Teaching Committee report to the Faculty Board

Examinations for Parts IA, IB and II of the Tripos, 2016

1 Introduction

The Committee met three times, for two hours each, to consider the 2016 undergraduate Tripos examinations. We looked, for each part of the Tripos, at:

- the examiners' report;
- the external examiners' reports;
- the examiners' comments on their questions;
- the examination statistics;
- the examination papers;
- the analysis of the paper lecture questionnaires;
- the responses to the on-line questionnaires;
- the report from the CATAM assessors (Parts IB and II).

We noted with pleasure that as usual the external examiners, without exception, commented favourably both on the examination process and also on the performance of the candidates. The six external examiners' reports include comments such as:

The Mathematical Tripos in Cambridge remains one of the most challenging Mathematics courses in the country, and the examination reflects this. (Part IA)

Overall, I am very satisfied with the standards of the examination and of the qualifications. The processes for assessment, examination and the determination of awards are sound and fairly conducted. (Part IB)

Cambridge's system of compendium examination papers is appropriate and commendable, and on the whole worked very well. (Part IB)

The standard of the examination papers remained gratifyingly high. The examinations have been conducted with scrupulous fairness. (Part II)

The overall standards exceedingly high – I saw many excellent answers to some very difficult questions on some very advanced material. In my view the standard is significantly higher than any other British University except, possibly, Oxford. (Part II)

Intellectual standards on Part II of the Mathematical Tripos are very high, comparing favourably both with other mathematics departments in the UK and with other departments in Cambridge. The Mathematical Tripos Part II is a course that is of great intellectual depth and rigour. (Part II)

There follows a summary of the points raised in the examiners' reports which the Committee believe need the attention of the Faculty Board. We have not generally highlighted points of a purely administrative nature: that is for the Chairs of this year's examiners and the Undergraduate Office to pick up rather than the Faculty Board.

2 General Matters

2.1 Errors

This year only two errors were reported, in a total of about 280 questions: no errors in Part IA; no errors in Part IB; two errors in Part II. In addition, two questions in Part IB were the subject of complaints after the examination (see §4.3). Overall, these numbers are significantly smaller than in recent years, and we congratulate the examiners, particularly the three chairs, for their diligent and meticulous checking of the questions.

Factors which may have contributed to the low error count are: (i) circulation to examiners of a letter from the General Board complaining about the number of errors on mathematics papers in the previous year; (ii) introduction of a list of typesetting conventions, which allowed examiners to focus more on the content of questions in their meetings; (iii) a possible reluctance of examiners to announce clarifications if not absolutely necessary in order to satisfy the General Board's concerns about the workload for the Registrary.

We **recommend** continued use of the typesetting conventions. We also **recommend** that all questions are produced using the LaTeX template from now on (rather than TeX), if necessary with assistance from the Undergraduate Office.

2.2 Rubric

The rubric on Part IA examination papers states:

> "*The examination paper is divided into two sections. Each question in Section II carries twice the number of marks of each question in Section I. Candidates may attempt* **all four** *questions from Section I and* **at most five** *questions from Section II. In Section II,* **no more than three** *questions on each course may be attempted.*
>
> **Complete answers are preferred to fragments.**"

Similar wording is used in Parts IB and II, though the exact rules are different.

[redacted] (Part IA external examiner) comments that "*Candidates may attempt at most N questions from Section X*" is potentially misleading if some candidates are aware that the actual policy is for examiners to use the best N answers while other candidates might believe that there is some sort of penalty for violating the rubric.

This traditional wording has been debated many times, the issue being a desire to discourage almost all candidates from wasting time on excess questions that will get no marks, while not penalising the few who really do have time to attempt an $(N+1)$th question as insurance. (Indeed [redacted] comments that some weaker candidates do hand in many fragments, to their disadvantage.) The wording is stated in the Schedules, and the Faculty policy to excess questions is clearly explained. We **recommend** that candidates are explicitly reminded of this policy this year in the email sent out at the start of the Easter Term that describes the rubric and cover sheets. We **recommend** that the sentence "The policy towards excess attempts is described on p.2." is added to the IA section of the Schedules.

Of course, "*Candidates may attempt at most N questions from Section X*" is not strictly true, because candidates may attempt more, and they will be marked, and then the best N marks will be used. It was suggested that "*Candidates may obtain credit on at most N questions from Section X*" would be a truer statement. We also noted that the statements about Section II questions carrying twice as many marks as Section I questions and complete answers being preferred to fragments, while strictly true, are not a very clear representation of the actual system involving alphas and betas. A wording which reflects these concerns is:

> "*The examination paper is divided into two sections. Each question in Section II carries twice the number of marks of each question in Section I. Section II questions also carry an alpha or beta quality mark and Section I questions a beta quality mark.*
>
> *Candidates may obtain credit from attempts on* **all four** *questions from Section I and* **at most five** *questions from Section II. In Section II, candidates may obtain credit from attempts on* **no more than three** *questions on each course.*

Part III has moved to the direct style "Attempt N questions" with no explanation about what happens if one does more! Does Faculty Board have views on how to balance accuracy, brevity, clarity, and direction?

2.3 Short questions

The beta threshold for short questions in 2015/16 was reduced from 8/10 to 7/10, and the target success rate for betas was been raised from 55–60% to 65–70%. The intention was to make short questions more attractive, particularly to weaker students who struggle to complete long questions. The outcome is shown in the following table.

	beta rate (%)			average mark			takeup in II	
	IA	IB	II	IA	IB	II	short	long
2016	59	59	57	6.8	6.8	6.5	17.4	17.1
2015	44	52	51	6.7	7.0	6.8	12.4	15.8
2014	60	51	48	7.2	7.0	6.6	12.9	15.5
2013	59	52	47	7.5	6.9	6.6	12.2	15.3

The success rate did increase, but not by as much as hoped. More encouragingly, in Part II the take-up of short questions increased by 50%, reflecting greater use of C courses.

We again **recommend** that pressure is maintained to keep short questions sufficiently short and easy.

2.4 Administrative and Computer Officer support

The examiners' reports remarked on the excellent support given by [redacted] in particular, and the administrative staff more generally. We **recommend** that the Chair of the Faculty Board expresses the appreciation of the Board for their hard work.

3 Part IA

3.1 Short questions

The examiners' report states incorrectly that their 59% beta rate (on average) on short questions was at the upper end of the Faculty recommended range. In fact, the recommended range is 65–70%, which was almost achieved by Pure, but significantly undershot by Applied.

3.2 Difficulty of questions

The 17% alpha rate for the long questions on Groups was well outside the Faculty recommended range of 40–45%. We ask this year's examiners to pay careful attention to previous years' statistics and the guidelines on question setting in the letter from the Chair of the Faculty Board.

4 Part IB

4.1 Class descriptors

[redacted] (external examiners) make a number of comments about the verbal class descriptors in the Schedules, which they seem to have understood to be the '*other factors besides marks and quality marks that may be taken into account*' and then tried to use them as secondary criteria to make decisions about individual cases near the boundaries. They found phrases such as 'a fair number of correct answers to straightforward and challenging questions' too vague to be useful and, if interpreted as a threshold on alphas and short betas, it was not clear where CATAM fitted in.

We agree that the verbal descriptors are too vague to be used to make detailed decisions about individual cases, and we believe this is inevitable because any class descriptor has to cover the considerable range of achievement within a given class, the variety of possibilities for accruing credit from a mixture of long and short questions and CATAM, and the possible variations in difficulty (and hence of borderlines) from one year to the next. Indeed, it was for this reason that the Faculty adopted the merit mark as the primary, and quantitative, classification criterion.

As the Schedules say, the intention is that *after* applying the classification criteria (merit mark, other factors, approximate percentages) to make detailed decisions, the classes can be characterised by the verbal descriptors. We **recommend** changing the wording to 'broadly characterised' to make this clearer. The Schedules also say that '*stronger performance on the Computational Projects may compensate for weaker performance on the written questions or vice versa*', which applies to all the descriptors.

What then are the 'other factors'? They are secondary factors, *other than* the values of m, α or β, which might tip the balance in marginal cases or which, in exceptional cases, might need to be considered in the interests of fairness. Examples might include whether the alphas are mainly 20s or 15s (or if there are a lot of 14s), whether the layout of the answers shows particularly clear or unclear thinking, or whether the questions attempted were on courses that were generally found difficult or easy (this is particularly relevant for fairness in Part II where candidates take very different selections of courses). It is anticipated that such factors are more likely to affect the location of the borderline than to cause a departure from merit-mark order. Only in exceptional cases should there be a significant departure from merit-mark order.

We are reluctant to add too much prescriptive detail to the Schedules in case this ties the examiners' hands in unusual situations and prevents them from exercising appropriate judgement. On the other hand, we **recommend** that the instructions to examiners in the Chair of Faculty Board's letter be expanded to describe, as above, the nature and role of other factors.

[redacted] also commented that the verbal descriptor for the 3rd class describes a lower level of achievement than would be usual for a 3rd class at other leading universities. We note that questions on Tripos examinations are probably more demanding than those elsewhere, but agree that the description is overly negative. The current wording is

Candidates placed in the third class will have demonstrated some knowledge but little understanding of the examinable material. They will have made reasonable attempts at a small number of questions, but will have lacked the skills to complete many of them.

We **recommend** that it be amended to

Candidates placed in the third class will have demonstrated some knowledge of the examinable material. They will have made reasonable attempts at a small number of questions, but will not have shown the skills needed to complete many of them.

4.2 Number of meetings

By force of circumstances, the IB examiners again only had two meetings to consider the draft questions, though they had intended to add a third meeting as in Parts IA and II. We strongly **recommend** that they do have a third meeting this year to improve the robustness of the setting process against divergences in difficulty and glitches in content.

4.3 Complaints about questions

Shortly following the examination, two complaints were received via the student feedback email about the content of two questions. One question on Metric & Topological Spaces started by asking for a definition of the p-adic metric on Q, though this had not been defined as an example in lectures in 2015. One question on Geometry asked for a definition of cross-ratio, which was not lectured in Geometry, but had been defined in IA Groups (where it is mentioned in the Schedules). Unfortunately, the last part of the question depended on using a different convention from the one used in Groups.

We are satisfied that the examiners took appropriate action to deal generously with those candidates who had got stuck on the last part of the second question by using the wrong convention. We are also satisfied that the questions could have been entirely suitable and on Schedule if either the question or the lecturer had given the relevant definitions. However, we are very sympathetic to the student complaint, and regard this as a failure of the examiner and the lecturer to communicate sufficiently about what students should be expected to know. Of the three undergraduate years, Part IB is the most vulnerable to a gap between what the examiner expects and what students have actually been taught. We **recommend** that the IB examiners take particular care to discuss their questions with lecturers, especially for the Easter Term courses where **both** years of possible lecturing should always be taken into account.

4.4 Syllabi and content of courses

[redacted] (external examiner) comments that the syllabi provided in the Schedules are a bare list of topics and results, and did not give him sufficient indication what students are expected to be able to do. (This comment may well be connected to the complaints about the questions above.)

The Teaching Committee believes that students should be familiar with all the material lectured that is on the Schedules, and with the content of the example sheets, and should be able to use this information to attempt examination questions, which may contain both bookwork and novel problems, of the type found on past papers. Both example sheets and 16 years of past papers are available online for examiners and students to use as a guide, and students should have worked through at least 3 or 4 years worth of past papers. (It is **not** expected that students should be familiar with any of the recommended text books.)

4.5 Availability of paperwork

[redacted] (external examiner) asks if some of the paperwork, such as the statistics on questions, could be made available by email on the evening that the external examiners arrive in Cambridge. The timetable is very tight, and sometimes careful cross-checking between the scripts, the cover sheets and the mark list is still in progress. Moreover, the support staff are very busy processing results for all Parts of the Tripos almost concurrently. Neverthless, we **recommend** that, if possible, the papers be sent electronically to the external examiners before their day of examining scripts.

He also asks about statistics on questions from previous years. These are sent to all examiners in November, and serve an essential purpose at the time of setting the questions. It was not clear to us why they might be useful at the time of classifying the candidates.

4.6 Rubbish sacks

The Part IB examiners recommend reconsideration of the policy of searching the rubbish sacks if a candidate claims a question on the master cover sheets but there is no attempt in the bundled script. They had five such cases, and search of the sacks produced three fragments with matching handwriting all worth zero marks. The Part II examiners had three such cases, and judged that two were clearly mistakes in filling in the cover sheets, while the third candidate was so far from a border that it would make no difference. A fourth candidate in Part II had failed to fill in their cover sheet, but subsequently appealed because their mark breakdown didn't show a question they claimed to have done. A search of the sacks did discover the candidate's answer which was marked 18/20, though this did not change the candidate's class. In recent years, there have been cases where candidates have claimed questions on the cover sheet, and a search of the sacks did produce substantive answers.

While some of us felt that it is the candidates' fault if they don't hand in an answer, the University policy of retaining the sacks suggests that we are expected to be more sympathetic. We **recommend** that examiners should (as in Part II) search the sacks if it is possible that full marks on a missing attempt would change the candidate's class, but otherwise wait to see if the candidate makes an appeal. The examiners should, of course, always check that the missing attempt has not simply been overlooked or put in the wrong bundle.

4.7 CATAM: plagiarism

New guidelines on plagiarism allowing for stricter penalties came into force this year, which were very much welcomed by the Part II external examiners who had been arguing strongly for this for several years. These guidelines were clearly advertised to the students, and it is thus very disappointing that a case came to light in Part IB for which it was appropriate to impose the maximum penalty of removing all marks for CATAM. Plagiarism is a serious offence and an assault on the integrity of the examination, and we **recommend** that examiners impose the maximum penalty appropriate whenever it is discovered.

(We note, as an aside, that the submission rates in Part IB (98%) and Part II (96%) continue to rise.)

5 Part II

5.1 Mark schemes

The examiners' reports contain conflicting comments on the level of detail appropriate for mark schemes. We do not wish to be overly prescriptive and simply state two principles: the draft mark scheme must be sufficiently detailed for the external examiner to judge the level of the draft questions; the examiners are responsible for the final mark scheme, and are at liberty to modify, refine or coarsen the suggestions made to them by the lecturers.

5.2 Difficulty of questions

Most of the courses identified last year as having an anomalously large number of alphas were moved back towards the average, the exception being Number Fields (still up at 79%). It was, however, again the case that more of the lowest scoring courses, as revealed by the bar charts of marks *after adjustment* for student effect, were Applied and more of the highest scoring courses were Pure.

[redacted] (external examiner) was concerned about the effect of disparity between courses, and recommends consideration of adjusted marks during classification. There are various reasons why this should be approached with great caution, in particular because good students will choose easier questions and should be rewarded for that, and also because it is far from clear how statistically reliable the separation of course effect and student effect actually is. We suggest that an exploratory analysis of past results might be considered if the computer-officer time and statistical expertise are available.

Meanwhile, we again **recommend** that this year's examiners make every effort to reduce differences in difficulty between questions on various courses, and that they inform the relevant lecturers of last year's issues and the desired direction of travel towards the middle of the range. The letter from the Chair of the Faculty Board to the 2017 examiners has been strengthened to emphasise this point.

5.3 C-course long questions

[redacted] (external examiner) observes that he understood the guidance in the Schedules that '*no distinction be made, for classification purposes, between quality marks obtained on the Section II questions for C courses and those obtained on D courses*' to mean that these questions should be of the same standard. We agree. On the other hand, he has twice encountered examiners who thought that even the long questions on C courses ought to be easier. We disagree with these examiners. We **recommend** either that the phrase 'and that these should thus be of comparable difficulty' be added to the Schedules, or that something be added to the instructions to examiners.

5.4 Athena SWAN

The examiners' report did not include a breakdown of results by gender as requested by Faculty Board. We remind this year's Chair of Examiners to do so.

6 Summary of recommendations

(The exact recommendation is described in the section indicated.)

2.1 Use typesetting conventions. Use LaTeX rather than TeX.

2.2 Email and Schedule additions concerning marking policy towards excess questions.

2.3 Pressure maintained to keep short questions genuinely short.

2.4 Thank [redacted] for their efforts last year.

3.2, 4.5, 5.2 Examiners and lecturers to actively consider previous years' statistics and adjustments necessary to achieve a more uniform level of difficulty.

4.1 Two changes to Schedules regarding class descriptors. Addition to Chair's letter regarding 'other factors'.

4.2 IB examiners to have an extra meeting to discuss questions.

4.3 Discuss draft questions carefully with all relevant lecturers.

4.5 Paperwork to be sent out electronically if available in time. Statistics to be sent to externals in November.

4.6 Search rubbish sacks for missing claimed questions only if it might make a difference to a class, or on appeal.

4.7 Impose the maximum appropriate penalty for plagiarism in CATAM.

5.3 C course long questions to be of comparable difficulty to D course questions.

[redacted]

November 9, 2016

Teaching Committee report to the Faculty Board

Examinations for Parts IA, IB and II of the Tripos, 2017

1 Introduction

The Committee met four times, for two hours each, to consider the 2017 undergraduate Tripos examinations. As usual we looked, for each part of the Tripos, at:

- the examiners' report;
- the external examiners' reports;
- the examiners' comments on their questions;
- the examination statistics;
- the examination papers;
- the analysis of the paper lecture questionnaires;
- the responses to the on-line questionnaires;
- the report from the CATAM assessors (Parts IB and II).

We noted with pleasure that as usual the external examiners, without exception, commented favourably both on the examination process and also on the performance of the candidates. The six external examiners' reports include comments such as:

The Mathematical Tripos in Cambridge remains one of the most ambitious Mathematics courses in the country, and the examination reflects this. (Part IA)

I am overall very happy with the examining procedure of which I was a part. The level of difficulty of the questions is appropriate, and the processes for assessment, examination and the determination of awards are sound and fairly conducted. (Part IB)

Compendium examination papers remain appropriate to the students at a world-class university. (Part IB)

As in my other two years, the standard of the examination papers remained very high and I was impressed by the performance of the best candidates. (Part II)

I gained an extremely favourable impression of the quality of the students, and of the fairness and rigour of the examination process. I am happy that Cambridge Part II mathematicians continue to represent the best of UK mathematics, and the course acts as a beacon of high standards, certainly matching or exceeding any other UK programme. (Part II)

The standards are markedly higher than at any other university where I have worked or served as an external examiner. (Part II)

There follows a summary of the points raised in the examiners' reports which the Committee believe need the attention of the Faculty Board. We have not generally highlighted points of a purely administrative nature: that is for the Chairs of this year's examiners and the Undergraduate Office to pick up rather than the Faculty Board.

2 General Matters

2.1 Errors

This year only six errors were discovered during or after the examinations, in a total of about 280 questions: no errors in Part IA; one error in Part IB; five errors in Part II. In addition, three errors were discovered before the examination and announced at the very start. Overall, these numbers are smaller than most recent years, though larger than last year.

The Part II examiners recommend that all questions are made to conform to the Faculty's typesetting conventions *in advance* of the third meeting of examiners. This should be done by the Chair (and perhaps their deputy from the other department). This would allow the examiners to focus on the mathematical content and difficulty in the meeting, rather than being distracted by issues of style.

We strongly **recommend** imposition of the typesetting conventions *before* the second or third examiners' meeting, if necessary by the Chair.

2.2 Short questions

Last year we noted that the success rate for betas on short questions was still a bit short of the target 65–70% and recommended that pressure was maintained to keep short questions both short and easy. We are pleased to see progress as shown in the following table:

	beta rate (%)			average mark			takeup in II	
	IA	IB	II	IA	IB	II	short	long
2017	62	65	59	7.1	7.2	6.7	14.6	17.1
2016	59	59	57	6.8	6.8	6.5	17.4	17.1
2015	44	52	51	6.7	7.0	6.8	12.4	15.8
2014	60	51	48	7.2	7.0	6.6	12.9	15.5

[redacted] (Part IB externals) and [redacted] (Part II external) all mention the importance of short questions as a means for giving even the weakest students opportunity to demonstrate success in learning. We entirely agree, and hope that examiners will continue to push in this direction.

2.3 Comments on questions

Faculty Board has agreed that the examiners' comments on questions will now be put online in the January following. Several examiners' reports requested guidance regarding the intended length and substance of these comments.

We **recommend** that the comments should describe how the candidates performed compared with the examiners' expectation of the question, which parts were done well and what were the most common mistakes. Two or three sentences would be appropriate for most questions. We also **recommend** that, if possible, the comments are written and made available to the external examiners at the time they are considering scripts.

2.4 Procedures in the examination halls

There were a number of annoying glitches at Mill Lane, such as students not being permitted to take the question papers away, and examiners having difficulty finding the space or the tools to write corrections on whiteboards. [redacted] will compile a list of issues and discuss the matter with the Student Registry. We **recommend** that the Undergraduate Office includes a copy of the maths-specific invigilators' script with the Examiners' Memorandum, so that examiners know what the students should be told.

2.5 CATAM plagiarism

We are delighted that there were again no cases of plagiarism in CATAM last year. We believe that the robust line taken by the Faculty, and supported by the University and external examiners is acting as a deterrent, and hope that the University will continue to support a robust line.

2.6 Timing of corrections

The IB examiners announced a correction about 15 minutes before the end of the examination; the Part II examiners chose not to announce a clarification 5 minutes before the end. We thought that both decisions were correct. We did not agree with the IB examiners' recommendation that any correction in the last 30 minutes of the exam should not be announced, because withholding a known correction is likely to increase the risk of student grievance and complaint. The timings of corrections should be noted, and markers should bear in mind that corrections will take longer to reach candidates in colleges.

2.7 Administrative and Computer Officer support

The examiners' reports remarked on the excellent support given by [redacted] in particular, and the administrative staff more generally. We **recommend** that the Chair of the Faculty Board expresses the appreciation of the Board for their hard work.

3 Part IA

3.1 Extra attempts

Last year's change to the rubric led, as expected, to an increase in the number of extra attempts at long questions from 116 to 188. Compared to the total of 4385 attempts, this is not currently a significant addition to the marking load. We do not agree with the suggestion that students who have 6 long answers should have to choose which 5 to submit. We **recommend** that the new rubric should continue to be used without alteration until further notice.

3.2 Difficulty of questions

The success rates for alphas on the long questions on Groups and Statistics were well outside the Faculty recommended range of 40–45%. We again ask all this year's examiners to pay careful attention to previous years' statistics and the guidelines on question setting in the letter from the Chair of the Faculty Board.

3.3 Draft questions on secure server

We are not persuaded that draft examination questions should be exchanged between examiners and the Undergraduate Office using a secure server rather than hard copy.

3.4 Reporting partial marks

The Chair asks for guidance on whether partial marks (e.g. for physics practicals, or for CATAM) for withdrawn candidates who did not sit the written papers should be reported on the Faculty List. The University guide to examiners states unambiguously that they should be. This is indeed our current practice; such candidates are listed at the end of the list as 'Not Classed' (which is distinct from 'Failed').

4 Part IB

4.1 Difficulty of questions

[redacted] (externals) note that some variations in difficulty between questions on different courses is inevitable, but ask whether it is the case that some courses consistently over the years have easier or harder questions. This is one of the issues that the Teaching Committee looks out for, and the answer in Part IB is no. Examiners are instructed by the Faculty to make every effort to aim for a common level of difficulty, which is specified by target alpha and beta success rates, and to use statistics from recent years to judge the desired direction of travel. For example, as the external examiners note, the questions on some pure and applicable courses were found rather difficult last year, but were found easier this year after the examiners made appropriate adjustments. We agree with [redacted] that there must be a recurrent effort every year, in the interests of fairness for that cohort.

We have an impression that the variability in the difficulty of the applicable questions appears to have been rather greater (in both directions) over a number of years than that of the pure and applied questions. We wondered whether this is because the applicable examiner can be more isolated so that there is less critical discussion of their questions. If so, we **recommend** that the other examiners try to pay more attention to the difficulty of the applicable questions. We also **recommend** that the Director of the Stats Lab consider whether it might be advantageous to create, as a formal role, a second 'assistant' examiner (perhaps also for IA Probability) to assist in setting questions, to provide oversight of difficulty, to attend some of the examiners' meetings, but not to do any marking. We would welcome constructive suggestions from this year's examiners.

4.2 Class boundaries

We are a little surprised by [redacted] comments regarding the apparent uncertainty over the 3rd/Fail and 2.2/3rd boundaries. While typically about half of the examiners are new each year, the other half are there to give continuity. Moreover, the present classification criteria have been in place for quite a while, are commented on each year by the Teaching Committee and discussed by Faculty Board. Most examiners should be aware of the intention of Faculty Board by now, and abide by that rather than finding their own maverick interpretations.

We reiterate some of the classification criteria. There should be no distinction between marks obtained on the CATAM projects and marks obtained on other courses. Examiners only *may* (not *will*) take into account whether candidates have obtained most (presumably more than half) of their marks on only one or two (and no more!) courses (which certainly includes CATAM in Part IB, where the projects are not tied to other courses) – this condition therefore does not apply to three of those who failed. The primary classification criterion is the merit mark, and there should be no separate consideration of the number of alphas and betas. In our recommendations last year, we clarified the possible sorts of secondary criteria, and made it clear that only in exceptional circumstances should there be a significant departure from the merit-mark order. (We have no objection to the small transposition that the examiners did decide on.) There is no mention at all in the classification criteria regarding the candidates' prospects in Part II, and there is no requirement of competence in more than one or two courses (including CATAM).

