Key messages from Ofsted's recent mathematics reports

This distance learning provides an overview of:

- *Good practice in primary mathematics*, a specially commissioned report, published in November 2011
- *Mathematics: made to measure*, based on the annual programmes of mathematics survey inspections, published in May 2012.

The aims of the distance learning are:

- to share key messages from *Good practice in primary mathematics* and *Mathematics: made to measure*
- to help inspectors to:
  - gather evidence and make judgements about mathematics
  - support schools in improving their work in mathematics.

This document contains the main materials for the distance learning and includes information, activities and an appendix. A separate four-page booklet contains answers to the activities. You may wish to print out both documents but, if you choose to work through them on screen, you will need paper copies of pages 7, 12 and 21 of this document.

The distance learning is intended for you to work through at your own pace. It should take you roughly an hour and a half to complete. The first part is based on *Good practice in primary mathematics* and the second part on *Mathematics: made to measure*. Primary and secondary school inspectors should complete both parts of the distance learning as the findings of each report are important for both phases.

You may find it helpful to complete this distance learning before the mathematics training, delivered in spring 2013.

Good practice in primary mathematics

This good practice survey was conducted at the beginning of the Coalition Government's review of the National Curriculum, which takes into account the curricula of high performing countries in mathematics, many of which focus strongly on conceptual understanding of number. The survey followed a ministerial request for Ofsted to provide evidence on effective practice in the teaching of early arithmetic. It focused on identifying characteristics of effective practice in building pupils' secure knowledge, skills and understanding of number so that they demonstrate fluency in calculating, solving problems and reasoning about number.

Prior to the good practice survey, Ofsted's findings from regular survey inspections had identified a lack of:

- conceptual understanding
- progression that is coherently developed and ensures links between interim and formal methods.

Ofsted had not previously looked in depth at specific topics within mathematics, such as arithmetic.

The aims of the good practice survey were to:

- identify characteristics of effective practice in building pupils' secure knowledge, skills and understanding of number so that they demonstrate fluency in calculating, solving problems and reasoning about number
- explore the choices of methods that middle to lower attaining pupils in these successful schools make when presented with calculations and problems to solve.

It is timely to discuss this report because the current review of the National Curriculum is nearing the stage of formal consultation. An informal consultation on the draft primary programmes of study was held in June 2012.

The aims of the draft primary mathematics National Curriculum are for fluency, problem solving, and mathematical reasoning. They align closely with the findings of the good practice report. The definition of fluency incorporates conceptual understanding.

The draft programme of study for mathematics is ambitious. It emphasises traditional formal calculation methods. However, the draft programme of study and accompanying guidance that were published in June 2012 did not reflect all three aims equally well and attracted considerable criticism.

Activity 1

Here is some mathematics for you to do:

Work out: 35 + 17 and 45 + 27

Some people take much less time to work out the second answer once they have worked out the first.

How do you think they do this?

What about 55 + 37?

Before moving on, refer to the answer booklet for the comments on Activity 1.

Key findings from *Good practice in primary mathematics*

The schools had identified some essential precursors for learning traditional vertical algorithms (column methods) for addition, subtraction, multiplication and division. These were:

- understanding of place value
- fluency in mental methods
- recall of number facts such as multiplication tables and number bonds.

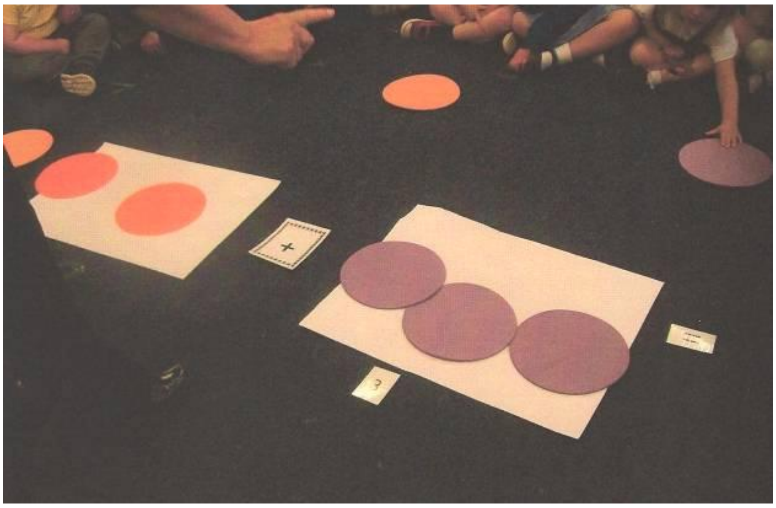
Practical, hands-on experiences of using, comparing and calculating with numbers and quantities and the development of mental methods are of crucial importance in establishing the best mathematical start in the Early Years Foundation Stage and Key Stage 1.

The schools visited coupled this with plenty of opportunities for developing mathematical language so that pupils learn to express their thinking using the correct vocabulary.

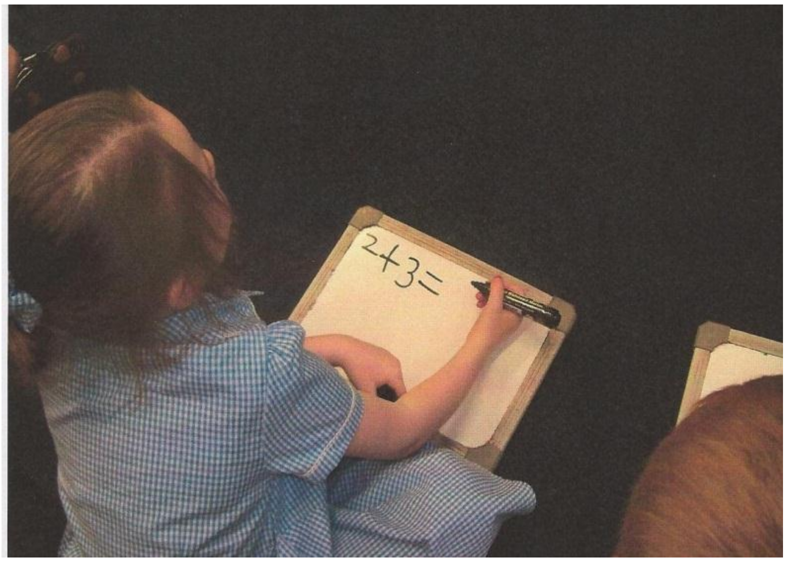
The schools were careful to link mental, practical and written methods to support the development of pupils' understanding.

Look at the photographs below. They show Reception children carrying out simple calculations.

Note the use of practical equipment and cards which show numbers and the = and + signs. Placed next to the objects, they help early recording of number sentences with correct use of symbols.



Here more able children are recording independently on whiteboards.



An interlude on arithmetic

The following are the most common written methods in arithmetic for:

- addition and subtraction
  - partitioning
  - empty number line
  - column method
- multiplication
  - grid method
  - short multiplication
  - long multiplication
- division
  - chunking
  - short division
  - long division

The next section explains these in more detail.

1. Addition and subtraction

Addition by partitioning

'Partitioning' is the process of splitting a number into parts, often (but not always) into units, tens and hundreds.

e.g. 123 = 100 + 20 + 3 56 = 50 + 6

To add 32 + 26 by partitioning: first, partition 32 as 30 + 2 and 26 as 20 + 6. Then add the tens (50) and the units (8), and recombine to make 58. This models a sensible mental process for adding the numbers as well as providing an early written method:

$$\begin{array}{rrr} & 32 & (30 + 2) \\ + & \underline{26} & \underline{(20 + 6)} \\ & \underline{58} & \underline{(50 + 8)} \end{array}$$

Written vertically, this method can be linked directly to the process of column addition.

Subtraction by partitioning

Pupils often use partitioning effectively for addition but not when subtracting. A common incorrect answer to 43 – 27 is 24:

$$\begin{array}{rr} 43 & (40 + 3) \\ -\underline{27} & -\underline{(20 + 7)} \\ ?? & (20 + ?) \end{array}$$

The reason for this error is that the pupil subtracts the smaller units digit from the larger units digit: e.g. (40 – 20) and (7 – 3) to get 20 and 4, giving 24.

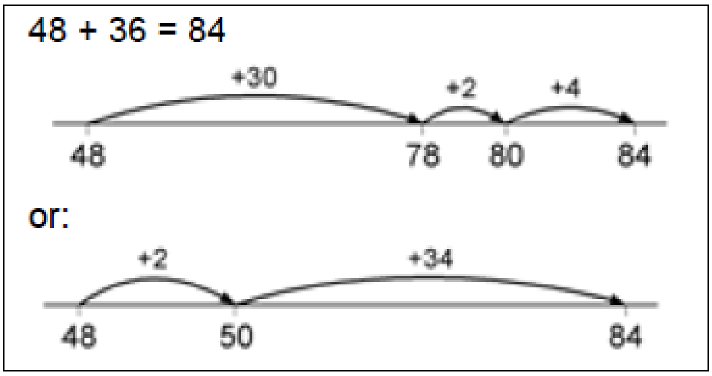
Teachers often place much more emphasis on partitioning into tens and units than other ways of partitioning such as 56 = 40 + 16. This is of particular importance in subtraction.

