O_t must satisfy the identity

$$1 = O_t + U_t + N_t.$$

The labour force itself is then simply given by $L_t = U_t + N_t$. The law of motion for N_t and U_t are given by

$$\begin{aligned} N_{t+1} &= U_t f + (1-\delta)N_t \\ U_{t+1} &= U_t(1-h-f) + \delta(1-U_t), \end{aligned}$$

where f, h, and δ are some constant parameters. A steady-state is defined as a situation in which $X_{t+1} = X_t = X$, for $X \in \{U, N, O\}$; that is, a situation in which all variables are constant.

(a) What is the implied law of motion for O_t? Provide an intuitive explanation. What is the interpretation of the parameter h?

(b) What are the steady state values of U_t, N_t, and O_t. Explain intuitively how these depends on the parameters f, h, and δ.

(c) What is the steady state unemployment *rate*? How is the unemployment rate related to the parameter h? Explain intuitively why this relationship emerges.

(d) Figure 1 illustrates the transitional dynamics from doubling the value of h, using the steady-state values of N_t, U_t, and O_t as initial conditions. As can be seen from the figure, the employment-to-population ratio declines to a permanently lower value, while the unemployment rate temporarily declines but subsequently recovers. Provide an intuitive explanation for why this pattern is observed. In this case, is the unemployment rate an accurate indicator of how well the economy is performing?

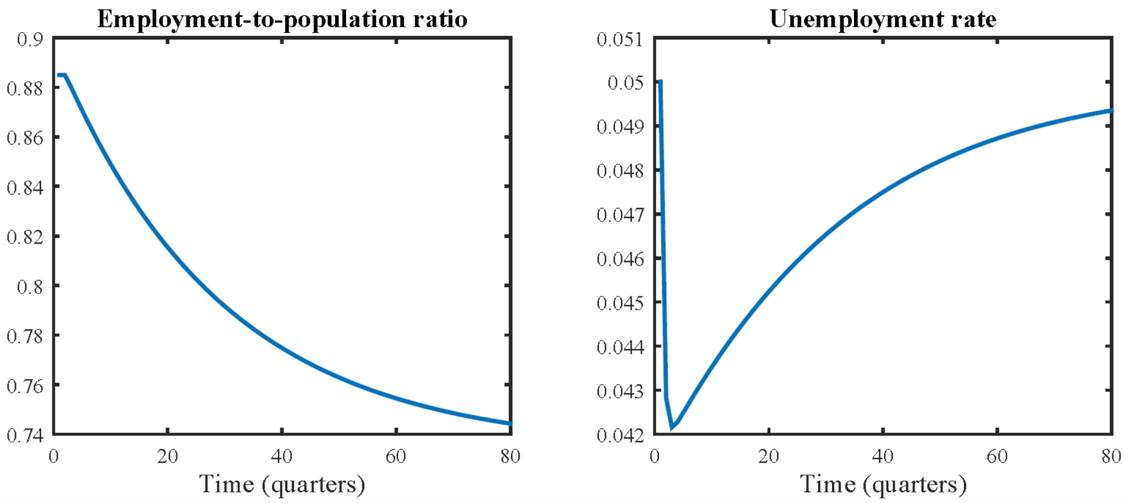


Figure 1: The transitional dynamics of the employment-to-population ratio and the unemployment rate arising from a doubling of the value of h.

B3 A small open economy with a floating nominal exchange rate and a fixed price level is hit by a large nominal demand shock that increases the demand for nominal money balances. The government wants to stabilise the economy by cutting taxes.

(a) Analyse the effect of this policy diagrammatically and evaluate how effective it is likely to be.

(b) The tax cut is financed through borrowing. International investors become concerned about the level of government debt and demand a higher risk premium to be willing to invest in the country. How does this affect your evaluation of the tax cut?

SECTION C

C1 Suppose that the new president of the United States decides to appoint governors to the Federal Reserve Board who have an inflation target of 2% and a target for real GDP growth of 4%, while current longer-run Federal Reserve forecasts for US real GDP growth are about 2%. Use the Barro-Gordon model to explain what effect that is likely to have on US inflation and output. Discuss how the result is likely to be affected if the Federal Reserve governors are given personal incentives to achieve the real GDP growth target of 4%, such as a contract that includes a bonus that is increasing in real GDP growth, or the risk of being fired if the real GDP growth target is not achieved.

C2 The United Kingdom plans to exit the European Union (‘Brexit’) in about two years. Suppose that the implementation of Brexit will lead to a lasting increase in barriers to international trade that will reduce the relative demand for UK goods and services. Analyze the effects that the *anticipation* of Brexit is likely to have on the UK economy in the short run, including aggregate output, consumption, investment, the current account, the nominal interest rate and the nominal exchange rate. Compare this to the likely future effects of the *implementation* of Brexit on the UK economy. Specify and discuss the assumptions behind your results.

C3 A government wishes to fund some of its expenditures through debt instead of (lump-sum) taxes. However, in order to encourage the private sector to lend the required resources to the government the interest rate must rise, which may crowd out private investments. Discuss the merits of this argument in the context of the neoclassical model with and without the presence of liquidity constrained consumers.

C4 Suppose there is a large increase in the supply of labour due to immigration. Under the assumption that the immigrants are identical to domestic workers in all characteristics, discuss how this increase in labour supply could affect the unemployment rate in the context of:

(a) A standard demand-supply framework in which a minimum wage prevents market clearing.

(b) A generic model of efficiency wages.

C5 Oil prices collapsed from around \$114 in June 2014 to \$31 in January 2016. Are low oil prices good for the world economy?

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECAD1
Advanced Diploma in Economics

Tuesday 5 June 2018 9:00am - 12:00pm

Paper 2
MACROECONOMICS

This paper comprises three Sections, A, B and C.
Answer **ALL FIVE** questions from Section A.
Answer **ONE** question from Section B.
Answer **ONE** question from Section C.

Section A will carry 50% of the marks, with each question weighed equally.
Sections B and C will each carry 25% of the total marks for this paper.

Write your candidate number (not your name) on the cover of each booklet.

Candidates are asked to note that there may be a reduction in marks for scripts with illegible handwriting.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A.1 Using the simplification $\beta(1+r) = 1$, the consumption function according to the Permanent Income Hypothesis is given by

$$c_t = \frac{r}{1+r}\left(\sum_{s=t}^{\infty} \beta^{s-t} y_s + (1+r)a_t\right),$$

where c_t denotes consumption in period t, $\beta \in (0,1)$ the discount factor, r the (real) interest rate, y_s (real, non-financial) income in period s, and a_t wealth in period t.

(a) Compute the marginal propensity to consume (MPC) out of current income.

(b) Define permanent income, $\bar{y}$. Compute the MPC out of permanent income.

(c) Explain what the MPC out of transitory income is.

A.2 Consider the following Shapiro-Stiglitz model in which the value of working is $w - \bar{e}$, where w denotes the real wage, and $\bar{e}$ the utility cost of exerting effort. The value of shirking is given by

$$(1-\pi)w + \pi p w^* + \pi(1-p)b,$$

where π denotes the probability of being detected shirking, p the probability of finding a new job at outside wages, w^*, and b the level of unemployment benefits. In equilibrium, $p = 1 - u$ and $w = w^*$, where u is the unemployment rate.

(a) Use the above expressions and relationships to derive the no-shirking condition. Provide a brief intuitive explanation of the relationship between the unemployment rate, u, and the wage, w.

(b) Assume there are N firms in the market. The first order condition for each firm yields

$$F'(\bar{e}L)\bar{e} = w,$$

where L denotes labour demand, and the production function is given by $F(\bar{e}L) = (\bar{e}L)^\alpha$, with $\alpha \in (0,1]$. Provide an intuitive explanation of the relationship between wages and labour demand.

(c) The unemployment rate equals

$$u = \frac{\bar{L} - NL}{\bar{L}},$$

where $\bar{L}$ denotes the labour force. Show how an expansion of the labour force from $\bar{L}$ to $\bar{L}'$ affects the equilibrium wage and unemployment rate in this model for (i) $\alpha \in (0,1)$, and (ii) $\alpha = 1$.

A.3 'In the Bernanke-Blinder model with a liquidity trap, an open market purchase by the central bank could lead to a larger increase in aggregate output than without a liquidity trap, ceteris paribus.' True or false? Explain your answer carefully and illustrate it graphically.

A.4 Suppose the economy of Euroland is described by the DD-AA model, with the nominal interest rate at its effective lower bound, i_{ELB}. Carefully explain what this implies about the shape of the AA curve. In addition, analyze the short-run effect of a sudden, permanent increase in Euroland's money supply (e.g. due to quantitative easing) on its nominal exchange rate and aggregate output.

A.5 Consider a closed economy where all prices are fixed. The marginal propensity to consume is 0.5 and the income tax rate is 0.5. The government runs a balanced budget and it spends the revenue from the income tax on goods and services. By how many units would income fall if consumption demand falls by 10 units? Give an intuitive explanation.

SECTION B

B.1 Consider the following optimization problem for a representative consumer:

$$\max_{c_1,c_2,b_2} \{\ln(c_1) + \beta \ln(c_2)\},$$
$$\text{s.t.} \qquad y_1 = c_1 + b_2,$$
$$c_2 = y_2 + (1 + r_2)b_2,$$

where c_t denotes consumption in periods $t = 1, 2$; b_2 savings from period 1 to period 2; y_t (real, non-financial) income in periods $t = 1, 2$; r_2 the real interest rate received in period 2; and $\beta \in (0, 1)$ the discount factor.

(a) Derive the Euler equation for the above problem. Provide an intuitive explanation of the relationship between c_1 and c_2.

(b) Use the market clearing condition, $b_2 = 0$, to find the equilibrium interest rate, r_2, in terms of β, y_1, and y_2. Explain intuitively how the equilibrium interest rate depends on the ratio y_1/y_2.

For the remainder of this question assume that $y_2 = z_2 k_2^\alpha$, where z_2 and k_2 denote productivity and capital in period 2, respectively, and $\alpha \in (0, 1)$. In addition

$$k_2 = (1 - \delta)k_1 + I_1,$$
$$r_2 + \delta = \alpha z_2 k_2^{\alpha-1},$$

where I_1 denotes investment in period 1, and δ the depreciation rate. Assume for simplicity that $\delta = 1$, and that income in period one, y_1, is still treated as a known parameter.

(c) Use the market clearing condition, $b_2 = I_1$, to derive the equilibrium values of I_1, c_1, r_2, y_2 and c_2 in terms of the parameters α, β, z_2 and y_1. Explain how they are affected by a rise in y_1.

(d) Illustrate the effect of a rise in y_1 on I_1, c_1 and r_2 using a graph with y_1 on the horizontal axis and r_2 on the vertical axis. Explain your answer.

B.2 Suppose the central bank of Americana minimises the loss function

$$L_t = \frac{1}{2}(\pi_t - \pi^*)^2 + \frac{1}{2}(Y_t - Y_t^*)^2$$

where π_t denotes inflation, π^* the central bank's inflation target, Y_t aggregate output, Y_t^* the central bank's output target, and the subscript t indicates the time period. The economy is described by the Phillips curve

$$\pi_t = \pi_t^e + \alpha\left(Y_t - \bar{Y}\right) + \varepsilon_t$$

where π_t^e denotes private sector inflation expectations, $\bar{Y}$ potential output, ε_t is an i.i.d. cost-push shock with $\mathrm{E}[\varepsilon_t] = 0$, and α is a positive parameter. The timing in each period t is as follows. First, the private sector rationally forms its inflation expectations π_t^e. Subsequently, the cost-push shock ε_t is observed. Finally, the central bank sets Y_t, resulting in π_t. Assume that initially $Y_t^* = \bar{Y}$ for all periods t.

(a) Derive the central bank's targeting rule for any Y_t^*. Give an intuitive explanation of the result.

(b) Derive private sector inflation expectations π_t^e, and solve for output Y_t and inflation π_t. Provide an economic interpretation of their properties.

(c) Now suppose that in period $t = 1$, *after* the private sector has formed its expectations π_1^e, the new President of Americana suddenly appoints new central bankers with a higher output target, such that $Y_1^* = \bar{Y} + \beta$, where β is a positive parameter. Compute output Y_1 and inflation π_1. Compare the results to part (b).

(d) Suppose that the new central bankers are appointed for two periods, so that $Y_2^* = \bar{Y} + \beta$. Compute private sector inflation expectations π_2^e, output Y_2 and inflation π_2. Compare the results to parts (c) and (b), and explain intuitively how output and inflation in Americana change over time.

B.3 Suppose that a country is hit by a hurricane which destroys physical capital. Use the AD-AS model to answer the following questions. In each case explain your answer making use of diagrams as appropriate.

(a) What is likely to happen to the price level?

(b) What is likely to happen to the demand for money?

(c) How would your answer to (a) change if the hurricane destroys the infrastructure of the banking system as well as capital more generally, making it more difficult to convert bonds into money?

SECTION C

C.1 'Ricardian equivalence implies that – in the context of the neoclassical framework – expansionary fiscal policy has no effect on the economy.' Discuss the merits of this argument using the neoclassical framework with and without liquidity constrained consumers.

C.2 The theory of efficiency wages reconciles optimizing behaviour with wages that are above the market-clearing level. Discuss the ideas underlying efficiency wages, and discuss evidence for and/or against this theory.

C.3 During the first six weeks of 2018, 10-year government bond yields in the United States, United Kingdom and Germany rose by about 40, 40 and 30 basis points, respectively, while the Federal Reserve, Bank of England and European Central Bank maintained their policy rate settings. Analyze what factors may have contributed to this bond market sell-off.

C.4 The Trump administration has embarked on large tax cuts and protectionist trade policies. Analyze the short-run effects of each of these measures on the US economy, including the nominal interest rate, the nominal exchange rate, aggregate output, consumption, investment and the current account deficit. Discuss any assumptions you make.

C.5 The debt-growth nexus has received renewed interest among academics and policy makers alike in the aftermath of the recent global financial crisis and the subsequent euro area sovereign debt crisis. Discuss whether there exists a tipping point, for public indebtedness, beyond which economic growth drops off significantly; and more generally, whether a build-up of public debt slows down the economy in the long run.

END OF PAPER

EXAMINATION FOR THE DIPLOMA IN ECONOMICS

Wednesday 6 June 2012 1.30 to 4.30

Paper 3

ECONOMETRICS

Answer ***four*** *questions from Part A and* ***two*** *questions from Part B.*
Each Section carries 50% of the total marks.
You are permitted to use your own calculator where it has been stamped as approved by the University. Cambridge Elementary Statistical Tables, standard graph paper, Durbin-Watson and Dickey-Fuller Tables are provided.
Credit will be given for clear presentation of relevant statistics.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
20 Page booklet x 2	Durbin-Watson and Dickey Fuller Tables
Metric Graph Paper	New Cambridge Elementary Statistical Tables
Rough work pads	Approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

A1 A researcher studies factors determining the quality of UK secondary schools. The following regression model is run:

$$PS_i = \beta_0 + \beta_1 EXP_i + \beta_2 LON_i + u_i, \; i = 1, ..., n, \tag{1}$$

where

PS_i	is the performance score for school i,
EXP_i	is the difference between school i's actual expenditure (in thousands of pounds per pupil) and the national average expenditure of £5000 per pupil, and
LON_i	is a dummy variable equal to 1 if school i is located in London.

Suppose that a random sample (PS_i, EXP_i, LON_i), $i = 1, \ldots, n$ is available, and that the unobserved error term u_i satisfies

$$E(u_i \,|PS_i, EXP_i, LON_i) = 0.$$

(a) Interpret the coefficients β_0, β_1, and β_2.