In the context of these criteria, it seems to us that the only notable ambiguity is what merit mark is sufficient for a 3rd (or a 2.2), which can sensibly vary a little from year to year if there is variation in the difficulty of the exam. The borderline for this year's examination (where three candidates with merit mark in the range 200–250 were failed) is rather high in comparison with recent years, and the examiners' report gives no insight into their reasoning. Hence we **recommend** that examiners are provided with a printout of the bottom of the class list for the last 5 years as a way of providing comparisons and guidance. We **recommend** that examiners are encouraged to focus on the classification criteria, as printed in the Schedules, and as amplified in the letter from the Chair of Faculty Board, and not on the flexible verbal descriptors or anything else. We **recommend** that examiners minute all decisions pertaining to individual candidates, and their reasons, in conformity with university guidelines (https://www.student-registry.admin.cam.ac.uk/files/examiners_meetings.pdf).

As last year, we are reluctant to add too much prescriptive detail to the Schedules in case this ties the examiners' hands in unusual situations and prevents them from exercising appropriate judgement. On the other hand, unless the situation is very unusual, we urge examiners to give the highest priority to the criteria as set out in the Schedules.

According to our student representatives, students pay no attention to the verbal descriptors and understand that they should simply pay attention to maximising their merit mark (which is the correct strategy), though occasionally, ill-advised by urban myths, some still worry about numbers of alphas and betas (which is incorrect, except insofar as they contribute to the merit mark). We are not therefore worried about trying to fine-tune the verbal descriptors. We agree with [redacted] that the criteria and descriptions should be kept under review.

4.3 Achievement at lower boundaries

We agree with [redacted] that it is important to give opportunities for weaker students to show some level of achievement. It is for this reason that we have been exhorting examiners to keep the short questions genuinely short and straightforward, and we are pleased to see year-on-year improvements in this regard (see §2.2). We also note that the Computational Projects in Parts IB and II are intended as opportunities for candidates to show achievement in ways other than on examination questions, and that the classification criteria explicitly say that marks achieved on CATAM should be regarded in the same way as marks achieved in the examination. Our own experience as external examiners suggests that imposition of credit frameworks is as likely to distort and lower standards of assessment, as it is to raise standards of attainment.

4.4 Model answers and marking conventions

We were unconvinced that solutions with crossings-out are unlike what a good student would produce under exam conditions. Nevertheless, we emphasise the model solutions should be legible, and that the intended solution and mark scheme should be clear to any examiner or supervisor who looks at them. We agree with [redacted] (external) that there should be some indication on scripts where marks are lost (e.g. a ring, underline, $\times$, $\neq$, or $\not\Rightarrow$) and we note that the instructions on script marking already specify that this should be the case.

4.5 Presentation of CATAM

[redacted] (external) suggests that the proportion of marks for presentational or mathematical clarity in CATAM reports, be increased from the present 10% to perhaps 30–40% for a range of report-writing skills. We had mixed feelings. On the one hand, we agree that these are transferable skills, and worth recognition and encouragement; on the other, we worried that assessment would be seen as subjective, and that students already express concern that they don't know what the "right" answers are for full marks. We referred the matter to the Computational Projects Assessment Committee, since its members have relevant experience of assessing projects.

5 Part II

5.1 Difficulty of questions

Last year, we noted that more of the lowest scoring courses, as revealed by the bar charts of marks *after adjustment* for student effect, were Applied and more of the highest scoring courses were Pure. We recommended that this year's examiners make every effort to reduce differences in difficulty between questions on various courses, and that they inform the relevant lecturers of last year's issues and the desired direction of travel towards the middle of the range. The letter from the Chair of the Faculty Board to the 2017 examiners was strengthened to emphasise this point. We are therefore dismayed that the same problem has arisen again.

While some lecturers clearly did make appropriate adjustments, four of the nine highest-scoring courses in 2016 got even higher scores in 2017 when they should, instead, have been moving back towards the middle of the distribution. Comparing the bar charts before and after adjustment for student effect, we think very little of the differences between courses can be written off as due to differences in the quality of the students taking that course; the dominant effect is due to the differences in the difficulty of the questions being set.

The examiners' reports detail the extensive efforts they went to look at the range of questions attempted by each candidate in the light of the statistics, with the result that they moved the borderlines lower down the list so that borderline Applied candidates were treated generously and not disadvantaged by their course selection. Nevertheless, we **recommend** that much greater efforts are made towards homogenization of difficulty. In particular, we **recommend** the following measures:

- Writing specifically to the lecturers of the courses that were significantly out-of-line with the Faculty guidelines in the previous year.
- Requiring all lecturers to sign a sheet saying they have considered the statistics from previous years when handing in their questions to the Undergraduate Office.
- Asking lecturers to aim for the Faculty's target alpha rate rather than assuming that any discrepancies are due to differences in the quality of the students.
- Heads of Departments should try to ensure that the slate of examiners has significant expertise across the full range of courses.
- Examiners and checkers need to take more responsibility, and challenge lecturers if they think that the questions are not appropriate.
- Examiners are reminded of their freedom to tweak mark schemes in the light of the answers in a way that distinguishes more clearly between good and adequate understanding.
- New lecturers to Cambridge should be given explicit guidance and mentoring regarding the importance, in the Mathematical Tripos, of setting questions at the appropriate level of difficulty.

5.2 Model answers

[redacted] (external examiner) commented that most of the model solutions he was sent were illegible, and some solutions were incomplete. We weren't persuaded by his suggested remedy of having solutions typed in LaTeX – we believe that a hand-written solution showing what a good student would write gives a much better indication of the length of the question. We totally agree with him that solutions should be legible and kept up-to-date with any changes in the question.

The lecturer is responsible for providing the *first* drafts of questions and solutions, and for making every effort, as the specialist, to use previous statistics to provide questions set at the appropriate level of difficulty. The examiners are unequivocally responsible for the questions on the examination paper, for comparing questions on different courses, and for making sure that the solutions and mark schemes are accurate and clear to any examiner or supervisor who looks at them. If necessary, solutions should be written out again by the examiner. While the preparation timetable is often tight, we **recommend** that the Chair encourages examiners to exercise quality control on what is sent to the external examiners.

5.3 Mark sheets

We thought the suggestion of adding the average mark for a question beside each individual's mark for that question would probably clutter the mark sheet and obscure the information it currently presents. The statistics on questions are, of course, routinely provided to examiners in a separate document.

6 Summary of recommendations

(The exact recommendation is described in the section indicated.)

2.1 The Chair to impose typesetting conventions between examiners' meetings.

2.3 Expectation of the content of the comments on questions. Available to externals.

2.4 Inclusion of invigilators' script with examiners' memorandum.

2.7 Thank [redacted] for their efforts last year.

3.1 Continued use of the new rubric.

4.1 Increase scrutiny of the applicable questions in IA and IB.

4.2 Provision of history of decisions at the bottom. Focus on the classification criteria as printed. Minuting of all decisions.

5.1 Various measures to improve homogenization of difficulty.

5.2 Greater effort to improve the quality of solutions sent to external examiners.

[redacted]

November 10, 2017

Teaching Committee report to the Faculty Board

Examinations for Parts IA, IB and II of the Tripos, 2018

1 Introduction

The Committee met four times, for two hours each, to consider the 2018 undergraduate Tripos examinations. As usual we looked, for each part of the Tripos, at:

- the examiners' report;
- the external examiners' reports;
- the examiners' comments on their questions;
- the examination statistics;
- the examination papers;
- the analysis of the paper lecture questionnaires;
- the responses to the on-line questionnaires;
- the report from the CATAM assessors (Parts IB and II).

We noted with pleasure that as usual the external examiners, without exception, commented favourably both on the examination process and also on the performance of the candidates. The six external examiners' reports include comments such as:

The academic standards set by the examination were very high. The calibre of the best students means that the exams have to have substantial, hard questions while at the same time including questions accessible to weaker students. (Part IA)

The standards of the examinations and of the qualifications are very high. The processes for assessment, examination and the determination of awards are sound and fairly conducted. (Part IB)

The examiners take a great deal of care in both the setting and marking of examination scripts and the Faculty of Mathematics has robust systems for checking and compiling marks. (Part IB)

The standard of the examination papers was very high, and the performance of the candidates was typically exceptionally strong. In a period of grade inflation across the sector, it is reassuring that Cambridge continues to hold strong and not devalue its own currency. (Part II)

I was extremely impressed with the quality of the students, and with the rigour of the examination process. Part II Maths continues to maintain the highest standards in terms of both the breadth and depth of its courses, and to provide a worthy and challenging hurdle for the UKs strongest students to compare themselves against. (Part II)

The level and breadth of Pure Mathematics courses offered was excellent, and the standard of the exam questions was high. (Part II)

There follows a summary of the points raised in the examiners' reports which the Committee believe need the attention of the Faculty Board. We have not generally highlighted points of a purely administrative nature: that is for the Chairs of this year's examiners and the Undergraduate Office to pick up rather than the Faculty Board.

2 General Matters

2.1 Errors

This year only nine errors were discovered before, during or after the examinations, in a total of about 280 questions: no errors in Part IA; two errors in Part IB; seven errors in Part II.

The examiners followed last year's recommendation that typesetting conventions should be applied before the second or third examiners' meeting, if necessary by the Chair. They comment that this worked well, allowing the examiners to focus on the mathematical content and difficulty in the meeting, rather than being distracted by issues of style. We strongly **recommend** that this practice continue.

2.2 Reuse of old questions and model answers

The examiners in Part IB were notified by some concerned students after Paper 2 that all the questions in Quantum Mechanics and Variational Principles were exact duplicates of questions from previous years (2007 & 2011). Anticipation that this might recur in Papers 3 and 4 was shared via social media, particularly in some larger colleges, and concern was expressed that this might disadvantage those students not in the know. The questions on Papers 3 and 4 were indeed duplicates from previous years, but not years that might easily have been anticipated. There is no evidence, statistically speaking, that any students were disadvantaged. Nevertheless, this incident should serve as a wake-up call to any examiners thinking it is safe in an internet age to recycle multiple questions.

We **recommend** that: (i) examiners do not reuse any question from the previous at least 6–8 years (ii) examiners be aware that it is very easy to identify reused questions on early papers from the online resources available (iii) examiners should notify the committee at the second meeting of any prior instances of the questions they are proposing to set (iv) the majority, preferably all, of the questions on any course should be new, though appropriateness should be given much greater priority than novelty. The guidance for examiners will need updating.

A number of student complaints in the end-of-year questionnaires alerted us to the fact that a complete set of LaTeXed answers to IA Probability questions 2001–2017 was circulated by social media shortly before the Part IA examinations. The answers were not copies of those held in the undergraduate office, and were presumably prepared by a supervisor and given to their supervisees. There are issues with fairness and relative advantage within the student body. We **recommend** that the Faculty reiterates its policy that undergraduates should not be given model answers to questions, beyond the usual going through recent questions in supervisions, and perhaps providing solutions to questions the students themselves have attempted. For students to work to gain an advantage by doing more questions under their own steam is one thing, but we strongly deprecate, on educational grounds, the idea that students should be prepared for exams by being fed a set of answers to learn.

2.3 Administrative and Computer Officer support

The examiners' reports remarked on the excellent support given by [redacted] in particular, and the administrative staff more generally. We **recommend** that the Chair of the Faculty Board expresses the appreciation of the Board for their hard work.

3 Part IA

3.1 Difficulty of examination

This year's examination was found significantly more difficult than that in recent years, with marks about 20% down at each border and the number of questions attempted down by about 5%. In IA an unexpectedly hard exam is, of course, the same for everybody (except the physicists), but some may find it significantly more disconcerting than others. We **recommend** that the IA exam briefing include the advice 'expect the unexpected'. Though marks were down across the board, the criterion for a 3rd was similar to last year ($2\alpha + \beta > 9$), with the result that there were 9 failures. While we agree it is not appropriate to allow candidates to progress who are unequipped to do so, we are somewhat surprised by where the line was drawn.

While the overall 42% alpha rate on long questions was within the target range of 40–45%, the 49% beta rate on short questions fell well short of the target 65–70%. Prof. Watts (external examiner) comments if the short questions were more straightforward (easier) then it would simplify matters at the lower end of the scale while barely affecting the better students. We agree that short questions are very important as a means for giving even the weakest students opportunity to demonstrate success in learning.

We **recommend** that this year's examiners pay careful attention to the statistics from previous years and the guidelines on question setting in the letter from the Chair of the Faculty Board.

3.2 Draft questions on secure server

We are still not persuaded that draft examination questions should be exchanged between examiners and the Undergraduate Office using a secure server rather than hard copy. Any breach of security, perhaps thorough human error, would be catastrophic.

4 Part IB

4.1 Typographical conventions

There seems to be some confusion about typographical conventions, with some examiners remembering that, once upon a time, (a), (b), (c) was used to denote logically distinct parts of questions and (i), (ii), (iii) was used for related parts of a question. Precisely because students, and more recent examiners, were completely unaware of this obscure historical practice, this convention was formally abandoned in 2015, and the current conventions for labelling parts are those that are circulated to examiners.

4.2 Queries during the examination

The examiners' report notes that 'there seemed to be an excessive number of queries requiring no action this year and some instances of students wanting to engage in discussion about their queries'. The Teaching Committee was very much against suggestions that queries should be written, and delivered via an examination administrator, noting that it is vital that real mistakes are identified and disseminated easily and accurately. At the same time, we encourage examiners to give a simple 'the question is correct as written' response to any query that has not identified either a mistake or a genuine ambiguity that **requires** clarification to all students.

4.3 Difficulty of questions

There was good balance in the difficulty of Section II questions, but the Section I questions on pure and statistics were on average not as accessible as they might have been. We **recommend** that next year's examiners again pay attention to the need to keep Section I questions accessible.

Last year we noted that the variability in the difficulty of the applicable questions appears to have been rather greater (in both directions) over a number of years than that of the pure and applied questions. This year, with a very experienced examiner, the variability was less, but we remain concerned that the applicable examiner is potentially more isolated with less critical discussion of their questions. We again **recommend** that the Director of the Stats Lab consider whether it might be advantageous to create, as a formal role, a second 'assistant' examiner (perhaps also for IA Probability) to assist in setting questions, to provide oversight of difficulty, and to attend some of the examiners' meetings, but not to do any marking.

4.4 Number of questions

The examiners suggest that Faculty Board might consider reducing the number of questions per course (currently 4 long and 3 short for a 24 lecture course; 3 long and 2 short for a 16 lecture course; 2 long and 2 short for a 12 lecture course). The Teaching Committee had a lot of sympathy with this view, noting that there are more questions per course that in either Part IA or Part II, and that fewer questions might encourage students to take a slightly broader range of courses. It was not obvious to us how to reduce the number of questions while maintaining an appropriate balance in both short and long questions between courses of different lengths. However, bearing in mind the different purposes of long and short questions, we **recommend** that the number of short questions on a 24 lecture course be reduced to 2. All courses will then have two short questions, while the number of long questions (4, 3 and 2 respectively) continues to be reflective of the course length.

4.5 Borderline processes and question structure

[redacted] (external examiner) notes that while students do know the merit mark formula, the approximate borderlines from previous years, the percentage guidelines and the classification descriptors, they may not know the precise process by which examination boards decide borderlines. The student representatives on the Teaching Committee were completely satisfied that they already knew more than enough to guide their examination strategy.

[redacted] (external examiner) welcomed the fact that classification is not entirely algorithmic and leaves space for some academic discretion and judgement, but was concerned that the basis on which discretion is exercised should be clear. We agree! In 2016 Faculty Board agreed a description of when and how 'other factors' might be taken into account, which was communicated to the 2017 examiners, but was omitted by mistake from the information given to the 2018 examiners. The omission has been rectified, and future examiners will receive this guidance. The guidance is very much along the lines recommended by [redacted]

[redacted] also wondered whether questions should have a uniform structure. In part, this may be prompted by the misunderstanding regarding typographical conventions described above. The Committee was firmly of the view that imposing a uniform structure would be an unnecessary constraint that would have a negative impact on the design of suitable questions for each course.

4.6 Marking conventions

[redacted] suggests denoting partial marks as fractions, e.g. 3/5, to show where marks are lost, and that examiners should comment explicitly that a solution was or wasn't worth an alpha when awarding 14/20 and 15/20. The Teaching Committee noted that examiners are already instructed to show where marks are lost by means of crosses, underlines, rings, etc. and that student answers are often not in sequence so that the '3/5' may be dispersed through the script. Examiners are very aware of the big difference between 14/20 and 15/20. We were unpersuaded that there was added value in increasing the amount of writing, and thought it might be a distraction during marking.

4.7 Transferable skills

We note with pleasure the positive comments by [redacted] regarding the high uptake of CATAM, and the opportunity it provides to develop computational and written communicational skills to complement broad mathematical skills. Cambridge supervisions develop oral and written communication skills, while challenging assignments provide opportunities for team work with a supervision partner or friendship group. Colleges and societies also provide opportunities to develop a wide range of social and organisational skills. Our belief, supported by strong employment statistics, is that students are acquiring transferrable skills without need for summative assessment in place of some mathematical content.

4.8 Advice on strategy

[redacted] comments that some candidates made poor tactical choices about how to balance short and long questions. Students are generally advised to optimise their merit mark. First-year students are given more specific advice in a pre-examination briefing at the start of the Easter Term. All students receive an email containing examination advice, in which they are invited to take note of the straightforward nature of short questions when considering their strategy. Individuals should receive tailored advice, appropriate to their abilities, from their Director of Studies. Unfortunately, some candidates may not listen to advice.

5 Part II

5.1 Difficulty of questions

In recent years, there have been persistent differences in the difficulty of questions on various courses, with a handful of Pure courses, in particular, repeatedly having alpha rates well in excess of the target range. A number of new measures were put in place following last year's report on examinations, in particular putting a greater emphasis on the responsibility of lecturers to consider Faculty guidelines regarding difficulty and the desired direction of travel from previous years' statistics. We are very pleased to note that these measures were largely successful, both in terms of the examiners reporting 'more uniformity in difficulty between different areas of the Tripos than in previous years' and in terms of most previously out-of-line courses producing results much closer to Faculty targets.

We **recommend** that all the new measures remain in place. Like the letter to examiners, the letter to lecturers should come from the Chair of the Faculty. We emphatically agree with the examiners' **recommendation** that similar vigilance be shown next year for courses which may have been slightly too easy or too hard this year, and special vigilance shown towards courses that have been systematic outliers over several years.

5.2 Queries during the examination

Despite the best efforts of examiners, the size and complexity of Part II makes it almost inevitable that more errors will slip through into the Part II papers than in Parts IA and IB; this year's figures are quite typical. There will thus be more queries to address in the examination halls in Part II, with the additional complication of possibly needing to contact the responsible examiner. We **recommend** that the Part II examiners consider whether having three duty examiners at each examination might mitigate the stress of a cluster of queries, and facilitate a more rapid response. We note that the Board is larger than in Parts IA and IB.

5.3 Paperwork

[redacted] (external examiner) suggests that it would have been easier to judge the suitability of draft questions, if he had been sent the statistics on questions from previous years. We **recommend** that it should be standard practice to send two years of past statistics with the bundle of draft questions to external examiners in all Parts of the Tripos.

The three external examiners all commented that their consideration of scripts on the Monday was delayed and made less efficient by the unavailability of statistics and box plots until later in the day. In part, this was caused by at least one examiner missing the Thursday deadline for the return of scripts, and the subsequent unavailability of a mark checker. It should be a matter of both professionalism and consideration for others that deadlines are not allowed to slip, and that the external examiners should always have a full set of paperwork available to them at the very start of their day's work. We **recommend** that procedures and instructions to examiners are tightened up. We also **recommend** that when checkers are assigned by the undergraduate office the expectations and timetable should be clearly explained.

5.4 Merit mark

Perhaps associated with the chaotic start on Monday, the external examiners expressed some uncertainty about the ranking order presented to them. The concern is in our view misplaced. Even in the extreme example constructed by [redacted] of a candidate with 15 alphas but few marks otherwise, the possible merit marks would be either $M_1 = 555$ or $M_2 = 450$. The former is not even close to a First and the candidate would therefore be assessed on the basis of M_2 for whether they should get a 2.1 or a 2.2. In practice, there is no ambiguity about which borderline a candidate might be close to, and the marks processing program ensures that candidates are ranked in the appropriate order near each borderline. We **recommend** that chairs and external examiners are provided with more details of the algorithm.

5.5 Equality, diversity and inclusion

[redacted] (external examiner) felt uncomfortable that the examination board was all male. As in 2015, the Teaching Committee did not believe that the gender balance of the examiners was relevant to the performance of the candidates. All candidates are considered anonymously, with their gender unknown. Moreover, previous statistical analyses have failed to show any appreciable difference in the strategies pursued by male and female candidates, in contrast to stereotypical assumptions that are too easily made. The Faculty is fully committed to providing a level playing field with respect to all issues of equality, diversity and inclusion, and has a number of committees and working groups examining detailed statistical evidence related to these issues.

5.6 Model answers

We are pleased that [redacted] (external examiner) saw some improvement in the quality of the solutions sent to him with draft questions. There is still room for improvement, and we support him in his request for more carefully prepared solutions. As last year, we are not in favour of having solutions typed in LaTeX, since a hand-written solution showing what a good student would write gives a much better indication of the length of the question.

6 Summary of recommendations

(The exact recommendation is described in the section indicated.)

2.1 The Chair to continue imposing typesetting conventions between examiners' meetings.

2.2 Restrictions on examiners' use of past questions.

2.2 Reiterate restrictions on providing model answers to students.

2.3 Thank [redacted] for their efforts last year.

3.1 IA exam briefing for undergraduates to include 'expect the unexpected'.

3.1,4.3 Attention to the need to keep short questions short and straightforward.

4.3 Possible increase of scrutiny of the applicable questions in IB (and IA).

4.4 Reduction in the number of short questions on a 24 lecture course in Part IB from 3 to 2.

5.1 The new measures to even difficulty of questions to remain in place, vigilance to be maintained.

5.2 Part II examiners to consider whether to have 3 duty examiners at each examination.

5.3 Statistics to be sent with the draft questions to external examiners in all Parts. Tightening of procedures for getting everything ready for the external examiners.

5.4 Details of the marks-processing algorithm for the chairs and external examiners.

[redacted]

November 8, 2018

Teaching Committee report to the Faculty Board

Examinations for Parts IA, IB and II of the Tripos, 2019

1 Introduction

The Committee met three times, for two hours each, to consider the 2019 undergraduate Tripos examinations. As usual we looked, for each part of the Tripos, at:

- the examiners' report;
- the external examiners' reports;
- the examiners' comments on their questions;
- the examination statistics;
- the examination papers;
- the analysis of the paper lecture questionnaires;
- the responses to the on-line questionnaires;
- the report from the CATAM assessors (Parts IB and II).

We noted with pleasure that as usual the external examiners, without exception, commented favourably both on the examination process and also on the performance of the candidates. The six external examiners' reports include comments such as:

The exams distinguished well between the good students and the standards achieved were very high. The changes to the shorter questions this year meant that that the exams were also very good at distinguishing between the weaker students. (Part IA)

The examination process is both fair and rigorous in assessing performance of the candidates. The examiners take a great deal of care in both the setting and marking of examination scripts and that the Faculty of Mathematics has robust systems for checking and compiling marks. (Part IB)

Discussion of borderline cases and the precise location of class boundaries was the main subject of the final meeting. Having spent the previous day examining scripts of the potential borderline candidates we were able to have a rigorous discussion that ensured the final classification was both fair and appropriate for the qualification. (Part IB)

The standard of the examination papers was very high, and the performance of the candidates was typically exceptionally strong. That Cambridge continues to hold strong in a time of grade inflation and not devalue its own currency is to be encouraged. (Part II)

The final papers represent a high, but not insurmountable, challenge for students, testing material at a challenging and appropriate level that matches or exceeds all other institutions in the UK (and, I suspect, internationally). (Part II)

My fellow external examiners and I met on the day before the final meeting to discuss borderlines and scrutinise the scripts. We found that the marking schemes had been applied consistently and the marking had been fairly and carefully done. (Part II)

There follows a summary of the points raised in the examiners' reports which the Committee believe need the attention of the Faculty Board. We have not generally highlighted points of a purely administrative nature: that is for the Chairs of this year's examiners and the Undergraduate Office to pick up rather than the Faculty Board.

2 General Matters

2.1 Errors

This year only seven errors were discovered before, during or after the examinations, in a total of about 280 questions: no errors in Part IA; one error in Part IB; six errors in Part II, two fewer than last year. The Board of Examinations data suggests there were also four minor clarifications (which they count as errors) in Part II and one in Part IB, and they are much exercised by the total number of announcements. While the Board of Examinations may not appreciate the distinctive character of mathematics papers, we note that corrections, particularly in Part II, increase the likelihood of requests for Examination Review. (Questions with corrections are, of course, always marked generously by the examiner, and the final meeting will always consider whether border-line candidates might have been affected.)