The key to the subtraction 43 – 27 is to partition 43 as 30 + 13, allowing 20 + 7 to be subtracted from 30 + 13 to give 10 + 6 = 16. This links clearly to the column subtraction:

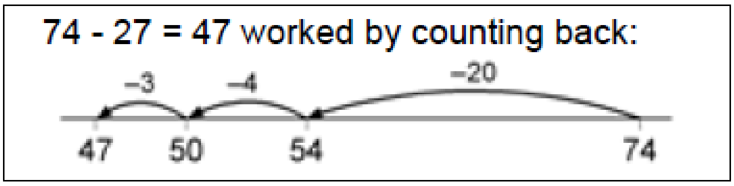
$$\begin{array}{rr} & {}^{3}\cancel{4}\,{}^{1}3 \\ - & \underline{2\ \ 7} \\ & 1\ \ 6 \end{array} \qquad\qquad \begin{array}{rl} & (30 + 13) \\ - & \underline{(20 + \ \ 7)} \\ & (10 + \ \ 6) \end{array}$$

Addition and subtraction using the empty number line

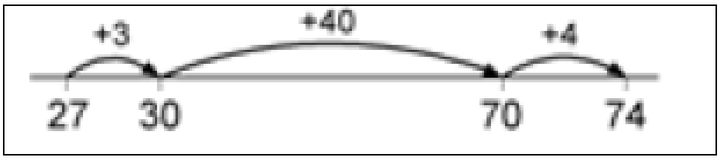
Pupils use an empty number line (which has no pre-marked numbers on it and no fixed scale) to record numbers and steps in the calculation. Each calculation can be represented in different ways. In the examples below, the pupil has made use of multiples of 10 in carrying out the addition, in the first case 'bridging through 80'.



For subtraction, pupils usually count back:



but sometimes they count on, in this case from 27 up to 74:



As with addition, a calculation can be represented in many alternative ways.

Addition and subtraction using column methods

The emphasis in the draft National Curriculum programme of study is on ensuring that pupils progress quickly to efficient methods.

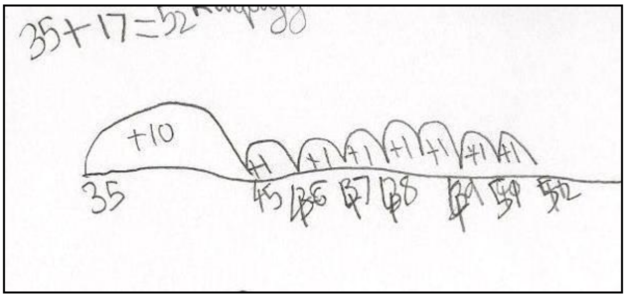
Column addition	Column subtraction	
		or very occasionally
$\begin{array}{rr} & 5\,6\,7 \\ + & \underline{2\,2\,6} \\ & \underline{7\,9\,3} \\ & {}_{1}\ \ \ \end{array}$	$\begin{array}{rr} & 3\ {}^{5}\cancel{6}\,{}^{1}2 \\ - & \underline{2\ \ 2\ \ 6} \\ & \underline{1\ \ 3\ \ 6} \end{array}$	$\begin{array}{rr} & 3\ \ 6\ {}^{1}2 \\ - & \underline{2\ \ 2_{1}\ 6} \\ & \underline{1\ \ 3\ \ 6} \end{array}$

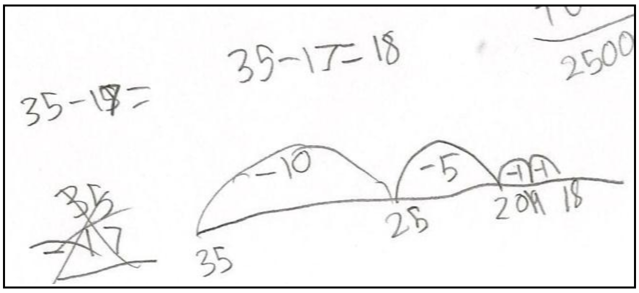
Activity 2

Look at the following examples of work from three pupils

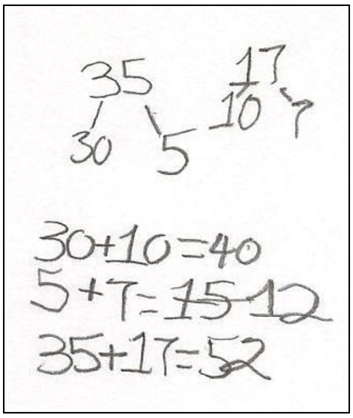
- What methods do they use?
- What mistakes have they made (if any)?

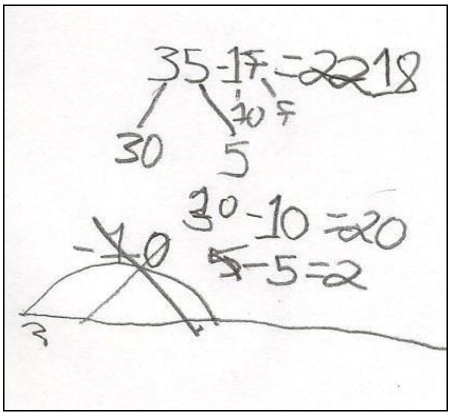
Pupil A's work



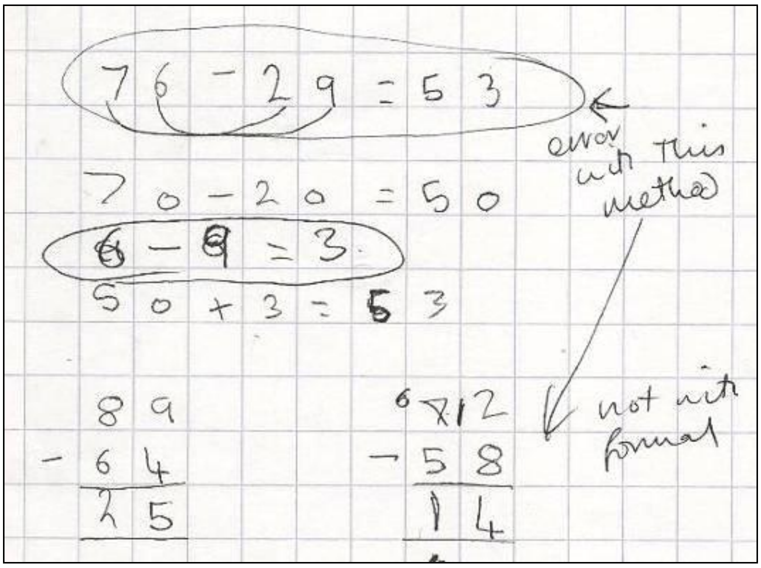


Pupil B's work





Pupil C's work



Before moving on, refer to the answer booklet for the comments on Activity 2.

2. Multiplication

Activity 3 A multiplication for you! Work out 99 x 8 Can you do it by another method?
Before moving on, refer to the answer booklet for the comments on Activity 3.

Multiplication by a mental method using partitioning

This involves partitioning the larger number and multiplying each part before recombining:

e.g. 36 x 4 = 30 x 4 + 6 x 4 = 120 + 24 = 144

Short multiplication

The method above has clear links to short multiplication:

$$\begin{array}{rr} & 3\;6 \\ \times & \underline{\;\;4} \\ & \underline{1\;4\;4} \\ & {\scriptstyle 2}\;\; \end{array} \qquad \text{expanded method:} \quad \begin{array}{rr} & 36 \\ \times & \underline{\;\;4} \\ & 24 \\ & \underline{120} \\ & \underline{144} \end{array}$$

Multiplication by the grid method and long multiplication

The grid method calculates the products of the partitioned numbers and then adds them. For 248 x 34, this involves six products:

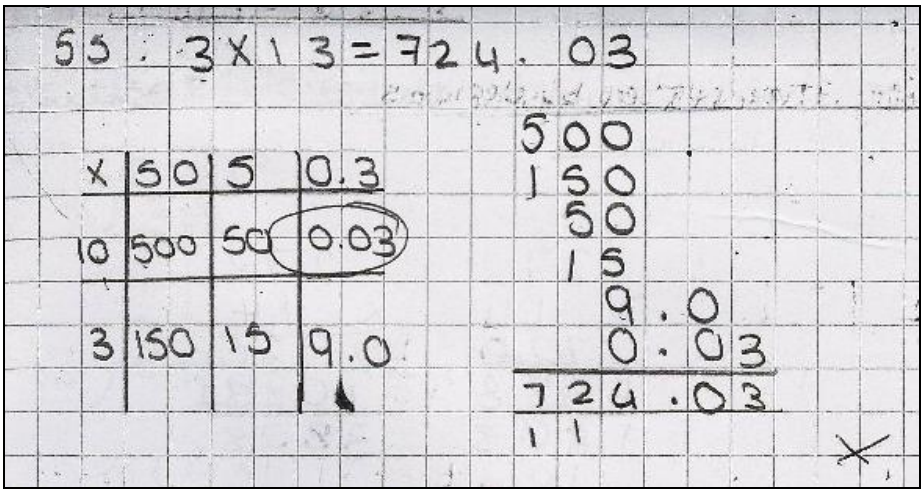
x	200	40	8	Row sum
30	6000	1200	240	7440
4	800	160	32	992
			total	**8432**

The sums of the three products in each row match the figures in the long multiplication method:

$$\begin{array}{rr} & 2\;\;4\;\;8 \\ \times & \underline{\;\;\;\;3\;\;4} \\ & 7\;{}_{1}4\;{}_{2}4\;\;0 \\ + & \underline{9\;{}_{1}9\;{}_{3}2} \\ & \underline{8\;\;4\;\;3\;\;2} \\ & {\scriptstyle 1\;\;1}\;\;\;\;\;\; \end{array}$$

Activity 4

This example from a recent mathematics inspection shows a pupil’s difficulty using the grid method for decimals. The product being calculated was 55.3 x 13



The pupil’s understanding of decimal place value was not secure, as shown by the product of 10 and 0.3 being given as 0.03, which the teacher has ringed.