(b) A researcher suggests using the product $EXP_i * LON_i$ as an extra explanatory variable in (1). Discuss a possible rationale behind such a suggestion.

(c) Another researcher suggests including dummy variables for twelve different geographical regions of UK. Discuss why you may prefer *adjusted* R^2 to the regular R^2 as a measure of the goodness of fit of such a regression.

A2 Consider the model

$$y_i = \beta_0 + \beta_1 x_i + u_i, \; i = 1, ..., n,$$

where y_i denotes output and x_i measures workforce size. Suppose that a random sample (y_i, x_i), $i = 1, \ldots, n$ is available. Assume further that the unobserved error term u_i satisfies $E(u_i \,|x_i) = 0$, and $E(u_i^2 \,|x_i) = f(x_i)$ for some unknown positive function f.

(a) Show that the OLS estimator of β_1 is unbiased.

(b) How would you test for heteroskedasticity in u_i? Outline the main advantages/disadvantages of the proposed test procedure.

(c) Suppose now that the form of the heteroskedasticity is known to be

$$E\left(u_i^2 \,|x_i\right) = \exp(\delta_0 + \delta_1 x_i).$$

How would you use this information to improve upon the OLS estimator of β_1?

A3 A regression model specifies how the conditional mean $E[y|\mathbf{x}]$ depends on $\mathbf{x}$. In the most general sense we write

$$y = g(\mathbf{x}, \boldsymbol{\beta}) + \varepsilon,$$

where $g(\mathbf{x}, \boldsymbol{\beta}) = E[y|\mathbf{x}]$, and ε is a stochastic term with $E[\varepsilon|\mathbf{x}] = 0$.

(a) When y is binary show that

$$g(\mathbf{x}, \boldsymbol{\beta}) = \Pr(y = 1|\mathbf{x}).$$

(b) Let y be an indicator of the event "individual goes to college", with potential determinants of y

$\mathbf{x} = \{$high school grades; family income; parents' education.$\}$

Let

$$U_0 = \alpha_0 + \mathbf{x}'\boldsymbol{\beta}_0 + \varepsilon_0$$

denote the utility associated with choosing $y = 0$ (no college), and let

$$U_1 = \alpha_1 + \mathbf{x}'\boldsymbol{\beta}_1 + \varepsilon_1$$

denote the utility associated with choosing $y = 1$ (college). Show that

$$\Pr(y = 1|\mathbf{x}) = \Pr(\varepsilon_0 - \varepsilon_1 < \alpha_{10} + \mathbf{x}'\boldsymbol{\beta}_{10}|\mathbf{x}),$$

where $\alpha_{10} = \alpha_1 - \alpha_0$, and $\boldsymbol{\beta}_{10} = \boldsymbol{\beta}_1 - \boldsymbol{\beta}_0$.

A4 Consider the following bivariate linear regression

$$y = \alpha + \beta x + u$$

where x is a binary regressor, and u is an error term.

(a) An analyst has a suspicion that u is likely to be heteroskedastic. Explain why in such a situation the analyst should always test for heteroskedasticity, rather than simply adopting robust standard errors.

(b) An analyst is told that it is likely that x is endogenous, and that a single instrument z is available. Explain why in such a situation one should always test for endogeneity, rather than simply adopting an instrumental variable estimator.

A5 A panel data model is written as

$$y_{it} = \alpha_i + \beta_1 x_{it} + \beta_2 z_i + u_{it},\ i = 1, ..., N,\ t = 1, ..., T,$$

where N is the number of individuals, T is the number of time periods,

x_{it} and z_i	are both scalar regressors,
u_{it}	denotes unobserved variables that change over time, and
α_i	is an unobserved variable that is time invariant.

The fixed effects estimator utilizes variation in y_{it} and x_{it} around mean values, respectively $\overline{y}_i = \frac{1}{T}\Sigma_{t=1}^{T} y_{it}$ and $\overline{x}_i = \frac{1}{T}\Sigma_{t=1}^{T} x_{it}$.
In what sense does this transformation

(a) circumvent the problems of endogeneity in a panel data model with fixed effects,

(b) remove both exogenous and endogenous variation in y_{it}?

A6 Suppose that u_t, $t = 1, 2, ...$ are independent and identically normally distributed random variables with mean zero and finite positive variance σ_u^2. Let y_0 be fixed at zero ($y_0 = 0$) and let y_t be generated from the equation

$$y_t = \beta y_{t-1} + u_t,\ t = 1, 2, ...,$$

where $\beta \neq 0$.

(a) What are the variances of y_1 and y_2?

(b) Is the process y_t, $t = 1, 2, ...$ covariance stationary? Why or why not? Would your answer change if β were equal to zero?

(c) What assumptions of the Gauss-Markov theorem are violated for the regression of y_t on y_{t-1}, $t = 1, ..., T$? What are the implications for the t-test of the hypothesis that $\beta = 0.5$?

A7 An investigator analysing industrial demand for energy hypothesises the following relationship

$$\begin{aligned} z_t &= \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 y_{t-2} + \beta_4 z_{t-1} + \beta_5 z_{t-2} \\ &\quad + \beta_6 p_t + \beta_7 p_{t-1} + \beta_8 p_{t-2} + \varepsilon_t, \end{aligned} \qquad (\clubsuit)$$

where
- z_t is the log of industrial demand for energy
- y_t is the log of an index of industrial output
- p_t is the log of the real price of energy
- ε_t is a serially uncorrelated disturbance with zero mean

(a) Rewrite equation ($\clubsuit$) in the error correction form. Suppose that, in the long-run, the following equilibrium relationship holds

$$P_t Z_t = \alpha Y_t,$$

where α is a constant, and Z_t, Y_t, P_t are the level variables corresponding to z_t, y_t, p_t respectively. What restrictions would this equilibrium relationship imply for the parameters of equation ($\clubsuit$)?

(b) Describe how the investigator could test the validity of the assumption that ε_t is serially uncorrelated.

SECTION B

B1 Consider a Cobb-Douglas production function

$$Y = e^{\beta_0} K^{\beta_1} L^{\beta_2} e^{u}, \qquad (\spadesuit)$$

where the variables Y, K and L denote output, capital and labor respectively, variable u quantifies an unobserved factor (such as plant-specific technology) that affects the production process, and β_0, β_1 and β_2 are unknown parameters of interest. Suppose that you have a random sample (Y_i, K_i, L_i), $i = 1, 2, ..., 100$ that satisfies ($\spadesuit$) and that u_i satisfies $E(u_i|K_i, L_i) = 0$.

(a) Transform the model appropriately so that it is linear in the parameters, and provide the interpretation of β_1 and β_2.

(b) An expert suggests that the production function exhibits constant returns to scale in capital and labor. Rewrite the model from part (a) so that the hypothesis of constant returns to scale is equivalent to a hypothesis that one of the coefficients in the rewritten model equals zero.

(c) Some experts believe that the contributions to the production technology due to capital and labour inputs are *equal* in the sense that $\beta_1 = \beta_2 = 0.5$. Explain how this belief *differs* from the constant returns to scale claim. How can you use R^2 from two different regressions to test the hypothesis $\beta_1 = \beta_2 = 0.5$?

(d) Suppose that the first 60 observations are obtained from factories that use older generation machines, whereas the last 40 observations are obtained from factories that use brand new machines. The following table lists the sums of squared residuals (SSR) obtained from running the regression in part (a) using different portions of the sample.

	Data used	SSR
Model A	whole sample	127.653
Model B	first 60 observations	70.588
Model C	last 40 observations	54.132

Test the hypothesis that the technologies used by factories with old and new machines are different. State any additional assumptions you are making.

B2 Consider the population of all first-year students in a college. Let $GYM_i = 1$ if student i regularly goes to gym and let $GYM_i = 0$ otherwise. Further, let $GEN_i = 1$ if student i is female and $GEN_i = 0$ if student i is male. Finally, let AG_i be the average exam grade of student i. You are given the following results of a probit regression:

$$\widehat{\Pr}\left(GYM_i = 1|GEN_i, AG_i\right) = \Phi\left(0.3 - 0.5GEN_i + 0.005AG_i\right)$$

(a) What would be the predicted change in probability of regular gym attendance for a female student if her AG drops from 70 to 64? What is the corresponding change in probability for a male student? Why do your answers for female and male students differ? Would they be different for

(i) a logit model,

(ii) a linear probability model?

(b) The value of the maximized log likelihood for the above probit regression is -97.4. When you run a probit regression with GEN_i excluded from the set of explanatory variables, the value of the maximized log likelihood becomes -103.0. Using these results, test the hypothesis that gender has no effect on the probability of regular gym attendance.

(c) The linear probability model estimated from the same data is

$$\widehat{\Pr}\left(GYM_i = 1|GEN_i, AG_i\right) = 0.62 - 0.19GEN_i + 0.0018AG_i.$$

As you see, the coefficient estimates are very different from those in the probit regression. Does this suggest an error in the calculations?

(d) Suppose that you learned that the above probit regression was incorrectly estimated because a researcher who ran the regression used variable $1 - GYM_i$ instead of GYM_i. What are the coefficients $\hat{\alpha}$, $\hat{\beta}_1$, and $\hat{\beta}_2$ in the correctly specified regression

$$\widehat{\Pr}\left(GYM_i = 1|GEN_i, AG_i\right) = \Phi\left(\hat{\alpha} + \hat{\beta}_1 GEN_i + \hat{\beta}_2 AG_i\right)?$$

B3 An investigator analyzing the relationship between house prices and disposable income estimates a number of equations based on the following Vector Autoregression (VAR) using quarterly data for the UK covering the period 1969Q3-2010Q4 (166 observations)

$$\begin{aligned}\Delta p_t &= \alpha_0 + \sum_{j=1}^{4} \theta_j \Delta y_{t-j} + \sum_{k=1}^{4} \phi_k \Delta p_{t-k} + \varepsilon_t \\ \Delta y_t &= \beta_0 + \sum_{j=1}^{4} \gamma_j \Delta y_{t-j} + \sum_{k=1}^{4} \pi_k \Delta p_{t-k} + \upsilon_t,\end{aligned}$$

where y_t denotes the log of real disposable income in period t, p_t denotes the log of the real house price in period t, and Δ is the first difference operator such that $\Delta y_t = y_t - y_{t-1}$. The disturbances ε_t and υ_t are assumed to be serially uncorrelated, normally distributed random variables with zero mean and constant variance.

(a) Suppose that Δy_t and Δp_t equal zero for all four quarters of 2010. What are the values of Δy_t and Δp_t predicted by the above VAR for the first and for the second quarter of 2011?

(b) Results for two alternative versions of the VAR equations are given in Table B1 at the end of the question. Using the results given in Table B1, test the following hypotheses at 5% significance level:

(i) Changes in real income Granger-cause changes in real house prices.

(ii) Changes in real house prices Granger-cause changes in real income.

(c) Discuss the limitations of the tests you have conducted in part (b) above, explaining in particular how you would interpret a finding that real house prices Granger-caused income over this period.

(d) The investigator now re-estimates the equation for the change in real house prices (equation (1) in Table B1) over the period 1969Q3-2007Q4. The sum of squared residuals from this regression is 0.0800. Using this information, conduct a 5% significance level test of the hypothesis that the equation estimated over this shorter period can predict the behaviour of house prices over the period 2008Q1-2010Q4.

Table B1

	(1)	(2)	(3)	(4)
Dependent variable	Δp_t	Δp_t	Δy_t	Δy_t
Intercept	-0.0042	0.0019	$0.0081^{(*)}$	$0.0075^{(*)}$
Δp_{t-1}	$0.3950^{(*)}$	$0.4766^{(*)}$	0.0534	
Δp_{t-2}	0.0563	0.0413	0.0752	
Δp_{t-3}	-0.0128	0.0127	-0.0006	
Δp_{t-4}	$-0.1957^{(*)}$	$0.2149^{(*)}$	-0.0477	
Δy_{t-1}	$0.2514^{(*)}$		$-0.2919^{(*)}$	$-0.2633^{(*)}$
Δy_{t-2}	$0.5470^{(*)}$		-0.0048	0.0227
Δy_{t-3}	$0.3435^{(*)}$		-0.0710	-0.0155
Δy_{t-4}	-0.0227		-0.0167	0.0535
R^2	0.4636	0.3843	0.1119	0.0798
SSR	0.0861	0.0991	0.0397	0.0411

Coefficients shown with superscript (*) are significantly different from zero at the 5% significance level. R^2 is the coefficient of multiple determination and SSR is the sum of squared residuals.

END OF PAPER

UNIVERSITY OF
CAMBRIDGE

ECD1
Diploma in Economics

Wednesday 5 June 2013 1:30 - 4:30pm

Paper 3

ECONOMETRICS

The paper is divided into two Sections - A and B.

Answer **four** questions from Section A and **two** questions from Section B.

Each Section carries equal weight.
Credit will be given for clear presentation of relevant statistics.

Write your **candidate number** not your name on the cover of each booklet.

This written exam carries 60% of the marks for Paper 3.

STATIONERY REQUIREMENTS
Durbin Watson and Dicky Fuller Tables
New Cambridge Elementary Statistical Tables
20 Page booklet x 1
Rough work pads

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so

SECTION A

1 The following figure shows the weekly time series p_t for the price of the IBM share during 144 weeks from January 2001 to December 2012.

(a) How would you model this time series, and what is the economic justification for your model?

(b) A linear regression of p_t on a constant and p_{t-1} yields the following OLS results

$$\hat{p}_t = \underset{(1.958)}{0.900} + \underset{(0.017)}{0.997} p_{t-1},$$

where the standard errors of the coefficient estimates are given in parentheses. Test the hypothesis that p_t is integrated of order one.

(c) The sample standard deviation of $p_t - p_{t-1}$ is 7.15. Would you be surprised if the price of the IBM share in the last week of June of 2013 turns out to be 270 U.S. dollars? Explain your answer.

2 A production function for farms is

$$\log Y_{it} = \alpha + \beta \log X_{it} + \gamma \log S_{it} + u_{it}, \tag{1}$$

where Y_{it} is the crop produced at farm i during period t, X_{it} is the amount of labour employed, S_{it} describes the quality of soil, and u_{it} consists of random factors such as weather. Suppose that you observe Y_{it} and X_{it} for $i = 1, ..., n$ and $t = 1, ..., T$, but that you do not observe S_{it}.

(a) Under what conditions is the OLS estimator of β, from the regression of Y_{it} on a constant and X_{it}, consistent for β? Are these conditions plausible from the microeconomic perspective? Explain.

(b) Describe the fixed effects estimator of β. Is it consistent? Explain.

(c) It was observed that producing an excessively large quantity of the crop during period t leads to lower productivity of plants during the next period. How would you modify equation (1) to model this phenomenon?

3 Person i, who is caught speeding, can choose to attend a day-long speed awareness course instead of taking three points on his or her license (12 or more penalty points disqualify you from driving). Let $Y_i = 1$ if person i is caught speeding again within the next 3 years, and let $Y_i = 0$ otherwise. Further, let $D_i = 1$ if individual i takes the course, and let $D_i = 0$ otherwise.

(a) Suppose that

$$Y_i = \begin{cases} Y_{0i} & \text{if} \quad D_i = 0 \\ Y_{1i} & \text{if} \quad D_i = 1 \end{cases}, \tag{2}$$

where Y_{1i} (Y_{0i}) denote the potential outcomes if individual i takes (does not take) the course. Assuming that the individual treatment effect of taking the course is constant and equal to β, show that equation (2) can be written as

$$Y_i = \alpha + \beta D_i + \eta_i, \text{ where } E(\eta_i) = 0.$$

(b) Let $E(Y_i|D_i) = \gamma + \delta D_i$. Explain why δ may not be equal to β.