We **recommend** that Examiners should refrain from making trivial clarifications and restrict announcements to substantive corrections. (This might have eliminated about three announcements this year.)

We also **recommend** for Part II that the detailed checking of questions by another examiner be moved to between the second and third meetings, by which time the questions will be closer to final form.

2.2 Mark-lists

In response to comments from examiners in Parts IA and II, we **recommend** some changes, as described in a separate paper, in the formatting of the computer-generated mark lists that are provided at the final examiners' meeting. The new formatting clarifies the presentation of information to the examiners, but the classification criteria and the merit mark, ranking and UMS percentage issued to candidates are unaffected.

2.3 Ranking

The University now requires students to be provided with an official ranking, which is posted on CamSIS. We **recommend** that the ranking be that on the Faculty results list, namely determined by merit-mark order within each class. Faculty Board previously decided not to put rank on the mark breakdown out of consideration for candidates whose results were disappointing.

2.4 Reprographics

The IB examiners strongly **recommend** negotiation of a later submission date for camera-ready copy to Reprographics. They point out that the current date requires an examiners' meeting and subsequent editing within the first week of Full Easter Term, which is full of other commitments. If the deadline could be delayed until the end of the second week, that might reduce the chance of errors. They note that the papers were, in fact, not copied until 31 May, two weekdays before the IB exams began.

2.5 Administrative and Computer Officer support

The examiners' reports remarked on the excellent support given by [redacted] in particular, and the administrative staff more generally. The external examiners were likewise very complimentary about the administrative arrangements and the detailed information that was ready for them to assist their scrutiny of the borderline scripts. We **recommend** that the Chair of the Faculty Board warmly expresses the appreciation of the Board for their hard work.

3 Part IA

3.1 Difficulty of examination

This year's examination was found significantly easier than last year's, and perhaps an overcompensation overall when compared to the four previous years or to the target alpha rate on long questions. Nevertheless, the 60% beta rate on short questions was a welcome increase towards the 65–70% target. [redacted] (external examiner) commented very positively that the accessibility of the short questions gave weaker students a good chance to show what they knew.

We **recommend** that this year's examiners pay careful attention to the statistics from previous years and the guidelines on question setting in the letter from the Chair of the Faculty Board.

3.2 Dynamics and Relativity

Various negative comments were received in student feedback about the quirkiness of this year's questions, and the marks were lower than those on other courses. We wonder whether some of the stories built around the problem element were found over-elaborate or off-putting.

3.3 Re-use of old questions

[redacted] (external examiner) suggests that it would be helpful if the Faculty issued some guidelines on re-using questions verbatim. We thought we had provided some very clear guidelines in last year's report, and these were incorporated into the letter from the Chair of Faculty Board to examiners. We **recommend** some minor clarifications of the wording, and hope that the guidelines will be followed this year.

4 Part IB

4.1 Difficulty of questions

There was good overall balance in difficulty between the long questions in Pure, Applied and Statistics. We agree with the recommendation in the examiners' report regarding the need to keep the short questions accessible in all subject areas, and we **recommend** that this year's examiners pay attention to the comments of [redacted] (external examiner) regarding two outlying courses.

4.2 Number of questions

[redacted] (external examiners) comment that the structure of the examination gives students a large choice of questions to attempt and hence the flexibility to decide on the breadth of topics to attempt. We agree that for almost all students this is a good thing, and it allows students to optimise their strategy in line with their interests and abilities. We agree that the candidate who (just) achieved a 2.ii by answering questions on only two courses well (and CATAM) is a highly unusual anomaly (and a high-risk strategy) , and we note that they will almost certainly have attended lectures and supervisions on other courses during the year. [redacted] suggests that the Faculty might consider slightly reducing the number of questions available. This has, in fact, been done this year with the reduction of the number of short questions on 24-lecture courses from 3 to 2 and with the incorporation of Metric and Topological Spaces into Analysis & Topology (a net loss of 3 questions).

5 Part II

5.1 Difficulty of questions

We were very pleased to note that there were again no significant differences in difficulty between the major areas of Pure, Applied and Statistics. The measures introduced in 2018, in particular putting greater emphasis on the responsibility of lecturers to consider Faculty guidelines regarding difficulty and the desired direction of travel from previous years' statistics, seem to be broadly effective at avoiding systematic variation between areas and reducing variation between courses. The examiners' report **recommends** continuation of all procedures and we again **recommend** continued vigilance regarding individual courses which may have been slightly too easy or too hard this year, and special attention to courses that have been systematic outliers over several years.

5.2 Late submission of questions

Lecturers are required to submit draft questions with model answers to the Undergraduate Office by the Monday of the week before Full Lent Term. Of 36 lecturers, 23 were on time or within 1 week i.e. by the start of Full Term. However, 7 provided questions only within 2 weeks, 4 within 3 weeks and 2 within a month! Late submission of this magnitude significantly complicates the task of both the examiners and the Undergraduate Office. It prevents the questions being considered properly and checked in advance of the second meeting, it makes it difficult for the examiners to moderate difficulty between courses, and it probably contributes to the number of errors (either directly, or by distracting the examiners from other courses).

The quality of our examinations is, of course, of great importance to the Faculty because it is important to our students (and the University), and this is a needless spanner in the works. The Undergraduate Office can prompt lecturers about upcoming or missed deadlines, but, in our opinion, it is not fair to expect them to pursue unresponsive lecturers. Accordingly, we **recommend** that Faculty Board follow through with its previous decision to name at its first meeting of Term all those who are still late. We also **recommend** that the Chair of the Faculty Board email all those whose questions are still outstanding on the first Friday in Term, and cc. their Head of Department. And we **recommend** that the Chair of Faculty Board email those who were significantly late in the previous year (if still lecturing) reminding them of their responsibilities.

5.3 Reporting of errors

The examiners' report should detail all errors and clarifications, and not just give the corrected versions of significant errors.

5.4 Number of duty examiners

The Part II examiners decided to follow last year's precedent and have three duty examiners present at each examination rather than the traditional two. This was again found beneficial in dealing with the larger volume of queries that are almost inevitable in Part II. We **recommend** that Faculty Board confirm that having three duty examiners should be standard practice in Part II.

5.5 Equality, diversity and inclusion

In response to the concerns of [redacted] (external examiner), we repeat that statistical analysis has failed to show any appreciable difference in the strategies pursued by male and female candidates, in contrast to stereotypical assumptions that are easily made. The Faculty is committed to addressing all issues of equality, diversity and inclusion, and has a number of committees and working groups thinking hard about these issues.

5.6 CATAM

[redacted] (external examiner) notes that the CATAM projects are of significant benefit to students and, similarly to last year, the 5 students who did not submit projects all got 3rds or Failed. The Faculty does email all IB students with graphs showing the very clear contribution that a CATAM mark typically makes to a student's overall performance. The median CATAM mark was 115/150.

This year candidates were allowed to submit more than 30 units of projects, with an algorithm for discarding or scaling the weakest projects down to 30 units of credit. Six students went more than one project over 30 units, and one of these went more than two projects over. The Teaching Committee considers that (i) the new rules are fairer and have been a success (ii) the number of excess projects is small and at an acceptable level (iii) attempting excess projects, like doing excess questions in an examination, is to be discouraged as not in the students' interests, but that is not the same as trying to prohibit it. We were unpersuaded by a suggestion from CPAC that the number of submittable units should be capped, but will review the situation next year.

6 Summary of recommendations

(The exact recommendation is described in the section indicated.)

2.1 Examiners to refrain from trivial clarifications.

2.1 Detailed checking of questions in Part II to occur between the second and third examiners' meetings.

2.2 Change the format of the preliminary mark lists.

2.3 Ranking defined to be that on the Faculty results list.

2.4 Negotiate a later submission date of camera-ready copy to Reprographics.

2.5 Thanks the individuals who provided administrative and computer officer support.

3.1,4.1 This year's examiners to give attention to the statistics and comments on the difficulty of last year's examination.

5.1 The new measures to even difficulty of questions to remain in place, vigilance to be maintained.

5.2 Various measures to put more pressure on lecturers to submit their draft questions on time.

5.4 Part II examiners to have 3 duty examiners at each examination as standard practice.

[redacted]

November 12, 2019

Teaching Committee report to the Faculty Board

Examinations for Parts IA, IB and II of the Tripos, 2020

1 Introduction

The examinations this year were abnormal in many ways because of the arrival of the Covid-19 pandemic in March and the consequent requirement to move the June assessments online. These online examinations were taken remotely and uninvigilated, and candidates accessed the papers and submitted scans of their scripts via Moodle.

The online examinations in Parts IA and IB were formative, with no on-the-record results. The number of papers was reduced from 4 to 2, but the structure of the papers was similar to the usual format. Candidates were asked to take the papers in closed-book timed conditions, and sign a declaration that they had done so.

The Part II online examination in June was Pass/Fail only, with each of the two papers containing one Section I question on each course. Candidates were asked to take the papers in timed conditions, but open-book, again with a signed declaration. Those who Passed, and all did, were entitled to graduate with their class from Part IB. (A similar 'second-sit' examination was provided in August for the benefit of one candidate who was unable to take the June assessment.)

A full set of in-person invigilated Part II examinations were provided in September for those candidates who were able to return to Cambridge and wished to improve on their IB class. They were taken by 83 candidates and 30 were successful in improving their class. The candidates scripts were handled with gloves and scanned on receipt by the undergraduate office to avoid infection.

The Part II internal examiners and the Part II external examiners and, indeed, the Part II lecturers, have thus, in some sense, done double duty and we **recommend** that the Faculty express its thanks to them.

The scanned scripts from all these examinations were sorted electronically using information captured either from Moodle or from new machine-readable coversheets. A candidate's script was split into separate PDFs for each question and labelled with the blind grade number; all attempts at a given question were collected and sent to the relevant examiner via SharePoint; marking and checking was done electronically with exchange via SharePoint; the marked, checked attempts were then resorted into candidate folders; coversheet information was used to provide all the usual checks against the entries in the mark book.

The external examiners, without exception, commented very favourably on the way that the Faculty adapted its examinations to the pandemic situation. The six external examiners' reports include comments such as:

The care exercised around online exams this year was really excellent. The dangers of cheating are indeed not to be underestimated and the decision to make IA purely formative was the best choice possible. It certainly will have reduced student stress at a time of general great uncertainty. (Part IA)

The speed and flexibility the department showed in moving to formative assessments for Part 1B was excellent. They offered the students a virtually standard exam experience, assessing the majority of course material, while recognising that some students would find themselves in unsuitable situations for regular summative assessments. (Part IB)

I am happy that the adaptions made to the assessment for part 1B were appropriate, given the short time available. It provided the students with an opportunity to test their understanding of the part 1B material, but without making this part of their official record. Students appear to have adapted well to the change in the assessment process. (Part IB)

I must commend the Cambridge examiners on their excellent handling of the crisis caused by Covid-19. I believe that their solution has given the students the best experience they could have hoped for of the examination process, and that their experience of the whole course has been excellent, despite the crisis and its obvious impact on their course and assessment. (Part II)

The revised examinations became pass/fail, which ensured the integrity of the degrees remained fully intact. Students were informed about the revised exam procedures in advance. It is my impression that the students had a positive experience of the course and assessment, and the excellent liaison with the students about the new exams process is an example of good practice. (Part II)

It is clear that the professional and administrative support for the examinations has been outstanding, and I think this has also contributed greatly to the success. It has been a magnificent all-round endeavour by everyone involved. (Part II)

The efficiency of the examination marking and collation of the results has been remarkable and I believe many things can be learnt from this process when the crisis is over. Given this immense success, I have every confidence that the team's approach to the September examinations will be appropriate. (Part II)

There follows a summary of points raised in the examiners' reports which the Committee believe need the attention of the Faculty Board.

2 General Matters

2.1 Errors

In line with University guidance, no means was provided for candidates to raise queries during the online examinations and, instead, they were instructed to write any queries on their scripts. Fortunately, there were no errors discovered in any of the online examinations: for Parts IA and IB, this is not unexpected; for Part II, this perhaps reflects the fact that the Pass/Fail examination questions were largely based on old, and tested, examination questions.

Five errors were discovered in the Part II in-person examination papers, one fewer than last year. (Careful attention was paid during marking to ensure that no candidate was disadvantaged.) In line with last year's recommendation, Examiners refrained from making clarifications and restricted announcements to substantive corrections.

2.2 Administrative and Computer Officer support

These reports regularly pass on the thanks of the Chairs, and the commendations by all the examiners, for the superb support provided by the Undergraduate Office and the Computer Officers. Those thanks and commendations are again forthcoming from the Chairs, but it is hardly possible to set down in words just how much the Faculty owes its support staff, who worked in difficult circumstances, under incredible pressure, for long and unsociable hours to put together from scratch an online system that worked behind the scenes, that made possible the task of examiners and, most importantly, served our students well. We **recommend** that particular thanks are due to [redacted]

2.3 Online setting, marking and script handling

The examiners in all three parts thought that the online processes introduced as emergency measures for setting the examinations via SharePoint, sharing and editing the questions by examiners, and for the distribution and marking of scripts via SharePoint all worked very well. The Part II examiners strongly recommend that online marking be continued. [redacted] (Part IA external examiner) recommends that exam setting should remain online to avoid transfer of drafts on paper. The Part II examiners note that additional security measures would be desirable if everything remains on SharePoint.

The Teaching Committee agrees that things worked remarkably well last year, and **recommends** that similar processes be adopted this year and, moreover, they are likely to be necessary. We **recommend** that the IT and UGO teams urgently be asked to investigate this term possible ways[1] of improving the security of examination setting on SharePoint, to inform examiners of procedures before new questions are submitted in January. Noting that setting up and operating

[1]e.g. two-factor authentication, encryption, IP address for access, audit trail, . . .

the online procedures required an enormous investment of human resources last year, we **recommend** that IT and the UGO be asked to comment on the feasibility of repeating something like the procedures used for the in-person Part II examinations and on ways they might be improved if implemented across all three Parts. We **recommend** that there be a review of the glitches that were discovered in the processes, with a view to improving the robustness of the checks in place.

2.4 Excess question attempts

This year, in an attempt to alleviate the burden on examiners, candidates in Parts IA and IB were told that any attempts in excess of the numbers allowed by the rubric would be discarded at random and hence that they should submit only what they considered their best attempts. This proved remarkable effective at almost eliminating excess attempts, with just two candidates ignoring these limits.

Nevertheless, the Teaching Committee did not feel it would be appropriate to take such a draconian approach in summative classed examinations, and we **recommend** a return to the usual rubric and policy that the best attempts will count. In a normal year, the extra marking is not a significant addition.

2.5 Alternative assessment

Other than CATAM, our assessment focuses on end-of-year invigilated examinations as the best means of assessing learning outcomes, in particular, the ability to understand and recall advanced mathematics and apply it in limited time to the solution of unseen problems. [redacted] (Part IB external examiner) wonders whether we should explore alternative assessment methods that are better suited to being done at a distance such as extended open-book assignments.

The Teaching Committee thought that there were considerable risks to quality associated with trying to make such adaptations at short notice, and potential unfairness to students who have been trained via lectures, example sheets and supervisions for traditional examinations. We believe that the successful example of the in-person Part II examinations gives ground for optimism that in-person examinations will be possible next June. If not, the Faculty is actively exploring the possibility of online proctoring, as successfully used in STEP examinations and to be trialled for the left-over Part III examinations, as a fall-back option.

Nevertheless, aside from coping with the immediate practicalities of the pandemic, the Committee had some sympathy towards the idea of somehow giving a little more space to assessing, say, presentation and writing skills. Ideas such as having mini-CATAM projects associated with IA courses, more marks for presentation in IB CATAM, or giving a talk were mentioned in passing. Similar ideas have not found favour in the past, partly because of practicalities of resourcing and fair assessment, but we wonder if there is any enthusiasm to refer this to the Curriculum Committee.

2.6 CATAM

The take-up of CATAM in Part II was slightly down on previous years, and 31 candidates in Part IB who had submitted projects in January did not submit further projects in April. In the circumstances, this is perhaps not surprising, but we **recommend** that the Part IB examiners compare the Lent and Easter submission rates this year to see if they are more in-line.

2.7 Previous recommendations

Some of the recommendations from the 2019 examinations may not have been implemented or taken effect before the pandemic turned everything upside-down. We **recommend** that Chairs and the UGO review last year's report to make sure that no recommendations have been lost. The UGO should send both last year's report and a slightly longer range of historical statistics to the examiners.

3 Part IA

3.1 Difficulty of examination

This year's examination was found very much easier than last year's, which itself was found easy when compared to the four previous years. The average alpha success rate was 60%, well in excess of Faculty guidelines and two courses approached a success rate of 80%. The average mark was at least 100 more than is typical. This outcome is almost certainly because, to accommodate the reduction to two papers, examiners were asked to select 'mainstream' questions on the central ideas in each course. There is no element of criticism in this comment, simply a drawing of attention to the observation.

We **recommend** that this year's examiners pay careful attention to the statistics from previous 'normal' years and to the guidelines on question setting in the letter from the Chair of the Faculty Board.

3.2 Merit formulae

[redacted] (external examiner) is concerned that the 1 mark difference between 14 and 15 on a long question can make a difference of 26 to the merit mark and, if this applies to several questions, candidates may move a long way from a boundary. The Teaching Committee noted that examiners know the significance of alphas and thus think carefully about whether to give 14 or 15. The difference is thus a conscious decision and not a random fluctuation. The student representatives observed that the merit mark and the significance of alphas are clearly understood by students, and they are not unhappy. We did not think highlighting 'unlucky' students as possible 'borderline cases' would be helpful or consistent with the advertised classification criteria. We also note that there are mark checking and review procedures as protection against any small possibility of mistakes.

4 Part IB

4.1 Difficulty of questions

[redacted] (external examiner) notes that the marks on Numerical Analysis have been anomalously high for all three years of his tenure. He recommends that there is a need to make the long questions on this course more challenging, and also suggests that the syllabus may need revising.

The Teaching Committee agrees that there is a systemic problem here that has been present and persistent since Numerical Analysis was moved from the Easter term to the Lent Term. The content of the course is appears lighter than other courses at this level, and the examination questions are typically more formulaic and closely related to the notes and example sheets. We **recommend** that this matter be referred to the Curriculum Committee. One avenue to explore is whether the course should be returned to the Easter Term, filling the slot left vacant by Metric and Topological Spaces, where it would be seen as accessible to IA students and useful to them as a lead in to CATAM.

5 Part II

5.1 Appeals

The Chair notes a discrepancy between the way that the EAMC makes recommendations that a class be revised on the basis of evidence provided to the committee, and the relevant regulation in Statutes and Ordinances which requires examiners to re-assess the examination performance. The Teaching Committee agrees that it would be odd for examiners to be asked to rubber-stamp a decision made by another committee on the basis of evidence that it has not seen. It is quite possible, however, that this situation was largely the product of the peculiar arrangements that were necessitated by the Covid-19 pandemic. We will keep this under review.

[redacted]

November 17, 2020

Annual Report of the Teaching Committee 2020–2021

1. Teaching and examining this year have been significantly affected by the pandemic. In brief:

    (a) The examinations in June 2020 were taken online with no invigilation. Those in Parts IA and IB were only formative. Those in Part II were made pass/fail only, which allowed students to graduate with their class from Part IB. The number of papers was reduced from 4 to 2 in all Parts and their structure adjusted.

    (b) In-person invigilated examinations were offered to Part II students in September 2020, to give students who wished a chance to upgrade their class. They were taken by 83 candidates, of whom 30 did achieve a better class.

    (c) Both the setting and the marking of all examinations moved online. This has involved the use of Sharepoint to host and exchange documents, and in-house development of new electronic procedures to collect, sort, distribute, mark and resort scripts. These procedures are being used again this year.

    (d) All undergraduate lectures in 20/21 were given online and not in-person. A few courses in Part II were live-streamed via Zoom and also recorded. The vast majority were pre-recorded by the lecturer, uploaded to Moodle course pages, and watched by students on Panopto. The undergraduate experience of lectures has been one of watching videos all year.

    (e) All supervisions in Easter Term 2020 and Lent Term 2021 were given remotely, because the vast majority of students were sent home during lockdowns. About 80% of the supervisions in Michaelmas Term 2020 were also given online because of both personal and institutional concerns over safety.

    (f) This year's examinations will be classed and have the normal structure. Planning has proceeded in parallel both for in-person examinations if the pandemic permitted and for online examinations invigilated using ProctorExam if it did not. At the time of writing, it appears that we will be running a dual system with about 85% of candidates in-person in the Sports Hall and about 15% online under remote invigilation.

    (g) A survey of undergraduate welfare and mental health issues by the student representatives on Faculty Board has highlighted just how challenging the last 15 months have been for many students for a whole host of reasons, including factors such as stress, anxiety and isolation on top of the need to adapt learning habits to the online environment.

    (h) Nevertheless, surveys by the student representatives, both last year and this, have given strong support to the Faculty's approach to maintaining the integrity of examinations in mathematics, and the decision to hold in-person examinations this year if at all possible. Key indicators such as comprehensibility and the quality of notes have held up well in the usual 2nd-week and end-of-term questionnaires. An online-teaching questionnaire at the end of Michaelmas Term supported the value of traditional line-by-line development of a mathematical argument.

    In conclusion, it has been a very difficult and challenging year for all concerned. The educational environment provided to our students has differed markedly from what we would normally offer, but it seems to me that the Faculty has provided the best possible teaching and assessment that were achievable in the circumstances. I am thus very grateful to the teams in the undergraduate office and IT, to all those who have adapted their teaching to an online environment, and to the examiners, from all of whom immense effort, flexibility and commitment has been required to keep things on the road and to maintain the standards expected of Cambridge mathematics. I am also grateful to our students, who have responded with great patience, maturity and forbearance to the changes that have been thrust upon us all.

2. The need to respond rapidly and agilely to sudden changes of external circumstances and of University policy, has meant that many decisions have necessarily had to be taken by 'cobra' groups formed from the Director of Undergraduate Education, the Chair of the Teaching Committee, the Chair of Faculty Board, the Heads of Department and Part III equivalents. While students, Directors of Studies and Faculty Board have been kept fully informed of developments, I hope that 2021/22 might allow for a return to the more usual consultative and measured form of governance.

3. The Teaching Committee met online 3 times in the Michaelmas Term to consider reports on exams and the early feedback on online lecturing. As usual, we provided a detailed report on the undergraduate examinations to the Faculty Board based on reports of the Examiners and External Examiners. Agreed recommendations were transmitted to this year's Examiners. The full report on examinations is available online.

4. The members of the Teaching Committee this academic year have been

[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]

* Student members

[redacted]
May 25, 2021

Annual Report of the Teaching Committee 2018–2019

1. The members of the Teaching Committee this academic year have been

   [redacted] [redacted] [redacted] [redacted]
   [redacted] [redacted] [redacted] [redacted]
   [redacted] [redacted] [redacted] [redacted]

   * Student members

   Expressions of interest for the vacant student position were sought via an e-mail to all upcoming Part IB students before the start of the year, and the new representative was appointed from a field of 5 on the basis of their applications and comments from Directors of Studies.

2. The Teaching Committee met a total of 6 times in the academic year 2018/19, and dealt with some items by circulation. As usual, the major item on the agenda was the review of the undergraduate Tripos examinations.

   We provided a detailed report on the undergraduate examinations to the Faculty Board based on reports of the Examiners and the External Examiners, on examination statistics and the actual questions, and on the information from the e-mail questionnaires. This report was considered by the Board at its November meeting, having previously been received by the meeting of Directors of Studies, and was used as the basis of the Faculty Board's response to the General Board's Education Committee. Agreed recommendations were transmitted to this year's Examiners. The full report on examinations is available online.

3. This year's NSS response rate (22%) was well below the 50% threshold response and we were not asked to comment. In previous years the Teaching Committee has anyway found our own system of second-week questionnaires, end-of-term questionnaires and end-of-year questionnaires, more informative and useful to us since they focus on the teaching of mathematics on a course-by-course basis.

4. We provided a report to Heads of Department (and to the Director of the Statslab) on the teaching of their departments, based on the paper and on-line questionnaires. A small number of courses/lecturers were referred to the Heads of Department for possible action. In general, the feedback from students was very positive and we drew attention to 13 courses where the feedback was outstanding.

   In this year's report we drew the attention of the Heads of Department to a total of 8 courses (out of 62) where the feedback was negative: in some cases the lecturer is no longer lecturing and no action was necessary; in others we took action ourselves as well as alerting the HoDs.

5. Part IA lectures moved from the Cockcroft to the Babbage Lecture Theatre in 2018/19. Use of two visualizers, each projecting 2 sheets of paper, appears to be working quite well as a means of laying out a mathematical exposition (though not perhaps as dynamically as on a set of blackboards). Severe constraints were placed on the IA timetabling by both the limited availability of timeslots in the Babbage and the scheduling of IA Physics. As a result, students of Mathematics with Physics were unable to get to the (non-examinable) lectures in Numbers & Sets and Dynamics & Relativity. The Faculty introduced 8 "lecture classes" for each course, to provide these students with enough of a catch-up to follow the main ideas of the course should they wish to proceed into IB Mathematics.