- What else do you notice about this calculation?

Before moving on, refer to the answer booklet for the comments on Activity 4.

3. Division

Short division

Short division is often called the 'bus-stop' method:

e.g. $$3\overline{)2^{2}9^{2}1}^{\;\;97}$$

$$\begin{array}{r} \underline{\;\;9\;7} \\ 3)2^{2}9^{2}1 \end{array}$$

The method involves working from the left, so that the division is broken down into a sequence of simpler divisions:

- 2 ÷ 3 = 0 remainder 2 (the remainder 2 is carried into the next column)
- 29 ÷ 3 = 9 remainder 2 (the remainder 2 is carried into the next column)
- 21 ÷ 3 = 7

Division by chunking

Chunking is based on repeated subtraction or addition. When chunking is carried out efficiently, the first chunk that is subtracted is the largest multiple of 10 times the divisor. In the example below, 16 is the divisor and the biggest multiple of 10 x 16 that can be subtracted is 2 x 10 x 16, which is 320. Subtracting 320, leaves 64. The next chunk is the largest multiple of the divisor, in this case 4 x 16. The multiples used are then summed, which are 20 and 4 in the example below, giving the answer of 24.

$$\begin{array}{rrl} & \underline{\;\;24} & \\ 16\,) & 384 & \\ - & \underline{320} & (20\text{x}16) \\ & 64 & \\ - & \underline{64} & (4\text{x}16) \\ & 0 & \end{array}$$

More usually, pupils subtract simpler multiples repeatedly, say 160 then 32 or 16. They often make subtraction errors too.

$$\begin{array}{rrl} & \underline{\;\;24} & \\ 16\,) & 384 & \\ - & \underline{160} & (10\text{x}16) \\ & 216 & \\ - & \underline{160} & (10\text{x}16) \\ & 64 & \\ - & \underline{32} & (2\text{x}16) \\ & 32 & \\ - & \underline{32} & (2\text{x}16) \\ & 0 & \end{array}$$

$$384 \div 16 = 10 + 10 + 2 + 2 = 24$$

Long division

While efficient chunking looks not dissimilar to long division, chunking is conceptually different to short and long division in that it works on the complete number being divided. Thus chunking does not link directly to the algorithms for short or long division, as the comparison below shows.

Short division	*Chunking*	*Long division*	*(thinking - not generally recorded)*
2 4	24	24	
16) 3 8^{6}4	16) 384	16) 384	*(16 into 38 goes 2; 2x16 = 32)*
	– 320 (20x16)	– 32↓	*(38 – 32 gives remainder 6)*
(thinking: 16	64	64	*(bring the 4 down)*
into 38 goes 2 r 6;	– 64 (4x16)	– 64	*(16 into 64 goes 4; 4x16=64)*
16 into 64 goes 4)	0	0	*(no remainder)*

In most schools, pupils are taught short division and chunking, with only the more able being taught the long-division algorithm. Consequently, many pupils use chunking when the divisor is a two-digit number.

Note that chunking does not extend easily to division of decimals.

Key findings from Good practice in primary mathematics

Few of the schools were completely satisfied with the methods they were teaching for long division.

Those that taught chunking for long division acknowledged that some pupils have difficulty spotting large multiples and that errors creep in with the repeated subtractions.

Some schools use short division accompanied by jottings of multiples for dividing by two-digit numbers, so do not teach long division:

2 4	*1* x *16 = 16*
16) 3 8^{6}4	*2* x *16 = 32*
	3 x *16 = 48*
	4 x *16 = 64* …

Key findings from *Good practice in primary mathematics*

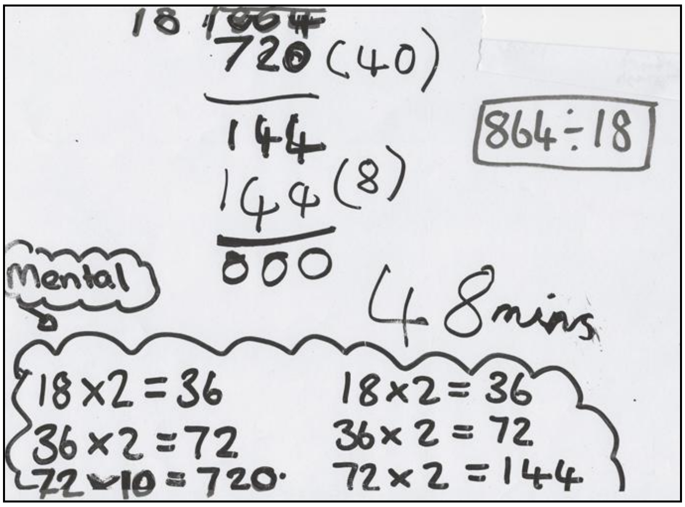
Pupils often chose traditional algorithms over other methods. This occurred in lessons and in meetings with inspectors. When encouraged, most pupils showed flexibility in their thinking and approaches, enabling them to solve a variety of problems as well as calculate accurately.

Activity 5

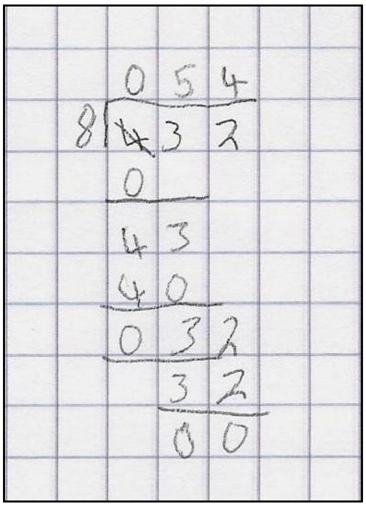
Look at the following examples of divisions calculated correctly by three pupils.

- What methods have they used?
- How efficient are their methods?