(c) Suppose that the course can only be taken by individuals who live in randomly chosen areas within the UK. Suppose that you know post codes of these areas. How would you use data on the post codes of the speed limit violators to consistently estimate β?

4 A researcher studies the effect of water pollution on the mortality rate in a fish population. He runs extensive laboratory experiments and determines that the 0.1% and 0.05% concentrations of a pollutant in water lead to 50% and 23% mortality rate, respectively.

(a) Assuming that the probability of death is described by a probit model, determine the mortality rate of fish in absolutely clean water.

(b) Let $\Phi(x)$ be the cumulative distribution function of $N(0,1)$. The values of the standard normal density at x_1 and x_2 such that $\Phi(x_1) = 0.5$ and $\Phi(x_2) = 0.23$ are 0.399 and 0.303, respectively. Using this information, compare the marginal effects of reducing the concentration of the pollutant from the initial values of 0.1% and 0.05%.

(c) In the context of the problem above, discuss potential pitfalls of the linear probability model.

(TURN OVER)

5 An investigator analysing the relationship between wages and unemployment starts by estimating the following static linear regression model using aggregate time series data

$$w_t = \beta_0 + \beta_1 p_t + \beta_2 U_t + \varepsilon_t,$$

where w_t is the logarithm of the nominal wage, p_t the logarithm of the price level, and U_t the unemployment rate (measured as a percentage).

(a) A colleague criticises the investigator's estimates of this regression on the grounds that, since wages and prices are trended, the resulting relationship is 'spurious'. Explain what is meant by a spurious regression, and how you might detect it.

(b) The investigator now reformulates his regression to allow for dynamic effects, so that it takes the form

$$w_t = \theta_0 + \theta_1 p_t + \theta_2 p_{t-1} + \theta_3 U_t + \theta_4 U_{t-1} + \theta_5 w_{t-1} + \varepsilon_t$$

Explain how you could use estimates of the coefficients of this regression to compute the long run effect of a higher price level on nominal wages.

(c) A second colleague now argues that the regression should be reformulated so that nominal wages are allowed to respond to movements in the expected price level p_t^e, so that it becomes

$$w_t = \theta_0 + \theta_1 p_t^e + \theta_2 p_{t-1} + \theta_3 U_t + \theta_4 U_{t-1} + \theta_5 w_{t-1} + \varepsilon_t$$

Suggest two strategies which you might employ to estimate such a regression.

6 Consider a simple linear regression framework

$$y_t = \beta_0 + \beta_1 x_t + u_t \text{ with } t = 1, \ldots, T,$$

where $E(u_t|x_1, \ldots, x_t) = 0$ for all t. Suppose the error term follows a stationary AR(2) process, say $u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$, where $\{\varepsilon_t, t = 1, ..., T\}$ are i.i.d. with distribution $N(0, \sigma^2)$.

(a) In the above regression, what are the advantages of GLS relative to OLS?

(b) Describe in detail how you would compute the feasible GLS estimators for β_0 and β_1.

(c) In general, would you always prefer feasible GLS to OLS? Discuss.

7 Let $hy6_t$ be the three-month holding yield from buying a six-month U.S. Treasury bill at time $t-1$ and selling it at time t as a three-month Treasury bill. Further, let $hy3_t$ be the three-month holding yield from buying a three-month Treasury bill at time $t-1$. The expectation hypothesis implies that the slope coefficient in the regression of $hy6_t$ on $hy3_t$ should not be statistically different from one.

(a) Assuming that there is evidence of a unit root in $hy3_t$, discuss problems with standard OLS analysis of the above regression.

(b) Rewrite the following model in the error correction form

$$hy6_t = \theta_0 + \theta_1 hy6_{t-1} + \theta_2 hy3_t + \theta_3 hy3_{t-1} + u_t. \tag{3}$$

What constraint on the parameters $\theta_1, ..., \theta_3$ is implied by the expectation hypothesis?

(c) Assuming that the constraint holds, explain how you would estimate the error correction form of equation (3). How would you test for serial correlation in u_t?

SECTION B

8 A study of wage determination based on a random sample of 1472 individuals from the working population of Belgium in 1994 contains 893 males and 579 females. Table 1 reports OLS results from four different specifications of linear regression of log ($wage_i$) (in Belgian francs) on the explanatory variables defined as follows:

- $male_i$ is 1 if male, 0 if female;
- $educ_i$ is 1 if primary school, 2 if lower vocational training, 3 if intermediate level, 4 if higher vocational training, 5 if university level;
- $edu(j)_i$ is 1 if $educ_i = j$, 0 otherwise;
- $exper_i$ is experience in years.

(a) Suppose that a woman is expected to earn five million Belgian francs. According to the OLS results for specification (1), how much would a man with the same amount of education and experience be expected to earn?

(b) From an economic point of view, explain why you may prefer specification (2) to specification (1). Test specification (1) against specification (2) at the 1% significance level.

(c) Use results from Table 1 to perform the Chow test (at the 1% significance level) of a hypothesis that wage determination is the same for men and women.

(d) The results in Table 1 do not take into account a potential heteroskedasticity problem. What effects would heteroskedasticity have on the results reported? In particular, comment on the validity of the estimates, their standard errors, R^2, adjusted R^2 and the F-statistic.

(e) Consider specification (2). You suspect that the source of heteroskedasticity is related to gender alone. Describe in detail how you would test for this. Propose an alternative estimator that may be preferred to the OLS estimator in case your suspicion is correct.

Table 1

OLS results for four specifications of wage regression. Standard errors are in parentheses. F-statistic corresponds to the test of the regression's significance.

Regressor	(1)	(2)	(3)	(4)
constant	1.145 (0.041)	1.272 (0.045)	1.372 (0.040)	1.216 (0.078)
$male_i$	0.118 (0.016)	0.128 (0.015)		0.154 (0.095)
$\log(educ_i)$	0.437 (0.018)			
$\log(exper_i)$	0.231 (0.011)	0.230 (0.011)	0.310 (0.021)	0.207 (0.017)
$edu(2)_i$		0.144 (0.033	0.113 (0.021)	0.224 (0.068)
$edu(3)_i$		0.305 (0.032)	0.343 (0.024)	0.433 (0.063)
$edu(4)_i$		0.474 (0.033)	0.374 (0.025)	0.602 (0.063)
$edu(5)_i$		0.639 (0.033)	0.469 (0.026)	0.755 (0.065)
$male_i \times edu(2)_i$				−0.097 (0.078)
$male_i \times edu(3)_i$				−0.167 (0.073)
$male_i \times edu(4)_i$				−0.172 (0.074)
$male_i \times edu(5)_i$				−0.146 (0.076)
$male_i \times \log(exper_i)$				0.041 (0.021)
Summary Statistics				
R^2	0.376	0.398	0.388	0.403
adjusted R^2	0.375	0.395	0.385	0.399
F-statistic	294.96	161.14	166.24	89.69

(TURN OVER)

9 A researcher is investigating the relationship between oil price 'shocks' and the business cycle in the United States over the period 1988Q1-2011Q4. She estimates the following pair of regressions

$$\widehat{\Delta p_t} = \underset{(0.1408)}{0.3343} + \underset{(0.0008)}{0.0020t} - \underset{(0.0471)}{0.1175}p_{t-1} \text{ with } AIC = -0.9944,$$

and

$$\begin{aligned}\widehat{\Delta p_t} &= \underset{(0.1470)}{0.2844} + \underset{(0.0008)}{0.0017t} - \underset{(0.0491)}{0.1002}p_{t-1} + \underset{(0.1026)}{0.3330}\Delta p_{t-1} - \underset{(0.1090)}{0.3662}\Delta p_{t-2} \\ &\quad + \underset{(0.1052)}{0.2164}\Delta p_{t-3} - \underset{(0.1038)}{0.1844}\Delta p_{t-4} \text{ with } AIC = -1.1040,\end{aligned}$$

where p_t is the log of the real oil price at date t, Δ is the first difference operator such that $\Delta p_t = p_t - p_{t-1}$, AIC is the Akaike Information Criterion, and standard errors are in parentheses.

(a) Using these equations, conduct a test of the null hypothesis that the log of the real oil price is a random walk with drift. Explain the reasoning behind your choice of test equation.

(b) The researcher now estimates a Vector Autoregression (VAR) of the form

$$\begin{aligned}\Delta p_t &= \alpha_0 + \sum_{j=1}^{4} \beta_j \Delta gdp_{t-j} + \sum_{k=1}^{4} \gamma_k \Delta p_{t-k} + \varepsilon_{1t} \\ \Delta gdp_t &= \psi_0 + \sum_{j=1}^{4} \theta_j \Delta gdp_{t-j} + \sum_{k=1}^{4} \phi_k \Delta p_{t-k} + \varepsilon_{2t}\end{aligned}$$

where gdp_t is the log of real US GDP at date t. The disturbance terms $\{\varepsilon_{1t}, \varepsilon_{2t}\}$ are jointly normally distributed with mean zero and variance-covariance matrix Ω, and are serially uncorrelated.

The results of these regressions and their constrained versions are given in Table 2 below. Using these results, and a 5% significance level, test the hypothesis that movements in real oil prices do not Granger-cause movements in real US GDP.

(c) The researcher now re-estimates her equations over two selected subperiods: 1988Q1-2007Q4 (the 'Great Moderation') and 2008Q1-2011Q4 (the 'Great Recession'). The residual sums of squares from these subperiod regressions are shown in the lower part of Table 2. Using these results, and a 5% significance level, test the hypothesis that the coefficients of the equation relating growth in real GDP to the lagged growth in GDP and real oil prices are structurally stable.

(d) Comment on the implications of the additional results from part (c) for your conclusions in part (b) of this question.

Table 2
OLS estimates of the VAR from question 9. Standard errors are in parentheses

	A	B	C	D
Dependent	$\Delta oilp_t$	Δgdp_t	$\Delta oilp_t$	Δgdp_t
$\Delta oilp_{t-1}$	0.2455 (0.1075)	−0.0073 (0.0043)	0.3001 (0.1018)	
$\Delta oilp_{t-2}$	−0.4175 (0.1112)	−0.0027 (0.0044)	−0.4256 (0.1053)	
$\Delta oilp_{t-3}$	0.2088 (0.1093)	−0.0079 (0.0044)	0.1965 (0.1057)	
$\Delta oilp_{t-4}$	−0.2143 (0.1088)	−0.0060 (0.0043)	−0.2308 (0.1020)	
Δgdp_{t-1}	3.6963 (2.6955)	0.3692 (0.1072)		0.3845 (0.1035)
Δgdp_{t-2}	2.8905 (2.8177)	0.2951 (0.1121)		−0.4256 (0.1099)
Δgdp_{t-3}	−4.0373 (2.8008)	−0.1018 (0.1114)		−0.1627 (0.1099)
Δgdp_{t-4}	1.1741 (2.6931)	0.1186 (0.1071)		0.0772 (0.1038)
Intercept	−0.0094 (0.0236)	0.0021 (0.0009)	0.0136 (0.0141)	0.0026 (0.0009)
R^2	0.2332	0.3471	0.1870	0.2490
SSR $(88Q1-11Q4)$	1.60851	0.00254	1.70532	0.00293
Results for subsamples				
SSR $(88Q1-07Q4)$	0.95080	0.00148	1.09563	0.00161
SSR $(08Q1-11Q4)$	0.33948	0.00043	0.48177	0.00072

(TURN OVER)

10 An analyst wishes to examine the effect of family size on the employment status of mothers. Data have been taken from the 1980 census on married women aged 21-35 with at least two children. Columns of Table 3 marked as (2), (4) and (6) report OLS and IV (Wald) estimates of the effects of a third birth on labour supply using Twins and Sex Composition as instruments. Employment is a binary indicator equal to one if the woman is in the labour force and zero otherwise. Twins is a binary indicator equal to one if the first two children are twins, and Sex Composition is a binary indicator equal to one if the first two children are of the same sex.

(a) In what sense might an observed association between family size and employment be difficult to interpret?

(b) Write down the regression equation for the structural, first-stage and reduced form equation. Discuss how each of these equations can be used to estimate the impact of family size on labour supply.

(c) Explain the basis for the use of Sex Composition as an instrument for labour supply. Interpret the first stage estimates for this instrument.

(d) Compare the results for the OLS and the Wald estimates based upon the use of Twins and Sex Composition as instruments.

(e) In what sense are the individual Wald estimates inefficient?

Table 3

Estimates of the effects of family size on labour supply. All models include controls for mother's age, age at first birth, dummies for the sex of first and second births, and dummies for race.

			IV Estimates Using			
			Twins		Sex Composition	
Dependent Variable	Mean (1)	OLS (2)	First Stage (3)	Wald Estimates (4)	First Stage (5)	Wald Estimates (6)
Employment	0.528	−0.167	0.625	−0.083	0.067	−0.135
		(0.002)	(0.011)	(0.017)	(0.002)	(0.029)

END OF PAPER

ECD1
Advanced Diploma in Economics

Wednesday 4 June 2014 1:30pm to 4:30pm

Paper 3

Econometrics

This paper is in <u>two</u> sections – A and B. Candidates are required to answer ***FOUR*** *questions from Section A and* ***TWO*** *questions from Section B.*

Each section carries equal weight.

Answers from each section must be written in separate booklets with the letter of the section written on each cover sheet.

Credit will be given for clear presentation of relevant statistics.

This written exam carries 60% of the marks for Paper 3.

Write your number ***<u>not</u>*** *your name on the cover sheet of* ***each*** *booklet.*

STATIONERY REQUIREMENTS
20 Page Answer Booklet x 2
Metric Graph Paper
Rough Work Pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Durbin Watson and Dicky Fuller Tables
New Cambridge Elementary Statistical Tables
Calculator – students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

A.1 A researcher is interested in analysing the relationship between inflation and unemployment. She starts by considering the regression model:

$$\Delta\pi_t = \beta_0 + \beta_1 U_t + \varepsilon_t$$

where $\Delta\pi_t$ is the first difference of the inflation rate at time t and U_t is the unemployment rate. ε_t is a disturbance term which the researcher assumes to be independent and identically distributed over time. The researcher obtains ordinary least squares estimates of β_0 and β_1 and their associated standard errors.

(a) The researcher suspects that the disturbance term might be generated by an AR(1) process, i.e. $\varepsilon_t = \phi\varepsilon_{t-1} + \widetilde{\varepsilon}_t$ where $\widetilde{\varepsilon}_t$ is an independent and identically distributed (over time) disturbance. Explain what implications, if any, this would have for the original least squares estimates and standard errors.

(b) Describe in detail one formal test statistic that can help the researcher in deciding whether or not her initial assumption was correct.

(c) Suppose the researcher concludes that the disturbance term is serially correlated as described in (a) above. The true model is now assumed to be:

$$\begin{aligned}\Delta\pi_t &= \beta_0 + \beta_1 U_t + \varepsilon_t\\ \varepsilon_t &= \phi\varepsilon_{t-1} + \widetilde{\varepsilon}_t.\end{aligned}$$

Show that this model can be rewritten as an autoregressive distributed lag (ADL) model. Does the original model impose any parameter restrictions on your ADL model? Ignoring any such restrictions, discuss why least squares standard errors for the (unrestricted) ADL model are valid.