6. The University has replaced the old Regulations 5–7 governing appeals against examination results with a new system, overseen by OSCCA, which has more limited grounds for appeal. The Faculty has thus formalised a procedure by which a candidate may request, via their Director of Studies, a mark check on particular questions if there are good reasons to believe an error has occurred.

7. It was disappointing that a nomination for a Pilkington Teaching Prize, originating from a special committee of all our student representatives, was unsuccessful, perhaps because 'excellence in teaching' is now low on the University's stated prize criteria. The Faculty does

believe in rewarding excellence in teaching, and in listening to the student voice on such matters, and hopes to institute a Faculty prize.

8. The Teaching Committee advised the Faculty Board regarding the appropriate way to advertise non-examinable courses, such as the History of Mathematics and Ethics in Mathematics, for which the Faculty is not responsible for the content.

9. Members of the Teaching Committee were involved in various miscellaneous activities (for some of which the Board has received an independent report), including:

   - Attending early lectures of all new lecturers and some other lecturers, and discussing their lecturing;
   - Monitoring electronic student feedback on the Feedback line and in second-week questionnaires;
   - Advising on the choice of lecturers;
   - Advising on the timetable for lectures;
   - Responding to the report of the Learning and Teaching Review
   - Email communication to students regarding course information;
   - An extensive and ongoing statistical analysis of Tripos performance in relation to a number of factors relevant to admissions and widening participation;
   - An induction session for first year students;
   - Induction sessions for second and third year students;
   - An examination briefing session for Part IA students;
   - Supervisor training sessions.

[redacted]
May 11, 2019

Annual Report of the Teaching Committee 2017–2018

1. The Teaching Committee is required by its terms of reference to provide an annual report to the Faculty Board.

2. The members of the Teaching Committee this academic year have been

   [redacted]

   * Student members

   Expressions of interest for the vacant student position were sought via an e-mail to all upcoming Part IB students before the start of the year, and the new representative was appointed from a field of 14 on the basis of the applications and comments from Directors of Studies.

3. The Teaching Committee met a total of 5 times in the academic year 2017/18, and dealt with some items by circulation. As usual, the major item on the agenda was the review of the undergraduate Tripos examinations, but other items have also needed attention.

   We provided a detailed report on the undergraduate examinations to the Faculty Board based on reports of the Examiners and the External Examiners, on examination statistics and the actual questions, and on the information from the e-mail questionnaires. This report was considered by the Board at its November meeting, having previously been received by the meeting of Directors of Studies, and was used as the basis of the Faculty Board's response to the General Board's Education Committee. Agreed recommendations were transmitted to this year's Examiners, and the guidance to lecturers on question-setting in Part II was strengthened. The full report on examinations is available online.

4. We reported on the National Student Survey findings and in general on other forms of feedback. The NSS showed a significant drop in overall student satisfaction, which was not mirrored in our own internal feedback. Many of the NSS scores were highly discrepant relative to previous years. We concluded that the most plausible causes were the circumstances around the 2017 survey created both by the NUS boycott and by media coverage about possible student fee increases under TEF.

5. We provided a report to Heads of Department (and to the Director of the Statslab) on the teaching of their departments, based on the paper and on-line questionnaires. A small number of courses/lecturers were referred to the Heads of Department for possible action. In general, the feedback from students was very positive and we drew attention to 14 courses where the feedback was outstanding.

   In this year's report we drew the attention of the Heads of Department to a total of 6 courses (out of 62) where the feedback was negative: in some cases the lecturer is no longer lecturing and no action was necessary; in others we took action ourselves as well as alerting the HoDs.

6. A new C course was introduced in Part II on Quantum Information and Computation and appears to have been received well. The lecturer, [redacted] coordinated provision of supervisory capacity in this specialist new area, which in the event proved more than sufficient.

   By contrast, an unexpected 50% increase in the demand for supervisions of some Part II Applicable courses caused considerable difficulties. It is not clear how much was due to last year's IB exams and how much to a significant shift in student behaviour in an evolving job market.

7. The arrangements by which the new Director of Undergraduate Education and the Chair of the Teaching Committee work together, with complementary but overlapping roles, appear to both parties to be working smoothly in this their first year of operation. We suggest a brief review this summer.

8. A major headache throughout the year has been the availability of suitable lecture theatres for Parts IA and IB. This year's central allocation was initially unsuitable and would, for example, have involved students having to change sites for the middle lecture of three. Next year's demolition of the Cockcroft threatened to displace Part IA to Lady Mitchell Hall. After lengthy negotiations, it appears that we have secured slots in the Babbage instead for next year, but longer term plans by the University remain a concern.

9. In addition, members of the Teaching Committee were involved in various miscellaneous activities (for some of which the Board has received an independent report), including:

   - Attending early lectures of all new lecturers and some other lecturers, and discussing their lecturing;
   - Monitoring electronic student feedback on the Feedback line and in second-week questionnaires;
   - Advising on the choice of lecturers;
   - Advising on the timetable for lectures;
   - Preparation for, and participation in, the Learning and Teaching Review
   - Email communication to students regarding course information;
   - Induction session for first year students;
   - Induction sessions for second and third year students;
   - Examination briefing session for Part IA students;
   - Supervisor training sessions.

[redacted]
May 11, 2018

Annual Report of the Teaching Committee 2016–2017

1. The Teaching Committee is required by its terms of reference to provide an annual report to the Faculty Board.

2. The members of the Teaching Committee this academic year have been

   [redacted]

   * Junior members

   Junior members are nominated by the Chair and appointed by the Faculty Board. Expressions of interest were sought via an e-mail to all upcoming Part IB students before the start of the year, and the new representative was appointed on the basis of the applications and comments from Directors of Studies.

3. A new post of Director of Undergraduate Education was established to complement the role of the Chair of the Teaching Committee, strengthening oversight of the undergraduate Mathematical Tripos, and taking the lead for the Faculty in communications with students and with the wider University on teaching matters. The new DUE, [redacted] is *ex officio* a member of the Teaching and Curriculum Committees, Faculty Board and other bodies.

4. The Teaching Committee met a total of 6 times in the academic year 2016/17. As usual, the major item on the agenda was the review of the undergraduate Tripos examinations, but other items have also needed attention.

   We provided a detailed report on the undergraduate examinations to the Faculty Board based on reports of the Examiners and the External Examiners, on examination statistics and the actual questions, and on the information from the e-mail questionnaires. This report was considered by the Board at its November meeting, having previously been received by the meeting of Directors of Studies, and was used as the basis of the Faculty Board's response to the General Board's Education Committee. Agreed recommendations were transmitted to this year's Examiners. The report is available online.

   We reported on the National Student Survey findings[1] and in general on other forms of feedback.

   We provided the Faculty response to a University consultation on overall degree classification, which argued unequivocally that the Part II result is the only sensible option for the Mathematical Tripos.

   We provided the Faculty response to the Senior Tutors' Education Committee regarding the number and nature of College-based pre-sessional courses. We reported that they help students settle into College and form supportive relationships, but do not convey an unfair academic advantage and are not academically necessary.

5. A new Part II course on Analysis of Functions was introduced, and the revamped schedule for Part II Cosmology was lectured for the first time. The rubric for undergraduate examinations was revised to clarify the treatment of 'excess' questions and the allocation of quality marks. Examiners' comments on questions will now be published online.

6. We provided a report to Heads of Department (and to the Director of the Statslab) on the teaching of their departments, based on the paper and on-line questionnaires. A small number of courses/lecturers were referred to the Heads of Department for possible action. In general, the feedback from students was very positive.

[1] The Guardian's league tables https://www.theguardian.com/education/ng-interactive/2017/may/16/university-guide-2018-league-table-for-mathematics, drawn from NSS results, show that Mathematics in Cambridge has much higher student satisfaction ratings than our major competitors in all three categories (for the course, for teaching and for feedback).

In this year's report we drew the attention of the Heads of Department to a total of 7 courses (out of 61): for 2 of these we recommended some action be taken by HoDs; for 3 we took action ourselves.

We also drew attention to 9 courses where the feedback was outstanding. One of the two Pilkington Prizes assigned to the School of Physical Sciences was awarded to [redacted]

7. In addition, members of the Teaching Committee were involved in various miscellaneous activities (for some of which the Board has received an independent report), including:

   - Attending early lectures of all new lecturers and some other lecturers, and discussing their lecturing;
   - Monitoring electronic student feedback on the Feedback line and in second-week questionnaires;
   - Advising on the choice of lecturers;
   - Advising on the timetable for lectures;
   - Email communication to students regarding course information;
   - The Pilkington Teaching Prize Committee;
   - A working group, led by [redacted] on improving supervisor training and feedback;
   - Induction session for first year students;
   - Induction sessions for second and third year students;
   - Examination briefing session for Part IA students;
   - Supervisor training sessions.

[redacted]
May 17, 2017

Annual Report of the Teaching Committee 2015–2016

1. The Teaching Committee is required by its terms of reference to provide an annual report to the Faculty Board.

2. The members of the Teaching Committee this year have been

   [redacted] [redacted] [redacted] [redacted]
   [redacted] [redacted] [redacted] [redacted]
   [redacted] [redacted] *denotes Junior members

   Junior members are nominated by the Chair of the Committee and appointed by the Faculty Board. Expressions of interest were sought by means of an e-mail to all Part IB and Part II students before the beginning of the academical year, and one representative was appointed from each year on the basis of the applications and comments from Directors of Studies.

3. We met a total of 6 times in the academic year 2015/16, and dealt with some items by circulation. As usual, the major item on the agenda was the review of the undergraduate Tripos examinations, but other items have also needed attention.

   We provided a detailed report on the undergraduate examinations to the Faculty Board based on reports of the Examiners and the External Examiners, on examination statistics and the actual questions, and on the information from the e-mail questionnaires. This report was considered by the Board at its November meeting, and was used as the basis of the Faculty Board's response to the General Board's Education Committee. Agreed recommendations were transmitted to this year's Examiners.

   This year there has been an unusual flurry of consultation papers from the central bodies. We provided the Faculty response to questionnaires on student workload, on examinations and assessment, and on publication of class lists.

   We reported on the National Student Survey findings and in general on other forms of feedback.

   We reviewed the classification criteria for the Mathematical Tripos, and reported that standards are appropriately high, and employment rates and starting salaries for Cambridge mathematicians are better than for our main competitors.

4. A new Part II course on Automata and Formal Languages was introduced and had very good attendance. Part IB Numerical Analysis was front-loaded in the Lent Term. The 'short-beta' threshold has been reduced to 7/10. Procedures for treatment of absent candidates and illegible scripts in examinations have been clarified. Guidance on workload has been added to the Schedules. Solutions to sample Tripos questions have been put online.

5. We provided a report to Heads of Department (and to the Director of the Statslab) on the teaching of their departments, based on the paper and on-line questionnaires. A small number of courses/lecturers were referred to the Heads of Department for possible action. In general, the feedback from students was very positive.

   In this year's report we drew the attention of the Heads of Department to a total of 7 courses (out of 61): for 3 of these we recommended some action be taken by HoDs; for 3 we took action ourselves; the Part II course Cosmology was referred to the Curriculum Committee, and a significantly revised schedule has now been approved for 2016/17.

   We also drew attention to 10 courses where the feedback was outstanding.

6. In addition, members of the Teaching Committee were involved in various miscellaneous activities (for some of which the Board has received an independent report), including:

   - Attending early lectures of all new lecturers and some other lecturers, and discussing their lecturing;

- Monitoring electronic student feedback on the Feedback line and in second-week questionnaires;
- Advising on the choice of lecturers;
- Advising on the timetable for lectures;
- Email communication to students regarding course information
- Induction session for first year students;
- Induction sessions for second and third year students;
- Examination briefing session for Part IA students;
- Supervisor training sessions.

[redacted]
May 9, 2016

COMMENTS ON QUESTIONS

MATHEMATICAL TRIPOS PART IA, 2020

Course: ANALYSIS I

Paper no. 1 Question no. 3E

Comments: Not terribly popular, but those who attempted it, did well. A significant few were shaky on the precise definition of an improper Riemann integral.

Paper no. 1 Question no. 9D

Comments: I was pleasantly surprised how many students had learnt the bookwork for (a). An alternative proof (involving applying Rolle $N+1$ times) was also quite popular. There were still a few students who skipped this part, or attempted a proof by induction that got nowhere. The numbered parts of (b) were found easy, and students who missed the idea of going via $s(x+2k)$ for the last part, were able to complete the question using the addition rule for cos (imitating the proof of, or differentiating, (b)(ii)). It is puzzling how many students think that in Rolle's theorem the common value taken at the end points has to be zero, but they did not lose any marks for this.

Paper no. 1 Question no. 10F

Comments: This question was very well done in general, both the bookwork and the problem part. I had expected that the complexity of the notation necessary for the result in the final part of the question would be a source of difficulties, but in practice a large majority of candidates dealt with it very well and saw clearly what was going on.

Course: DIFFERENTIAL EQUATIONS

Paper no. 1 Question no. 2A

Comments: Those that wrote the differential equation terms of $x = x(y)$ got the answer very quickly using an appropriate integrating factor. Others made progress by finding a scalar function $f = f(x, y)$ for which the ODE was equivalent to $\mathrm{d}f = 0$.

Paper no. 1 Question no. 7A

Comments: Not a tremendously popular question. Many students lost marks by simply asserting that $\lim_{t\to 0^+} K_t(x) = \delta(x)$, rather than computing the weak limit directly using the hint. Relatively few managed to produce the solution to Burgers' equation in terms of the solution to the heat equation.

Paper no. 1 Question no. 8A

Comments: Another popular question attempted by the vast majority of candidates. Those who noticed the (x, z) equations decoupled solved the problem most efficiently. Those that solved the 3×3 system directly also did well, with most marks lost due to algebraic error.

Course: DYNAMICS AND RELATIVITY

Paper no. 2 **Question no. 4C**

Comments: The first part of this orbits question was done well by those who could quickly apply conservation of energy and angular momentum. The second part was an unseen variation and several attempts made invalid assumptions.

Paper no. 2 **Question no. 11C**

Comments: The first part of the question should have been straightforward, but a significant number of answers incorrectly set the tensions equal to the weights. The first order ODE was generally solved correctly, but there were exceptions. The integrals for the moment of inertia were generally done correctly, but there were some small errors.

Paper no. 2 **Question no. 12C**

Comments: This relativity problem was straightforward if done using 4-momenta. Many people avoided 4-momenta, some with success, some not. Several candidates incorrectly assumed in the first part that the photons both travel in the same direction as the initial state particle, in which case they could infer that the two photons had equal energy. A few candidates forgot the γ in $p = \gamma mv$. The question had a lot of aborted attempts, with several people writing that they had run out of time.

Course: GROUPS

Paper no. 2 **Question no. 1E**

Comments: Most people went through the calculations within the symmetric group proficiently enough. The final part involving double transpositions caused a few minor problems (usually double or under-counting).

Paper no. 2 **Question no. 5E**

Comments: A very straightforward and popular question on fixed points of Möbius transformations; both the (substantial) bookwork and the easier true/false questions were done well. A non-trivial minority of people believed complex conjugation was in fact Möbius.

Paper no. 2 **Question no. 6E**

Comments: Unpopular, despite the proof of the Cauchy–Frobenius lemma appearing fairly recently and with a substantial hint given. There were even fewer good attempts at the rider, though some did produce insightful attempts on using the lemma to find formulae for the number of necklaces and bracelets under the D_{10}-action.

Course: NUMBERS AND SETS

Paper no. 2 Question no. 2D

Comments: Mostly well done, but (i) and (ii) were found harder than (iii) and (iv), so perhaps the order was unkind. The most common mistake was to attempt to prove transitivity in (i) and (ii) by algebraic manipulation rather than to think geometrically. There were significant minorities who thought they could prove transitivity in (i), thought the inequality in (i) was always satisfied, or thought (iii) was not reflexive.

Paper no. 2 Question no. 7D

Comments: A predictably popular question that was generally well done. Derivations of the formula in (a) were extremely variable in length and quality, but it seems (b) and (c) were found quite easy. In hindsight I should have made the numbers at the end of (c) a bit larger to force the use of Euclid's algorithm. It was surprising how many students lost marks in (c) for not saying how the earlier parts of the question are used.

Paper no. 2 Question no. 8D

Comments: The hint for this question was almost universally misunderstood. I had hoped the students would apply (b) with $X = \mathbb{N} \times \mathbb{Z}$, $Y = \mathbb{N}$, fixing σ and varying f. However most students took $X = Y = \mathbb{N}$, fixed f and varied σ. This works, but it requires some care to arrange that different σ give rise to different elements of T. I gave sufficient partial credit for substantial progress on (c)(ii) that students with an incomplete proof still had a chance to obtain an alpha. Between them the students found many different and creative ways to solve the problem.

Course: PROBABILITY

Paper no. 1 Question no. 4F

Comments: A routine application of standard results on branching processes. Most of the attempts at this question were good.

Paper no. 1 Question no. 11F

Comments: This question proved difficult. Most candidates were confident in proving the first of the two results from lecture (inclusion-exclusion). However, the second (the simplest case of the Bonferroni inequalities) was done much less well; in particular, many candidates either mistakenly claimed that the magnitudes of the terms in the inclusion-exclusion formula are decreasing or gave an incorrect expression as a single probability for the quantity obtained by deleting the initial terms. There were very few successful attempts at the problem part of the question, most of which were by an unanticipated method, suggesting that the question would have worked better with a hint added for this part.

Paper no. 1 Question no. 12F

Comments: This was the much better done of the two long probability questions. Most candidates dealt well with the calculations. Many were unable to complete the final part applying the continuity theorem, but there were also several good solutions to this part.

Course: VECTOR CALCULUS

Paper no. 2 Question no. 3B

Comments: Part (a) was reasonably well done although there were quite a few errors made inserting the curve parametrisation into the integrand (in both cases) and simplifying the calculation down to the final integral. Part (b) was done very well.

Paper no. 2 Question no. 9B

Comments: This is a straightforward question testing the evaluation of surface and line integrals. For the surface integral, there were a lot of errors generating an expression for the surface normal, evaluating the curl of the vector field (surprisingly) and parameterizing the surface. The line integrals were done much better although there was some confusion with signs (evaluating both line integrals in an anticlockwise sense means they need to be subtracted not added to give the surface integral result).

Paper no. 2 Question no. 10B

Comments: Parts (a) and (b) are straightforward and were uniformly well done. Part (c) proved to be more difficult with some students failing to see that the mixed boundary condition could be handled much as the Dirichlet or Neumann condition in lectures. There was some confusion solving for the spherically symmetric solution (getting r and b mixed up) and some students insisted on guessing the solution rather than deriving it.

Course: VECTORS AND MATRICES

Paper no. 1 Question no. 1C

Comments: This question on complex numbers was generally done well. Frequent errors were writing $|z| = x^2 + y^2$, errors in $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$, and forgetting that there were multiple solutions to $w = \log z$.

Paper no. 1 Question no. 5C

Comments: This question on vectors was generally done well. A notable number of answers used circular logic to argue that L_{AB} is perpendicular to $\overrightarrow{AB}$, claiming that it was obvious because the line is in a plane normal to $\mathbf{n}_{AB}$. In the final part of the question, a few candidates forgot to consider the case where the two lines are parallel and hence $|\mathbf{u}_1 \times \mathbf{u}_2| = 0$.

Paper no. 1 Question no. 6A

Comments: A simple exercise on Hermitian matrices and unitary diagonalisation that was answered well by most. A very kind question – probably too kind.

COMMENTS ON QUESTIONS

MATHEMATICAL TRIPOS PART IB, 2020

Course: ANALYSIS AND TOPOLOGY

Paper no. 1 Question no. 10E

Comments:
This question went largely as expected. A common mistake, when calculating the derivative of f, was to forget that matrix multiplication is not commutative. The bookwork was generally well executed, as well as the part about f having a local inverse. For the final part (f not having a global inverse), a lot of sketchy arguments were given; some rigorous arguments include explaning that the derivative of f should be invertible everywhere; and showing directly that f is not injective by considering the images of scalings of the identity.

Paper no. 2 Question no. 2E

Comments:
This question was very well executed in general, with the exception of the final part, for which there was a range of common mistakes, for instance: mis-identifying the product topology on $\mathbf{C}^2$; and claiming that a complex polynomial in two variables over $\mathbf{C}$ has finitely many solutions in $\mathbf{C}^2$.

Paper no. 2 Question no. 10E

Comments:
This question was generally well executed, with an alpha rate close to 50 percent. (Both long analysis questions were equally popular, with around 50 attempts.) The construction of the Lipschitz bijection with Lipschitz inverse was generally well done, albeit with some approximate arguments regarding bounds to get an inequality for the inverse. Many more students cited (correct) results from Numerical Analysis than I had anticipated; there was no penalty for this. For the very last past (labelled (ii)), a small, single digit number of candidates noticed that the claim followed immediately from the Topological Inverse Function Theorem. My guess that is because this was lectured as part of the new Analysis and Topology course, which started this year, but didn't feature as prominently in AII or MTS. (A number of students gave direct proofs instead.)

Course: COMPLEX ANALYSIS OR COMPLEX METHODS

Paper no. 1 **Question no. 3G**

Comments:
The standard solution consists of three steps. The first uses a Möbius map (as suggested by the question). Since the region D is bounded by a straight line and an arc of a circle which meet at 0 and 1, it is hard to see what else one can do other than mapping 0 and 1 to 0 and ∞ which then maps D onto an infinite sector. Very few attempts were able to do this. Those who did, were able to complete the question only losing a few marks, but not the beta, for saying nothing about why the maps are conformal equivalences.

Paper no. 1 **Question no. 12G**

Comments:
Less than half the attempts were serious, the others only gave few fragmentary answers earning few marks. For the more decent attempts, the issues varied: some had difficulty finding an analytic branch of $\sqrt{1-z^2}$ or justifying it (although rigorous proof was not required here); others had problems finding the Laurent series of $\phi(1/z)$; the integral at the end was often done well except for a frequent sign error the came from not being careful with the change of variable.

Paper no. 2 **Question no. 12B**

Comments: Very popular, presumably both with those who had studied Complex Analysis and Complex Methods, and a surprising variety of answers. Not too easy, although some examiners thought it was, and it compensated for the relative difficulty of the other joint questions on the Complex courses. The Laurent series were best evaluated using binomial series, but some made algebraic errors. The hint of finding the real integral by integrating the given f (imaginary part of the contour integral round the unit circle) was taken up by a minority. Many expressed $\cos(\theta)$ in terms of $z + z^{-1}$ which needs two residues rather than one to be found, and some used real half-angle formulae. These other methods were longer and there were many algebraic errors.

Course: ELECTROMAGNETISM

Paper no. 1 **Question no. 16D**

Comments: A straightforward question, but with a broad spread of marks. There was a factor of 2 mistake in the energy, which a couple of students picked up on but it didn't hinder the last part of the question.

Paper no. 2 **Question no. 5D**

Comments: A straightforward question that was mostly done well. Marks were lost because students forgot that the potential should be continuous, or forgot how to integrate over $\mathbf{R}^3$.

Paper no. 2 **Question no. 15D**

Comments: I thought that this was a challenging question on a difficult part of the course. However, those students that were brave enough to attempt it clearly knew what they were doing and the vast majority got close to full marks.

Course: FLUID DYNAMICS

Paper no. 1   **Question no. 17C**

Comments: This problem was done quite well by the students. It was rather simplified from its original version, in which the students were supposed to find the velocity potential themselves. Giving them the explicit form for the velocity potential (which they needed to substitute into the condition of the potential flow), apparently, greatly simplified the solution for many. The part which caused most difficulty seems to be the determination of the streamline shape, which required them to come up with a calculation method.

Paper no. 2   **Question no. 6C**

Comments: This question caused certain challenges for some as students first needed to figure out the properties of each flow, and then to carry out an actual calculation.

Paper no. 2   **Question no. 16C**

Comments: Most students did very well, as they have seen something similar in the lectures, although with a different geometry. Perhaps, providing them with a solution for substitution into an equation made the problem too easy for many.

Course: GEOMETRY

Paper no. 1 **Question no. 2E**

Comments:
This question had a disappointingly low number of attempts, even though essentially all of those attempts were correct. I think that this is to be attributed to the fact that the question covers a concept which is new to the schedules (the Gauss map), as opposed to intrinsic difficulty. (Students and supervisors reading these comments in future years should note that there was a 'revision sheet' available with sample questions covering the new parts of the schedules, including several on the Gauss map; this should be available from the DPMMS webpages under past examples sheets.)