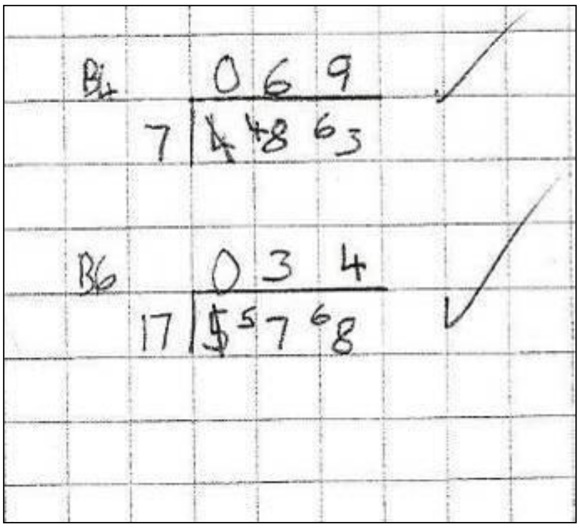
Pupil D's work

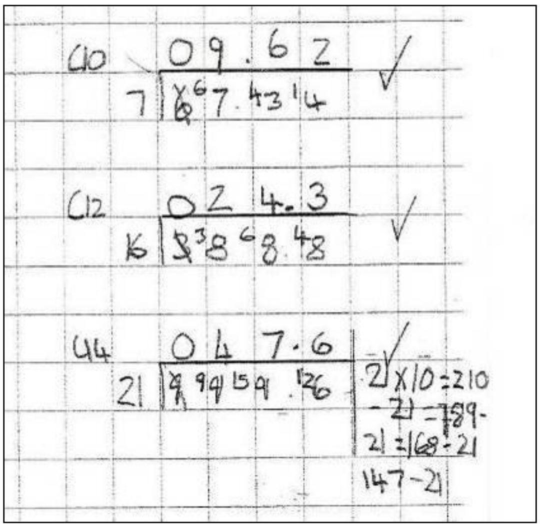


Pupil E's work



Pupil F's work



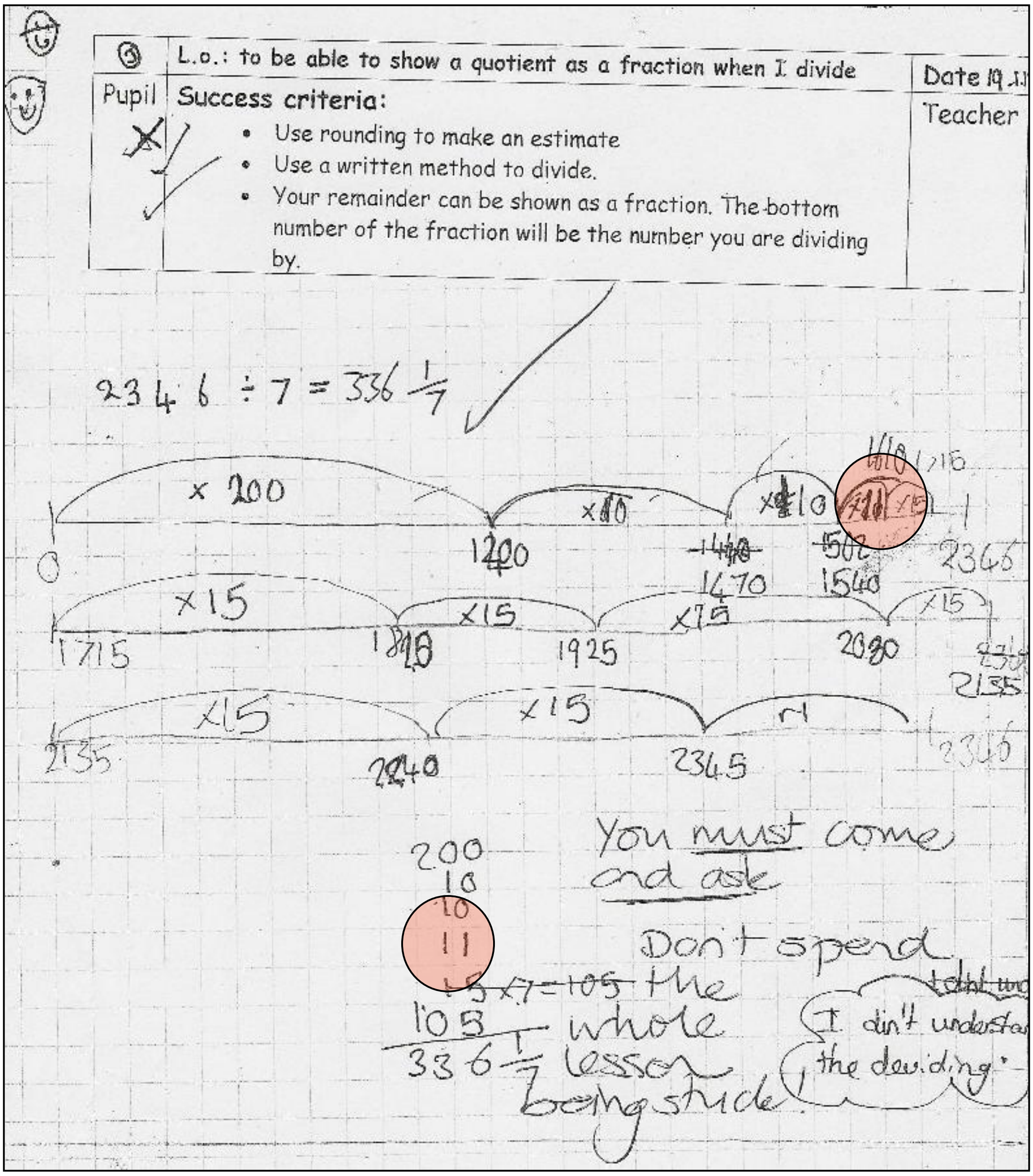


Before moving on, refer to the answer booklet for the comments on Activity 5.

Activity 6

Look at the example below, which is from a recent mathematics inspection. It shows use of chunking by a high-attaining Year 6 pupil. The division being attempted was 2346 ÷ 7. The pupil's answer is incorrect: the minor slip that led to this is highlighted.

- How efficient is the use of chunking in this example?
- Calculate the correct answer by short division. Compare its efficiency with the chunking method used by the pupil.



Before moving on, refer to the answer booklet for the comments on Activity 6.

Key findings from *Good practice in primary mathematics*

Many of the most successful schools have reduced the use of 'expanded methods' and 'chunking' in moving towards efficient methods. This is because they find that too many steps in methods confuse pupils, especially the lower attaining pupils.

Several of the schools do not teach the traditional long-division algorithm by the end of Year 6 (age 11) and most of those that do say that a large proportion of pupils do not become fluent in it.

Clear, coherent calculation policies and guidance, tailored to the particular school's context, ensure consistent approaches. They also ensure that teachers use visual images and models that secure progression in pupils' skills and knowledge lesson by lesson and year by year.

The schools recognise the importance of good subject knowledge and subject-specific teaching skills and seek to enhance these aspects of subject expertise.

Building progression and understanding

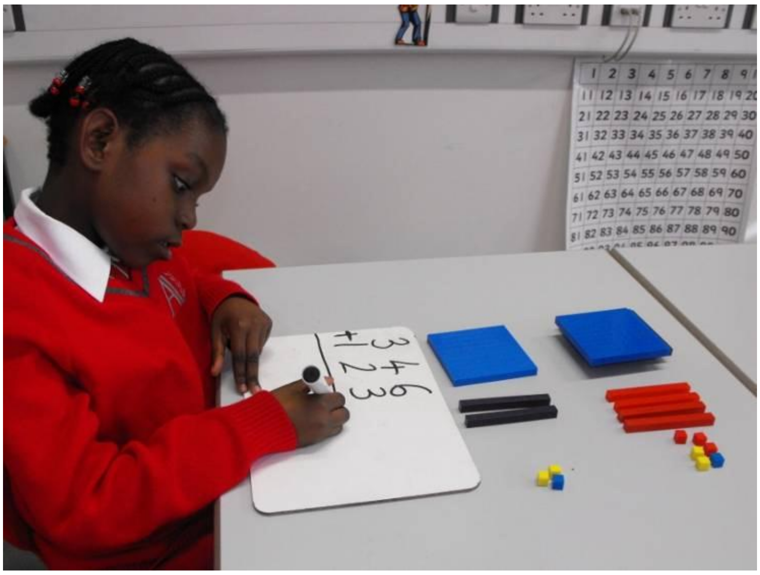
The successful schools followed a similar pattern for developing skills in the four operations (+, −, x, ÷)

- secure pre-requisite knowledge of the number system
- calculate in practical contexts using hands-on resources and link these to recording
- mental methods supported by jottings and visual images
- written forms of recording, moving to efficient methods over time.

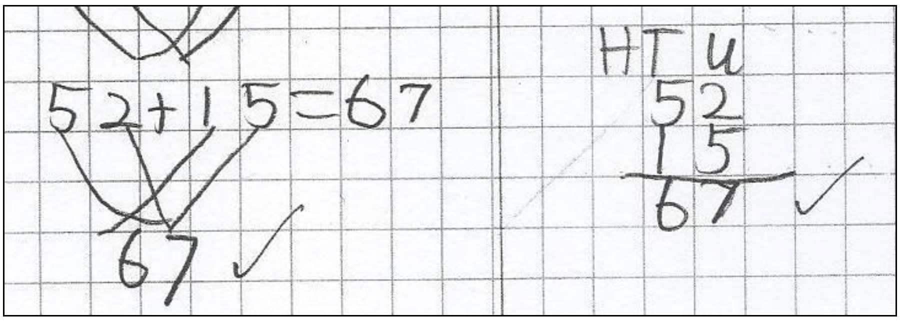
The schools are quick to recognise and intervene in a focused way when pupils encounter difficulties, ensuring that misconceptions do not impede the next steps in learning.

Linking mental, practical and written methods

The photo shows a pupil using practical equipment to support column addition. This base-10 equipment enables pupils to count the units, the tens and the hundreds, and supports the recording of the calculation. It also supports the concept of exchange – for instance exchanging 10 cubes for a stick of 10, and vice versa, in addition and in subtraction.



The extract below of a pupil's work shows the link being made between partitioning and column addition. Year 3 pupils in this lesson were encouraged to see and discuss the connection.



Key findings from *Good practice in primary mathematics*

High-quality teaching secures pupils' understanding of structure and relationships in number, and secures progress in developing increasingly sophisticated mental and written methods.

Subtraction is generally introduced alongside its inverse operation, addition, and division alongside its inverse, multiplication.

The schools are quick to recognise and intervene in a focused way when pupils encounter difficulties. This ensures misconceptions do not impede the next steps in learning.

Using and applying mathematics: problem solving

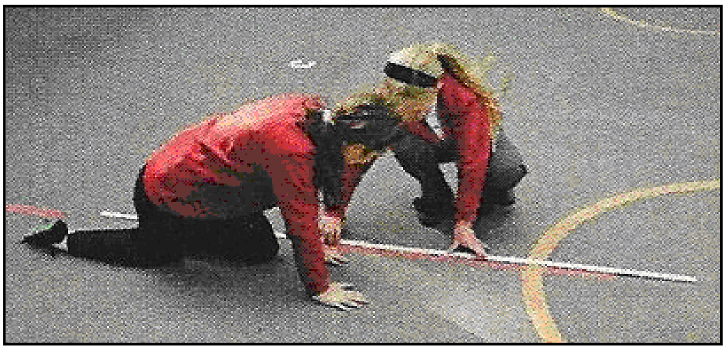
Key findings from *Good practice in primary mathematics*

Pupils' confidence, fluency and versatility are nurtured through a strong emphasis on problem solving as an integral part of learning within each topic (and across topics).

Skills in calculation are strengthened through solving a wide range of problems, exploiting links with work on measures and data handling, and meaningful application to cross-curricular themes and work in other subjects.