A.2 A researcher wants to estimate the impact of terrorism on tourism in Nigeria. However, she has reasons to believe that high levels of tourism can themselves trigger terrorist activities and decides to analyse the two series jointly using the following system of equations:

$$\begin{aligned}n_t &= \beta_0 + \beta_1 i_t + \beta_2 n_{t-1} + \beta_3 i_{t-1} + \varepsilon_{nt}\\ i_t &= \gamma_0 + \gamma_1 n_t + \gamma_2 i_{t-1} + \gamma_3 n_{t-1} + \varepsilon_{it},\end{aligned}$$

where n_t is the number of tourists visiting Nigeria in quarter t and i_t is the number of terrorism incidents during the same period. The terms ε_{nt} and ε_{it} are normally distributed random variables with zero means, finite variances σ_n^2 and σ_i^2, and covariance σ_{ni}. Further, the researcher has ascertained that both series are stationary.

(a) Explain why the least squares estimates of the parameters in the system of equations above will be biased.

(b) A colleague suggests that she can still forecast each variable by using the associated reduced form. Explain how to obtain this reduced form from the above system and why OLS estimates would be consistent in this case.

(c) Suppose the researcher wants to test the hypothesis "terrorism Granger-causes tourism in Nigeria". Explain what is meant by this hypothesis and how the researcher can test it.

A.3 Let Y_i denote an observed measure of health for individual i, and Y_{1i} (Y_{0i}) potential health with (without) health insurance. The causal effect of insurance on health is given by the difference $Y_{1i} - Y_{0i}$.

(a) Consider the following potential outcome data for two individuals k and l:

$$\begin{aligned} Y_{1k} &= 4, \quad Y_{0k} = 3 \\ Y_{1l} &= 5, \quad Y_{0l} = 5. \end{aligned}$$

You are told that one of the individuals purchases insurance. Which of the two would you expect to buy insurance?

Show that $Y_k - Y_l$ can be written as additive in a causal effect of insurance on k and a selection effect.

(b) For a group of N individuals let $D_i = 1$ (0) denote that individual i is insured (not insured). A colleague suggests computing the following difference in group means, namely $\bar{Y}_{D_i=1} - \bar{Y}_{D_i=0}$.

In what sense does this average difference in means estimate the causal effect of insurance on health?

(c) If insurance status is randomly assigned, show that the difference in expectations by insurance status identifies the causal effect.

If insurance status is *not* randomly assigned discuss how the causal effect of insurance can be identified if all differences across the insured and uninsured individuals were measurable.

A.4 Consider the following two regression models

$$Y_i = \alpha + \beta X_{1i} + \gamma X_{2i} + e_i \qquad (1)$$

$$Y_i = \alpha^s + \beta^s X_{1i} + e_i^s \qquad (2)$$

where (1) represents the true model in that Y is a linear function of X_1 *and* X_2. (2) represents the model specified by the analyst who does not observe X_2. e_i and e_i^s are error terms. Assume that e_i is an i.i.d. error. α, α^s, β, β^s, and γ denote unknown parameters.

(TURN OVER)

(a) Show that the bias from omitting X_2 can be written as

$$\beta^s = \beta + \pi_{21}\gamma, \tag{3}$$

where π_{21} is the coefficient on X_{1i} in a regression of X_{2i} on X_{1i}.

(b) As a possible solution one analyst proposes to use an imperfect measure of X_2, say X_2^*. Another analyst proposes an approach based upon the use of one or more instruments for X_1.

Explain the key distinction between these two estimation strategies.

A.5 Consider the following panel data model

$$y_{it} = x_{it,1}\beta_{i1} + x_{it,2}\beta_{i2} + x_{i,3}\beta_{i3} + \alpha_i + \varepsilon_{it}, \tag{4}$$

where $i = 1, ..., N$ and $t = 1, ...T$. $x_{it,1}$ and $x_{it,2}$ denote regressors that vary over i and t; $x_{i,3}$ is a time invariant *observed* regressor. α_i denotes an *unobserved* time invariant effect, and ε_{it} is an error term.

Parameters β_{i1}, β_{i2}, and β_{i3} vary across i.

(a) What restrictions placed upon (4) would justify the use of a pooled OLS estimator?

What restrictions placed upon (4) would justify the use of a fixed effects (FE) estimator?

Explain why the FE estimator might be preferable to OLS. In what sense is the FE estimator restricted?

(b) An economic model for describing gross firm investment for the i^{th} firm in year t, denoted INV_{it}, is written as

$$INV_{it} = f(V_{it}, K_{it}), \tag{5}$$

where V_{it} denotes the stock market value of the firm at the beginning of the year. K_{it} is a measure of the capital stock.

In Table 1 we present the results based upon the use of a Pooled OLS estimator using $T = 20$ years of data (1935-1954) and $N = 2$ firms. SSE is the sum of squared residuals, and $\widehat{\sigma}^2$ denotes the estimated variance of the error term.

In Table 2 we present the results from an alternative model where $D_i = 1$, a binary indicator for firm 1, has been used to allow for heterogeneity in partial effects at the level of the firm.

i. Test whether the restrictions implied by the use of the Pooled OLS estimator are justified.

ii. Write down a regression equation which underlies the model in Table 2. Explain how the Pooled OLS estimator with multiplicative effects is different from the FE estimator.

In what sense is this regression equation restricted?

Table 1: Pooled Ordinary Least Squares Estimates of Investment Equations

Dependent Variable: INV		
Variable	Coefficient	p-value
constant	17.872	0.015
V	0.0152	0.019
K	0.1436	0.000

$SSE = 16563.00 \quad \widehat{\sigma}^2 = 447.65$

Table 2: Pooled Ordinary Least Squares Estimates from the Dummy Variable Model

Dependent Variable: INV		
Variable	Coefficient	p-value
constant	-9.956	0.676
D	9.447	0.745
V	0.027	0.030
$D \times V$	0.026	0.448
K	0.152	0.000
$D \times K$	-0.059	0.615

$SSE = 14989.82 \quad \widehat{\sigma}^2 = 440.877$

A.6 An investigator analysing the behaviour of real personal consumption in the USA over the period 1960Q1-2012Q4 (212 observations) estimates the following series of regressions using Ordinary Least Squares

$$\begin{aligned} \widehat{\Delta c_t} &= \underset{(0.0014)}{0.0100} - \underset{(0.0414)}{0.0599} r_t \\ R^2 &= 0.0099 \qquad SSR = 0.009905 \end{aligned} \tag{6}$$

$$\begin{aligned} \widehat{\Delta c_t} &= \underset{(0.0014)}{0.0066} + \underset{(0.0525)}{0.2547} \Delta y_{t-1} + \underset{(0.0525)}{0.1193} \Delta y_{t-2} - \underset{(0.0391)}{0.0455} r_t \\ R^2 &= 0.1316 \qquad SSR = 0.008687 \end{aligned} \tag{7}$$

$$\begin{aligned} \hat{u}_t &= \underset{(0.0015)}{0.0013} - \underset{(0.0600)}{0.0879} \Delta y_{t-1} - \underset{(0.0586)}{0.0604} \Delta y_{t-2} - \underset{(0.0385)}{0.0023} r_t \\ &\quad + \underset{(0.0772)}{0.1569} \hat{u}_{t-1} + \underset{(0.0801)}{0.1776} \hat{u}_{t-2} \\ R^2 &= 0.0427 \qquad SSR = 0.008316 \end{aligned} \tag{8}$$

c_t denotes the log of real consumption, y_t the log of real personal disposable income, and r_t is a measure of the real return on an investment in the US stock market. Δ is the first difference operator such that $\Delta c_t = c_t - c_{t-1}$. Figures in parentheses are standard errors, R^2 is the coefficient of determination and SSR is the sum of squared residuals. In (8) the variable $\hat{u}_t$ is the estimated residual from (7).

(a) Using the information provided, and assuming that the disturbances in (6) and (7) are not serially correlated, test the hypothesis (using a 5% significance level) that the growth rate of consumption is unaffected by past growth rates of disposable income. Comment on your findings.

(b) Explain how the results of equation (8) can be used to construct a test of whether the disturbances associated with (7) are serially correlated, and conduct such a test using a 5% significance level.

Explain the reason for including the regressors Δy_{t-1}, Δy_{t-2}, and r_t when conducting such a test.

(c) Comment on your findings, indicating any further regressions which you would estimate using this dataset.

A.7 Consider the first-order autoregressive process

$$x_t = \rho x_{t-1} + \varepsilon_t,$$

where ε_t is a Normally distributed white noise process satisfying

$$\begin{aligned} E\left[\varepsilon_t\right] &= 0 \\ E\left[\varepsilon_t^2\right] &= \sigma^2 < \infty \\ E[\varepsilon_s \varepsilon_t] &= 0, \quad s \neq t. \end{aligned}$$

(a) Derive expressions for the mean, variance and autocovariances of x_t, and set out the conditions for x_t to be a stationary time series process.

(b) An investigator analysing the behaviour of the sterling-dollar exchange rate using monthly data for the period Jan 1979-Dec 2012 (408 observations) estimates the following regression

$$\begin{aligned} \widehat{\Delta} \mathrm{exr}_t &= \underset{(0.0185)}{0.04475} - \underset{(0.0109)}{0.02729} \mathrm{exr}_{t-1}, \end{aligned}$$

where exr_t is the logarithm of the sterling-dollar exchange rate, and Δ is the first difference operator such that $\Delta \mathrm{exr}_t = \mathrm{exr}_t - \mathrm{exr}_{t-1}$. Figures in parentheses are standard errors.

Using this information, test (at the 5% significance level) the null hypothesis that the logarithm of the sterling-dollar exchange rate is a non-stationary time series. Comment on your findings and the investigator's analysis.

SECTION B

B.1 A researcher is interested in testing the theory of Purchasing Power Parity (PPP) using UK and US quarterly data. Let e_t, p_t^* and p_t denote the logarithms of the Pounds (Sterling) per Dollar exchange rate, the US price level and the UK's price level in quarter t. Define $f_t = e_t + p_t^*$, such that f_t gives the logarithm of the Pound sterling value of the US price level. The researcher considers testing for two distinct versions of PPP theory. The *strong* version of PPP theory requires $f_t - p_t$ to be a stationary, zero mean, process. The *weak* version of PPP requires $f_t - \beta p_t$ to be a stationary process for some positive constant, β. The researcher supplies you with three results:

i Based on unit-root tests, she has concluded that f_t is an integrated stochastic process of order 1. Additionally, she has studied the time series properties of p_t and obtained the following estimated regression

$$\Delta p_t = \underset{(0.188)}{0.212} + \underset{(0.009)}{0.011}t - \underset{(0.0286)}{0.0312}p_{t-1} + \widehat{v}_t, \; 1976Q1-1998Q4 \text{ (92 observations)}$$

where t is a linear time trend, Δp_t is the first difference of the logarithm of the UK's price level and $\widehat{v}_t$ denotes least squares residuals. The figures in parentheses give the associated standard errors.

ii A least squares regression of f_t on p_t and a constant yields:

$$f_t = \underset{(0.041)}{0.012} + \underset{(0.026)}{0.988}p_t + \widehat{u}_t, \; 1976Q1 - 1998Q4 \text{ (92 observations)}$$

where $\widehat{u}_t$ denotes least squares residuals. The figures in parentheses give the associated standard errors.

iii In addition the researcher estimated the following equation:

$$\Delta f_t = \underset{(0.004)}{0.019} - \underset{(0.032)}{0.106}\widehat{u}_{t-1} + \widehat{\varepsilon}_t, \quad SSR = 0.0421, \; 1976Q1-1998Q4 \text{ (92 observations)}$$

where Δf_t is the first difference of f_t and $\widehat{\varepsilon}_t$ denotes least square residuals. SSR gives the residual sum of squares under this sample. The figures in parentheses give the associated standard errors.

(a) The researcher is concerned that the regression of f_t on p_t in the second regression above might be spurious. What is the spurious regression problem and what property of p_t would justify this concern? Using only the first estimated regression (i.e. the regression for Δp_t), test, at the 5% level, whether this is indeed a valid concern. State clearly your null and alternative hypothesis.

(b) Assume that the spurious regression problem is indeed a valid concern. What information would you need regarding $\widehat{u}_t$ (in the second estimated regression) in order to ignore this problem and what does this imply for f_t and p_t? Would this information also allow you to test whether the *weak* version of PPP theory finds support in the data? Describe your proposed testing strategy for the weak version of PPP theory.

(c) Suppose the researcher was only interested in testing the strong version of PPP theory. Suggest an alternative testing strategy that allows the researcher to test whether the strong version of PPP holds in the data.

(d) Assume that the weak version of PPP theory holds. Give an economic interpretation of the slope parameter in (iii) estimated by the researcher.

B.2 An investigator analysing household expenditure on healthcare (defined as the sum of out-of-pocket expenses and health insurance premiums) in the US has access to a sample of data for 6771 randomly selected households. She starts by estimating the following regression equation using Ordinary Least Squares

$$\ln \mathrm{hx}_i = \beta_0 + \beta_1 \ln \mathrm{inc}_i + \beta_2 \ln \mathrm{size}_i + \varepsilon_i, \tag{9}$$

where hx_i is total expenditure on healthcare by household i, inc_i is a measure of disposable income for household i, and size_i is the total number of persons in household i. ε_i is assumed to be a Normally distributed random disturbance with mean zero.

(a) Provide an economic interpretation for the parameters of equation (9). What restriction would you need to impose on these parameters to test the hypothesis that (i) the share of income spent on healthcare is the same for all income levels and (ii) the amount spent *per household member* is independent of household size?

Using the results shown in column (1) of Table 3, conduct separate tests of these two hypotheses using a 1% significance level.

The investigator now adds a number of additional variables to the regression. These are defined in Table 4.

She then runs a series of further regressions: the results of these regressions are given in columns (2-5) of Table 3.

(b) Using the results in column (3) of Table 3, derive an expression for the marginal effect of an additional year of age (as defined in Table 4) on healthcare expenditure. What do your results suggest about how healthcare expenditure varies with age?

(c) The investigator now splits the sample into two sub-samples, distinguished by whether the household has health insurance ($hins_i = 1$) or not ($hins_i = 0$).

Results for these two sub-samples are given in columns (4) and (5) of Table 3.

Using a 1% significance level conduct a formal test of the hypothesis that the parameters determining healthcare expenditure are the same for both groups of households. Describe how you would specify a regression equation to test the hypothesis that the parameters relating to income, household composition and age differed between subsamples, while the remaining parameters (relating to education, region and urban status) were the same.

(d) Comment on the investigator's methods, and her principal findings.

Table 3: Results

Dependent Variable: ln hx

	(1)	(2)	(3)	(4) uninsured	(5) insured
ln inc	1.526**	1.566**	1.449**	1.013**	0.488**
	(0.039)	(0.037)	(0.040)	(0.057)	(0.023)
ln size	−0.530**	0.037	0.137*	−0.011	0.290**
	(0.050)	(0.060)	(0.061)	(0.090)	(0.032)
babies		0.806*	0.796*	−0.060	0.193
children		−0.346*	−0.350*	−0.334	−0.171*
age		0.319**	0.031**	0.052**	0.017**
agesq		0.0002**	0.0003**	−0.0003*	0.000
ed2			0.286**	0.040	0.156**
ed3			0.617**	0.305**	0.182**
ed4			0.586**	0.199	0.185**
MidWest		0.240**	0.229**	0.027	0.102*
South		−0.037	−0.021	−0.216	0.072*
West		−0.344**	−0.347**	−0.235*	−0.006
Other city		0.298**	0.274**	0.103	−0.064
Big town		0.297**	0.292**	0.044	−0.036
Small town		0.499**	0.485**	0.413**	0.058
Rural		0.350**	0.348**	0.163	0.061
Intercept	−7.860**	−11.100**	−10.590**	−7.650**	1.260**
R^2	0.185	0.353	0.360	0.207	0.230
SSR	32742	25999	25716	7534	2710
N	6771	6771	6771	2427	4344

Standard errors are given (in parentheses) for the variables ln inc and ln size. R^2 denotes the coefficient of determination, SSR the sum of squared residuals, and N the sample size. ** (*) denotes significance at 5% (10%).