Paper no. 1 **Question no. 11E**

Comments:
This question went largely as expected. I was dissapointed with the overall quality of the sketching for the first part. A number of techniques were used to prove that the surface is smooth; note that it was far quicker to do so using tools that are 'new' to the 24h version of the course as opposed to the 16h one. As for the later parts of the question, showing that the Gauss curvature is zero at $r = 1$ was reasonably well done (though sometimes needlessly long, with e.g. pages spend re-deriving formulae from lectures). However, most answers for the very final part (zero curvature elsewhere) were quite weak; many students tried to use a cylindrical coodinate patch over the origin; note that candidates weren't required to find exactly where the curvature vanishes, so could e.g. appeal to continuity of the curvature and the intermediate value theorem.

Paper no. 2 **Question no. 11F**

Comments: This question on hyperbolic geometry had a good number of attempts, but some answers showed limited progress and only earned betas. Most candidates did not have any real problem with the bookwork part and with a (counter-)example when the infimum was not attained, but in the main exercise part were only able to do a straightforward application of the hyperbolic Pythagoras theorem and (more-or-less) proved the existence of the common perpendicular. Proving that it realizes the distance between two ultraparallel lines was a challenge. Some candidates were able to complete this proof or make substantial progress spotting e.g. that the dilations are isometries and earned alphas.

Course: GROUPS RINGS AND MODULES

Paper no. 1 Question no. 9G

Comments:
This question was fairly well attempted. Most candidates knew the general theory and were able to apply it to obtain the rational canonical form of a linear map. Some difficulties arose when candidates found themselves on more unfamiliar grounds: for example, expressing the minimal and characteristic polynomials in terms of invariant factors or finding the invariant factors of a linear map given the information on kerθ.

Paper no. 2 Question no. 1G

Comments:
There were disappointingly few attempts on this question despite requiring little more than IA Groups. Those who did not do well usually stumbled on the first part showing that H-orbits have the same size.

Paper no. 2 Question no. 9G

Comments:
There was a pleasing number of attempts and candidates generally did well on what was mostly a bookwork question. There were a fair number of incorrect arguments showing that an algebraic integer generated a principal ideal. Some had difficulty showing that the given polynomial f is irreducible. Finally, the last part required some careful, albeit brief, reasoning referring to a series of results from the course, and hence this part ended up another source of marks lost.

Course: LINEAR ALGEBRA

Paper no. 1 Question no. 1F

Comments: A basic question on the Jordan normal forms; most candidates did not have problems with it. Typical errors in weaker attempts were trying to use row/column transformations to find the JNF or numerical errors affecting the final conclusion.

Paper no. 1 Question no. 8F

Comments: The question was popular and largely well done. Almost everyone attempting the question managed to put together the sensible answer to the first, bookwork part; some weaker candidates did not progress too much further. The middle part, about a basis consisting of non-singular matrices, proved to be the most challenging and worked well to test the stronger students. Many of those attempting the last part were able to justify the given upper bound but sometimes did not quite justify the example where this bound was attained.

Paper no. 2 Question no. 8F

Comments: Another popular question with most candidates earning an alpha or a solid beta. As expected, the clause (iv) proved to be more challenging and worked well to test the stronger candidates. Generally, one fault of some weaker candidates was confusing a direct sum complement of a vector subspace with the complement of the set of all vectors in that subspace. Some overlooked that endomorphisms do not in general commute or that the product of two matrices can be zero without either factor being zero.
The intended solution to (iii) implicitly assumed that the field of coefficients does *not* have characteristic 2. The question makes good sense with and without this assumption, but if characteristic 2 is allowed, then (iii) becomes false rather than true with a counterexample $\alpha = \beta = \mathrm{id}$ (the identity map). Only a handful of candidates (up to 12 out of 133 attempting this question) noticed. Noone appears to have been disadvantaged; candidates were not penalized for overlooking the characteristic 2 possibility.

Course: MARKOV CHAINS

Paper no. 1 Question no. 20H

Comments: This question had a fairly decent number of attempts and overall the answers were better than expected. A large proportion of students were however not able to correctly define a stopping time or the strong Markov property, but often they went on to solve the unseen part of problem reasonably well (several students incorrectly defined a stopping time as a random variable independent of the future).

Paper no. 2 Question no. 7H

Comments: This question attracted a large number of attempts. It was almost entirely easy bookwork, and was perhaps a bit too standard even for a Section I question. The answers were mostly very good with the majority of students who attempted this getting 10/10.

Course: METHODS

Paper no. 1 Question no. 14B

Comments: Standard separation of variables combined with Fourier series expansion. Rather unusual jump condition along a line to deal with delta function. Special case of this, and simple translation needed to find Green's function. Question straightforward if steps followed but several opportunities to slip up.

Paper no. 2 Question no. 4B

Comments: Those who knew the formula for a convolution and the basic theorem concerning their Fourier transform had little difficulty, but the integral of a product of overlapping discontinuous functions needed care. It helps if students understand that the convolution theorem is an implication both ways (it was sometimes quoted in the wrong direction, though this was not penalised).

Course: NUMERICAL ANALYSIS

Paper no. 1 Question no. 5C

Comments: Most students did reasonably well on this question, which required them to apply their knowledge of a particular method. Some struggled with the arithmetic mistakes.

Paper no. 1 Question no. 18C

Comments: Many students seemed to be very familiar with the interpolation concepts exploited in this question, thus they have done very well. Most of the lost points were in the last part of the problem, which required students to do an actual calculation, albeit an easy one.

Paper no. 2 Question no. 17C

Comments: This question also ended up being on the easy side. Students seemed to know this topic very well, demonstrating in general 2 different pathways towards the correct solution, as well as the knowledge of the useful theorems.

Course: OPTIMISATION

Paper no. 1 Question no. 7H

Comments: This was a standard simplex algorithm question, the type of which is at the core of the course, but received very few attempts. A couple of students failed to use the simplex algorithm to solve the question and instead solved it graphically. As the wording of the question specifically mentioned the simplex algorithm was to be used, some marks were deducted for this. There were also several students who made numerical errors in their implementation of the simplex algorithm, but provided the method was applied correctly, marks were awarded appropriately.

Paper no. 2 Question no. 19H

Comments: This question had a much larger number of attempts than the short question. However, a large proportion of the solutions seemed to be more inspired by the Variational Principles course rather than the Optimisation course, and the answers were somewhat muddled. The reasonably high average mark is due to the first 5 marks of the question being awarded for proving the Lagrangian sufficiency theorem, which almost everyone was able to do.

Course: STATISTICS

Paper no. 1 Question no. 6H

Comments: This question on deriving a posterior distribution was mainly a minor modification of some bookwork, but was evidently found a bit hard perhaps as the course does not have too much material on Bayesian Statistics.

Paper no. 1 Question no. 19H

Comments: This question had a large number of attempts and was done fairly well. Several solutions at some point took $\max_i X_i$ rather than $\max_i X_i/2$ as the MLE which led to some confusion. Very few solutions correctly specified the confidence interval in the last part of the question, with most putting the lower endpoint at 0 rather than the MLE thus giving an interval with a larger expected length than required (some students as a result questioned on their answer sheets the correctness of expected length given).

Paper no. 2 Question no. 18H

Comments: This question was quite well done and perhaps slightly on the easy side. A large number of solutions took a more algebraic approach to solving (c) rather than computing with matrices directly, and marks were awarded for either approach; the algebraic approach typically required more writing.

Course: VARIATIONAL PRINCIPLES

Paper no. 1 Question no. 13D

Comments: There was something of a bi-modal distribution for this question, entirely correlated with the ability of the student to work in polar coordinates. The examiners realised in earlier meetings that this would be the case, but felt that the question had enough clues to suggest that polar coordinates would be advantageous.. A surprising number of students thought that time-independence of the Lagrangian bought them two conserved quantities (one for each coordinate) rather than just one.

Paper no. 2 Question no. 3D

Comments: A straightforward question that was mostly done well. There were two classes of solutions to the equation, giving 4+5=9 stationary points in total, although most came in +/- pairs. Most marks were lost because students missed one or other class.

COMMENTS ON QUESTIONS

MATHEMATICAL TRIPOS PART II, 2020

Course: ALGEBRAIC GEOMETRY

Paper no. 1 **Question no. 25F**

Comments: This was an easy question; each subsequent question was slightly harder. All were appropriate, and well done by the students who attempted them. The cohort who usually attempt this exam were mostly absent, having already received 1sts; this explains the small number of attempts in Q1-4.

Paper no. 2 **Question no. 24F**

Comments: As above.

Paper no. 3 **Question no. 24F**

Comments: As above.

Paper no. 4 **Question no. 24F**

Comments:
This was a reasonable question, slightly more difficult then the previous; appropriate for the top half of the 1sts, and not for the 1st/2 borderline.

Course: ALGEBRAIC TOPOLOGY

Paper no. 1 **Question no. 21F**

Comments: Good question, but too hard for this cohort.

Paper no. 2 **Question no. 21F**

Comments: Good question, and well done by all who attempted it. Even those who didn't understand the formalism precisely did understood the basic examples.

Paper no. 3 **Question no. 20F**

Comments: Solid question, probably a little too hard for the II/I border.

Paper no. 4 **Question no. 21F**

Comments: Good question, a little too hard for this cohort and the II/I border.

Course: APPLICATIONS OF QUANTUM MECHANICS

Paper no. 1 **Question no. 35C**

Comments: Straightforward question for those who have prepared the material.

Paper no. 2 **Question no. 35C**

Comments: Standard not easy question. Few attempts but most got it right.

Paper no. 3 **Question no. 34C**

Comments: Standard not easy question. Few attempts but most got it right.

Paper no. 4 **Question no. 34C**

Comments: A bit more challenging that other questions for this course reflected on the small number of alphas.

Course: ASYMPTOTIC METHODS

Paper no. 2 **Question no. 31D**

Comments: Attempts at this question were satisfactory though few did well. For part (a), a number of candidates lacked coherence in carefully establishing that there was an asymptotic sequence in (i) and linear independence in (ii). Most had the right idea for part (b) but few obtained the first four terms correctly.

Paper no. 3 **Question no. 30D**

Comments: This had a good uptake and most candidates were comfortable approximating the integrals as an asymptotic expansion. With so few terms to calculate, fortuitous shortcuts meant most got the right answer.

Paper no. 4 **Question no. 31A**

Comments: The level of the question was appropriate.

Course: APPLIED PROBABILITY

Paper no. 1 **Question no. 28K**

Comments: Appropriate question, perhaps a bit on the easier side.

Paper no. 2 **Question no. 27K**

Comments: Appropriate question, perhaps a bit on the easier side.

Paper no. 3 **Question no. 27K**

Comments: This question had a bit too much bookwork compared to the original question element.

Paper no. 4 **Question no. 27K**

Comments: Appropriate question.

Course: AUTOMATA AND FORMAL LANGUAGES

Paper no. 1 **Question no. 4F**

Comments:

These were excellent questions, which tested the entire syllabus. They were routine, and almost everyone who attempted them had a thorough and substantial understanding of the material; there were almost no partial answers. The course is probably the most intellectually threadbare of any of the Part II courses, it functions as a lifevest against failing.

Paper no. 1 **Question no. 12F**

Comments: As above.

Paper no. 2 **Question no. 4F**

Comments: As above.

Paper no. 3 **Question no. 4F**

Comments: As above.

Paper no. 3 **Question no. 12F**

Comments: As above.

Paper no. 4 **Question no. 4F**

Comments: As above.

Course: CLASSICAL DYNAMICS

Paper no. 1 **Question no. 8B**

Comments: In principle straightforward, but candidates found algebra off-putting. Few really good attempts.

Paper no. 2 **Question no. 8B**

Comments: Seemed a fair short question, reasonably well done.

Paper no. 2 **Question no. 14B**

Comments: Generally well done. Main issue concerned making consistent expansion in powers of ϵ in last part.

Paper no. 3 **Question no. 8B**

Comments: Very unpopular question.

Paper no. 4 **Question no. 8B**

Comments: No significant attempts.

Paper no. 4 **Question no. 15B**

Comments: Reasonable question - candidates who struggled with this mostly did so in part c).

Course: COSMOLOGY

Paper no. 1 Question no. 9D

Comments: Manipulating the Friedmann equation seemed a familar task that almost all did well. Establishing there must have been a big bang also appeared to be transparent to most.

Paper no. 1 Question no. 15D

Comments: Again finding solutions for the Friedmann equation seemed familiar territory for most candidates. They were able to identify the correct scaling laws for the different matter components and obtain the appropriate scale factor solution.

Paper no. 2 Question no. 9D

Comments: This variant with a scalar degree of freedom in the slow-roll limit appeared to be straightforward for most candidates.

Paper no. 3 Question no. 9D

Comments: Relating the fermionic and bosonic distributions required a minor insight with partial fractions, which meant a lower uptake mainly by those who recognised this route.

Paper no. 3 Question no. 14D

Comments: Despite this question having an easy route in, uptake was fairly low, and few had a clear idea what they were doing. Physical intuition about the Jeans length was also relatively poor, with a number confusing which were gravity- and pressure-dominated regimes.

Paper no. 4 Question no. 9D

Comments: The easy manipulations here were performed well and recollection of the quantities in the Saha equation were excellent.

Course: ELECTRODYNAMICS

Paper no. 1 Question no. 37D

Comments: There were few attempts at this question, like the remainder of electrodynamics, however, some did well. Few understood the effect of the gauge transformation on the action. A number failed to include the Lorentz factor required to obtain the proper angular frequency.

Paper no. 3 Question no. 36D

Comments: Still few attempts though more satisfactory. The candidates were able to do the vector calculus rearrangements necessary in the first part. Most understood the manner by which to calculate the radiation pressure.

Paper no. 4 Question no. 36D

Comments: This question was not answered satisfactorily and few candidates achieved much traction. Most attempts were unable to obtain both junction conditions correctly, meaning the latter parts of the question eluded them as well. Few got to the end to demonstrate the critical value for reflection. With the benefit of hindsight this question was certainly too long.

Course: FLUID DYNAMICS

Paper no. 1 Question no. 39B

Comments: Quite a challenging question, but well done by those who attempted it.

Paper no. 2 Question no. 38B

Comments: The last two parts of the question proved the most challenging.

Paper no. 3 Question no. 38B

Comments: Latter parts of question proved challenging.

Paper no. 4 Question no. 38B

Comments: Straightforward question, mostly book-work.

Course: GALOIS THEORY

Paper no. 1 Question no. 18

Comments: This question was fairly well done in general. Many stated an imprecise definition of a splitting field, forgetting to mention that it is the smallest extension in which the polynomial splits. Part (d) of the question should have said that the polynomial should be irreducible - no correction was made as this part was only worth 2 marks but it did make the question easier.

Paper no. 2 Question no. 18

Comments: This question was the hardest of the Galois theory questions, with a relatively small uptake. Some people seemed unsure of which results to quote in part (d).

Paper no. 3 Question no. 18

Comments: This question was very well done. The last part of the question was trivial due to an error in the statement of the question. Due to the very small number of marks attributed to this part, no correction was made, and any students querying this were told to answer the question as stated.

Paper no. 4 Question no. 18

Comments: A missing irreducibility assumption in part (b) was not queried during the exam, but was taken into account during marking. Most candidates proceeded as if the error was not there. The question had a relatively small uptake.

Course: GENERAL RELATIVITY

Paper no. 1 Question no. 38D

Comments: Presumably this question was perceived to be difficult, so few attempted it. The first parts (i) was answered satisfactorily, but candidates got lost in part (ii) not recognising that derivative of S wrt T was equal to the opposite. The later sections were very straightforward but few got to attempt them.

Paper no. 2 Question no. 37D

Comments: This proved to be a challenging question, requiring some physical intuition to understand the orbit configurations. There were non-trivial algebraic calculations involved, which led a number astray through errors. Few were able to calculate the time intervals, let alone in the correct order.

Paper no. 3 Question no. 37D

Comments: Generally attempted very well. A number did not recognise the Kronecker delta defined at the outset and how it transforms. The remaining algebraic manipulations were long but very straightforward and most candiates were able to reach the end.

Paper no. 4 Question no. 37D

Comments: The last GR question was also answered well. In most attempts, the linearized equations were manipulated well. The remaining parts with the cosmological constant proved fairly straightforward.

Course: GRAPH THEORY

Paper no. 1 Question no. 17

Comments: This question was well done, with most candidates displaying an impressive knowledge of the material.

Paper no. 2 Question no. 17

Comments: This question was generally well done. Some candidates were unsure of the bookwork proof of Menger's theorem.

Paper no. 3 Question no. 17

Comments: This question was slightly harder than other Graph Theory questions. Several candidates missed the case of $\Gamma(u) \cap \Gamma(v) < 2n$ in the final part of the question.

Paper no. 4 Question no. 17

Comments: A small correction was made to the last part, specifying that n should be > 1. This did not affect any candidates in a significant way. The question was generally well done, with candidates showing a thorough knowledge of the chromatic index of graphs.

Course: INTEGRABLE SYSTEMS

Paper no. 1 Question no. 33C

Comments: Straightforward question.

Paper no. 2 Question no. 33C

Comments: Maybe it was more challenging than expected judging by the very few attempts and no alphas.

Paper no. 3 Question no. 32C

Comments: Straightforward question.

Course: LOGIC & SET THEORY

Paper no. 1 Question no. 16H

Comments: Despite much of it appearing in previous years, this question wasn't as easy for students as expected. With so many parts to the question, the students were mostly able to provide enough work to obtain a beta, but many of them didn't seem to understand what they were really doing, leaving the more subtle steps unjustified or making logical errors. Many points lost were in the details and from lack of care.

Paper no. 2 Question no. 16H

Comments: Unsurprisingly, the students found parts (a) and (b)(i) of this question mostly straightforward, and struggled significantly on (b)(ii). It was also quite difficult to mark (b)(ii), and for similar questions in the future it seems reasonable to scaffold that part, asking first for an axiomatization of the problem, for example.

Paper no. 3 Question no. 16H

Comments: This question had low attempts, and there was no seeming pattern to which portion of the question that students found difficult. The question could use some tweaking to see more diversity in difficulty of the parts rather than a large collection of small medium-difficulty pieces.

Paper no. 4 Question no. 16H

Comments: I expected this question to be very easy, and there were a significant number of alphas, but the other half of the students attempting the question struggled quite a bit. Unsurprisingly, part (c)(iii) posed difficulties, but there were also many scattered issues, even including the details of the bookwork stating Zorn's lemma.

Course: MATHEMATICAL BIOLOGY

Paper no. 1 **Question no. 6B**

Comments: Unpopular question, what answers there were seemed very insecure on this material.

Paper no. 2 **Question no. 6B**

Comments: Straightforward question, many good attempts.

Paper no. 3 **Question no. 6B**

Comments: Very straightforward question, many good attempts.

Paper no. 3 **Question no. 13B**

Comments: Quite a number of very disappointing attempts on this question. Few were able to use the Fourier Transform to correctly solve the PDE in part (i).

Paper no. 4 **Question no. 6B**

Comments: Seemed to be a fair question, but part (iii) proved tricky.

Paper no. 4 **Question no. 14B**

Comments: Many poor attempts - few candidates appeared comfortable with this material.

Course: MATHEMATICS OF MACHINE LEARNING

Paper no. 1 **Question no. 31**

Comments: The manipulations in part (b) of this questions were, in hindsight, probably too difficult to memorise for an exam even though they were done in lecture. Many students did not go beyond part (a). Five students completed parts (a) and (b) almost perfectly and obtained betas. However, the generalisation in part (c) proved too difficult which is why there were no alphas in this question. In general the take up for Mathematics of Machine Learning was low relative to the number of people attending the lectures, probably because it was a new course without many model exam questions.

Paper no. 2 **Question no. 30**

Comments: Despite the low take up, I think this was an interesting question with a suitable level of difficulty.

Paper no. 4 **Question no. 30**

Comments: This question was perhaps on the long side, but the number of alphas and betas was near the target.

Course: NUMBER FIELDS

Paper no. 1 Question no. 20

Comments: The question was not very well done in general. Many candidates misquoted Minkowski's theorem.

Paper no. 2 Question no. 20

Comments: This question was badly done. Many candidates used Dedekind even though they were not supposed to, and hardly any were able to do (b) part (ii).

Paper no. 4 Question no. 20

Comments: This question was done better than other Number Fields questions this year. Many candidates disppointingly forgot μ_k in the statement of Dirichlet's Unit theorem.

Course: NUMBER THEORY

Paper no. 1 Question no. 1H

Comments: This question is a standard one for the course, and students who attempted the whole question did well. The average was brought down somewhat by a number of students picking up only the couple of points of bookwork in the first part of the question.

Paper no. 2 Question no. 1H

Comments: This question had a minor error (not explicitly disallowing rational angles) but it did not seem to affect any student's work. Most students were able to pick up marks from the bookwork (defining the convergents) and the computation of the continued fraction expansion at the end of the question; the betas were picked up by those students (just a few) who attempted to prove the inequality in the middle part of the question.

Paper no. 3 Question no. 1H

Comments: This question is significantly bookwork, and students picked up most of the marks on any part they attempted. However, many of them did not attempt the final part of the question, and a smaller number only picked up the initial couple of points from the definition in the first part of the question.

Paper no. 3 Question no. 11H

Comments: This was a good question for this course, and the students did well on most of it. As expected, there were struggles with the counting on the final part. It is worth noting that this question was very long - many of the students took six pages or more to answer the question, and a similar question in the future might benefit from slimming down.

Paper no. 4 Question no. 1H

Comments: I was surprised by the difficulties students had with this question, which seems utterly basic for NT and has occurred in various forms in many past exams. There was a lot of nonsense and (false) hand-waved conclusions in the attempts to prove the final part of the question.

Paper no. 4 Question no. 11H

Comments: The students did well with this question. Unsurprisingly, many found part (c) tricky, even with the hint, and this tended to separate the alphas from the betas.

Course: PRINCIPLES OF QUANTUM MECHANICS

Paper no. 1 **Question no. 34A**

Comments: The level of the question was appropriate and a satisfactory fraction of the students got an alpha. It was important to have the hint in part b on how to prove the equation.

Paper no. 2 **Question no. 34A**

Comments: The level of the question was adequate. Part a was quite straightforward but at the same time difficult to mark because students gave the most disparate set of derivation, with various assumptions, starting and even ending points. It was not easy to accurately attribute the points. Part b i went well although a perhaps suprizing fraction of students switched the excitation and the relaxation terms. The final part, b ii was very challenging and few students got full marks. Perhaps breaking it further down into two part would have helped both the students and make the marking easier.

Paper no. 3 **Question no. 33A**

Comments: Actually there were very few serious attempts (the number is artificially larger perhaps because some students tried to harvest a couple of points from the opening of the question and the abandoned). The last part of the question, about indistinguishable particles was probably too hard. Student did not seem to have been practicing that type of manipulations and tricks to extract symmetric and linearly independent states. This last part carried a proportionally large number of points, which, in combination with a few non serious attempts to the question, made the final average score very low,

Paper no. 4 **Question no. 33A**

Comments: The level was probably slightly high, but not too far off the mark. Perhaps a bit of guidance on how to approach the problem (e.g. the evolution equation for ρ) would have been appropriate and would have allowed students to collect a few more points from textbook material.

Course: PRINCIPLES OF STATISTICS

Paper no. 1 **Question no. 29**

Comments: This question was close to an example sheet problem and might have been a tad easy. However, a surprising number of students noted that the estimator had constant risk and tried to use, incorrectly, the characterisation of minimax rules as Bayes rules with constant risk.

Paper no. 2 **Question no. 28**

Comments: A fair number of students obtained betas, but there was only 1 student who completed the question in full. Some students made a reference to Wilks' theorem but no one pointed out why the theorem proved in lecture does not apply (composite null).

Paper no. 3 **Question no. 28**

Comments: This question had a suitable level of difficulty considering it's the first time this topic is covered in the exam.

Paper no. 4 **Question no. 28**

Comments: This question was of a suitable level of difficulty.

Course: PROBABILITY AND MEASURE

Paper no. 1 **Question no. 27K**

Comments: Overall okay, but part (d) too difficult.

Paper no. 2 **Question no. 26K**

Comments: Very appropriate question.

Paper no. 3 **Question no. 26K**

Comments: Appropriate question. A minor concern was that (c) (ii) is tedious to write out without adding any conceptual difficulty.

Paper no. 4 **Question no. 26K**

Comments: Very appropriate question.

Course: QUANTUM INFORMATION AND COMPUTATION

Paper no. 1 **Question no. 10C**

Comments: Probably more difficult than expected for a short question.

Paper no. 2 **Question no. 10C**

Comments: Very easy question.

Paper no. 2 **Question no. 15C**

Comments: Straightforward. Probably easier than expected.

Paper no. 3 **Question no. 10C**

Comments: Few attempts and few betas. Low overall performance.

Paper no. 3 **Question no. 15C**

Comments: Standard question. Marks well distributed.

Paper no. 4 **Question no. 10C**

Comments: Very easy question.

Course: REPRESENTATION THEORY

Paper no. 1 **Question no. 19F**

Comments: A lovely question, well done by those who attempted it.

Paper no. 2 **Question no. 19F**

Comments: A great question, again with few partial attempts. Correct answers here indicate genuine understanding of most of the course.

Paper no. 3 **Question no. 19F**

Comments: A more difficult question then the previous, well done by those that attempted it.