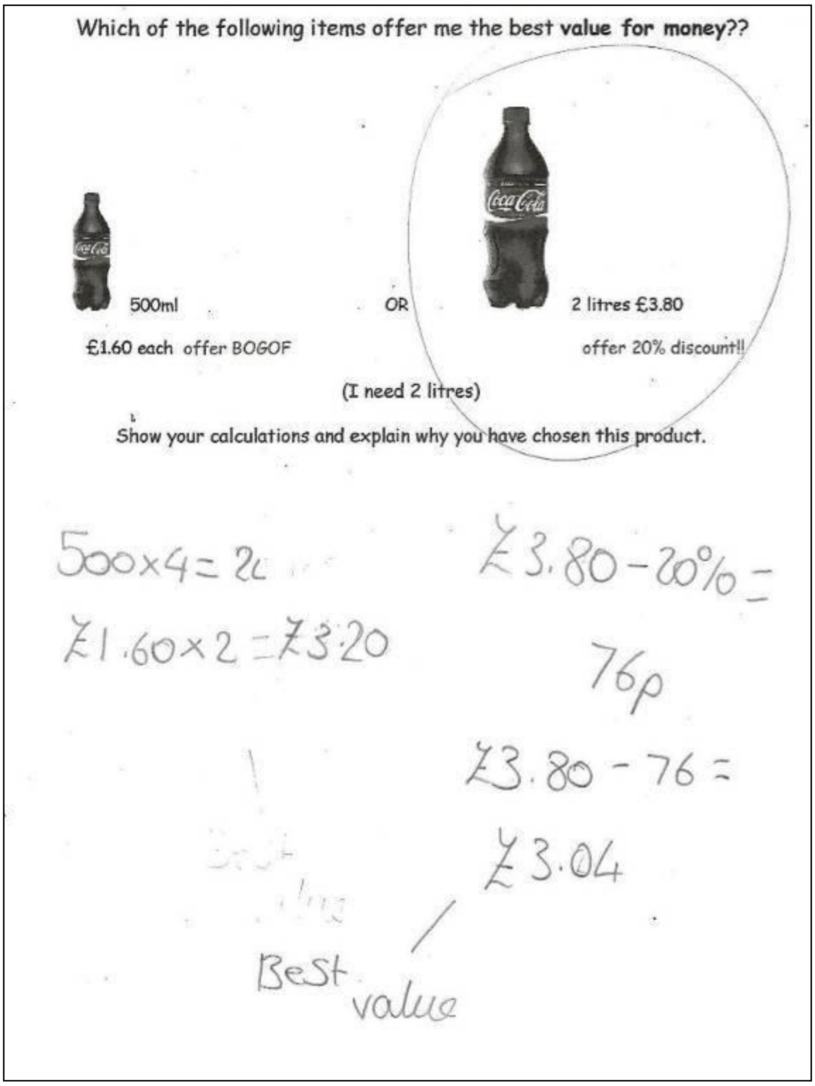
The following two examples illustrate problems set in real-life contexts. The first is a substantial multi-step activity.

The photographs show pupils taking measurements in order to prepare an estimate for the cost of new tarmac for the playground which is an irregular shape.



This activity is a good example of a meaningful real-life problem that enables pupils to use and apply their mathematical knowledge.

The second problem (below) involves proportional reasoning. It was one of a set of short problems, each of which required a different approach. This set of problems made pupils think hard about how to solve them because they did not follow a predictable format.



Problem solving does not have to be rooted in real-life contexts or involve practical work. Problems can also be abstract or set in purely mathematical contexts.

More generally, as reported in *Mathematics: made to measure*, pupils have too few opportunities to solve a wide range of problems across the mathematics curriculum.

Mathematics: made to measure

Mathematics: made to measure was published in May 2012. It was based on mathematics inspections carried out between 2008 and 2011. In his foreword to the report, HMCI stated that Ofsted would:

- produce support materials to help schools identify and remedy weaknesses in mathematics
- raise ambition for the mathematics education of all pupils by placing greater emphasis in school inspection on:
  - how effectively schools tackle inconsistency in the quality of mathematics teaching
  - how well teaching fosters understanding
  - pupils' skills in solving problems
  - challenging extensive use of early and repeated entry to GCSE examinations.

The starting point for the report was its predecessor, *Mathematics: understanding the score*, which was published in 2008. It reported that:

- although standards were rising at GCSE, they had stalled in the primary key stages
- around half of teaching was no better than satisfactory, with too much 'teaching to the test' which explained how to carry out methods but not why, so did not put enough emphasis on understanding.

Part B of *Mathematics: understanding the score* provided support for the development of teachers' subject knowledge and pedagogy with examples and descriptors of what good teaching looks like.

The key message from *Mathematics: made to measure*

The responsibility of mathematics education is to enable all pupils to develop:

- conceptual understanding of the mathematics they learn, its structures and relationships
- fluent recall of mathematical knowledge and skills

in order to equip them to solve familiar problems as well as tackle creatively the more complex and unfamiliar ones that lie ahead.

That responsibility is not being met for all pupils – in fact, for some pupils in almost every school.

Achievement: the national picture

Attainment – national trends to 2011

- rising at GCSE grades A*-C due to a strong emphasis by schools
- rising at AS/A level, with a huge increase in uptake
- in primary schools:
  - at Key Stage 2, it has stalled at L4+, slight rise at L5+
  - at Key Stage 1, it has stalled at L2+, declining trend at L3+
  - in the EYFS, risen a bit but calculation still weaker

Key concerns:

- high attainers are not challenged enough
- at all ages, there is not enough problem solving and application of mathematics – pupils are not being made to think hard enough for themselves

Progress – 2011 figures

- 82% of Year 6 pupils made the expected 2 levels of progress KS1→KS2; 86% of L2b reached L4 but only 58% of L2c
- 62% of pupils made expected 3 levels of progress KS2→KS4 but:
  - 48% of L4c reached grade C (up from 33% in 2010)
  - only 30% of low attaining pupils made expected progress
  - over 37000 pupils who attained L5 at primary school got no better than grade C at GCSE.

Key concerns:

- reaching the expected level in one key stage does not ensure success at the next, due to the focus on meeting thresholds rather than securing essential foundations for the next stage
- potential high attainers are being lost to post-16 study (few GCSE grades B and C pupils opt for or succeed with AS/A level).

Widening gaps across key stages

The following figures from 2011 are typical of the situation in recent years.

- Overall, 10% of pupils did not reach L2 at age 7, 20% did not reach L4 at age 11, and nearly 40% did not reach grade C at GCSE.
- FSM pupils did markedly worse than their peers on attainment at expected levels and on progress.

Attainment	**FSM (%)**	**Non-FSM (%)**
Key Stage 1 L2+	81	92
Key Stage 2 L4+	67	83
GCSE grade C+	42	68
Expected progress	**FSM (%)**	**Non-FSM (%)**
Key Stage 1 - 2	75	84
Key Stage 2 - 4	45	67

Gaps for high attainers - 2011

- FSM pupils did markedly worse than their peers on attainment at higher levels
- too few FSM pupils are enabled to progress to AS/A level and into mathematically related careers because they do not attain GCSE grades A*/A

Attainment	**FSM (%)**	**Non-FSM (%)**
Key Stage 1 L3	9	23
Key Stage 2 L5	19	38
GCSE A*/A	6	21

Achievement: quality of learning

A key discriminator in evaluating the quality of learning is the extent to which pupils' understanding is developed (alongside the gains they make in knowledge and skills). The grade descriptors used in the mathematics survey expanded on those in the evaluation schedule. Those for achievement and teaching are on pages 30 and 31 in this document.

One of the key ways for teachers and inspectors to know whether pupils understand is by seeing how they solve familiar and unfamiliar problems in a variety of contexts.
How pupils think gives insight into their understanding as well as their ability to solve a range of problems.

The following extracts relate to understanding and its link with problem solving.

Mathematics grade descriptors: extracts linked to understanding

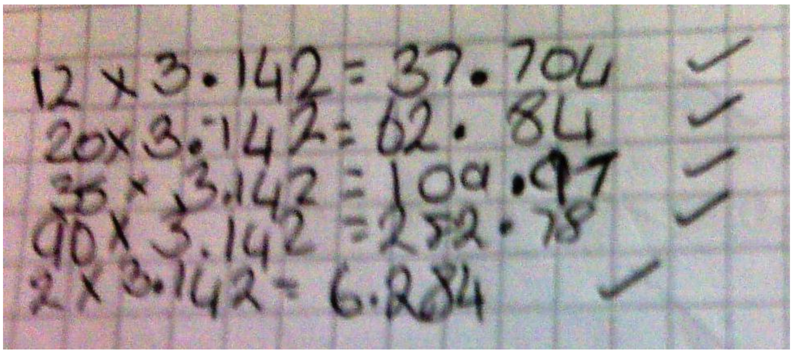
Grade	Description
1	Pupils understand important concepts and make connections within mathematics … They think for themselves …
2	Pupils understand some important concepts and make some connections within mathematics … many show a developing ability to think for themselves …
3	Pupils use techniques correctly, often through emulating the teacher’s methods, but their understanding of the underpinning concepts is insecure … pupils are generally dependent on procedural prompts from examples, resources or staff
4	Pupils’ lack of understanding impedes progress. Although they can carry out taught techniques, their learning over time is fragmented and lacks adequate breadth and depth.

Mathematics grade descriptors: extracts linked to problem solving

Grade	Description
1	They show exceptional independence and take the initiative in solving problems in a wide range of contexts, including the new or unusual. They think for themselves, and are prepared to persevere when faced with challenges.
2	They are able to work independently, and sometimes take the initiative in solving problems in various contexts. Many show a developing ability to think for themselves, and are willing to try when faced with challenges.
3	They are able to solve routine problems set in various contexts. Pupils are generally dependent on procedural prompts from examples, resources or staff and tend to seek help rather than persevere when faced with challenges
4	They have difficulty in solving problems other than the most routine … they give up too readily, or wait for others to provide answers

Activity 7

This is all of a pupil’s recent work on the circumference of circles.



Above it, the pupil’s notes showed a circle with the diameter marked ‘d’ and the formula $C = \pi \times d$ followed by a worked example
$C = 8 \times 3.142 = 25.136$

- What do you notice about the pupil’s work?
- Which of the mathematics grade descriptors for understanding and problem solving might this evidence be consistent with?
- What questions might you ask this pupil to probe her/his understanding and problem-solving skills?

Before moving on, refer to the answer booklet for the comments on Activity 7.

Activity 8

In probing this further, inspectors might ask the pupil:

- How did you learn about this formula?
- Did the teacher explain why it works?
- Did you do any work on this apart from the questions you answered in your book?

Below are some typical responses from pupils p, q, r, s and t.