Table 4: Variable Definitions

age_i	mean age of *adults* in the household
$agesq_i$	$(age_i)^2$
$babies_i$	*proportion* of household members aged less than 2
$children_i$	*proportion* of household members aged 2-16
$ed2_i$	1 if household head completed high school
	(but has no college education)
$ed3_i$	1 if household head has some college education
$ed4_i$	1 if household head is college graduate
$region_i$	regional dummy variables - North East is omitted
$urban_i$	dummy variables for urban status - 'big cities' omitted
$hins_i$	1 if household has health insurance

B.3 We consider the determinants of choice of school faced by students in an urban district of Uttar Pradesh, India. There are three types of schools: two are private, and one is government (G). The private schools are aided (PA) and unaided (PUA). Although privately managed, PA schools do not charge fees and are almost entirely funded by the state government and heavily regulated. PUA schools charge fees, are autonomous and self-financed. Government funded schools do not charge fees.

(a) Let $U_{i,\text{PA}}$, $U_{i,\text{PUA}}$, and $U_{i,\text{G}}$ denote, respectively, the utility received by the i^{th} student from choosing each school. Assuming utility maximisation, write down the observational rule based upon the binary choice of whether or not to send a student to a private unaided (PUA) school.

Write down a general expression for $E(y_i = 1|\mathbf{x}_i)$, where $y_i = 1$ indicates that the i^{th} student attends a PUA school, and $\mathbf{x}_i$ denotes a vector of exogenous variables that influence school choice.

(b) Data on the schooling choice, alongside a number of explanatory variables, were collected from 928 students aged 13-14 years in 30 schools across the different types. Table 5 presents variable definitions. In Table 6 we present results for a simple binary choice model where School Type = 1 if an individual went to a private unaided school. P-values are reported in parentheses.

i. Summarise the principal results for the linear probability model (LPM) in column (1).

ii. Compare the results for the LPM model with those for the probit model in columns (2) and (3). APE denotes the Average Partial Effect.

iii. In what sense is the use of the OLS estimator restrictive?

If all the explanatory variables are binary, how would this change your response?

(c) In another probit model where both Ability and Wealth were excluded, the log likelihood statistic (LR) is -451.08. Do the data support these restrictions?

(d) What are the limitations of the observational rule you found in part (a)?

Table 5: Definitions of Variables

Siblings	Number of siblings
Ability	Score on the ability test
Wealth	Index of monetary value of assets in the household, divided by 10
Male	male = 1, female = 0
Lowcaste	Belongs to the low caste? yes =1, no = 0
Muslim	Religion Muslim? yes = 1, no = 0
MEducation	Mother's education in years, divided by 10
MEducation^2	Square of MEducation
Sikh	Religion Sikh or Christian? yes = 1, no = 0

Table 6: Results

Dependent Variable: School Type			
	(1)	(2)	(3)
	OLS	Probit $\widehat{\beta}$	Probit APE
Siblings	-0.002 (0.007)	-0.100 (0.009)	-0.038 (0.009)
Ability	0.008 (0.000)	0.030 (0.000)	0.011 (0.000)
Wealth	0.010 (0.000)	0.046 (0.000)	0.018 (0.000)
Male	0.173 (0.000)	0.858 (0.000)	0.317 (0.000)
Lowcaste	-0.141 (0.000)	-0.550 (0.003)	-0.193 (0.001)
Muslim	-0.139 (0.000)	-0.723 (0.000)	-0.251 (0.000)
MEducation	-0.025 (0.002)	-0.126 (0.001)	-0.048 (0.001)
MEducation^2	0.002 (0.001)	0.008 (0.001)	0.003 (0.001)
Sikh	0.220 (0.002)	0.888 (0.007)	0.340 (0.002)
Constant	0.004 (0.939)	-1.883 (0.000)	–
LR statistic	-	-340.44	-

END OF PAPER

ECAD1
Advanced Diploma in Economics

Wednesday 3 June 2015 1:30pm to 4:30pm

Paper 3

Econometrics

*This paper is in two sections – A and B. Candidates are required to answer **FOUR** questions from Section A and **TWO** questions from Section B.*

Each section carries equal weight.

Answers from each section must be written in separate booklets with the letter of the section written on each cover sheet.

Credit will be given for clear presentation of relevant statistics.

This written exam carries 60% of the marks for Paper 3.

*Write your number **not** your name on the cover sheet of **each** booklet.*

STATIONERY REQUIREMENTS
20 Page Answer Booklet x 2
Metric Graph Paper
Rough Work Pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Durbin Watson and Dickey Fuller Tables
New Cambridge Elementary Statistical Tables
Calculator – students are permitted to bring an approved calculator

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

A1 A researcher, who has access to a dataset containing observations on labour demand, capital, output and average wages for a sample of 569 firms, computes the following regressions using Ordinary Least Squares (OLS):

$$\begin{aligned}
\log(labour) &= \underset{(0.071)}{6.177} + \underset{(0.026)}{0.990}\log(output) - \underset{(0.019)}{0.004}\log(capital) - \underset{(0.071)}{0.928}\log(wage) + \widehat{u} \\
SSR &= 122.24 \qquad SST = 779.09 \qquad R^2 = 0.8430
\end{aligned}$$

and

$$\begin{aligned}
\hat{u}^2 &= \underset{(3.003)}{2.545} - \underset{(0.560)}{0.904}\log(output) + \underset{(0.376)}{1.142}\log(capital) - \underset{(1.753)}{1.299}\log(wage) \\
&\quad + \underset{(0.036)}{0.138}\,(\log(output))^2 + \underset{(0.014)}{0.090}\,(\log(capital))^2 + \underset{(0.259)}{0.193}\,(\log(wage))^2 \\
&\quad - \underset{(0.037)}{0.192}\log(output) * \log(capital) + \underset{(0.163)}{0.138}\log(output) * \log(wage) \\
&\quad - \underset{(0.105)}{0.252}\log(capital) * \log(wage) + residual \\
SSR &= 404.53 \qquad SST = 450.93 \qquad R^2 = 0.1029
\end{aligned}$$

For each regression SSR is the residual sum of squares and SST is the total sum of squares. Figures in brackets are standard errors. The dependent variable $\hat{u}_i^2$ in the second regression is the squared residual from the first.

(a) Using this information, conduct a statistical test (at the 5% level) of the null hypothesis that the disturbances in the first of the two regressions above are homoscedastic. Explain the reasoning behind your choice of test statistic.

(b) What are the implications of the result which you obtained in part (a) for the properties of the OLS estimators in the first regression and their associated standard errors?

(c) Outline two alternative strategies which the researcher could adopt in response to a finding of heteroscedasticity.

A2 A researcher, who has access to monthly data on London house prices from February 1996 to December 2014 inclusive (227 observations), obtains the following estimates by OLS :

$$\begin{aligned} \Delta \widehat{\log(price_t)} &= \underset{(0.0230)}{0.0539} + \underset{(0.00003)}{0.00002t} - \underset{(0.0048)}{0.0089} \log(price_{t-1}) \\ SSR &= 0.02411 \qquad SST = 0.02564 \qquad R^2 = 0.0599 \end{aligned}$$

$$\begin{aligned} \Delta \widehat{\log(price_t)} &= \underset{(0.0198)}{0.0482} + \underset{(0.00003)}{0.00005t} - \underset{(0.0042)}{0.0094} \log(price_{t-1}) \\ &\quad + \sum_{j=1}^{12} \widehat{\gamma}_j \Delta \log(price_{t-j}) \\ SSR &= 0.01584 \qquad SST = 0.02564 \qquad R^2 = 0.3825, \end{aligned}$$

where $\Delta \log(price_t)$ is the first difference of $\log(price)$, that is $\Delta \log(price_t) = \log(price_t) - \log(price_{t-1})$, and the $\widehat{\gamma}_j' s$ in the second equation are (unreported) regression coefficients. For each regression, SSR is the residual sum of squares, SST the total sum of squares and figures in brackets are standard errors.

(a) Explain why the researcher has included both a constant and a trend term in the first equation.

(b) Explain why the researcher has included the additional lagged differences in the second equation. Are they statistically significant? (You may assume that the two regressions are estimated with the last 213 observations).

(c) Using your preferred equation, conduct a statistical test (at the 5% level) of the null hypothesis that the coefficient of lagged $\log(price_t)$ is zero. What do you conclude about the model specification?

 (TURN OVER)

A3 (a) State the conditions under which a univariate stochastic process, z_t, is stationary.

(b) Write out the moving average representation of the autoregressive process

$$z_t = \rho z_{t-1} + \varepsilon_t \ , \qquad \varepsilon_t \sim NID(0, \sigma^2),$$

where $NID(0, \sigma^2)$ denotes normally and independently distributed with mean zero and variance σ^2. Hence show it is stationary provided that $|\rho| < 1$.

A4 A researcher interested in the effect of military service in Vietnam on wages collects data from a random sample of 4000 workers aged 40 and runs an OLS regression on the model

$$y_i = \alpha + \beta x_i + u_i, \qquad i = 1, ..., 4000,$$

where y_i is the i-th worker's annual wage, x_i is a binary variable that is equal to 1 if the i^{th} individual served in the military and is equal to 0 otherwise, α and β are unknown parameters and u_i is an error term.

(a) Write down an expression for the OLS estimator $\widehat{\beta}$.

(b) During the Vietnam War there was a draft, where priority for the draft was determined by a national lottery based upon the day of birth, db, of the potential conscript $(1 \leq db \leq 365)$.

For a given draft year, a variable $e_i = \mathbf{1}(db_i < \tau)$ was constructed and used as an instrument for x_i. The integer τ is chosen based upon the demand for conscripts in a given year. Setting $e_i = \mathbf{1}$ (0) indicates that the i^{th} individual is draft-eligible (ineligible).

i. Explain the requirements for e_i to by a valid instrument.

ii. Show that the IV estimator of β may be written as

$$\widehat{\beta}_{IV} = (\bar{y}^e - \bar{y}^n)/(\hat{p}^e - \hat{p}^n),$$

where $\bar{y}^e$ ($\bar{y}^n$) is mean earnings for the draft-eligible (ineligible) sample and $\hat{p}^e$ ($\hat{p}^n$) is the probability of being a veteran conditional upon being draft-eligible.

A5 A regression model, which uses panel data with T observations for each of N individuals, seeks to explain the total variation of a set of observations, y_{it}, $i = 1, .., N$, $t = 1, ..., T$ around a global mean $\overline{y}$. The total variation can be written:

$$\begin{aligned} T_{yy} &= \sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it} - \overline{y})^2 \\ &= \underbrace{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it} - \overline{y}_i)^2}_{T_{wy}} + \underbrace{\sum_{i=1}^{N}\sum_{t=1}^{T}(\overline{y}_i - \overline{y})^2}_{T_{by}} \end{aligned}$$

(a) With reference to the above decomposition, explain the rationale for basing alternative panel data estimators on variation in T_{yy} and T_{wy}.

(b) For the panel data estimator based solely on T_{wy}, explain the rationale for discarding T_{by}. In what sense is there a cost to excluding this variation?

A6 Suppose a researcher is interested in using time series data to understand how private consumption expenditures (C_t) are affected by aggregate disposable income (Y_t). In particular the researcher wants to know the long run effect on consumption of a permanent change in income. The researcher initially considers the following ADL(1,1) model:

$$c_t = \beta_0 + \beta_1 y_t + \beta_2 y_{t-1} + \beta_3 c_{t-1} + \varepsilon_t,$$

where c_t and y_t are the natural logarithms of C_t and Y_t, respectively and ε_t is a disturbance term.

(a) Show that the ADL(1,1) model under consideration can be equivalently represented by an Error-Correction Model (ECM). Which parameter in your ECM representation gives the long-run effect the researcher is interested in? Explain.

(b) Despite overwhelming evidence for non-stationarity of consumption and income, the researcher is considering simply regressing c_t on y_t in order to estimate the long-run relationship between them. Under what conditions would this strategy be valid? Give an economic reason that makes this condition more likely to be met and briefly discuss one way to test for it.

(TURN OVER)

A7 Suppose a researcher is trying to estimate whether the price of electricity, p_t, responds to the price of oil in world markets, o_t. The researcher has access to time series data and finds that both series are stationary in first differences, but not in levels. Therefore the researcher decides to estimate the following equation relating the growth in electricity price to the growth in oil prices:

$$\Delta p_t = \alpha + \beta \Delta o_t + \varepsilon_t,$$

where ε_t is assumed to be an independently and identically distributed disturbance with mean zero and variance σ^2.

(a) Having estimated the model by OLS, the researcher is concerned that the residuals might be serially correlated. If this is the case, what are the consequences for OLS estimation and inference? How can the researcher formally test for this serial correlation?

(b) Suppose that, when making their pricing decisions, electricity firms would prefer responding fully to current oil prices changes. However, adjusting electricity prices is costly because it breaks legal contracts with households who have signed a fixed-price agreement based on prices last period. Suppose therefore that firms set their price by minimizing the quadratic cost function

$$C_t = \lambda(\Delta p_t - \Delta p_t^*)^2 + (1 - \lambda)(\Delta p_t - \Delta p_{t-1})^2,$$

where Δp_t^* is the profit maximizing price-growth rule, assumed to be given by $\Delta p_t^* = \alpha + \beta \Delta o_t$. Show that the solution to the cost-minimization problem implies an Autoregressive Distributed Lag (ADL) model. Give a brief economic interpretation of this model.

SECTION B

B1 An investigator analysing the determinants of earnings in Belgium has access to a dataset containing information on 1472 individuals (579 females, 893 males). The investigator starts by estimating the following regression using OLS:

$$\begin{aligned}
\ln(\widehat{wage}) &= \underset{(0.0347)}{5.0123} + \underset{(0.0153)}{0.1145} male + \underset{(0.0064)}{0.1643} educ + \underset{(0.0025)}{0.0343} exper - \underset{(0.00006)}{0.00049} exper^2 \\
SSR &= 116.15 \qquad SST = 193.34 \qquad R^2 = 0.3992
\end{aligned}$$

where the variables are defined as follows:

wage	hourly wage rate
male	dummy variable - 1 if male, 0 female
educ	educational level attained (1 is lowest, 5 highest)
exper	work experience in years

SSR is the residual sum of squares, SST is the total sum of squares and figures in brackets are standard errors.

(a) Using these results, compute the marginal effect of an extra year of experience on the wage of a worker with 20 years of experience. What is the number of years after which this marginal effect becomes negative?

(b) The investigator now wishes to assess whether the determinants of earnings are different for males and females and so estimates the regression

$$\ln(\widehat{wage}) = \widehat{\beta}_0 + \widehat{\beta}_1 educ + \widehat{\beta}_2 exper + \widehat{\beta}_3 exper^2$$

for the whole sample, and for males and females separately. The table below gives summary statistics (coefficients and standard errors are not reported). Using these results test, at the 5% significance level, the null hypothesis that the determinants of earnings are the same for males and females.