Paper no. 4 **Question no. 19F**

Comments: Similar level of difficulty to the previous question.

Course: RIEMANN SURFACES

Paper no. 1 Question no. 24F

Comments: This course is aimed at the top 2/3 of the First cohort; few attempted it. The questions were excellent.

Paper no. 2 Question no. 23F

Comments: As above.

Paper no. 3 Question no. 23F

Comments: Easy first half of question; surprisingly few attempts.

Course: STATISTICAL MODELLING

Paper no. 1 Question no. 5

Comments: This question was fairly easy but perhaps too long for a short question.

Paper no. 1 Question no. 13

Comments: This question was very appropriate for the course. Some of the attempts were nearly empty.

Paper no. 2 Question no. 5

Comments: It is surprising that the take up and scores were so low in this question as it was fairly easy and typical for the course.

Paper no. 3 Question no. 5

Comments: This question was very easy and any student attending the course should have been able to solve it.

Paper no. 4 Question no. 5

Comments: I don't think this question was too difficult for a short question, but there were no attempts, so it is difficult to judge.

Paper no. 4 Question no. 13

Comments: This question was wholly unsuitable for an exam, mainly because of the length. In actual fact, it was not difficult to obtain at least 15 points, but getting there required internalising some definitions which were unusual for the course, and more importantly, committing to a question that *looked* too difficult. Whilst the question is not harder than an average Section II question, I think questions for C courses (certainly Statistical Modelling) tend to be more straightforward.

Course: STATISTICAL PHYSICS

Paper no. 1 Question no. 36A

Comments: Appropriate level and results.

Paper no. 2 Question no. 36A

Comments: The level was again appropriate and the question was effective at distinguishing among students.

Paper no. 3 Question no. 35A

Comments: This question turned out to be too easy. The main issue was that the final parts of the question are very straightforward (solving an ode, etc) and so the students who started it managed to get almost full points. Perhaps a more challenging small last question for a handful of points would have allowed to differentiate better among the candidates.

Paper no. 4 Question no. 35A

Comments: Same as previous question. The level more on the easier side and was missing a more challenging final part to effectively rank the students.

Course: STOCHASTIC FINANCIAL MODELS

Paper no. 1 Question no. 30K

Comments: Appropriate question, no problem with marking.

Paper no. 2 Question no. 29K

Comments: Appropriate question, no problem with marking.

Paper no. 3 Question no. 29K

Comments: Overall appropriate question, though few correct answers for (b) (ii).

Paper no. 4 Question no. 29K

Comments: Appropriate question, no issues with marking.

Course: TOPICS IN ANALYSIS

Paper no. 1 Question no. 2H

Comments: Most students who attempted this question answered the question well and obtained full or nearly full marks. The hint for Part (iii) defining the auxiliary function may not have been needed.

Paper no. 2 Question no. 2H

Comments: The students seemed unprepared on this material. To my surprise, very few students even attempted the bookwork first part of the question.

Paper no. 2 Question no. 11H

Comments: This question went better than I expected - clearly the students realized the importance of this proof in their studies, and many picked up most of the points on the first part of the question. Very few, however, made a reasonable attempt at the final part of the question, and some who did failed to notice that they were working now with open sets (the point of the question).

Paper no. 3 Question no. 2H

Comments: The students struggled with this question, with a low number of attempts, and some of those only picking up the easy points at the beginning of the question. This was very surprising as it nearly identically matched a pass/fail question that they had done well on a few months before! I don't have an explanation for this disparity.

Paper no. 4 Question no. 2H

Comments: This question was well-pitched, and the second part distinguished the students who understood the concepts. Most attempts on the second part were either full marks, or completely absent or incorrect.

Paper no. 4 Question no. 12H

Comments: This question had a minor error in the directive at the end of the question (an incorrect index) but it was quickly corrected and didn't seem to affect any scores. The low scores are primarily explained by a lack of attempts on parts (b) (ii) and (iii) of the question. Given the low attempts, the students did not seem to anticipate that this material was significant.

Course: WAVES

Paper no. 1 Question no. 40B

Comments: Straightforward question, but only 2 attempts.

Paper no. 2 Question no. 39B

Comments: Straightforward question, but only 2 attempts.

Paper no. 3 Question no. 39B

Comments: The difficulty here seemed to be with the complexity of the stationary phase calculation.

Paper no. 4 Question no. 39B

Comments: Only one attempt, which was very good indeed.

MATHEMATICAL TRIPOS PART IA, 2019

COMMENTS ON QUESTIONS

Course: ANALYSIS I

Paper no. 1 Question no. 3E
Stating Bolzano-Weierstrass and the easy example in part (i) posed no problems. Part (ii) received a surprisingly large number of attempts from students who hadn't learnt the bookwork. The most common mistake was not to realise that the sequence needs to be bounded *below* in absolute value. Part (iii) was found more challenging than I expected, there being a clear division between students who saw what was going on, and those who made claims like "every unbounded real sequence must tend to infinity".

Paper no. 1 Question no. 4F
The bookwork part of this question was done poorly by many candidates. Many of the attempted proofs were completely wrong, in several cases attempting to use the ratio test. However, the calculations for the two examples were done well.

Paper no. 1 Question no. 9D
Generally well answered with most students embracing the mood of the question.

Paper no. 1 Question no. 10D
A significant number of students completed the bookwork then didn't look at the second half of the question. A shame, as it was very doable.

Paper no. 1 Question no. 12F
This question had only a small number of attempts and several of these did not go beyond defining the Riemann integral and dealing with the known function in (a). Part (b) was generally found most difficult.

Paper no. 1 Question no. 11E
The students generally did quite well in finding suitable inequalities to answer this question. The deduction of part (c) from the hint attracted answers of wildly different lengths, with some just quoting the Cauchy criterion, and others giving a page of explanation. It seems the hardest parts were finding the counterexample at the end of (b), which lots of students still managed, and the proof at the end of (c), which attracted some overly complicated answers.

Course: DIFFERENTIAL EQUATIONS

Paper no. 2   Question no. 1C
The challenge of this question was to remember the form of the particular integral needed and be able to carry through the algebra to find the required numerical coefficients. Mostly well done but many algebraic slips.

Paper no. 2   Question no. 2C
Generally well done but some lack of clarity on when it is possible to solve $(\mathbf{B} - \lambda\mathbf{I})\mathbf{u} = \mathbf{x}$ for $\mathbf{u}$.

Paper no. 2   Question no. 5C
A somewhat unfamiliar question and so relatively few attempts. Not many answers concluded that $\alpha = 2$. b) was generally well done but c) less so. Again there were errors carrying through the algebra to find the discrete and exact solutions consistent with the initial conditions.

Paper no. 2   Question no. 6C
The first part of the question was well done including finding the lowest order polynomials. Errors crept in when attempting the second part of the question. The majority of answers did not follow the 'hence' route instead taking the 'otherwise' route by posing a general 4th order polynomial and deriving conditions on all the coefficients (lots of errors in doing this).

Paper no. 2   Question no. 7C
This was a very well done question up until part e). Lots of errors taking the $a \to 0$ limit of t^* with 2 being the most common and incorrect answer despite this making no physical sense as noted by a few students.

Paper no. 2   Question no. 8C
Parts a) and b) were very well done although the phase portraits varied quite a bit in quality with some answers rotating things by 90 degrees for some reason. Some confusion also about how the special contour $H = 1/4$ fitted into the picture. Quite a few errors calculating the time in part c). This involves a partial fraction integral which was a common source of errors as was manipulating the resulting logarithm.

Course: DYNAMICS AND RELATIVITY

Paper no. 4 Question no. 3A
Bookwork on Tsiolkovsky rocket equation, with a simple problem seen in lectures on calculating the speed after half the mass has been burnt. On the whole it was very well done: perhaps too well done; the question wasn't a great discriminator (but then again candidates did get to demonstrate their knowledge, one of the stated objectives of Section A questions).

Paper no. 4 Question no. 4A
Quadratic drag. There are several routes to the final answer, which can take different looking forms, so the question as written wasn't the easiest to mark. Many candidates only partially completed the question. Several could find the time taken to fall in terms of the speed but stumbled to integrate once more to find time taken to fall in terms of the height.

Paper no. 4 Question no. 9A
Pendulum and corrections due to the gravitational field changing. Nearly everyone could conserve energy and solve the pendulum in a constant gravitational field. The rest of question was answered with varying success: candidates finding the Taylor expansion at the end particularly difficult.

Paper no. 4 Question no. 10A
The first part was bookwork about motion of a charge under a constant magnetic field and was well done. The second part is about equilibria under Coulomb repulsion. Candidates found the equilibrium point at the origin, but very few successfully showed that it was stable. In hindsight, an explicit instruction to calculate the Taylor expansion of the potential a small distance from the equilibrium point may have helped some. A few more managed to calculate the frequency of small oscillations, but on the whole the question was poorly done.

Paper no. 4 Question no. 11A
Relativity: constant acceleration up to relativistic speeds, time dilation and energy-momentum conservation. There were a few easy marks for writing dimensions of force and of electric field and checking the dimensions of the obtained speed were correct. Very few candidates obtained the speed as a function of time in order to proceed further. This unpopular question was poorly done on the whole.

Paper no. 4 Question no. 12A
This popular question on rotating reference frames and the Coriolis force was very well done on the whole, many of the marks being for bookwork. It was sometimes difficult to discern which direction the force pointed from the answers because they were drawn ambiguously.

Course: GROUPS

Paper no. 3 Question no. 1D
Bookwork was well done. Suprising number of students didn't know how many distinct conjugacy classes A_5 has.

Paper no. 3 Question no. 2D
This question was well answered.

Paper no. 3 Question no. 5D
A relatively easy question so I marked quite strictly. Careless marks lost due to sloppy, incomplete proofs.

Paper no. 3 Question no. 6D
Part (a) $C_3 \times C_9$ caused the most problems, with 4 and 9 being common answers.

Paper no. 3 Question no. 7D
Some people had cleary seen the second isomorphism theorem before. Finding a counterexample to the final part caused more issues than anticipated.

Paper no. 3 Question no. 8D
This question was set up differently to notes. Some students embraced the question, some reordered and reproduced notes, others did a mixture. All approaches were fine, as long as they were clear and correct.

Course: PROBABILITY

Paper no. 2 Question no. 3F

The most difficult part of this question turned out to be the bookwork proving that $\log(n!) \sim n \log n$, but there were still many good accounts of this proof. The problem part of the question, applying Stirling's approximation, was done very well overall.

Paper no. 2 Question no. 4F

This was identical to an example from lectures (22.2) with some numbers changed. However, many candidates believed incorrectly that the variables were independent, either through not considering at all the region on which the joint pdf was supported or alternatively through falsely asserting that it was rectangular.

Paper no. 2 Question no. 9F

This question was generally well done. A small number of candidates stated Bayes' theorem as $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$ rather than what I intended as given in lectures; on checking, I found some sources claiming the former as Bayes' theorem so I gave full credit for this. Incomplete attempts generally went wrong on the final problem part, but many of these were able to complete the earlier, easier problems and gain enough marks for a β.

Paper no. 2 Question no. 10F

This question was also well done. The two most common flaws in solutions were (i) omitting the case $p = \frac{1}{2}$ in the bookwork; and (ii) failing to produce a valid argument for monotonicity of the function needing to be optimized for the problem part.

Paper no. 2 Question no. 11

This question was by far the least popular on this course, with only 30 attempts. Many of the attempts were good, but there were also several candidates who believed that any sum of normal random variables is normal and/or that if X and Y are normal then (X, Y) is bivariate normal.

Paper no. 2 Question no. 12

The proofs of the expectation and variance formulae and calculation were very well done. The part found most difficult was the application of Chebychev at the end, but around half of the attempts still managed to do well on this part.

Course: VECTOR CALCULUS

Paper no. 3 Question no. 3B

This question was found straightforward by most candidates. Most of those that did not make it to a β failed on the evaluation of the surface integral.

Paper no. 3 Question no. 4B

I was a bit surprised that most candidates identified a function f such that $df = u\,dx + v\,dy + w\,dz$ rather than setting out conditions on u, v and w, but I think that this was because they had tried something like integrating $v(x, y, z)$ with respect to y, realised that there was a simple answer and then verified consistency with u and w. A significant number of candidates presented a function f, but then proceeded to evaluate a path integral, rather than simply calculating the change in f along the path.

Paper no. 3 Question no. 9B

A disappointing number of good solutions. Many that were less good lost marks because of a poor explanation of the rule for change of integration variables, e.g. some contained no mathematical argument at all. For the last part of the question two or three different choices of change of variable were possible, with each working fairly straightforwardly if applied correctly. But candidates were often very imprecise about defining the domain in the space of new variables over which the integration had to be performed and it was therefore difficult for them to make further progress with the calculation.

Paper no. 3 Question no. 10B

Few attempts on this question and the number that obtained substantial credit was very small indeed. This might have been a general lack of confidence with tensors, but certainly the problem element in the second half of the question needed significant insight and probably looked daunting to many candidates. Of the candidates who did reach the second half of the question, many then failed to combine productively change of co-ordinate axes and change of integration variable.

Paper no. 3 Question no. 11B

There were a large number of attempts, many complete or close-to-complete. Some candidates failed to find $\mathbf{A}(\mathbf{x})$, which was slightly non-standard, but which should have been straightforward given the vector identity at the beginning of the question. Others seemed confused about the application of Stokes theorem to a curve spanned by two surfaces which taken together form a closed surface, and deduced that the integral along the curve must be zero, in some cases then modifying incorrectly their evaluation of the integrals to be consistent with this.

Paper no. 3 Question no. 12B

This was another relatively popular question. Most answers were good on (a) and the first part of (b). Candidates who lost several marks typically did so in deriving the solution to the Poisson equation, e.g. incorrectly deducing that ϕ was constant in $b < r < a$ or failing to apply the correct continuity condition on the solution at $r = b$. These errors then almost inevitably led to problems in the final part of the question.

Course: VECTORS AND MATRICES

Paper no. 1 Question no. 1C
Quite a few silly mistakes on a) and b). c) was generally well done although claiming $\log i = \pi/2$ was a common mistake but d) was very well done.

Paper no. 1 Question no. 2A
A bookwork question about determinants of square matrices, mostly well done. There were several different methods/notations (permutations/Levi-Cevita symbol/antisymmetric product of columns), all of which were accepted, if valid.

Paper no. 1 Question no. 5C
This was a very straightforward question which was generally very well done. The hardest part appears to have been part b).

Paper no. 1 Question no. 6B
Answers to the first part on demonstrating the rule for the change in the matrix representing a linear transformation under a change of basis were variable in quality and in approach. Sometimes the logic wasn't clear, or was simply hopeful. But there were good answers too. The second part was designed so that calculating the inverse of the matrix P was not required, but many people did that. Some answers incorrectly assumed that basis vectors must be mutually orthogonal.

Paper no. 1 Question no. 7B
Most answers indicated decent understanding of this topic. Marks were lost because of vagueness of lack of precision in logic, e.g. only showing that eigenvalues were zeros of the characteristic polynomial and not vice versa. This particularly applied to part (c) regarding the reasoning in going from the hint to the required result.

Paper no. 1 Question no. 8A
On exponentials of matrices (inspired by Lorentz transformations and rotations). After advice from the lecturer, the question was made easier. An extremely common mistake in the 3 by 3 cases was to miss the identity term, yielding a row of zeroes. Some candidates wrote $\cosh(x)$ and $\sinh x$ in terms $\sin(ix)$ or $\cos(ix)$, which was also deemed acceptable. In hindsight, a harder element to this question would make it a better discriminator.

MATHEMATICAL TRIPOS PART IB, 2019

COMMENTS ON QUESTIONS

Course: ANALYSIS II

Paper no. 1 Question no. 11E

Overall as expected. Bookwork well executed, with occasional sloppiness in proof of chain rule with $o(||h||)$-type considerations. Unseen part: some carelessness with manipulations of power series / convergence issues; some confusion with derivative of r_C (needs right multiplication), and with application of the chain rule. Direct attempts ('otherwise' rather than 'hence') typically either very poor or excellent.

Paper no. 2 Question no. 3E

Somewhat dissapointing: lots of candidates ignored the instruction to 'stat[e] accurately any result that [they] require' – almost systematic for part (a), and for part (b) it was common to cite 'Inverse Function Theorem' without statement (small inaccuracies, e.g. C^1 hypothesis omitted, also common). Many candidates also disregarded 'continuously' in (a). Some issues with Df vs $(Df)^T$.

Paper no. 2 Question no. 12E

Overall as expected. (a) (i): surprisingly common to omit $a > 0$ for the smaller Lipschitz constant; (a) (ii): piece of bookwork they found most difficult, overall well executed; many forget to check $a > 0$; some 'chicken and egg' type confusions (e.g. asserting that the unit sphere for an arbitrary norm is compact). Unseen section generally well executed, small range of commonly chosen examples.

Paper no. 3 Question no. 2E

As expected. When a candidate argued that an example was uniformly continuous by splitting domain into adjacent two closed intervals with no open overlap, I decided not to subtract points because of 'briefly'. Some common mistakes on (ii) (lack of differentiability at 0 does not mean that it's not uniformly continuous) and on (iii) (don't spot that the interval starts at 1), though slightly more than I was expecting on (ii), and slightly fewer on (iii).

Paper no. 3 Question no. 12E

We strongly expected this to be the candidates' least favourite question, which turned out to be the case. The lecturer, checker and I agreed when setting the question that it is not in fact because the question is harder (if anything, it requires slightly less work than the other long ones), but rather tied to the general lack of popularity of that section of the course, exacerbated by the fact that the Picard-Lindelöf theorem is seldom examined. This is witnessed by the fact that as expected, comparatively few candidates could give a correct statement of the theorem. (b) (i) was fine; some weak scripts only answered that part, for a few points; some otherwise strong scripts didn't give the max and min of t^{-c} on $[1, b]$. (b) (ii) was generally reasonably attempted by those who chose to do so; common errors include confusion over which norms to use to use where, and in manipulating inequalities; omitting or failing to explain why the Contraction Mapping Theorem applies; and carelessness in concluding (which requires the FTC).

Paper no. 4   Question no. 3E
As expected. The first two parts were almost universally well executed; some slightly shaky arguments for (ii) (logical confusion: *if* the functions converge uniformly *then* they must converge pointwise to the same limit) and some slightly overly terse ones for (iii) (e.g. sketching a sequence of functions, not quite spelling out uniform convergence).

Paper no. 4   Question no. 12E
Most popular of the long Analysis II questions and very well executed , despite requiring relatively large volumes of writing. This is as expected, and typical of Analysis II questions on material which overlap with Met&Top (this is the final year that the overlap will be a concern, though I would expect that section of material to remain popular). All parts were overall well executed; I was comparatively strict with partial credit, including regarding omissions or unproved assertions. I decided to accept both definitions of compactness (the one given in Analysis II is sequential compactness, but Met&Top muddies the waters). The hint (which was added at one of the proof-reading / feedback stages, I can no longer figure out which one) ended up being little used, and also confused some candidates, and in hindsight would have been better omitted. The most significant conceptual errors were in (a) (ii), typically when the example given was not complete, or by asserting that 'closed and bounded implies compact' is true for arbitrary topological spaces; and in (b) (ii), most eggregiously when candidates picked a single value of ϵ for a finite cover of X by ϵ-balls, fixed a subsequence lying in one such ball, and then asserted, taking $\epsilon \to 0$, that that same subsequence was Cauchy. For (b) (iii) it was harder to spot that the space wasn't complete rather than that it wasn't totally bounded; perhaps there would have been merit in asking explicitly for the former (though I'm personally ambivalent about that).

Course: COMPLEX ANALYSIS

Paper no. 3 Question no. 13F

The problem part of this question had some good attempts, but not many candidates took the approach the examiner had intended. Many stated, and sometimes proved, Rouché's theorem, but often applied it to functions which violate the required inequality. A few candidates showed that the polynomial has two negative real roots, and used this to deduce that it has the required two roots in the right-hand half plane.

Paper no. 4 Question no. 4F

This piece of bookwork was reasonably well done. Marks were usually lost by stating the Cauchy Integral Formula imprecisely (e.g. without any hypotheses), or by passing too carelessly from information on a circle $|z - z_0| = \rho$ to information on $D(z_0; r)$.

Course: COMPLEX ANALYSIS OR COMPLEX METHODS

Paper no. 1 Question no. 2F

This question was unpopular and badly done. Most candidates could not accurately define Laurent series (for example, forgetting to mention that it should converge on the annulus). Many candidates tried to state their answer as "the product of the Laurent series of $\frac{10}{z+2}$ and of $\frac{1}{z^2+1}$", which is not a Laurent series, or tried to multiply this product out, which usually ended in chaos. Relatively few candidates used partial fractions, and those that did usually made work for themselves by going as far as $\frac{1}{1+z/2} + \frac{2+i}{1-iz} + \frac{2-i}{1+iz}$ rather than stopping at the wiser $\frac{1}{1+z/2} + \frac{4-2z}{z^2+1}$.

Paper no. 1 Question no. 13F

A surprising number of candidates wasted their time by proving Jordan's lemma twice, once under the assumption $|zf(z)| \to 0$ where it is a triviality, and again under the "more sensitive" assumption $|f(z)| \to 0$. They did not seem to notice that the second case implies the first.

Many candidates made a huge deal of showing that $\mathrm{res}_a(\frac{g(z)}{(z-a)^k}) = \frac{g^{(k-1)}(a)}{(k-1)!}$, and almost invariably wrote their answer as $\lim_{z\to a} \frac{1}{(k-1)!} \frac{d^{k-1}}{dz^{k-1}} g(z)$, for some reason worried about evaluating this function at $z = a$.

The integral was generally well done, but marks were typically lost by not being able to compute the derivative of $\frac{z^3 e^{iz}}{(z+1)^2}$ accurately, or to evaluate it at $z = i$.

Paper no. 2 Question no. 13D

The bookwork part of the question, on conformal maps, was largely done well. There were a wide variety of approaches to proving this, perhaps reflecting the fact that candidates could attempt this question having attended different lecture courses. In the second part of the question, marks were often lost by either not showing that the map was 1:1, or not showing it was conformal on each of the two regions. In the final part, many candidates realised they could construct the required conformal map by composing the map $h : z \mapsto 1/z$ with the map given in the earlier part of the question. However, many candidates did not check that the composition extends over the origin (required as part of the unit disc), where h itself is not defined.

Course: COMPLEX METHODS

Paper no. 3 Question no. 4D

Most candidates successfully found that the given transformation mapped the unit disc to the uhp, and found the appropriate boundary conditions on ϕ along the $\mathrm{Re}(w)$ axis. Many candidates then struggled to find an appropriate solution to Laplace's equation, with the most common incorrect answer being along the lines of $\phi_0 \,\mathrm{sgn}(\mathrm{Re}\, w)$. Credit was awarded for expressing $(x, y) = (\mathrm{Re}\, z, \mathrm{Im}\, z)$ in terms of $(u, v) = (\mathrm{Re}\, w, \mathrm{Im}\, \mathrm{w})$, even if this was done in the wrong form of solution (or no solution).

Paper no. 4 Question no. 14D

A popular question, but one with which candidates often struggled. A common error was in not distinguishing between the cases $t > 0$ and $t < 0$ when taking the inverse Laplace transform. In the later parts of the question, candidates often got bogged down in the algebra required to match their solution to the given initial conditions. I had hoped that giving them a final result to aim for would help motivate them to get through this, admittedly rather involved calculation, but few candidates made it all the way to the end.

Course: GEOMETRY

Paper no. 1 Question no. 3E
Largely as expected, perhaps slightly dissapointing. A flurry of small errors, both conceptual (D and its isometries), and computational (integration errors).

Paper no. 2 Question no. 14E
Slightly better than expected (from perhaps a pessimistic baseline). Lots of small inaccuracies or omissions in defining a smooth embedded surface. Surface sketch: most recognised a torus (with a number of mistakes at that stage though), with the sketch itself graded generously. Parametrisation: dissapointingly many failed to give bounds, or cover the surface. The Gaussian curvature calculation is significantly simplified by noting the first fundamental form is in geodesic polar form; either way the relevant sleugh of formulas is typically well cited, with various algebraic errors, but no more expected (candidates who had written down an incorrect parametrisation were not penalised at this stage). Final part: following the hint, most found the transformation (or its transpose, a common subtle mistake), though a non-trivial fraction didn't understand its significance (isometry, not just a diffeomorphism).

Paper no. 3 Question no. 5E
Comparatively popular and well executed, as expected. Common mistakes / omissions: failing to note that all values in range are attained (e.g. by IVT); allowing concave angles; answering the question in the hyperbolic world instead.

Paper no. 3 Question no. 14E
As anticipated: confirmed the expectation, shared with the lecturer and the checker, that the abstract smooth surface section often attracts only a comparatively cursory interest (as witnessed by the number of evidently made-up definitions of geodesic triangulations), but that questions involving the combinatorics of Euler numbers are relatively popular – with the latter trends slightly edging out the former one for this question. Moderate number of small algebra mistakes in the unseen part (which don't always affect the main conclusion).