- What supplementary evidence do they give you?
- How could they affect the match with the grade descriptors?

p. We worked in groups measuring different-sized circles to find if there was any link between the circumference and diameter. At first we thought C would be 10 more than d, but we found out that C was a bit more than three times d.

q. We wrote down our measurements for 10 different circles and we worked out C ÷ d like the teacher told us to. My numbers went from 2.4 to 3.9 but the teacher told us the right answer was 3.142. I'm not sure why there's only one right answer.

r. The formula was at the top of the worksheet. I don't know where it came from.

s. The teacher helped me to see why the circumference is definitely more than twice the diameter but less than four times the diameter, and is always a bit over three times it.

t. The teacher just told us the formula. He is really helpful; he always sets out the steps so we can follow them.

Before moving on, refer to the answer booklet for the comments on Activity 8.

Teaching

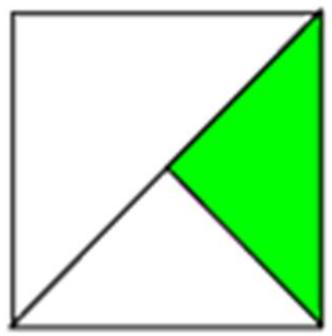
The best teaching develops conceptual understanding alongside pupils' fluent recall of knowledge and confidence in problem solving.

Too much teaching concentrates on the acquisition of disparate skills that enable pupils to pass tests and examinations but do not equip them for the next stage of education, work and life.

Activity 9 – Understanding fractions

Answer the following questions, which were put to primary pupils in recent mathematics survey visits.

- What fraction is shaded in the diagram?
- What does ¼ mean?
- Tell me a fraction that is bigger than ¼
- How do you work out one quarter of something?
- Can you work it out another way?



Before moving on, refer to the answer booklet for the comments on Activity 9.

The quality of teaching: pluses, minuses and inequalities

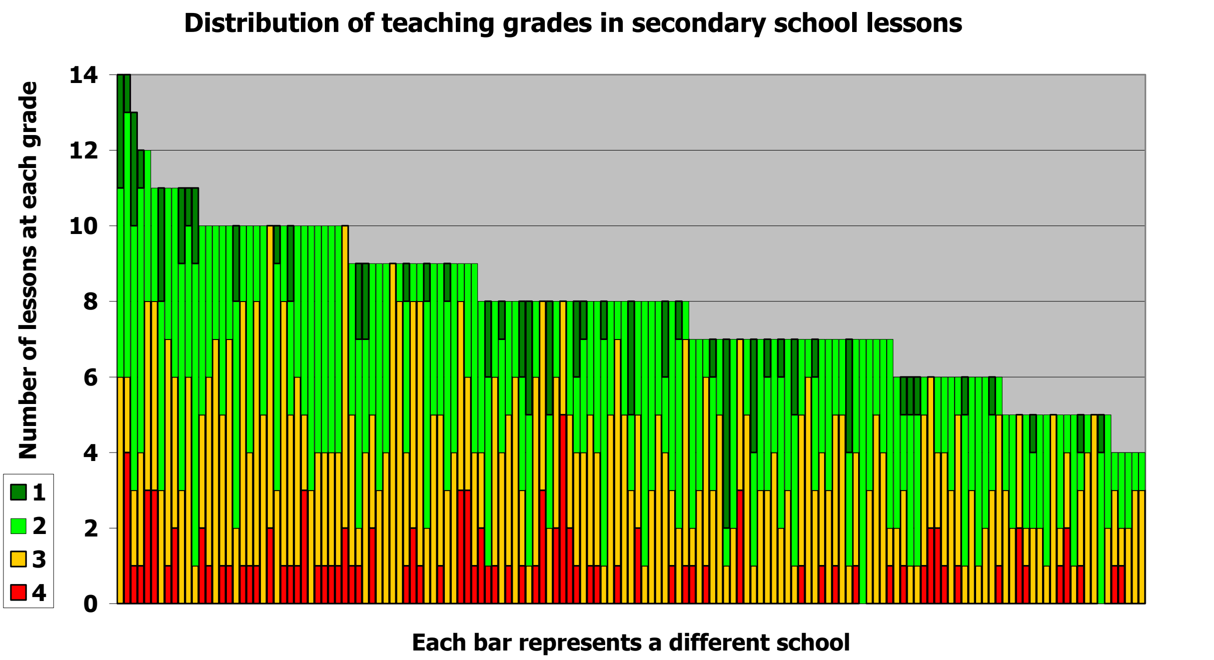
The mathematics survey found wide variations in the quality of teaching between key stages and for different sets within a key stage. The latter was most evident in Key Stage 4, where high sets received twice as much good teaching as low sets (for whom 14% of lessons were inadequate).

The weakest teaching was in Key Stage 3, with 38% good or better and 12% inadequate. The strongest teaching was in the Early Years Foundation Stage and in Years 5 and 6, with around three quarters good or outstanding. Teaching was weaker in Key Stage 1 than Key Stage 2.

Key concern:

- Wide in-school variation causes uneven progress and gaps in achievement, even in good and outstanding schools.

The graph below shows the in-school variation across 151 secondary schools visited for the mathematics survey:



- half of the schools had at least one lesson with inadequate teaching
- only two of the schools had consistently good or better teaching.

Activity 10 – Teaching inequalities Recall HMCI's commitment to a greater emphasis in section 5 inspections on how effectively schools tackle inconsistency in the quality of mathematics teaching. ■ How should inspectors obtain evidence of the extent of inequality in mathematics teaching?
Before moving on, refer to the answer booklet for the comments on Activity 10.

Findings on inequalities in teaching

Underlying factors or causes of inequalities in teaching include:

- deployment of stronger or more experienced staff to key test or examination classes, and less experienced/skilful staff to other classes
- absence of guidance for staff on approaches and resources that support conceptual understanding and progression lesson to lesson and over time.

The effectiveness of actions to improve the quality of teaching is impeded by:

- monitoring that omits some staff (e.g. non-specialists) or concentrates on English, say, in primary
- a focus on generic features during lesson observations and a lack of attention to mathematical details.

Schools rarely articulated their rationale for the deployment of teachers beyond an emphasis on maximising the performance of those pupils closest to external examinations and recognition that non-specialist teachers were unlikely to have the subject knowledge to teach higher-attaining sets.

Senior and subject leaders appeared to realise, perhaps too acceptingly, that ground would need to be made up in the future if the pace of learning of younger and lower-attaining pupils was affected by weaker teaching. This is one reason why it is so important that good-quality guidance and schemes of work are available to support teachers.

Curriculum findings

Key differences and inequalities extend beyond the teaching: they are rooted in the curriculum and the ways in which schools promote or hamper progression in the learning of mathematics. The extent to which schools focus on problem solving and understanding is a key discriminator between good and satisfactory provision.

Planning in primary schools is usually based on National Strategy materials. It is often detailed, but loses the big picture to support progression in strands of mathematics (see *Good practice in primary mathematics*).

In secondary schools, planning is frequently based on examination specifications at Key Stages 4 and 5 and on a variety of sources at Key Stage 3

Key concern:

- In many schools, a lack of guidance for teachers on what approaches and activities best support the development of conceptual understanding and progression over time means teachers, including the less experienced, non-specialists and temporary teachers, are often left to their own devices.

Teaching guidance for primary teachers

Primary schools often have calculation policies but many are not adapted from standard policies provided by their local authorities. The best show which interim methods, images and equipment to use, and are shared with all staff so that all:

- understand progression and not just what their year group needs to learn
- know how each concept is to be introduced.

However, schools rarely have similar policies for other areas of the mathematics curriculum.

Teaching guidance for secondary teachers

Secondary schemes of work are often based on examination specifications at Key Stages 4 and in the sixth form. They are not often adapted to the particular school or cohort. Key Stage 3 schemes vary widely.

The best secondary schemes of work:

- capture progression and indicate how far each topic should be developed for particular groups/sets
- include guidance on approaches and activities that support conceptual understanding, including ICT
- ensure that all pupils have diverse opportunities for using and applying mathematics, problem solving and mathematical reasoning.

Assessment data and intervention

The focus and timeliness of intervention in primary schools has improved – the best pick up on misconceptions, difficulties and gaps and intervene quickly to overcome them, so pupils don't fall behind.

Intervention in secondary schools tends to concentrate on practising topics for the next examination unit.

Key concern:

- Despite increasingly sophisticated tracking and analyses that identify pupils who are (in danger of) underachieving and topics/gaps where difficulties arise, schools rarely use such assessment information to improve quality-first teaching, accompanied by training and guidance for staff.

Early and repeated entry at GCSE and AS/A level

Many schools start teaching to the GCSE specification (and entering pupils for GCSE unit examinations) in Year 9. As a result, much teaching focuses on the next unit examination with lots of practice of exam-style questions. This approach is based on improving pupils' recall of the disparate facts and methods they have been taught.

Early entry is often at Foundation Tier in the summer of Year 10 or winter of Year 11. Re-sits are used to gain a better grade for pupils who are below grade C. Some, schools re-enter pupils who obtain a C grade at higher tier, but do not leave enough time for demanding topics such as algebra, geometry, trigonometry and graphs – the building blocks for AS/A level.

The most effective model, evident in good practice case studies is for GCSE to be taken at the end of Year 11, after a thorough coverage of the specification. For the most able, the best pattern is for any additional qualifications, such as Additional Mathematics, to be taken alongside GCSE in Year 11. Pupils are taught beyond the GCSE specification but not 'accelerated' onto AS courses.

Key concern:

- Early entry limits achievement and drives short-termism in teaching.