	All	*Males*	*Females*
SSR	120.5958	70.3979	44.5082
SST	193.5958	117.7076	71.2538
R^2	0.3762	0.4019	0.3754
Observations	1472	893	579

(c) Explain carefully how you would conduct a test of the hypothesis that the effect of experience of education on earnings differs between males and females, while the effect of experience is the same for both genders. You should give a detailed algebraic specification of any regressions you would conduct, and state the distribution of any test statistic you would use.

(d) Another researcher suggests that the education variable is misspecified, in that there is no reason to assume that an individual who has attained level 5 has five times as much 'educational capital' as someone who has only attained level 1. Explain how you could modify the specification of the original equation so that it incorporates a more general model of the returns to education, and indicate how you would chose between the two specifications.

(e) Comment on the design of the investigator's research, indicating in particular any additional data which you would wish to collect in order to improve the reliability of the parameter estimates.

B2 We are interested in the causal effect of fertility on the labour force participation of women. The population comprises married women and is restricted to couples who have at least two children, were married at the time of the 2000 Census, married only once, and married at the time of their first birth. The transition from two to three or more children is substantive - there are 38% of women with a third child in the sample; it is also likely to impart an impact on labour force participation.

The response variable, y_1, is binary, equal to 1 (0) if the woman was (not) working at the time of the survey. The key variable of interest, MOREKIDS, is unity if a woman has three or more children, and zero otherwise. Other controls used are: mother's age, AGEM, age at first birth, AGE FIRST, years of education of mother, EDUCM, first birth is a boy, BOY1ST, and second birth is a boy, BOY2ND. Race and ethnic dummy variables are also included; these are BLACK, HISP, and OTHERRACE.

(a) Write down a model for the probability of having more than two children conditional upon the available data. Do this assuming firstly that the effect of a change in any given regressor on the probability of having more than two children is constant, and then for the probit model, where the partial effects are individual specific.

(b) i. For the probit model write down an expression for the likelihood and log-likelihood for the i^{th} individual.

ii. Show that the partial effects for the probit model are heterogeneous. In what sense does this represent an advantage over the linear probability model?

iii. Write down an expression for the partial effect of a discrete and continuous variable on the probability of having more than two children.

(c) i. Summarise the principal findings in Table 1 overleaf. Why is the distinction between parameter estimates and marginal effects redundant in the case of the OLS estimator?

ii. Why might the OLS estimates be unreliable?

Table 1: Parameter Estimates

	OLS $\widehat{\beta}_{ols}$	Probit $\widehat{\beta}_p$	Marginal Effects
MOREKIDS	-0.161**	-0.422**	-0.160**
	(-78.77)	(-77.00)	(-79.91)
AGEM	0.0227**	0.0600**	0.0226**
	(72.40)	(70.69)	(72.92)
AGE FIRST	-0.0380**	-0.101**	-0.0380**
	(-96.04)	(-90.83)	(-95.80)
BOY1ST	0.000524	0.00139	0.000523
	(0.27)	(0.27)	(0.27)
BOY2ND	-0.00525**	-0.0139**	-0.00524**
	(-2.74)	(-2.73)	(-2.73)
BLACK	0.188**	0.521**	0.197**
	(46.87)	(42.94)	(43.41)
HISP	0.0477**	0.127**	0.0481**
	(11.34)	(11.34)	(11.35)
OTHERRACE	0.0602**	0.158**	0.0598**
	(12.95)	(12.77)	(12.78)
EDUCM	0.0272**	0.0726**	0.0274**
	(59.59)	(56.99)	(58.17)
Sample size	254654	254654	254654
R^2	0.0631		

t statistics in parentheses

$^{+}p < 0.10$, $^{**}p < 0.05$

B3 A Central Bank is interested in understanding the historical effect of money supply on inflation. Researcher A at the Central Bank proposes an Autoregressive Distributed Lag, ADL(1,1), model:

$$\Delta p_t = \beta_0 + \beta_1 \Delta m_t + \beta_2 \Delta m_{t-1} + \beta_3 \Delta p_{t-1} + u_t,$$

where Δp_t is the quarterly growth rate of the GDP deflator (that is, inflation), Δm_t is the quarterly growth rate of money supply and u_t is a zero mean disturbance with variance σ_u^2. This disturbance is assumed by Researcher A to be conditionally independent with respect to current money supply growth and past money growth and inflation. Researcher B at the Central Bank proposes instead the following bivariate Vector Autoregression Model (VAR) approach:

$$\begin{aligned}\Delta p_t &= \alpha_{p0} + \alpha_{p1}\Delta m_t + \alpha_{p2}\Delta p_{t-1} + \alpha_{p3}\Delta m_{t-1} + \varepsilon_{pt} \\ \Delta m_t &= \alpha_{m0} + \alpha_{m1}\Delta p_t + \alpha_{m2}\Delta p_{t-1} + \alpha_{m3}\Delta m_{t-1} + \varepsilon_{mt}\end{aligned}$$

where Δp_t and Δm_t are as in the ADL model and ε_{pt} and ε_{mt} are assumed to be zero mean disturbances with variances σ_p^2 and σ_m^2 respectively and covariance σ_{pm}. In both cases, disturbances are assumed to be independent of past disturbances and realizations of money growth and inflation. Additionally, both researchers have tested for non-stationarity and agree that Δp_t and Δm_t are stationary series.

(a) Researcher B claims that, if his VAR model for inflation and money supply growth is correct, the ADL(1,1) model for inflation proposed by Researcher A cannot possibly deliver consistent estimates of $\beta_0, \beta_1, \beta_2$. Under what condition on the Δm_t equation in the VAR is Researcher B's claim false? Justify your answer.

(b) Researcher A, in turn, claims that the parameters in the VAR - as proposed by Researcher B - cannot be estimated consistently by OLS either. Discuss (without proving) why Researcher A's claim is valid. If Researcher B is solely interested in forecasting inflation and money growth, is there a transformation of Researcher B's VAR that can be estimated consistently by OLS? If so, derive this transformation and justify why OLS estimates are now consistent.

(c) The Central Bank now discloses to the researchers that it will be using their model estimates to answer the following policy question: "If the Central Bank raises money supply growth by 2% permanently today what is the effect on long-run inflation?" Researcher B now claims that, if his VAR model for inflation and money supply growth is correct, the ADL model will not provide the correct answer to this question (even if B's previous concern in (a) is shown not to be valid). Under what condition on Δm_t is B's new claim invalid?

(d) Researcher B gathered data on quarterly inflation and money growth, estimated a reduced form VAR and obtained the following estimates:

$$\begin{aligned}\widehat{\Delta p}_t &= \underset{(0.03)}{0.02} + \underset{(0.11)}{0.65}\Delta p_{t-1} + \underset{(0.12)}{0.32}\Delta m_{t-1} \\ \widehat{\Delta m}_t &= \underset{(0.01)}{0.01} + \underset{(0.09)}{0.35}\Delta p_{t-1} + \underset{(0.17)}{0.51}\Delta m_{t-1},\end{aligned}$$

where the numbers in the equations are OLS estimates and the numbers in parenthesis give the associated OLS standard errors. Given these estimates, test whether Researcher B's claim described in question (c) is valid.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECAD1
ADVANCED DIPLOMA IN ECONOMICS

Wednesday 1 June 2016 1:30pm-4:30pm

Paper 3

THEORY AND PRACTICE OF ECONOMETRICS I

The paper is divided into two Sections A and B.

Answer **FOUR** questions from Section A and **TWO** questions from Section B.

Each Section carries equal weight.

Answers from each Section must be written in separate booklets with the letter of the Section written on each cover sheet.

Credit will be given for clear presentation of relevant statistics.

This written exam carries 60% of the marks for Paper 3.

Write your **candidate number** (not your name) on the cover of each booklet.

Write legibly.

If you identify an error in this paper, please alert the **Invigilator**, who will notify the **Examiner**. A **general announcement** will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator
Durbin Watson and Dickey Fuller Tables
New Cambridge Elementary Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

A1 Consider a three-period panel regression model

$$\begin{aligned} Y_{i1} &= \beta_0 + \beta_1 X_{i1} + \alpha_i + \varepsilon_{i1}, \\ Y_{i2} &= \beta_0 + \beta_1 X_{i2} + \alpha_i + \varepsilon_{i2}, \\ Y_{i3} &= \beta_0 + \beta_1 X_{i3} + \alpha_i + \varepsilon_{i3}, \end{aligned}$$

where Y_{it} and X_{it} denote the values of an outcome and a regressor repectively for the ith individual in period t, with $i = 1, ..., n$ and $t = 1, 2, 3$; α_i is an individual-specific unobserved effect, and ε_{it} is an unobserved error term. Assume that we have n individuals randomly sampled from the population.

(a) Suppose we pool all the observations so that there are 3 observations per individual corresponding to the 3 time periods and thus a total of $3n$ observations altogether. Now, suppose we regress Y_i on X_i (including an intercept) using all $3n$ observations. Under what assumption will this pooled OLS estimator give us consistent estimates of β_1?

(b) Now consider the *second*-differenced estimator obtained by regressing $(Y_{i3} - Y_{i2}) - (Y_{i2} - Y_{i1})$ on the regressor $(X_{i3} - X_{i2}) - (X_{i2} - X_{i1})$. Under what conditions will this estimator be unbiased for the causal effect of X on Y?

(c) Under what condition would you prefer the estimator in part (a) over the estimator in part (b)?

A2 Consider the regression model $Y_i = \beta_0 + \beta_1 X_i + U_i$ for an I.I.D. sample with $N = 1000$ observations. Suppose $U_i \sim$ I.I.D. $(0, \sigma^2)$ and X_i are I.I.D. for $i = 1, 2, ..., 1000$, and that the X_i is independent of U_i. Let $\hat{\beta}_1$ denote the OLS estimator of β_1, and consider another estimator $\tilde{\beta}_1$ of β_1, constructed in the following way:

$$\tilde{\beta}_1 = \frac{Y_3 + Y_1 - 2Y_2}{X_3 + X_1 - 2X_2}.$$

You can assume that X_i are continuously distributed and that $X_3 + X_1 - 2X_2$ never takes the value 0.

(a) Is $\tilde{\beta}_1$ an unbiased estimator of β_1? Why?

(b) Can $\tilde{\beta}_1$ be a better estimator than the OLS estimator? Why?

(c) Is $\tilde{\beta}_1$ a consistent estimator of β_1? Why?

 (TURN OVER)

A3 Consider the regression model for whether a student gets a first class mark in the final examination:

$$First_i = \begin{cases} 1 \text{ if } \beta_0 + \beta_1 Male_i + U_i > 0, \\ 0 \text{ otherwise,} \end{cases}$$

for an I.I.D. sample with 331 observations, where $Male_i$ is a dummy variable which equals 1 if student i is male and is zero otherwise, and $First_i$ is a dummy variable for whether student i got a first class mark, and is zero otherwise. Table 1 at the end of this question gives the distribution of First by gender.

(a) Suppose the unobserved component U_i is independent of $Male_i$, and follows a normal distribution with mean 0 and variance 1. What is the probability of getting a first class degree for male and for female students in terms of β_0 and β_1?

(b) Given your answer to part (a), how would you estimate β_0 and β_1 based on the information provided in Table 1? (Hint: remember that the cumulative distribution function for the normal distribution is strictly increasing).

(c) Is it reasonable to assume that U_i is independent of $Male_i$?

Table 1

	Female	Male	Total
First	19	34	53
Not First	97	181	278
Total	116	215	331

A4 The following problem concerns the phenomenon of "job-lock" in the United States, where employees cannot leave their present job because of employer provided health-insurance. If they leave the present job and take up a new job, then the new health insurance will not cover them for pre-existing health conditions, leading to a job-lock. In order to study this phenomenon empirically, a researcher estimates the following probit model on male workers in the construction industry who are between 35-45 years old and ethnically white:

$$\Pr\left(Y_i = 1 | HI_i\right) = \Phi\left(\alpha_0 + \alpha_1 HI_i\right)$$

where Y_i is a dummy variable which equals 1 if individual i has changed jobs in 2013 and is zero otherwise, HI_i is a dummy for whether individual i was covered by employer provided health insurance in 2012 and is zero otherwise, and $\Phi(\cdot)$ is the standard normal cumulative distribution function.

(a) How would you test whether there is any job-lock, based on this equation? Can you think of a reason why this test may not be a satisfactory indicator of job-lock?

(b) Now let P_i be a dummy variable taking the value 1 if individual i has a chronic medical condition which will not be covered by the new insurance plan if he changed jobs, and P_i is zero otherwise. Consider the equation

$$\Pr\left(Y_i = 1 | HI_i, P_i\right) = \Phi\left(\beta_0 + \beta_1 HI_i + \beta_2 P_i + \beta_3 P_i \times HI_i\right).$$

How would you test the presence of job-lock using estimates of parameters appearing in this equation? Why?

(c) Would you prefer the method based on the equation in part (b) or the one in part (a)? Why?

A5 A researcher considers the following expectations augmented Phillips curve

$$\pi_t - \pi_t^e = \beta\,(U_t - \mu) + e_t,$$

where π_t is inflation, π_t^e is expected inflation with the expectation formed in year $t-1$, U_t is the unemployment rate, μ is the natural rate of unemployment, and e_t is a supply shock. The researcher has data on UK annual CPI inflation and the unemployment rates for the period 1989-2014. OLS regression of $\Delta\pi_t = \pi_t - \pi_{t-1}$ on U_t gives

$$\Delta\widehat{\pi}_t = \underset{(0.89)}{1.61} - \underset{(0.12)}{0.25}U_t,\ R^2 = 0.1535,\ T = 25,$$

with standard errors in parentheses.

(a) Explain how the model estimated by OLS is related to the expectations augmented Phillips curve. What would be your estimate of μ?

(b) Under what assumptions is the OLS estimator of the coefficient on U_t unbiased? Discuss the validity of these assumptions.

(c) Test the hypothesis that U_t does not affect π_t using a 5% significance level test. Give two reasons why the results of your test might be unreliable.

A6 A researcher studies demand for cash in the UK. She runs a regression of the logarithm of the cash in circulation, $\log M_t$, on the logarithm of the nominal household final consumption expenditure, $\log C_t$, using quarterly data from 1985q1 to 2006q1. The OLS results (standard errors in parentheses) are:

$$\widehat{\log M_t} = \underset{(0.26)}{0.596} + \underset{(0.026)}{1.10} \log C_t, \;\; R^2 = 0.9570, \;\; T = 85.$$

(a) How would you interpret the estimated coefficient on $\log C_t$?

(b) A colleague points out that both $\log M_t$ and $\log C_t$ contain a clear time trend. What might be consequences of this fact for the validity of the above regression results?

(c) Reestimating the regression with time trend gives

$$\widehat{\log M_t} = \underset{(1.01)}{20.08} + \underset{(0.0015)}{0.03}\, t - \underset{(0.11)}{0.96} \log C_t, \;\; R^2 = 0.9923, \;\; T = 85, \qquad (1)$$

so that the researcher becomes very puzzled about the sign of the estimated coefficient on $\log C_t$. The colleague gets the residuals $\hat{u}_t$ from (1), and obtains the following OLS result:

$$\widehat{\Delta \hat{u}_t} = \underset{(0.001)}{0.0006} - \underset{(0.030)}{0.064} \hat{u}_{t-1},$$

where $\Delta \hat{u}_t = \hat{u}_t - \hat{u}_{t-1}$. What does this result tells us about the validity of (1)? Can you propose a better specification for the regression model describing the demand for cash?