Paper no. 4 Question no. 15E
Extremely dissapointing, both regarding number of attempts and quality thereof. This is especially true of the bookwork component, which was on average very poorly executed, despite being a core result of the hyperbolic geometry section of the course, and been broken up into steps. In particular, the number of students able to show that elements of $PSL(2, \mathbf{R})$ act by isometries strikes me as alarmingly low; and in latter stages of the bookwork proof it was often clear that the candidate was muddling through without feeling fully confident that they understood the proof they were nonetheless trying to reproduce. I suggest that this be brought to the attention of future lecturers for the (reformed) course. Compared with the bookwork, the unseen components (parts (c) and (d)) were relatively well executed, and I also chose to grade them comparatively leniently (which e.g. the instruction 'explain why' readily lends itself to anyway).

Course: GROUPS RINGS AND MODULES

Paper no. 1 Question no. 10G

The yearly question on Sylow's theorems is always popular, and this was no exception. Despite being fairly standard, this seems to have been at about the correct level of difficulty. The most common mistake candidates made was to try to reach a contradiction by counting elements of the group – it's difficult to make that argument work in this case. In fact, Sylow's third theorem gives arithmetic constraints on the numbers of p- and q-Sylow subgroups, which almost immediately lead to a contradiction.

Paper no. 2 Question no. 2G

This question was very close to a problem from an example sheet, simply rephrased to make it about a universal property. It must have looked unappealing, as it had the lowest uptake of any GRM question, but the candidates who did attempt it saw through it quickly, leading to a high success rate.

Paper no. 2 Question no. 11G

This question was based on some material that may have been familiar to students who had read ahead (splitting fields and finite fields), which may explain both the low take-up and the fairly high success rate (more than half of the attempts lead to α's). Candidates lost credit for asserting that $\binom{p^k}{i}$ is divisible by p for $0 < i < p^k$ without justification.

Paper no. 3 Question no. 1G

This question ought to have been completely routine. However, almost all attempts simply asserted that the given ideal was proper, without justification. These attempts lost marks (which explains the slightly low average mark), but still merited a β if they didn't make any further mistakes.

Paper no. 3 Question no. 11G

This question proved to be a good test of students' knowledge, seeing widely varying standards of attempts. Students with a strong grasp of the course produced some very creative answers.

Paper no. 4 Question no. 2G

This is a standard exercise in group theory, which had already appeared as part of a starred question on an example sheet. But it seems to have been slightly on the difficult side, although it did garner a decent number of attemtps.

Paper no. 4 Question no. 11G

The final part of this question admitted an (unintended) very short solution – since the minimal polynomial has a zero root, any such matrix must have zero determinant. But only a handful of students spotted it, and the question proved to be a good test of students' knowledge.

Course: LINEAR ALGEBRA

Paper no. 1 Question no. 1F

Some candidates think that bases need to be finite or countable. Some used notions (such as "dimension") to which this statement is logically prior, making their argument circular. Some candidates sparked joy by using Gaussian elimination rather than the Steinitz Exchange Lemma.

Paper no. 1 Question no. 9F

Candidates who were able to get over the unfamiliarity of this question usually found that it was not very hard. There were some very clever solutions.

Paper no. 2 Question no. 1F

The bookwork was usually well done, with only mistakes of logic in showing that the proposed basis of $U + W$ was linearly independent. While most candidates were able to show that $U \cap W$ is spanned by $(7, -5, 1, 0)$, much fewer were able to accurately produce $\ell = x_1 + 2x_2 + 3x_3 + 4x_4$. No candidates seemed to realise that it is precisely the same kind of problem, but in the dual space.

Paper no. 2 Question no. 10F

Parts (a) and (b) were usually well done, and (b) was sometimes done quite creatively. Part (c) was too easy. In part (d) candidates often forgot to carefully state results they used, but often made at least partial progress.

Paper no. 3 Question no. 10F

This question was surprisingly popular. Quite a few candidates misread the second part and took R to be the *maximal* subspace with this property. Used as they are to inner products, in the last part most candidates did not realise that $V = \mathrm{span}(e, f) \oplus \mathrm{span}(e, f)^{\perp}$ is a statement that requires proof: it does, and the crucial point is that $\varphi|_{\mathrm{span}(e,f)}$ is nonsingular. This is actually more involved than showing that the signature of $\varphi|_{\mathrm{span}(e,f)}$ is zero, and carried more marks.

Paper no. 4 Question no. 1F

Many candidates felt compelled to produce *explicit* bases of eigenvectors, taking them down a rabbit-hole of cases and calculations for which they received no marks.

Paper no. 4 Question no. 10F

The first three parts of this question were well done in general, but the last part less so. Most candidates did it by multivariable calculus, rather than by stating—or better deriving—the fact that the point $\alpha(v_0)$ on a plane $\mathrm{Im}(\alpha)$ closest to a given point w_0 occurs when $w_0 - \alpha(v_0)$ is orthogonal to $\mathrm{Im}(\alpha)$. Candidates who did it by multivariable calculus usually forgot to show that such a v_0 indeed *minimised*, or that this was an if and only if.

Course: METRIC AND TOPOLOGICAL SPACES

Paper no. 1 Question no. 12G

Both Met & Top long questions were very well done. It is, after all, a relatively elementary IB course. The lesson is that an examiner needs to be careful to set particularly tricky questions. But the lesson is moot, since this is the last year that Met & Top will be examined! (The original proposal for this question was significantly harder, but was judged too difficult by one of the lecturers.)

Paper no. 2 Question no. 4G

This question was bookwork, and was well done as one might expect. For full credit, candidates were expected to prove that an interval is connected. In retrospect, it might have been clearer to ask candidates to prove from first principles that a subset of the reals is connected if and only if it is an interval, and to deduce the Intermediate Value Theorem.

Paper no. 3 Question no. 3G

Considering this question was bookwork, it was surprisingly poorly done. Many otherwise correct answers were vague about why, if a sequence (x_n) has no convergent subsequence, every point of the space has an open neighbourhood that contains only finitely many x_n. However, these only lost one mark.

Paper no. 4 Question no. 13G

Both Met & Top long questions were very well done. It is, after all, a relatively elementary IB course. The lesson is that an examiner needs to be careful to set particularly tricky questions. But the lesson is moot, since this is the last year that Met & Top will be examined! For full credit on this question, the answer had to include a proof that a compact subspace of a Hausdorff space is closed.

Course: MARKOV CHAINS

Paper no. 1 Question no. 20H
This question had a decent number of attempts and a good spread of marks. Students had little difficulty with part (a). Those who remembered the formula for the expected return time in terms of the invariant distribution did well on part (b). There were a number of good starts on part (c), but many were not careful in their justification of the relationship between the time it takes the cop to catch the robber and their answer in part (b).

Paper no. 2 Question no. 20H
This question had a good number of attempts. Many students were correctly able to derive and use the formula in the hint for (b), though some made computational mistakes in their application of the formula. Students who made it to (c) had some difficulty deriving the analogous formula for the expected time to go from v_i to v_{i+1}. Some accidentally solved the problem with $n = 5$ (i.e., the case illustrated) instead of for general n but lost essentially no marks for this.

Paper no. 3 Question no. 9H
This question had a good number of attempts and most had little difficulty with it.

Paper no. 4 Question no. 9H
This question had a lower number of attempts but many of them were good. Most students did not make use of the hint and gave a different proof for part (a).

Course: OPTIMISATION

Paper no. 1 Question no. 8H
Although this was a theorem which was stated (but not proved) in the notes, this question had very few attempts.

Paper no. 2 Question no. 9H
This question had a good number of attempts. Many students did not correctly use the Lagrange sufficiency theorem to deduce the optimality of their solution.

Paper no. 3 Question no. 21H
This question had a good number of attempts. Most students had little difficulty with the bookwork. Students who got α's were thus often differentiated by their ability to apply the simplex algorithm correctly.

Paper no. 4 Question no. 20H
This question had a good number of attempts and was on the easy side. It was perhaps "too standard".

Course: STATISTICS

Paper no. 1 Question no. 7H

This question had a good number of attempts and spread of marks. Would have expected students to have performed even better given that this is perhaps the most standard example of an MLE being a biased estimator.

Paper no. 1 Question no. 19H

This question had a good number of attempts. It turned out to be on the easy side and marks were high. It was perhaps "too standard".

Paper no. 2 Question no. 8H

This question had a good number of attempts. Marks were low in part because many students gave a *random* answer for the expectation of $\widehat{\theta}$.

Paper no. 3 Question no. 20H

This question had a good number of attempts. Many students did not give proper justification as to why S_{XX} was σ^2 times a χ^2_{n-1} random variable (incorrectly arguing that a χ^2_n minus a χ^2_1 was χ^2_{n-1} without being careful about how the random variables were related). Also, a number of students gave half-infinite confidence intervals when it was not necessary.

Paper no. 4 Question no. 19H

This question had a good number of attempts. Students had little difficulty with the bookwork components (parts (a) and (b)). Many students tried to set part (c) up as an optimization problem with Lagrange multipliers which complicated their solution. Most who tried the question this way were not able to see it through and some inadvertently performed the optimization *inside* of the expectation. Fewer students than expected recognized that the expectation could be computed using the formula for the moment generating function for the Gaussian distribution.

Course: ELECTROMAGNETISM

Paper no. 1 Question no. 16A
This question was answered well by the majority of candidates, who showed good understanding of Gauss's Law and how to apply it. The sketching of E and ϕ was generally poor, however, and the interpretation asked for at the end produced a range of spurious comments. Overall, this was a fair test of largely standard material.

Paper no. 2 Question no. 6A
This short question on scalar and vector potentials and their far-field behaviour was entirely bookwork but it did not attract many attempts. Those who tried it did reasonably well.

Paper no. 2 Question no. 18A
A fairly standard kind of question on Faraday's Law and the resulting force on a conductor carrying an induced current. With most of the key answers given, clear explanations were essential to gain full credit and many candidates lost marks, to varying degrees, due to errors in signs (some seemed unperturbed at establishing a constant *upward* velocity). On the whole, the question was done well.

Paper no. 3 Question no. 17A
Candidates answered this question very well, showing assurance and fluency in applying Lorentz transformations to electric and magnetic fields. Most attempts got close to an alpha, but a few were held back by unclear and incomplete arguments for the final part.

Paper no. 4 Question no. 7A
This should have been straightforward, but most answers were over-complicated (deriving the wave equation for $\mathbf{E}$ and $\mathbf{B}$ wasn't necessary) and a bit confused: candidates failed to follow the steer given in the question, which would have helped in establishing a clear chain of implications. Very few seemed to appreciate that taking real parts was necessary before calculating the Poynting vector, and the interpretations offered tended to be imprecise.

Course: FLUID DYNAMICS

Paper no. 1 Question no. 5
A standard problem on parallel viscous flow with a slight extension calling for physical interpretation. Most of the serious attempts were substantially correct.

Paper no. 1 Question no. 17
A relatively difficult problem on irrotational flow with circulation, which had been covered in lectures. This produced a good number of excellent answers. A few candidates confused flow past a cylinder with flow past a sphere.

Paper no. 2 Question no. 7
A simple example of a 2D cellular flow. Some candidates thought the vorticity was zero and confused the streamfunction with the velocity potential. The last part of the question, requiring a sketch of the streamlines, seemed unexpectedly difficult for many.

Paper no. 3 Question no. 18
A question on interfacial waves in a cube, effectively combining two problems from an example sheet. Quite a few candidates didn't note that the pressure needed to be matched at the interface, and the gravitational term in the Bernoulli function often came with a sign error. Nevertheless, a good number of excellent solutions were forthcoming.

Paper no. 4 Question no. 18
A fairly easy question on geophysical fluid dynamics. The manipulation and solution of the differential equations was very straightforward for many, but relatively few candidates gave a good explanation of the assumptions behind the equations. Sketches of the solutions often had (wrongly) non-monotonic behaviour of $\eta(x)$ or a discontinuity in $v(x)$.

Course: METHODS

Paper no. 1 Question no. 14B
Most understood it was easiest by far to use Cartesians in the first part and then change to polars, and that it was acceptable to quote the formula for the Laplacian in polars. Various algebraic mistakes led to errors in the terms of the series expansions. This question was suitable for Section II, although a shorter version had been considered for Section I.

Paper no. 2 Question no. 5B
Most could quote the powers of r associated with Legendre polys, and could manipulate $\cos 2\theta$ in terms of Legendre polys. Some unnecessarily used P_1. The slightly awkward algebra defeated some, but this question was otherwise straightforward.

Paper no. 2 Question no. 16D
A popular question that was done very well by many candidates. Perhaps the most common omission was a proper explanation of why $(a, b) = (-1, 1)$. Many candidates seemed to guess this domain and assert that the operator would then be Sturm–Liouville (maybe through familiarity with the similar / special case of Legendre polynomials), rather than show this by a careful integration by parts. The final part of the question was often done well.

Paper no. 3 Question no. 7D
A disappointingly low number of attempts at this question, probably because most candidates assumed that Discrete Fourier Transforms would not 'come up'. This is despite it being in the schedules, in the lecture notes and on this year's problem sets.

Paper no. 3 Question no. 15D
This question was popular, but often found to be rather challenging. In the first part of the question, candidates were usually happy to differentiate the Heaviside step function. Most candidates rushed to differentiate twice, rather than consider simplifying the result of the first differentiation, and then had to consider integrating the resulting combination of $\delta(x)$ and $\delta'(x)$ against a test function to see that the ode was indeed solved. On the second part of the question, I was pleased both that candidates usually took the Fourier transform correctly, and that they spotted the connection between the resulting ode and the first part of the question. I was somewhat disappoitned that few candidates could take the (inverse) Fourier transform of $\sin(k|\mathbf{x}-\mathbf{y}|)$ wrt k; perhaps they were put off by the presence of $|\mathbf{x}-\mathbf{y}|$. Those candidates who made it through to the final part of the question usually did this part well.

Paper no. 4 Question no. 5D
The candidates who went ahead and manipulated the suggested integral usually arrived at the correct result $\lim_{\epsilon\to 0} g_\epsilon(x) = \delta'(x)$, with varying degrees of rigour in their justifications. However, several candidates seemed to decide in advance that the limit of the sequence 'must' be the Dirac δ-function. More marks were lost for getting the wrong type of distribution than for small numerical errors.

Paper no. 4 Question no. 17B
(a) Almost everyone could show self-adjointness for the given condition, but few did the integrations by parts concisely. (b) Many could do this, using (y, Ly), but a few got stuck considering the differential equation alone. (c) Most could find the general solution of the equation, and impose the boundary conditions, though the algebra was awkward if the hint was ignored. Many lost 1 mark by sketching $\tan x$ as having lesser slope than $\tanh x$ at $x = 0$.

Course: NUMERICAL ANALYSIS

Paper no. 1 Question no. 6
A bookwork question on divided differences. Generally well done.

Paper no. 1 Question no. 18
The bookwork part on multistep methods was generally well done. Most candidates could quote the Dahlquist equivalence theorem and apply it to the Adams–Moulton method. Curiously, some did not use the result of part (a) to determine the order of the method, but used an independent technique.

Paper no. 2 Question no. 19
A fairly standard problem on linear least squares and QR factorization. This produced many good answers. Candidates who calculated a 'reduced' QR factorization rather than one of the requested form could still obtain an alpha.

Paper no. 3 Question no. 19
A very popular question on orthogonal polynomials. Most candidates knew how to manipulate the inner products to show orthogonality, but there were logical flaws in some solutions, perhaps because the question was formulated in a different way from the lecture notes. Many candidates struggled to deduce the recurrence relation for Hermite polynomials from the Rodrigues formula using the Leibniz rule; a few found alternative methods.

Paper no. 4 Question no. 8
A straightforward question on LU factorization, which produced many correct solutions. The final answer needed to be correct to obtain a beta (except for some trivial errors such as obtaining the right answer and then copying it incorrectly). Candidates could easily have (and should have) checked their answers.

Course: QUANTUM MECHANICS

Paper no. 1 Question no. 15B
Popular, and well done, but answers got long if based on a general analysis of the energy eigenfunctions, rather than using the stated form of f as a polynomial of degree N. The logic about even parity was sometimes unconvincing, or relied on a general theorem that was not proved.

Paper no. 2 Question no. 17B
There was no question this year on the 3D harmonic oscillator or Coulomb problem, but this question was related. Many got to grips with the L_z eigenvalues and eigenstates, or at least some of them, and correctly quoted the range of L_z eigenvalues for given $\mathbf{L}^2$. But hardly anyone realised that there had to be a complete multiplet of five states with $l = 2$ hidden in the list given here. A few picked out the final combination with $l = 0$. Lecturers should stress that spherical symmetry implies an energy level has complete angular momentum multiplets.

Paper no. 3 Question no. 8B
Many did this well, although some slipped up calculating the commutator of operators with explicit functions multiplying single derivatives.

Paper no. 3 Question no. 16B
This was the Fourier series question on this year's exam, as the Methods questions didn't have one. It was surprisingly well done, despite the various and quite tricky integrals. The final probabilities were not given, so a range of results emerged. Many did not accurately convert their set of stationary states into the general, normalized non-stationary wavefunction, but the later part of the question didn't rely on this.

Paper no. 4 Question no. 6B
This looked both short and straightforward. (a) Most could show the current conservation, but many did not correctly derive its consequences for normalized and unnormalizable stationary states. (b) Most knew what they should calculate, but several were let down by algebraic mistakes, for example, not making use of the conjugate of the wavefunction.

Course: VARIATIONAL PRINCIPLES

Paper no. 1 Question no. 4A
A straightforward test of a standard method (Lagrange multipliers). The problem required only simple algebra, but it needed to be worked through carefully and systematically to avoid missing solutions (by dividing by something that might vanish, for instance) and a surprising number of candidates were rather careless in this respect.

Paper no. 2 Question no. 15A
This was a popular question and most attempts showed a good understanding of first integrals and how they could be combined to find an expression for dw/du. Far fewer were successful with the last part, however, resulting is a relatively low alpha rate. The hint was intended to reduce work and save time, but most candidates approached this too simplistically, trying to use the hint immediately rather than taking some easy preliminary steps to arrive at an integral of the given form.

Paper no. 3 Question no. 6A
A routine question with a standard example and application. Some slight subtlety was involved in determining the full domains of definition of f^* and $(f^*)^*$, and few did this completely correctly, but that by itself didn't prevent them from reaching the threshold for a beta.

Paper no. 4 Question no. 16A
This question worked well, on the whole. A number of people derived the first and second variations for a general lagrangian and then (inexplicably) failed to evaluate these for a lagrangian of the specific form given. Otherwise, most answers revealed a clear grasp and solid understanding of the material.

MATHEMATICAL TRIPOS PART II, 2019

COMMENTS ON QUESTIONS

Course: ANALYSIS OF FUNCTIONS

Paper no. 1 Question no. 23H

This was a relatively easy question. Most attempts were either very short or good enough for an α. The proof of the monotone convergence theorem was done well by almost all who attempted it.

Paper no. 3 Question no. 22H

This question had very few attempts, although I think this is more due to the course's reputation than to the actual difficulty of the question. The bookwork on the Banach-Steinhaus theorem tended to be sloppy but got the right idea across, and two out of three attempts gave a reasonable solution to the problem part.

Paper no. 4 Question no. 23H

This question was clearly too difficult, since there were no substantive attempts. (One candidate wrote down the statement of Banach-Steinhaus, without explaining how it could be used.)

Course: AUTOMATA AND FORMAL LANGUAGES

Paper no. 1 Question no. 4H
A good question, perhaps a little on the easy side. The three problem parts were generally done well. Answers to the last part tended to be much less detailed than the model answer, while still conveying that the candidate knew what to do.

Paper no. 1 Question no. 12H
A good question with a broad range of responses. The bookwork was more challenging than expected, although there were a number of good solutions. As with 4H, most solutions were substantially less detailed than that envisioned by the model answer. (Some candidates gave tautological responses, which I did not give much credit to.) The problem part of the question was generally done well.

Paper no. 2 Question no. 4H
A good question, perhaps a little on the easy side. Almost all candidates did well on the bookwork. The rider at the end attracted a variety of responses, but was worth so little it was virtually useless as a discriminator.

Paper no. 3 Question no. 4H
This was a straightforward question with many attempts, almost all of which resulted in betas. It might have been good to make the problem part a bit more complicated.

Paper no. 3 Question no. 12H
This was a good question. The last three parts of were done well by almost all candidates. The proof of Rice's theorem was more difficult. The most typical error was to write down the wrong function at the beginning of the proof, after which it is difficult to recover.

Paper no. 4 Question no. 4H
This question was more difficult than I expected. Errors were pretty much equal opportunity across the first three parts. Some candidates misread part 1a) and thought that any word of length ≥ 2 would be accepted.

Course: CODING AND CRYPTOGRAPHY

Paper no. 1 Question no. 3G
This question was either done very well, or not very well at all, i.e. there were no marks in the middle ground. Those who attempted the proof of Fano's inequality generally did it correctly.

Paper no. 1 Question no. 11G
Many were not able to correctly define the minimum distance in the first part of the question, forgetting to mention that the minimum distance should be achieved by some pair of codewords. Part (b) was where most marks were lost, there were many unconvincing attempts at proving the bound on the new minimal distance. Part (c) was generally done correctly, when attempted.

Paper no. 2 Question no. 3G
This question was generally done well. Some people worked using matrices. Several confused generator matrices and parity check matrices.

Paper no. 2 Question no. 12G
This question had a spread of marks. Many struggled to clearly describe the Huffman coding scheme. The true or false part was generally well done when attempted.

Paper no. 3 Question no. 3G
There were relatively few attempts of this question. The definition in the beginning part of the question was a problem for many. Those who attempted the whole question generally did well.

Paper no. 4 Question no. 3G
This question was very well done in general. There were many who made small errors in describing the Diffie–Hellman key exchange: for example, many did not mention that g should be a primitive root of a large prime p. There was some confusion over how much justification was needed on part (b).

Course: DIFFERENTIAL GEOMETRY

Paper no. 1 Question no. 26H

This was a good question, but harder than I expected. Most candidates could do the bookwork, but some were tripped up by the distinction between $SO(n)$ and $O(n)$. Many struggled with the first problem part, first because they tried to work with a map $SO(n+1) \to \mathbf{R}^{n+1}$ rather than S^n (and thus miscomputed the dimension), and second because they were unable to show that the derivative was surjective. I marked this part generously. On the second problem part, a number of candidates thought that transversal meant "tangent spaces intersect trivially" rather than "tangent spaces span." Most correct solutions to the problems started by recognizing that S_v is isomorphic to $SO(n-1)$.

Paper no. 2 Question no. 25H

This was a good question, with a good spread of responses. Most candidates handled the bookwork OK, although some were sloppy with the proof that $\tau \equiv 0$ implies the curve lies in a plane. Most candidate were able to tell that all three statements in the problem were false, but there was a wide variation in the quality of explanations.

Paper no. 3 Question no. 25H

This question was too difficult. There were few attempts. Only two candidates made any progress on the bookwork beyond the easy initial part, and only one gave a reasonable solution to the problem.

Paper no. 4 Question no. 25H

This question was done surprisingly poorly. The bookwork tended to be sloppy, and many candidates wound up with wrong answers due to computation errors. Some candidates assumed that the curve was parametrized by arclength, perhaps because the surface of revolution question on paper 3 assumed this was the case. (I marked generously and gave full credit for correct solutions that assumed this.) Most candidates who attempted the problem parts could remember a parametrization for a minimal surface, but few were able to check that it was minimal, or that the Gauss curvature was nonzero. No one gave anything resembling a complete solution for the last part, and I marked so that it was possible to get an α without it.

Course: GRAPH THEORY

Paper no. 1 **Question no. 17G**
Many struggled to complete part (d) of this question. While most tried to use part (c), this was often done incorrectly by continuing to consider an expression which was quadratic in the matrix A. There were also some difficulties with showing that B is a multiple of the "all ones" matrix J, with many concluding too quickly after finding that the image of B was spanned by e.

Paper no. 2 **Question no. 17G**
This question was difficult for many. Part (a) was done perfectly by most. The beginning of part (c) was sometimes done by a long, adding in one edge at a time argument. While most took the induction "bait" in the second part of (c), few completed the question correctly. The counterexamples were done well.

Paper no. 3 **Question no. 17G**
In part (b), many gave an incomplete proof by failing to check some details. In part (c), several students did not state Erdös–Stone correctly, by missing out the condition on the size of the graph. In (d), part (ii), many did not get the idea to consider choosing uniformly among sets of size $n/2$, and tried an ad hoc probability argument instead, which rarely led to a complete answer.

Paper no. 4 **Question no. 17G**
This question was done rather well and seemed the easiest out of the four papers. A small correction was made to the question early on in the exam, and shouldn't have seriously affected completion of the question. Many students stated Hall's theorem incorrectly, by missing out one of the implications in the "iff" statement. In the proof, some forgot to check Hall's condition for *all* subsets of a certain graph involved in the induction, with many simply checking that one of the bipartition classes had enough neighbours. The most difficult part of the question proved to be proving that a matching exists in the second part, with many getting stuck. The last part was generally done well, by those who had time to attempt it.