Leadership and management findings

Leadership and management were slightly weaker in primary schools in the survey published in 2012 than in the previous survey. However, management practices in secondary schools had strengthened:

- closer monitoring of teaching
- better use of data for intervention and other strategies
- increased use of performance management to drive higher exam results

Subject leaders in secondary schools tended to be among their school's most effective mathematics teachers. The picture was more varied in primary schools.
In all schools, it was important for teachers to work together, especially on guidance (e.g. primary staff on calculation policies - see good practice report).

Key concern:

- Leadership is not identifying and tackling the mathematical implications of weaker teaching and impact of curricular decisions.

All the key concerns identified above, and how they might be tackled, come back to leadership and management. However, monitoring tends to focus on generic characteristics rather than on pinpointing subject-specific weaknesses or inconsistencies.

Key concern:

- Strategic leadership, informed by mathematical insight, is lacking in too many schools.

Activity 11

Read the recommendations for schools in *Mathematics: made to measure* (page 33 of this document).

Select a recommendation.
Think about what weaknesses led to it.

- What inspection activities might enable you to identify and probe such weaknesses?

No answers are provided for this activity.

You have now completed the distance learning. We hope that you will find it helpful in pinpointing weaknesses in mathematics in the schools you inspect and in identifying recommendations for improvement.

Appendix

Key documents and links

Mathematics: made to measure (110159), Ofsted, May 2012; www.ofsted.gov.uk/resources/110159

Mathematics: understanding the score (070063), Ofsted, Sept 2008; www.ofsted.gov.uk/resources/070063

Good practice in primary mathematics: evidence from 20 successful schools (110140), Ofsted, Nov 2011; www.ofsted.gov.uk/resources/110140

Subject grade criteria; www.ofsted.gov.uk/resources/20100015

Ofsted's mathematics web page; www.ofsted.gov.uk/inspection-reports/our-expert-knowledge/mathematics

Early entry to GCSE examinations, Department for Education, 2011; www.education.gov.uk/publications/RSG/AllRsgPublications/Page7/DFE-RR208.

Guidance paper: calculation, The National Strategies, 2011; www.teachfind.com/national-strategies/guidance-paper-calculation

Draft National Curriculum programmes of study, DfE, 2012; www.education.gov.uk/schools/teachingandlearning/curriculum/nationalcurriculum/a00210036/sosletter

Grade descriptors: achievement of pupils in mathematics (Jan 2012 version)

	Generic	Supplementary subject-specific
1	**Outstanding** Almost all pupils, including, where applicable, disabled pupils and those with special educational needs, are making rapid and sustained progress in the subject over time given their starting points. They learn exceptionally well and as a result acquire knowledge quickly and in depth, including in the sixth form and areas of learning in the Early Years Foundation Stage. They develop and apply a wide range of skills to great effect, including reading, writing, communication and mathematical skills that will ensure they are exceptionally well prepared for the next stage in their education, training or employment. The standards of attainment of almost all groups of pupils are likely to be at least in line with national averages for all pupils with many above average. In exceptional circumstances, where standards of attainment of any group of pupils are below those of all pupils nationally, the gap is closing dramatically over a period of time.	Pupils understand important concepts and are able to make connections within mathematics. They develop a broad range of skills in using and applying mathematics. They show exceptional independence and take the initiative in solving problems in a wide range of contexts, including the new or unusual. They think for themselves, and are prepared to persevere when faced with challenges, showing a confidence that they will succeed. They embrace the value of learning from mistakes and false starts. When investigating mathematically, they reason, generalise and make sense of solutions. Pupils show high levels of fluency in performing written and mental calculations and mathematical techniques. They use mathematical language and symbols accurately in their work and in discussing their ideas with others. They develop a sense of passion and commitment to the subject.
2	**Good** Pupils are making better progress than all pupils nationally in the subject given their starting points. Groups of pupils, including disabled pupils and those with special educational needs, are also making better progress than similar groups of pupils nationally. Pupils acquire knowledge quickly and are secure in their understanding of the subject. They develop and apply a range of skills well, including reading, writing, communication and mathematical skills that will ensure they are well prepared for the next stage in their education, training or employment. The standards of attainment of the large majority of groups of pupils are likely to be at least in line with national averages for all pupils. Where standards of any group of pupils are below those of all pupils nationally, the gaps are closing. In exceptional circumstances, where attainment is low overall, it is improving at a faster rate than nationally over a sustained period.	Pupils understand some important concepts and can make some connections within mathematics. They develop a range of skills in using and applying mathematics. They are able to work independently, and sometimes take the initiative in solving problems in various contexts. Many show a developing ability to think for themselves, and are willing to try when faced with challenges. They are willing to learn from mistakes and false starts. When investigating mathematically, most are able to reason, generalise, and make sense of solutions. Pupils are generally fluent in performing written and mental calculations and mathematical techniques. Their use of mathematical language and symbols is mostly accurate when presenting their work and in discussing their ideas with others. They enjoy the subject and can explain its value.
3	**Satisfactory** Pupils are progressing at least as well in the subject as all pupils nationally given their starting points. Groups of pupils, including disabled pupils and those with special educational needs, are also making progress in line with similar groups of pupils nationally. Pupils generally learn well in the subject, with no major weaknesses. They acquire the knowledge, understanding and skills, including those in reading, writing, communication and mathematics that will ensure they are prepared adequately for the next stage in their education, training or employment. The standards of attainment of the majority of groups of pupils are likely to be in line with national averages for all pupils. Where standards of groups of pupils are below those of all pupils nationally, the gaps are closing overall. In exceptional circumstances, where attainment is low overall, it is improving over a sustained period.	Pupils use techniques correctly, often through emulating the teacher's methods, but their understanding of the underpinning concepts is insecure. Pupils develop some skills in using and applying mathematics. They are able to solve routine problems set in various contexts. Pupils are generally dependent on procedural prompts from examples, resources or staff and tend to seek help rather than persevere when faced with challenges. Many lack confidence and like to avoid making mistakes. When investigating mathematically, they can sometimes reason and make simple generalisations. Pupils are reasonably accurate in performing written and mental calculations and mathematical techniques, though sometimes slowed by hazy recall of number facts or over reliance on calculators. They often use mathematical language and symbols imprecisely. Most are ambivalent about the subject but recognise its value.
4	**Inadequate** Achievement in the subject is likely to be inadequate if any of the following apply. ■ Pupils' learning and progress, or the learning and progress of particular groups, is consistently below those of all pupils nationally given their starting point. ■ Learning and progress in any key stage, including the sixth form, lead to underachievement. ■ The learning, quality of work and progress of disabled pupils and those with special educational needs show that this group is underachieving. ■ Pupils' communication skills, including in reading and writing and proficiency in mathematics overall, or those of particular groups, are not sufficient for the next stage of education or training. ■ Attainment is consistently low showing little, fragile or inconsistent improvement, or is in decline. ■ There are wide gaps in attainment and in learning and progress between different groups of pupils and of all pupils nationally that are showing little sign of closing or are widening.	Pupils' lack of understanding impedes progress. Although they can carry out taught techniques, their learning over time is fragmented and lacks adequate breadth and depth. Pupils develop insufficient skills in using and applying mathematics. They have difficulty in solving problems other than the most routine. The accuracy of their mental and written work is affected by weak knowledge of number facts and incorrect use of mathematical techniques. They give up too readily, or wait for others to provide answers. Their lack interest in the subject is reflected in the low quality and limited quantity of their work.

Grade descriptors: the quality of teaching in mathematics (Jan 2012 version)