A7 In order to understand the relation between TV watching and obesity among children, a researcher estimates the following equation by OLS, with standard errors reported in parentheses:

$$ltvyest_i = \underset{(0.02)}{0.99} + \underset{(0.001)}{0.01}\, bmi_i + \underset{(0.011)}{0.018}\, age_i - \underset{(0.0005)}{0.0009} agesq_i - \underset{(0.012)}{0.034} female_i + U_i,$$

where $ltvyest_i$ denotes log of hours spent watching TV by the ith child on the day before the survey was taken, $female_i$ is a dummy which equals 1 if the child is female and is 0 otherwise, $agesq_i$ is the square of the child's age, bmi_i is the child's body mass index (weight for height) and U_i is an unobserved error term.

(a) Assuming the Gauss-Markov assumptions hold, what is the interpretation of the coefficient -0.034 on $female$?

(b) What do the above estimates imply about how watching TV varies with age, all else being equal?

(c) Provide an example of an omitted variable which would imply that the estimated coefficient of bmi in the above equation is biased for the causal effect of bmi on watching TV.

SECTION B

B1 The following question pertains to the effect of background characteristics on the probability that an applicant is admitted to study economics at a selective UK university. TSA is an aptitude test with two components – Critical Thinking and Problem Solving – in each of which the maximum possible mark is 100. We draw a random sample of 800 applicants and record their characteristics and whether they were admitted. The summary statistics are reported in **Table 2**, at the end of this question. Finally, a logit regression of admission on these background characteristics yields the output reported in **Table 3** which appears on the next page. Based on these output, please answer the following questions:

(a) A hypothesis test for the joint significance of the two dummy variables and their interaction yields a chi-square test-statistic with p-value equal to 0.0028. What can we infer from this?

(b) What is the predicted probability of being admitted for a male, independent school applicant who has scored exactly the average mark in the two TSA components? What is the predicted probability of being admitted for a female, non-independent school applicant who has scored exactly the average mark in the two TSA components?

(c) How would you test whether the differences in probabilities in part (c) are zero?

(d) If your test suggests that the admission probability is lower for male, independent school applicants, would this imply discrimination against this demographic group?

(e) What intercept and slope-coefficient estimates would we get if our dummy regressors were female, independent and their interaction, instead of male, independent and their interaction?

Table 2

Variable	Description	Mean	Min	Max
got_in	1 if admitted, 0 otherwise	0.37	0	1
tsa_critical	TSA Critical score	68.83	44	95
tsa_problem	TSA Problem-Solving score	61.68	36	95
indep	1 if from indep school, 0 otherwise	0.46	0	1
male	1 if male, 0 otherwise	0.61	0	1
indep_male	indep$\times$male	0.30	0	1

Table 3

Y=got_in	Coeff	Std_Error	Coeff/Std_Error
tsa_critical	0.10	0.01	8.86
tsa_problem	0.12	0.01	9.25
indep	0.002	0.30	0.01
male	-0.17	0.26	-0.66
indep_male	-0.66	0.38	-1.77
constant	-14.31	1.06	-13.35

Maximized log-likelihood = -372.1331, N=800,
LR chsq(5)=307.91, pvalue=0.00001.

B2 A researcher estimates the following ADL model using OLS:

$$\widehat{vc}_t = \underset{(138.7)}{675.6} + \underset{(0.11)}{0.49} vc_{t-1} + \underset{(0.006)}{0.015} cl_t - \underset{(0.007)}{0.018} cl_{t-1},$$

where vc_t is the monthly number of violent crimes in Cambridgeshire and cl_t is the number of people in East England who claim unemployment benefits during month t. The estimates of the standard errors are given in parentheses. The OLS estimates are based on 61 observations, starting from December 2010 up to December 2015. The average number of violent crimes and average number of unemployment benefit claims over this period were 738 and 89647, respectively.

(a) What is the estimated value of the long-run change in the expected value of the violent crimes given a permanent increase in the number of unemployment claims by 1000? Is this estimate economically significant?

(b) Under what assumptions are the OLS estimators of the coefficients of the ADL model consistent and asymptotically normal?

(c) Let $\hat{u}_t$ be the residuals from the above regression. The OLS regression of $\hat{u}_t$ on a constant, vc_{t-1}, cl_t, cl_{t-1}, and $\hat{u}_{t-1}$ yields

$$\widehat{\hat{u}}_t = \underset{(224.4)}{-109.2} + \underset{(0.176)}{0.086} vc_{t-1} - \underset{(0.008)}{0.003} cl_t + \underset{(0.008)}{0.004} cl_{t-1} - \underset{(0.217)}{0.171} \hat{u}_{t-1}.$$

What does this tell us about the validity of the original OLS results?

(d) The researcher next estimates the following OLS regression:

$$\Delta\widehat{vc}_t = \underset{(45.3)}{63.7} + \underset{(0.67)}{1.75} t - \underset{(0.074)}{0.156} vc_{t-1} - \underset{(0.13)}{0.25} \Delta vc_{t-1}$$

What does this result imply about the validity of the assumptions that you discussed in (b)?

(e) To forecast vc_t, the researcher estimates the following regression

$$\Delta^2\widehat{vc}_t = \underset{(12.5)}{2.92} - \underset{(0.106)}{0.597} \Delta^2 vc_{t-1},$$

where $\Delta^2 vc_t = \Delta vc_t - \Delta vc_{t-1} = vc_t - 2vc_{t-1} + vc_{t-2}$. The values of vc_t for December, November, and October of 2015 were, respectively 997, 1007, 1068. What is the forecast of vc_t for January 2016?

B3 We are interested in understanding how married couples' hours of work are related using data from n randomly sampled households. Consider the simultaneous structural equations

$$\begin{aligned} (1) \qquad hhours_i &= \beta_0 + \beta_1 whours_i + \beta_2 kids_i + U_i, \\ (2) \qquad whours_i &= \gamma_0 + \gamma_1 hhours_i + \gamma_2 kids_i + V_i. \end{aligned}$$

Here $hhours_i$ denotes the husband's weekly hours of work in the ith sampled household, $whours_i$ denotes the wife's weekly hours of work, $kids_i$ denotes the number of children the couple has, while U_i and V_i denote the error terms. Please answer the following questions.

(a) How would you interpret the coefficient γ_1? You do not need to discuss any economic model of utility maximization here.

(b) Solve for $hhours_i$ and $whours_i$ in terms of $kids_i, U_i$ and V_i by solving the two equations. The resulting equations are called the "reduced form" equations. Explain why an OLS of $hhours_i$ on $whours_i$ and $kids_i$ (and a constant) give us biased estimates of the causal effect of the wife's hours of work on the husband's hours of work.

(c) Under what conditions can we use $kids$ as an instrument for $whours$ and estimate β_1 by two-stage least squares? Can you write down one situation where these assumptions may fail?

(d) Finally, suppose that the covariance between U and V across households is zero, and suppose $kids$ is a valid IV for $whours$ in equation (1). Can you describe a way to consistently estimate γ_1, the cofficient of $hhours$ in the *second* equation, i.e., equation (2)? You will need to consult the reduced form for $hhours$ in part (b) of this question.

(e) Is it reasonable to assume that the covariance between U and V is zero?

END OF PAPER

ECAD1
ADVANCED DIPLOMA IN ECONOMICS

Monday 12 June 2017 1:30pm-3:30pm

Paper 3

ECONOMETRICS

The paper contains SEVEN questions.

Answer **any FOUR** questions.

Each question carries equal weight.

Credit will be given for clear presentation of relevant statistics.

This written exam carries 60% of the marks for Paper 3.

Write your **candidate number** (not your name) on the cover of each booklet.

Candidates are asked to note that there may be a reduction in marks for scripts with illegible handwriting.

If you identify an error in this paper, please alert the **Invigilator**, who will inform the **Examiner**. A **general** announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator
Durbin Watson and Dickey Fuller Tables
New Cambridge Elementary Statistical Tables
Engle-Granger Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. Suppose we are interested in the relationship between the number of police officers per capita and the number of reported crimes per resident in annual panel data on crime in a random sample of UK counties, collected from 2009-2010. The regression equation is

$$crmrte_{it} = \beta_0 \times \mathbf{1}(t = 2010) + \beta_1 police_{it} + \beta_2 poverty_{it} + a_i + u_{it},$$

where $crmrte_{it}$ is the crime rate per resident in county i in year t, $police_{it}$ is the number of police officers per resident, $poverty_{it}$ is the poverty rate, and a_i is the unobserved county specific effect while u_{it} is the idiosyncratic effect. The dummy $\mathbf{1}(t = 2010)$ is 1 if the observation comes from 2010 and is zero otherwise. This dummy is included to make it possible that the intercept in 2010 is different from that in 2009. Assume that u_{it} is uncorrelated with police and poverty in all years and is also uncorrelated with a_i.

(a) Under what condition(s) will a cross-sectional regression with just 2009 data give us *consistent* estimates of the causal effect of police on crime?

(b) Under what condition(s) will a first-differenced regression give us *consistent* estimates of the causal effect of police on crime?

(c) Given your answers to part (a) and part (b) above, can you suggest a situation where the cross-sectional regression might be preferable to the first-differenced regression for the above dataset?

2. The following is a system of simultaneous equations:

$$y_1 = \beta_{10} + \alpha_1 y_2 + \beta_{11} x_1 + u_1 \tag{2.1}$$
$$y_2 = \beta_{20} + \alpha_2 y_1 + \beta_{21} x_1 + u_2 \tag{2.2}$$

where y_1, y_2, x_1 are observed, and u_1, u_2 are unobserved zero mean homoskedastic disturbances satisfying the conditions $E(u_j x_1) = 0$, for $j = 1, 2$. You wish to estimate α_1.

(a) Derive the reduced-form for y_2 by solving the above equations for y_1 and y_2. Hence state conditions on the α and/or β parameters under which the OLS estimator of α_1 is consistent.

(b) Concerned that the necessary condition in part (a) may not hold, you estimate the model by 2SLS using x_1 as an instrument for y_2. Under what conditions on the α and/or β parameters is the 2SLS estimator of α_1 consistent?

(c) Do the conditions you derived in part (a) and in part (b) also imply 'unbiasedness' of the respective estimators? Explain your answer.

3. The following question concerns the effects of background characteristics and admission assessment scores on the eventual performance of students in the final university examination.
The names of the variables and a brief description are as follows:

Variable	Description	Mean	Std. Dev	Min	Max
finalsavg	Average finals score (the outcome)	64.98	4.44	25	77
tsa_critical	Pre-admission Reasoning Test-Score	75.16	9.53	49.9	95.6
tsa_problem	Pre-admission Quantitative Test-Score	67.78	9.31	45.6	95.6
interview	Interview Score	69	9.18	7.67	91
indep	= 1 if from independent school	0.48	0.5	0	1
male	=1 if male	0.64	0.48	0	1
indep_male	indep$\times$male	0.29	0.46	0	1

Finally, a linear regression of final university examination performance on these background characteristics using the same dataset yields the following output

Y=finalsavg	Coeff	Std Error	2-sided pvalue
constant	53.89	2.78	0.00
tsa_critical	0.03	0.04	0.29
tsa_problem	0.05	0.02	0.03
interview	0.06	0.02	0.00
indep	−0.04	0.78	0.96
male	0.67	0.66	0.31
indep_male	0.06	0.97	0.95

R^2=0.06, N=325

(a) Suppose all schools are classified as independent or state. What coefficients and intercept estimates would be obtained if we replaced male dummy by female dummy, indep dummy by state dummy, and include the interaction state$\times$female instead of indep$\times$male?

(b) How would you test the hypothesis that gender has no impact on final university examination performance, given the other covariates, in this regression model?

(c) If we do not include the interaction term indep_male in our regression model, what are we implicitly assuming about the effect of gender and school background on final university examination performance?

4. The following problem concerns the phenomenon of "job-lock", where employees cannot leave their present job because of health insurance provided by the employer. If they leave the present job and take up a new job, then the new health insurance will not cover them for preexisting health conditions, leading to a job-lock. In order to study this phenomenon empirically, a researcher collects data on male workers in the construction industry who are between 35-45 years old and ethnically white. Let Y_i be a dummy for whether worker i has changed jobs in 2013, let HI_i be a dummy for whether the individual was covered by a health insurance provided by the employer in 2012, and P_i be a dummy for whether the individual is covered by their spouse's health insurance plan in 2012 and 2013.

Suppose the true probability of job-change is correctly specified as a probit model. Precisely, suppose that

$$\Pr\left(Y_i = 1 | HI_i, P_i\right) = \Phi\left(\beta_0 + \beta_1 HI_i + \beta_2 P_i + \beta_3 P_i \times HI_i\right),$$

where $\Phi(\cdot)$ denotes the CDF for the standard normal distribution.

(a) How would you detect the presence of job-lock using estimates of parameters appearing in the above equation?

(b) What minimal assumption needs to be made about the unobserved characteristics of the sampled individuals in order that your method described in part (a) truly detects job-lock?

(c) Now suppose each worker included in the sample who has health insurance provided by the employer also happens to have spousal insurance cover; conversely, each worker included in the sample who does not have health insurance provided by the employer also happens not to have spousal cover. Can you still detect job-lock using the above equation? Why?

5. This question pertains to the purchase of a high-quality mosquito bednet in a developing country with prevalence of malaria. The data were generated by a randomized experiment across a cross-section of 2231 households. In particular, the (subsidized) price at which a household (hhd) could acquire a previously unavailable, high-quality mosquito net was varied randomly across households. The names and brief descriptions of the variables are as follows. The symbol 'R' stands for the local currency

Variable	Description	Mean	Std. Dev	Min	Max
net	1 if hhd bought net, 0 otherwise	0.34	0.48	0	1
price	Price faced by hhd (in R)	129.76	71.28	0	300
child	Number of children below 10 years	1.74	1.51	0	14
income	Hhd monthly income (in R)	138.45	309.79	0	5750

When fitted to these data, a probit model yields the following output:

Y=net	Coeff	Std Error	2-sided pvalue
constant	1.01	0.07	0.00
price	−0.02	0.0005	0.001
child	−0.065	0.02	0.00
income	−0.00008	0.0001	0.46

Pseudo-R^2=0.19, N=2231

(a) Consider a 'typical' household, defined as one with income R140 and having 2 children below 10 years. How low does the price have to be in order for the probability of bednet purchase for such a household to exceed 0.5?

(b) Suppose a household with income R140, having 2 children below 10 years and facing a price of R130, has its income increased to R1400. How much would its probability of buying a bednet change? Is this difference statistically significant? How can you tell?

(c) Can you suggest an intuitive explanation why the coefficient of 'child' is negative?

6. A researcher is studying the relationship between crime and unemployment using UK annual data from 1972 to 2002. Variables GC_t and ΔU_t equal, respectively, the annual growth of the number of recorded thefts (in percent) and the change in the unemployment rate from the previous year. The OLS regression of GC_t on ΔU_t and $\Delta U_{t\ 1}$ gives the following results (with the standard errors given in the parentheses)

$$\widehat{GC}_t = \underset{(2.30)}{8.77} + \underset{(3.01)}{5.07}\Delta U_t - \underset{(3.01)}{3.18}\Delta U_{t-1},\ R^2 = 0.095. \tag{6.1}$$

(a) What are the estimates of the impact and long-run multiplier? Interpret these estimates. Using a 5% significance level test, test the hypothesis that a one percentage point increase in the unemployment rate results in a ten percentage points increase in the theft growth rate in the same year.

(b) Under what assumptions are the reported OLS estimates consistent? Under what assumptions is the test you propose in (a) asymptotically valid? Briefly discuss the plausibility of all these assumptions.

(c) Let $\hat{u}_t$ be the OLS residuals from the above regression. The OLS regression of $\hat{u}_t$ on $\hat{u}_{t-1}$ yields

$$\widehat{\hat{u}}_t = \underset{(2.12)}{0.55} + \underset{(0.18)}{0.44}\hat{u}_{t-1},\ R^2 = 0.175.$$

The researcher says that these results invalidate those reported in equation (6.1) above and proposes to use the Cochrane-Orcutt method instead. Explain the researcher's logic and briefly describe how you would implement the Cochrane-Orcutt method. What may be a problem with this method in the present context?

7. Consider the following model for log of annual global carbon dioxide emissions, $lCO2_t$:

$$lCO2_t = \alpha_0 + \alpha_1 t + \alpha_2 lCO2_{t-1} + u_t, \qquad (7.1)$$

where u_t are i.i.d. shocks with mean zero and variance σ^2, t is time, and $|\alpha_2| < 1$.

(a) Assuming that the values of α_0, α_1, α_2 and σ^2 are known, what will be the variance of the forecast obtained using model (7.1) of log of carbon dioxide emissions 20 years from now? Name at least two potential problems with using model (7.1) for carbon dioxide forecasting.

(b) The OLS regression that uses data from $t = 1910$ to $t = 2013$ yields

$$\widehat{lCO2}_t = \underset{(1.65)}{-3.65} + \underset{(0.001)}{0.0022} \times t + \underset{(0.035)}{0.923} \times lCO2_{t-1}.$$

Using these results, test the hypothesis that $lCO2_t$ is a random walk with drift. Use a 5% significance level test.

(c) To study the effect of carbon dioxide emissions on global warming you regress average annual UK temperature, $Temp_t$, on $lCO2_t$. The corresponding OLS results (based on the data from 1910 to 2013) are

$$\widehat{Temp}_t = \underset{(0.54)}{10.35} + \underset{(0.068)}{0.225} \times lCO2_t, \; R^2 = 0.097.$$

Critically assess these results.

END OF PAPER

UNIVERSITY OF CAMBRIDGE

ECAD1
ADVANCED DIPLOMA IN ECONOMICS

Monday 11 June 2018 1:30pm - 3:30pm

Paper 3

ECONOMETRICS

The paper contains SEVEN questions.

Answer **any FOUR** questions.

Each question carries equal weight.

Credit will be given for clear presentation of relevant statistics.

This written exam carries 60% of the marks for Paper 3.

Write your **candidate number** (not your name) on the cover of each booklet.

Candidates are asked to note that there may be a reduction in marks for scripts with illegible handwriting.

If you identify an error in this paper, please alert the **Invigilator**, who will inform the **Examiner**. A **general** announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 1
Rough work pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator
Durbin-Watson, Dickey-Fuller and Engle-Granger Tables
New Cambridge Elementary Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1. A researcher has the following data for a sample of 64 US college towns for the years 1980 and 1990: average housing rental price *rent*, city population *pop*, average income *avginc*, and student population as a percentage of city population *pctstu*. The researcher defines a dummy variable $y90$ as 1 if the year is 1990 and 0 otherwise.

The researcher runs two regressions with $\ln(rent)$ as the dependent variable: an ordinary least squares (OLS) regression combining the observations for 1980 and 1990, and a first differenced (FD) regression based on these two years of data. The dependent variables used are a *constant*, $\ln(pop)$, $\ln(avginc)$, $pctstu$, and $y90$.

The slope coefficients from these regressions are shown in the following table; t statistics are given in parentheses.

Dep. variable: $\ln(rent)$	OLS	FD
$\ln(pop)$	0.041 (1.81)	0.072 (0.82)
$\ln(avginc)$	0.571 (10.76)	0.310 (4.66)
$pctstu$	0.005 (4.95)	0.011 (2.71)
$y90$	0.262 (7.54)	-
R^2	0.86	0.32
N	128	64

(a) The coefficient estimate on $\ln(avginc)$ is almost twice as large in the OLS regression relative to the FD regression. In contrast, the coefficient estimate on $pctstu$ is half as large in the OLS regression relative to the FD regression. Give an intuitive explanation of these findings.

(b) The change in $y90$ from 1980 to 1990 is the same for all towns. Explain why the table does not provide an estimate corresponding to $y90$ for the FD regression.

(c) Can you describe a scenario under which the OLS estimates are likely to be "better" than the FD estimates? In your answer clearly indicate what it means for the estimates to be "better" and discuss how reasonable you think this scenario is.

2. The following pair of simultaneous equations describe the total demand for and supply of the "Kingfisher" brand of beer across Indian restaurants in the UK in the month of June, 2017:

$$\text{Demand: } y_1 = \beta_{10} + \alpha_1 y_2 + \beta_{11} x + u_1 \quad (1)$$

$$\text{Supply: } y_2 = \beta_{20} + \alpha_2 y_1 + \beta_{21} x + u_2, \quad (2)$$

where y_1 and y_2 are the natural log of quantity and of price, respectively; the control variable x denotes the distance between the restaurant and the port where imported Kingfisher beer enters the UK; u_1 and u_2 are unobserved homoskedastic disturbances satisfying the conditions $Cov(u_j, x) = 0$, for $j = 1, 2$, and $Cov(u_1, u_2) = 0$.

(a) State conditions on the α and/or β coefficients under which we can consistently estimate the price elasticity of demand for Kingfisher beer by simply running an OLS of y_1 on a *constant*, y_2, and the control variable x.

(b) Suppose that you are concerned that the conditions stated in your answer to part (a) do not hold. Suppose that you wish to estimate the price elasticity of demand by 2SLS using only data for y_1, y_2, and x. How would you perform this exercise? Under what conditions on the α and/or β parameters is the 2SLS estimator consistent?

(c) Suppose the conditions stated in your answer to part (b) hold, so that we can consistently estimate the price elasticity of *demand*. Is that sufficient to also consistently estimate the price elasticity of *supply*? Give reasons for your answer.

3. Suppose that the county of Foxfordshire has passed a law requiring employers to provide 6 months of paid maternity leave. You are concerned that women's wages will drop in order to pay for this new benefit. You find a data set that samples men and women in Foxfordshire and in neighboring Wolfordshire and has information on wages. You pool two cross-sections, one from the year before the law took effect and one from the year after, and find that the average wage for various groups is as follows (the first row is women and the second is men):

Average wage	Wolfordshire		Foxfordshire	
	Before	After	Before	After
Women	9	12	8	10
Men	12	16	10	15

(a) Suppose you estimate the following model by OLS using only data from Foxfordshire:

$$wage = \beta_0 + \beta_1 after + \beta_2 women + \beta_3 (after * women) + u,$$

where $after$ is a dummy variable for the year after the change and $women$ is a dummy variable for being a woman. What is the numerical value of your estimate of β_3?

(b) Suppose instead you estimate the following model by OLS using all the data:

$$\begin{aligned} wage &= \gamma_0 + \gamma_1 after + \gamma_2 women + \gamma_3 (after * women) \\ &\quad + \gamma_4 F + \gamma_5 (after * F) + \gamma_6 (women * F) \\ &\quad + \gamma_7 (after * women * F) + u, \end{aligned}$$

where $after$ and $women$ are as before, and F is a dummy variable for Foxfordshire. What is the numerical value of your estimate of γ_7?

(c) Can you provide a concrete example of a scenario where the estimate $\hat{\beta}_3$ in part (a) will *not* yield a consistent estimate of the effect of the law on women's wages, but the estimate $\hat{\gamma}_7$ in part (b) will? Explain the reason(s) for the inconsistency of $\hat{\beta}_3$ and the consistency of $\hat{\gamma}_7$ in your example.

4. Consider the simple regression model

$$Y = \beta_0 + \beta_1 X + u,$$

where $E(u) = 0$ but $Cov(X, u) \neq 0$. Let Z be a valid instrumental variable for X. Further, let Z take on only two values: 0 and 1.

(a) Show that

$$\beta_1 = \frac{Cov(Z, Y)}{Cov(Z, X)}.$$

(b) Let $\pi = E(Z) = \Pr(Z = 1)$. Show that for any random variable W,

$$\begin{aligned} E(W) &= \pi E(W|Z = 1) + (1 - \pi) E(W|Z = 0), \\ Cov(Z, W) &= \pi (1 - \pi) [E(W|Z = 1) - E(W|Z = 0)]. \end{aligned}$$

(c) Using (a) and (b), show that

$$\beta_1 = \frac{E(Y|Z = 1) - E(Y|Z = 0)}{E(X|Z = 1) - E(X|Z = 0)}.$$

Suggest and interpret an IV estimator of β_1.

5. This question pertains to school attendance of children in rural India in 2005. The data were collected through a household survey, containing information on 17310 children. The names and brief descriptions of the variables are as follows. Price and income are in thousand Indian Rupees.

Variable	Description	Mean	Std. Dev	Min	Max
inschool	1 if child in school, 0 otherwise	0.34	0.48	0	1
price	Price per month of attending a local school	0.176	0.224	0	2
female	1 if child is female, 0 otherwise	0.44	0.49	0	1
income	Family income per capita per month	3.078	1.072	1.593	5.713
age	Age of child in years	11.77	4.77	5	18

When fitted to these data, a probit model for school attendance yields the following output:

Dep. variable: *inschool*	Coeff	Std Error
constant	3.76	0.114
price	-0.41	0.06
female	-0.65	0.14
age	-0.256	0.007
$age\_female = (age * female)$	0.018	0.009
income	0.29	0.01

Pseudo-R^2=0.33, N=17310

(a) Consider a female, 15 year old child with family income 3. How low does the price per month of attending a local school have to be in order for her probability of school attendance to exceed 0.5?

(b) Suppose instead of using the dummy variable for female, we use a dummy variable for male, i.e. $male = 1$ if and only if $female = 0$. Similarly age_female is replaced by $age\_male = age * male$. What will the coefficients be of a probit regression of school attendance on *constant*, *price*, *male*, *age*, *age_male* and *income*?

(c) How will you test whether *age* has any explanatory power for *inschool* in the original probit regression model above? Describe the test statistic and the criterion you will use to perform the test.

6. An economist estimates an AR(2) model for the UK employment rate using an OLS regression with quarterly data from 1971.q1 to 2017.q3 (187 observations). The regression results are

$$\widehat{empl}_t = \underset{(0.52)}{0.93} + \underset{(0.054)}{1.681} empl_{t-1} - \underset{(0.054)}{0.694} empl_{t-2}, \; R^2 = 0.99,$$

where the standard errors are given in parentheses, and $empl_t$ denotes the UK employment rate at time t, i.e., the proportion of people aged 16 to 64 years (in percent) who were in work at time t.

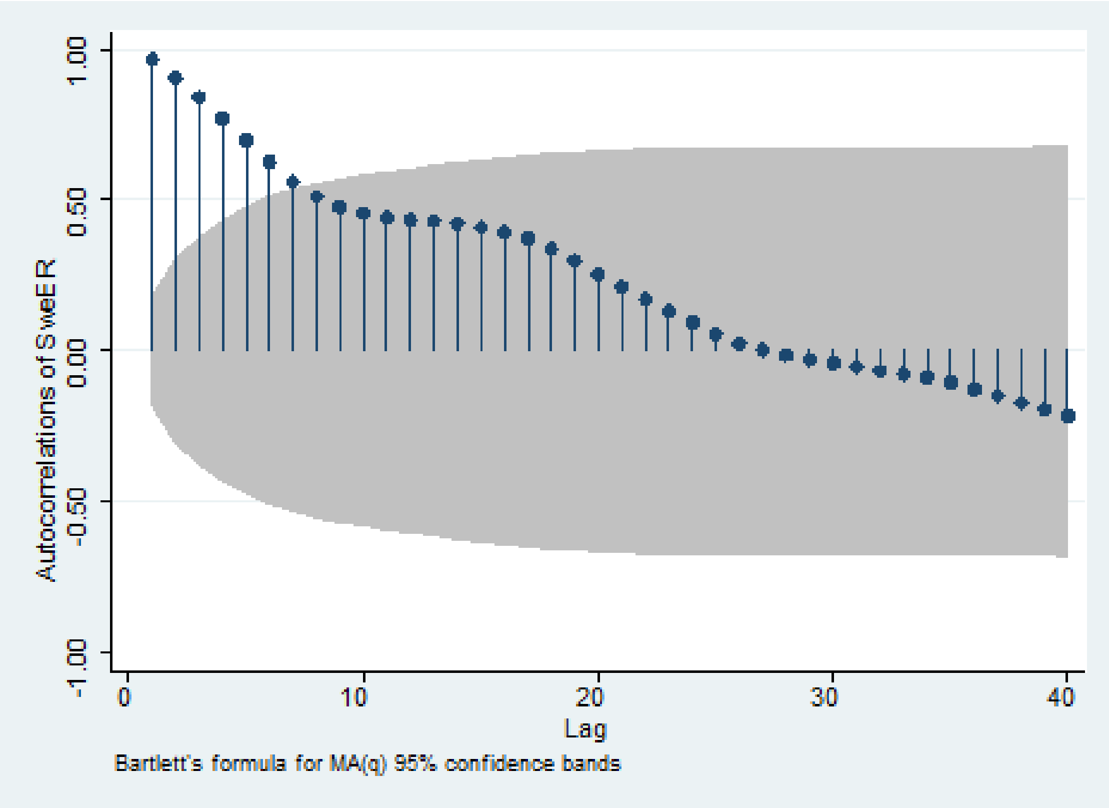
(a) Give the definition of a weakly stationary stochastic process. Assuming that $empl_t$ is weakly stationary and follows a stable AR(2) process, what do the above results tell us about the expected value of $empl_t$?

(b) Verify that the estimated AR(2) model is stable. Explain your reasoning. How would you test for the absence of serial correlation in the error term of the AR(2) model for the employment rate?

(c) The values of the UK employment rate in 2017.q2 and 2017.q3 were, respectively, 75.1 and 75.0. Given the above results, what is your forecast of the UK employment rate in 2018.q1? Explain how you would estimate the variance of your forecast error.

 (TURN OVER)

7. An economist studies monthly data on the Swedish exchange rate (kronor/US$) for the period from March 2008 to February 2018 (120 observations). This exchange rate is denoted as *SweER*. The sample autocorrelogram for *SweER* is as shown in the picture below.



(a) How would you characterize the behavior of *SweER* using the information in the above sample correlogram?

(b) The economist uses the data to obtain the following OLS results

$$\Delta \widehat{SweER}_t = \underset{(0.142)}{0.332} - \underset{(0.0191)}{0.0436} SweER_{t-1} + \underset{(0.084)}{0.421} \Delta SweER_{t-1},$$

where the numbers in parentheses are the standard errors, $SweER_t$ is the Swedish exchange rate at time t, and $\Delta SweER_{t-1} = SweER_{t-1} - SweER_{t-2}$. Using these results, test for a unit root in $SweER_t$. Use a 5% significance level test. Why is it important to have $\Delta SweER_{t-1}$ included in the regression for the purpose of testing for a unit root?

(c) Suppose that in addition to the data on the Swedish exchange rate, the economist has similar data on the Norwegian exchange rate. Describe a procedure that the economist can use to test a hypothesis that these two exchange rates are cointegrated. If we find evidence of cointegration, how could you use the cointegrating relationship for constructing a forecasting model for the Swedish exchange rate?

END OF PAPER