	Generic	Supplementary subject-specific
1	**Outstanding** Much of the teaching in the subject is outstanding and never less than consistently good. As a result, almost all pupils are making rapid and sustained progress. All teachers have consistently high expectations of all pupils. Drawing on excellent subject knowledge, teachers plan astutely and set challenging tasks based on systematic, accurate assessment of pupils' prior skills, knowledge and understanding. They use well judged and often imaginative teaching strategies that, together with sharply focused and timely support and intervention, match individual needs accurately. Consequently, pupils learn exceptionally well. Teaching promotes pupils' high levels of resilience, confidence and independence when they tackle challenging activities. Teachers systematically and effectively check pupils' understanding throughout lessons, anticipating where they may need to intervene and doing so with notable impact on the quality of learning. Time is used very well and every opportunity is taken to successfully develop crucial skills, including being able to use their literacy and numeracy skills. Appropriate and regular homework contributes very well to pupils' learning. Marking and constructive feedback from teachers and pupils are frequent and of a consistently high quality, leading to high levels of engagement and interest.	Teaching is rooted in the development of all pupils' conceptual understanding of important concepts and progression within the lesson and over time. It enables pupils to make connections between topics and see the 'big picture'. Teaching nurtures mathematical independence, allows time for thinking, and encourages discussion. Problem solving, discussion and investigation are seen as integral to learning mathematics. Constant assessment of each pupil's understanding through questioning, listening and observing enables fine tuning of teaching. Barriers to learning and potential misconceptions are anticipated and overcome, with errors providing fruitful points for discussion. Teachers communicate high expectations, enthusiasm and passion about their subject to pupils. They have a high level of confidence and expertise both in terms of their specialist knowledge and their understanding of effective learning in the subject. As a result, they use a very wide range of teaching strategies to stimulate all pupils' active participation in their learning together with innovative and imaginative resources, including practical activities and, where appropriate, the outdoor environment. Teachers exploit links between mathematics and other subjects and with mathematics beyond the classroom. Marking distinguishes well between simple errors and misunderstanding and tailors insightful feedback accordingly.
2	**Good:** As a result of teaching that is mainly good, with examples of outstanding teaching, most pupils and groups of pupils, including disabled pupils and those who have special educational needs, are achieving well in the subject over time. Teachers have high expectations of all pupils. Teachers use their well developed subject knowledge and their accurate assessment of pupils' prior skills, knowledge and understanding to plan effectively and set challenging tasks. They use effective teaching strategies that, together with appropriately targeted support and intervention, match most pupils' individual needs so that pupils learn well. Teaching generally promotes pupils' resilience, confidence and independence when tackling challenging activities. Teachers regularly listen astutely to, carefully observe and skilfully question groups of pupils and individuals during lessons in order to reshape tasks and explanations to improve learning. Teaching consistently deepens pupils' knowledge and understanding and teaches them a range of skills including literacy and numeracy skills. Appropriate and regular homework contributes well to pupils' learning. Teachers assess pupils' progress regularly and accurately and discuss assessments with them so that pupils know how well they have done and what they need to do to improve.	Teaching develops pupils' understanding of important concepts as well as their proficiency in techniques and recall of knowledge, equipping pupils to work independently. It helps pupils to see that topics are connected and form a 'big picture'. Many opportunities are provided for problem solving in various contexts, discussion and investigation, although these are not always integral to learning. Teachers focus on pupils' understanding when questioning, listening and observing. Barriers to learning and misconceptions are tackled well. Teachers have a clear understanding of the value of their subject which they communicate effectively to pupils, often with enthusiasm. They have a good level of specialist expertise which they use well in planning and teaching their subject. As a result, they use an appropriate range of resources and teaching strategies, including practical activities and, where appropriate, the outdoor environment. They make some links between mathematics and other subjects and with mathematics beyond the classroom. Marking identifies errors and misunderstanding and helps pupils to overcome difficulties.
3	**Satisfactory** Teaching results in most pupils, and groups of pupils, currently in the school making progress in the subject broadly in line with that made by pupils nationally with similar starting points. There is likely to be some good teaching and there are no endemic inadequacies across year groups or for particular groups of pupils. Teachers' expectations enable most pupils to work hard and achieve satisfactorily and encourage them to make progress. Due attention is often given to the careful assessment of pupils' learning but this is not always conducted rigorously enough and may result in some unnecessary repetition of work for pupils and tasks being planned and set that do not fully challenge. Teachers monitor pupils' work during lessons, picking up any general misconceptions and adjust their plans accordingly to support learning. These adaptations are usually successful but occasionally are not timely or relevant and this slows learning for some pupils. Teaching strategies ensure that the individual needs of pupils are usually met. Teachers carefully deploy any available additional support and set appropriate homework and these contribute reasonably well to the quality of learning for pupils, including disabled pupils and those who have special educational needs. Pupils are informed about the progress they are making and how to improve further through marking and dialogue with adults that is usually timely and encouraging. This approach ensures that most pupils want to work hard and improve.	Teaching focuses primarily on developing pupils' skills in mastering techniques and answering routine questions rather than understanding the underlying concepts. Accurate explanations give a piecemeal approach to learning a topic so that pupils are not helped to see the 'big picture'. Opportunities for problem solving are generally restricted to routine cases or are uneven, for example problems occur at the end of exercises so that not all pupils meet them. Pupils have some opportunities to investigate and discuss. Questioning tends to be closed rather than probing. Some barriers to learning and misconceptions are identified and tackled. Teachers understand the value of their subject which they communicate to pupils. They have adequate subject expertise which they use in their planning and teaching. As a result, they use a range of resources and teaching strategies, though one approach may dominate, for example, exposition by the teacher and practice by the pupils. They occasionally make links between mathematics and other subjects and with mathematics beyond the classroom. Marking is generally accurate and sometimes helps pupils to overcome difficulties.
4	**Inadequate :** Teaching in the subject is likely to be inadequate where any of the following apply. ■ As a result of weak teaching, pupils or groups of pupils currently in the school are making inadequate progress. ■ Teachers do not have sufficiently high expectations and teaching over time fails to excite, enthuse, engage or motivate particular groups of pupils, including those who have special educational needs and/or disabilities. ■ Pupils cannot communicate, read, write or use mathematics as well as they should, as appropriate, in the subject. ■ Learning activities are not sufficiently well matched to the needs of pupils so that they make inadequate progress.	Teachers are not able to engage pupils' interest in the subject and do not monitor their progress adequately. Weaknesses and gaps in the teacher's knowledge of mathematics or how pupils learn the subject hamper lesson planning, the choice of resources, or the quality of teachers' explanations so that, as a result, pupils make too little progress. Teaching focuses on pupils replicating techniques, and presents mathematics as a disparate set of skills and knowledge, resulting in a lack of adequate breadth and depth. Pupils have too few opportunities for problem solving, investigation or discussion. A narrow view of the subject isolates it from other subjects and the outside world. Marking is too irregular, inaccurate or unhelpful to pupils.

Good practice in primary mathematics

Key findings

The following key findings, taken together, reflect the 'what' and 'how' that underpin effective learning through which pupils become fluent in calculating, solving problems and reasoning about number.

- Practical, hands-on experiences of using, comparing and calculating with numbers and quantities and the development of mental methods are of crucial importance in establishing the best mathematical start in the Early Years Foundation Stage and Key Stage 1. The schools visited couple this with plenty of opportunities for developing mathematical language so that pupils learn to express their thinking using the correct vocabulary.
- Understanding of place value, fluency in mental methods, and good recall of number facts such as multiplication tables and number bonds are considered by the schools to be essential precursors for learning traditional vertical algorithms (methods) for addition, subtraction, multiplication and division.
- Subtraction is generally introduced alongside its inverse operation, addition, and division alongside its inverse, multiplication. Pupils' fluency and understanding of this concept of inverse operations are aided by practice in rewriting 'number sentences' like $3 + 5 = 8$ as $8 – 3 = 5$ and $8 – 5 = 3$ and solving 'missing number' questions like $\square – 4 = 5$ by thinking $5 + 4 = 9$ or $9 – 4 = 5$.
- High-quality teaching secures pupils' understanding of structure and relationships in number, for instance place value and the effect of multiplying or dividing by 10, and progress in developing increasingly sophisticated mental and written methods.
- In lessons and in interviews with inspectors, pupils often chose the traditional algorithms over other methods. When encouraged, most showed flexibility in their thinking and approaches, enabling them to solve a variety of problems as well as calculate accurately.
- Pupils' confidence, fluency and versatility are nurtured through a strong emphasis on problem solving as an integral part of learning within each topic. Skills in calculation are strengthened through solving a wide range of problems, exploiting links with work on measures and data handling, and meaningful application to cross-curricular themes and work in other subjects.
- The schools are quick to recognise and intervene in a focused way when pupils encounter difficulties. This ensures misconceptions do not impede the next steps in learning.
- Many of the schools have reduced the use of 'expanded methods' and 'chunking' in moving towards efficient methods because they find that too many steps in methods confuse pupils, especially the less able. Several of the schools do not teach the traditional long division algorithm by the end of Year 6 (age 11) and most of those that do say that a large proportion of pupils do not become fluent in it.
- A feature of strong practice in the maintained schools is their clear, coherent calculation policies and guidance, which are tailored to the particular school's context. They ensure consistent approaches and use of visual images and models that secure progression in pupils' skills and knowledge lesson by lesson and year by year.
- These schools recognise the importance of good subject knowledge and subject-specific teaching skills and seek to enhance these aspects of subject expertise. Some of the schools benefit from senior or subject leaders who have high levels of mathematical expertise. Several schools adopt whole-school approaches to developing the subject expertise of teachers and teaching assistants. This supports effective planning, teaching and intervention. Most of the larger independent preparatory schools provide specialist mathematics teaching from Year 4 or 5 onwards.

Recommendations

The Department for Education should:

- ensure end-of-key-stage assessments, and GCSE and AS/A-level examinations require pupils to solve familiar and unfamiliar problems and demonstrate fluency and accuracy in recalling and using essential knowledge and mathematical methods
- raise ambition for more-able pupils, in particular expecting those pupils who attained Level 5 at Key Stage 2 to gain A* or A grades at GCSE
- promote enhancement of subject knowledge and subject-specific teaching skills in all routes through primary initial teacher education
- research the uptake, retention and success rates in AS and A-level mathematics and further mathematics by pupils attending schools with and without sixth-form provision.

Schools should:

- tackle in-school inconsistency of teaching, making more good or outstanding, so that every pupil receives a good mathematics education
- increase the emphasis on problem solving across the mathematics curriculum
- develop the expertise of staff:
  - in choosing teaching approaches and activities that foster pupils' deeper understanding, including through the use of practical resources, visual images and information and communication technology
  - in checking and probing pupils' understanding during the lesson, and adapting teaching accordingly
  - in understanding the progression in strands of mathematics over time, so that they know the key knowledge and skills that underpin each stage of learning
  - ensuring policies and guidance are backed up by professional development for staff to aid consistency and effective implementation
- sharpen the mathematical focus of monitoring and data analysis by senior and subject leaders and use the information gathered to improve teaching and the curriculum.

In addition, primary schools should:

- refocus attention on:
  - improving pupils' progress from the Early Years Foundation Stage through to Year 2 to increase the attainment of the most able
  - acting early to secure the essential knowledge and skills of the least able.

In addition, secondary schools should:

- ensure examination and curricular policies meet all pupils' best interests, stopping reliance on the use of resit examinations, and securing good depth and breadth of study at the higher tier GCSE.