Course: LINEAR ANALYSIS

Paper no. 1 Question no. 22H

The initial parts were easy and done well, although a few people forgot to check why the limit used in definition of $\widetilde{T}$ exists/is well defined. The problem part was not very useful as a discriminator (T_1 to T_4 are all easy, T_5 was difficult, but only worth 2 marks) but the bookwork at the end served as a pretty good method of distinguishing α from non-α responses.

Paper no. 2 Question no. 22H

The question was also on the easy side. The problem part was generally done well (even the more difficult $\mathcal{F}_2$, for which I gave most of the marks. Most candidates had a good understanding of the bookwork, although some were sloppy proving that a piecewise linear function can be expressed as the sum of a linear function and an absolute value.

Paper no. 3 Question no. 21H

A good question which produced a reasonable range of responses. The individual parts were generally all or nothing — candidates either knew how to get started, in which case they could do the problem, or they didn't. (The exception was part a), where some solutions were a bit sloppy.) Parts c) and d) were done well by most, whereas parts a) and b) were more difficult.

Paper no. 4 Question no. 22H

This was a good question. The bookwork was generally done well, although people sometimes forgot the uniqueness part of the statement of the Riesz representation theorem. Many candidates had difficulty with part b). Part c) was generally easier, although some candidates gave up without trying to describe the approximate spectrum.

Course: LOGIC AND SET THEORY

Paper no. 1 Question no. 16I

The bookwork parts of this question were generally well done. The final problem element clearly divided the candidates into those who understood what was being asked and those who did not.

Paper no. 2 Question no. 16I

This question was pretty well done and found easier than I expected. Marks lost were generally a result of lack of care rather than lack of insight.

Paper no. 3 Question no. 16I

This question was found to be easier than I expected. Almost all candidates who attempted it could correctly answer the four 'proof or counterexample' parts.

Paper no. 4 Question no. 16I

Most parts of this question were generally found to be straightforward. However the proof that the square of an infinite cardinal is itself was generally not well presented. Although most knew the general proof strategy, a lack of care with the details often led to a signficant loss of marks.

Course: NUMBER FIELDS

Paper no. 1 Question no. 20G

This question had a range of marks. Many forgot to include the fact that the roots of unity form a finite cyclic group as part of Dirichlet's unit theorem. Students often did not give a complete justification of their answers for part (b) - it may have been better to say more explicitly what is required, either in lectures, or in the question. Very few stated that G was a subgroup of index 6 of $\mathcal{O}_K^\times$ by viewing it as the kernel of $\mathcal{O}_K^\times \to F_7^\times$.

Paper no. 2 Question no. 20G

There were a few small errors in the statement of Minkowski's upper bound, such as not defining the quantities involved. Many students did not justify their factorizations of the ideals in part (b) fully. In part (c), a common mistake was incorrectly finding the order of the ideal class group, which may show gaps in students' group theory background.

Paper no. 4 Question no. 20G

This question had relatively few attempts, but those who attempted it usually achieved close to full marks. The mark distribution was difficult to implement while marking this question, as a large number of marks were suggested for parts with a relatively small number of steps. In retrospect, this question could have been improved with a more difficult problem element at the end.

Course: NUMBER THEORY

Paper no. 1 Question no. 1I
I think this question was well pitched. Most could do it without much difficulty.

Paper no. 2 Question no. 1I
I think this question was well pitched. A few were confused about the statement of the Quadratic Reciprocity Law for the Legendre symbol, believing the syntactically meaningless statement that it applies whenever one of the entries is prime.

Paper no. 3 Question no. 1I
This question was well pitched. Lost betas generally were a result of slight errors in the definition of reduced form which led to errors in the list of reduced forms of discriminant -15.

Paper no. 3 Question no. 11I
There were lots of creative attempts at this problem. Many got stuck on the first part of (c).

Paper no. 4 Question no. 1I
This question was generally well done. A good section I question, I think.

Paper no. 4 Question no. 11I
The bookwork parts of this question were generally well done, although some lost marks through not explaining how parts (a) and (b) were connected. Part (c) was generally well done, often by induction rather than transposing matrices as intended. Part (d) was found hard by some.

Course: REPRESENTATION THEORY

Paper no. 1 Question no. 19I
I was a little suprised how hard many found it to work with real representations.

Paper no. 2 Question no. 19I
This question received the most good attempts of the Representation Theory questions despite errors in the last two parts which most candidates appeared not to notice.

Paper no. 3 Question no. 19I
This was not a popular question, perhaps inevitably for one about Mackey's Theorem.

Paper no. 4 Question no. 19I
This question was found to be a bit harder than I expected. Perhaps it was a little too long.

Course: TOPICS IN ANALYSIS

Paper no. 1 Question no. 2H
This question was too hard. There were very few attempts, most of them scrappy. Only two candidates got as far as stating the equiripple criterion. I marked generously and gave betas to those who did the the first part correctly.

Paper no. 2 Question no. 2H
This was a straightforward question with the right level of difficulty. It was done well by most who attempted it. Many candidates failed to realize that b) requires you to show that the intersection is nonempty, but it was possible to do this and still get a β.

Paper no. 2 Question no. 11H
In retrospect, this question was far too easy. All parts were roughly of equal difficulty (i.e. very little). Most attempts were done well and resulted in alphas.

Paper no. 3 Question no. 2H
Not many candidates attempted this question; very few answered it correctly. About half of the attempts gave up after differentiating player 1's expectation; many of those who kept going made a mistake in either algebra or interpretation.

Paper no. 4 Question no. 2H
This question was all bookwork, but was done poorly. Virtually all candidates knew they were supposed to integrate the given function by parts and get a polynomial with integer coefficients, but many missed the crucial factor of $n!$ in this polynomial.

Paper no. 4 Question no. 12H
This was a good question, with the right level of difficulty. The bookwork was challenging enough to produce a wide range of scores. Many candidates had difficulty explaining clearly why f could be uniformly approximated by a function of the form $\sum A_i/(z-\alpha_i)$. Another point which many attempts missed is that the geometric series converges uniformly on a closed disk of radius < 1, but not on an open disk of radius 1.

Course: APPLIED PROBABILITY

Paper no. 1 Question no. 28
A straight-forward question, perhaps too easy. Most attempts were successful.

Paper no. 2 Question no. 27
About right. A common mistake in part (e) was to use the equation for π_0 to derive the condition $(*)\quad \sum_n \rho_n \prod_{i=1}^{n}(1+\rho_i)^{-1} = 1$, which many listed as the necessary condition. Actually $(*)$ follows immediately from $\prod_{i=1}^{\infty}(1+\rho_i) = \infty$. The correct necessary condition is that the candidate invariant measure can be normalised.

Paper no. 3 Question no. 27
Too easy. Nearly all attempts were successful, modulo small calculation errors.

Paper no. 4 Question no. 27
A bit too easy. Most points were lost in clearly spelling out the bookwork in parts (a) and (b).

Course: PRINCIPLES OF STATISTICS

Paper no. 1 Question no. 29J

This question did not seem as hard to me as deemed to be by students (considerably low attempts). Part (a) is from an example sheet (many students stopped here) and part (b) is an immediate extension of it; many students did not realise that if $p > n$, there are infinitely many solutions to part (a) necessarily (cannot happen that there are none). Part (c) was immediate (matrix multiplication/manipulation) and parts (d) and (e) were a guided use of (c) together with PartIA Vectors and Matrices knowledge; some of these calculations were a sample version of what was seen in lectures on Principal Component Analysis, but not so many students attempted them, so parts (a) and (b) carried considerable weight when marking. In part (d) no one spotted that after normalising the sample PCs (so multiplying U by $\Lambda^{-1/2}$), the middle matrix is diagonal with entries $\Lambda_{i,i}/(\Lambda_{i,i} + \lambda)$.

Paper no. 2 Question no. 28J

The first half of this question was generally fine, although many students forgot to mention that $p_{\pi,\lambda}(k) \geq 0$ for all k so that it defines a distribution. Most students stopped here. In (c), several students found the right maximiser of the likelihood in π, but forgot to restrict it to $[0, 1]$ (only 5% did so). Less than half of the students who attempted (d) realised that the null corresponds to $\pi = 1$, so it is on the boundary of the parameter space. Lastly, no one got the limiting distribution right (by continuous mapping theorem it is the restriction to $[0, 1]$ of the limit of the unrestricted maximiser of the likelihood, so it is the positive part of a normal distribution), even though the same idea had been seen in an example sheet. I adapted to this when marking.

Paper no. 3 Question no. 28J

This question was on the easy side to compensate for the other slightly harder questions. It was well answered in general. Not so many students clearly stated the results they used.

Paper no. 4 Question no. 28J

This question was of appropriate difficulty looking at the statistics. Again, not so many students clearly stated the results they used. Some students used asymptotic normality of MLE in part (b,ii) to find the limit, which is wrong as there is no indication of the estimator being an MLE (the delta method should be used instead); and, still in that part, not everyone realised that the estimator is not necessarily biased, so CR does not necessarily apply. In part (iii), a, b were unknown so that Jackknife is used (question copied from example sheet), but this was not explicitly mentioned so it was accounted for when marking; given the wording there was no need to show that the bias was reduced, but most attempts showed it. A small fraction of students realised that by CLT and asymptotic normality of MLE, the two quantities mentioned in part (iv) are equal. It meant that no unbiased estimator can have better precision than MLE, but it may not have been so clear what was being asked so I accepted any reasonable insights.

Course: PROBABILITY AND MEASURE

Paper no. 1 Question no. 27
Surprisingly unpopular. Of the small number of attempts, a noticeable number tried to invoke invariance under orthogonal transformations and a longish calculation to show $f(r_1+r_2) = f(r_1)f(r_2)$. Of course, this conclusion is an immediate consequence of the assumed independence of X_1 and X_2.

Paper no. 2 Question no. 26
Unpopular. Even the bookwork in parts (a) and (b) was poorly done on average.

Paper no. 3 Question no. 26
Very unpopular. Perhaps a bit too long, though I had expected part (b), which involves exchanging the order of integration, to be seen to be an immediate consequence of Fubini, and I had thought (e) would be found completely straightforward calculation given (d). Also, no one had a correct solution to (d), though it follows immediately from (c) by setting $X' = e^{-X}$.

Paper no. 4 Question no. 26
Very unpopular. Admittedly, in part (a) showing (ii) implies (i) is tricky, but I had thought showing (i) implies (ii) would be found completely straightforward. Also (b) had few attempts, although the only thing to say is that L^2 convergence implies weak convergence of the law.

Course: STATISTICAL MODELLING

Paper no. 1 Question no. 5J
This question turned out to be on the hard side (low attempts), even though it came straight from the lecture notes. Maybe some guidance should have been given (e.g. hints) to show the first result, as the students got stuck in the difficulties that were pointed out (and addressed) in the lecture notes. It was marked generously.

Paper no. 1 Question no. 13J
A model both with and without an intercept were regarded as correct. Hardly any students attempted part (b), even though it was from an example sheet. In (c), not many remembered that a goodness-of-fit test assumes the fitted model to be in the null (in the same way that one always tests against the null and not the alternative; see part (d)); I was flexible with this and awarded some marks as long as the proposed tests were reasonable.

Paper no. 2 Question no. 5J
Nearly everyone answered the first part of this question correctly. However, no one realised that the randomised incentive means that its effects are likely to be independent of unobserved variables (unlike miles cycled), so in establishing a causal relationship it is more relevant. I gave just enough marks to get a merit to answers that were somehow close to grasping this (as long as the first part was fully correct).

Paper no. 3 Question no. 5J
This question was on the easy side, although it did not get many attempts. Not everyone found a genuine function of the means to identify W (e.g. the variance function).

Paper no. 4 Question no. 5J
I expected this question to be on the harder side of the short ones but it was the one with most alphas. Not everyone got the variance of $Y^* - \hat{\beta}^T x^*$ right.

Paper no. 4 Question no. 13J
This question was on the easy side but, surprisingly, got very few attempts. Parts (a) and (b) were easy short-question material, but were not necessarily answered correctly. There were varied answers to (b), but few realised the modelling limitation. Part (d) was harder but based on Part IB Statistics knowledge and it contained a hint and should have been quite helpful.

Course: STOCHASTIC FINANCIAL MODELS

Paper no. 1 Question no. 30
Straight-forward question with many good attempts. Points were lost for sloppy use of the conditional expectations notation, such as $\mathbf{E}(X_{n+1}|X_n \neq 0) = X_n$, and fuzzy statements such as M_τ is $\mathcal{F}_n$ measurable when $\tau \leq n$.

Paper no. 2 Question no. 29
Most candidates had the idea of trying to extend the hint for part (c) into a solution of part (d), but many struggled to cleanly explain why their measure $\mathbf{Q}$ was an equivalent martingale measure.

Paper no. 3 Question no. 29
Mostly bookwork, done well. Perhaps too easy.

Paper no. 4 Question no. 29
Straightforward question, but marks were deducted for not clearly commenting on the fact that S_T/S_t both has the same distribution as S_{T-t}/S_0 and is independent of $\mathcal{F}_t$.

Course: APPLICATIONS OF QUANTUM MECHANICS

Paper no. 1 Question no. 34B
Most of those who made a serious attempt at the question succeded in obtaining an α. There were also several fragmentary answers which received little credit. Part b) of the question involved a significant unseen element in the form of a background electric field. Most people who got to this point easily spotted that the new feature could be absorbed by completing the square in the Hamiltonian.

Paper no. 2 Question no. 34B
This question on the variational principle included a significant amount of bookwork which was sometimes incorrectly described. In their account of the variational principle, a significant number of introduced the Rayleigh quotentent and proved that is always larger than the groundstate energy, but then lost marks by failing to describe minimization over the parameters of a trial wavefunction. The application in the final part was staightforward but many candidates made errors in the evaluation of the integrals or in the subsequent

Paper no. 3 Question no. 34B
This question proved to be quite challenging. Most candidates correctly reporduced the proof of Bloch's theorem and found the basis for the dual lattice, but after this progress was more difficult. Relatively few attempts resulted in a correct description of the energy along the boundary of the Brilloin zone and the α rate was correspondingly low.

Paper no. 4 Question no. 33B
This question proved quite challenging despite the significant book-work content. Many found it hard to extract the S-matrix from the given wave-function in part b), and several candidates lost marks by writing down an r-dependent expression for S.

Course: ASYMPTOTIC METHODS

Paper no. 2 Question no. 30
This question was not particularly well-answered. Many students were imprecise with the definition of an *asymptotic expansion*, and fell into the trap of calculating the first three terms of the requested asymptotic expansion **without** providing the required justification that it was indeed an asymptotic expansion with a remainder term of the required form.

Paper no. 3 Question no. 30
This question was well-answered in general, with most students understanding the appropriate way to apply the complex version of Watson's lemma. Few understood the region of validity, but relatively few marks were lost.

Paper no. 4 Question no. 30
This question was very well-answered, with most students demonstrating a solid appreciation of the WKB approximation as well as solid manipulation skills. The final expression had a regrettable (minor) typographical error, with the bracket $(n+l+1)$ missing a superscript 2, which emerged too late in the examination to be announced. This part of the question (counting for very few marks) was marked generously.

Course: CLASSICAL DYNAMICS

Paper no. 1 Question no. 8E
This question was generally well answered, with most candidates able to do the bookwork and then correctly identify the correct symmetry transformation to apply Noether's theorem to the problem in part b).

Paper no. 2 Question no. 8E
This question was fairly straightforward, and was well answered on the whole. Where candidates lost marks it was typically by making the overly restrictive assumption that the constants of the motion in part e) did not depend explicitly on t.

Paper no. 2 Question no. 14E
This question was well answered. Some candidates struggled with the final part involving generating functions, although some managed to do this part by directly showing that the new Poisson brackets have the correct form, if they had been careful up to this point, the last part didn't cost them the alpha.

Paper no. 3 Question no. 8E
This question was a bit less popular than other short questions for this course, and was required a bit more work to complete. Marks were lost for finding I but not θ.

Paper no. 4 Question no. 8E
This was a fairly standard piece of bookwork, and on the whole it was well answered. Some lost marks by assuming $I_1 < I_2 < I_3$ without stating this anywhere, but generally well answered.

Paper no. 4 Question no. 15E
This was the least well answered of the questions for this course. In part a) candidates struggled to clearly relate the axes $1, 2, 3$ to the body, although those who drew a diagram fared better. Candidates often took very meandering routes to establish the inequality. Many marks were lost through carelessness when evaluating the (fairly straightforward) integrals in the second part.

Course: COSMOLOGY

Paper no. 1 Question no. 9B
This question proved harder than expected. Most candidates easily reproduced the required bookwork for sections (a) and (b), but stumbled on part (c).

Paper no. 1 Question no. 15B
This question proved straightforward for many of those who attempted it. Several candidates did not succeed in solving the equation of motion as they did not correctly implement the slow-roll approximation.

Paper no. 2 Question no. 9B
This question attracted many attempts but many were fragmentary, some of them focussing just on the first two parts which were bookwork. Very few candidates provided a correctly reasoned answer for part (c).

Paper no. 3 Question no. 9B
This question attracted few complete attempts. Only a few candidates were able to reproduce the derivation of the Friedmann equation.

Paper no. 3 Question no. 14B
This question proved quite accessible. The question required sorting through the information provided relating to the physical context of the problem. Only a few candidates managed the final part which required solving the Saha equation, for $\eta = 1$, to find an order of magnitude estimate of the recombination temperature.

Paper no. 4 Question no. 9B
There were very few attempts at this question and most of the answers were fragmentary.

Course: DYNAMICAL SYSTEMS

Paper no. 1 Question no. 31E
This was a nice question which tested understanding of basic ideas involving $\alpha-$ and $\omega-$limit sets. Most candidates were able to produce a correct definition, but some then produced answers that showed they did not fully understand this. Marks were mostly lost through being insufficiently systematic in analysing cases (a common theme with questions for this course).

Paper no. 2 Question no. 31E
This proved a good question to test understanding of notions of chaos. Most candidates could give the definitions, although there were many incorrect definitions for SDIC. The second part produced some good answers showing clear understanding of how the map acts. $G-$chaos was better treated than $D-$chaos. Some candidates attempted to copy a proof for the case $a = n$ involving $n-$ary expansions, which could be made to work, but without suitable care lost marks.

Paper no. 3 Question no. 31E
This question proved surprisingly difficult. Very few candidates correctly realised they could choose f, g such that $\dot{V} = 0$ for part *i*). Even fewer could then apply the energy balance method to do part *ii*). Dulac's criterion was fine, but again in part *iv*) the main cause for losing marks was being insufficiently systematic in analysing cases.

Paper no. 4 Question no. 31E
This question was mostly well answered, with the majority of candidates finding the fixed points and correctly identifying the values of a at which bifurcations occur. Many lost a few marks from not clearly indicating the ranges of a for which each fixed point exists. There were some haphazard guesses as to the nature of the bifurcations from some candidates. The ECM computations were not always attempted, but when an attempt was made it was mostly well done for $a = 0$ and $a = 3$, but more hit-and-miss for $a = 4$. Those with a good grasp on the overall picture were able to answer fairly succinctly, while some churned through a lot of algebra.

Course: ELECTRODYNAMICS

Paper no. 1 Question no. 36E

As with all the questions in this course, there were relatively few attempts, but those who did make a serious attempt at the question were often successful. Common mistakes included incorrectly converting the 3−velocity to a 4−velocity (hence missing factors of $\gamma(v_0)$) and some careless asymptotic expansions for large n.

Paper no. 3 Question no. 36E

This was a nice question on the dipole approximation for radiating sources. The bookwork was mostly very well answered, although marks were often lost for not accurately stating the assumptions for the approximation to be valid. The first part of the unseen material (calculating the time averaged power) was also well done. For the final part, some students did not realise that the radiated power is proportional to the energy, and so produced a linear decay in the energy (and incorrect half life) as a result.

Paper no. 4 Question no. 35E

There were fewer serious attempts at this question, but almost all of them produced essentially complete answers. The non-bookwork part could arguably have been a bit more difficult, but overall the question seemed suitable.

Course: FLUID DYNAMICS II

Paper no. 1 Question no. 38

This question was surprisingly badly answered. Many students struggled with formulating and scaling the axisymmetric problem, although stronger students demonstrated that the construction of the radial distribution of the pressure, (and hence the answer) should not have been enormously challenging.

Paper no. 2 Question no. 37

This question had quite a range of qualities of answers. It is possible that some students were confused by the notation, and also by the fact that the flow of interest is a simple variant of plane Couette flow, and thus does not require an imposed pressure gradient.

Paper no. 3 Question no. 38

This question also had a range in the quality of answers. Many students failed to appreciate that consideration of the mass (or volume) flux was required to demonstrate entrainment. Some students attempted to establish the (given) equation, suggesting that the wording might perhaps have made the fact that it was given even more explicit. The arguments of the streamfunction were unfortunately inconsistent in the printed version, (i.e. (x, y) rather than (x, y, z) or equivalently (r, z)). This error was corrected relatively early, and should not have affected the construction of the given expression for the similarity variable g.

Paper no. 4 Question no. 37

This question was exceptionally well-answered. The primary manipulations were typically done very well, although there were a few arithmetic slips in the last part. A clarification (that the rigid sphere has a specified radius a) was announced, although several students (naturally) assumed that a rigid sphere would have a constant finite radius.

Course: FURTHER COMPLEX METHODS

Paper no. 1 Question no. 7
This question was well-answered. Students typically demonstrated an understanding of the properties of the Gamma function, and the ability to manipulate expressions into the required form.

Paper no. 1 Question no. 14
This question was on the whole well-answered, with the students being able to manipulate P-symbols appropriately. A few found it difficult to construct the appropriate form of the second solution.

Paper no. 2 Question no. 7
This question was exceptionally well-answered, with only a few students misapplying the identation lemma.

Paper no. 2 Question no. 13
This question was solidly answered. Some students did not present a cogent argument for why the *Hankel representation* is an analytic continuation of $\zeta(z)$ for all $z \neq 1$. Also, not all students constructed and justified the "suitably modified Hankel contour" appropriately.

Paper no. 3 Question no. 7
This question proved more challenging than expected, with both a surprisingly large number of calculation errors for the first part, and also sometimes poor understanding of the required properties of the possible contour γ for the second part.

Paper no. 4 Question no. 7
This apparently straightforward question was very badly answered. In particular, the required construction of a multi-valued function for arcsin was often mangled.

Course: GENERAL RELATIVITY

Paper no. 1 Question no. 37

This Q worked well. Students used a few different tricks to compute the commutator of covariant derivatives on two-tensors, e.g. re-writing the tensor as $T_{\mu\nu} = \sum u_\mu u_\nu$ and using the result for vectors. Also many people used normal coordinates to simplify expressions setting all γ's to zero and arguing for general covariance.

Paper no. 2 Question no. 36

Students did too well on this questions and it was in hindsight a bit too easy. Mostly the reason that it was always pretty straightforward to map the physics question to a well-defined computational exercise. Also, probably this was a bit too similar to the Schwarzschild case. Maybe a some more advanced question on horizon to challenge and separate the most advanced students would help. Many students did not answer well the question about what happens at $f(r) = 0$.

Paper no. 3 Question no. 37

Q on Tolman-Oppenheimer-Volkoff equation. Probably it would have been sufficient to ask for less Christoffel symbols. The rest was very computational and most students did very well.

Paper no. 4 Question no. 36

This Q on commutators has many α's. This is due to the fact that the questions were well defined mathematically and many students were able to flawless execute the required calculations. Also, in hindsight, the final part of the question did not rise very steeply in difficulty as compared with the first part and so almost all students who seriously attempted the questions made it all the way to the end.

Course: INTEGRABLE SYSTEMS

Paper no. 1 Question no. 32C

Long paper 1 (concatenating int sys question): A story of three parts. Most candidates attempting this had no difficulty with the routine parts (a) and (b), and no problem with (d) if they got that far. Most candidates identified a suitable set of first integrals for (c) but many of those then either missed checking something, or did not adequately handle the extended coordinates. For part (e), many quoted Arnold-Liouville and then made an incorrect jump (that was unjustified and also untrue). A few students made the connection with the rest of the question and gained full marks. Missing part (e) still meant an alpha was possible so long as (c) had been done fairly carefully.

Paper no. 2 Question no. 32C

Long paper 2 (SL operator, scattering, Lax pair, Harry Dym): Most attempts were either abandoned early or went to near-completion. There was a wide range of spurious arguments for (b), but this did not cost the alpha if most of the rest was fine. For (c), some completed this very succinctly (as hoped) and others took pages of algebra by differentiating out every term then slowly matching things up. Generally (d) worked out, though some failed to use p goes to 1 as x goes to infinities.

Paper no. 3 Question no. 32C

Long paper 3 (prolongations): This was a tough question, requiring understanding of prolongations and careful work with some potentially messy algebra (or slick ways around it). However there were many very good attempts. Part (a) was generally very well explained, though sometimes forgetting the base case of the required recursion. Part (b) ranged from very succinctly done to many pages of algebra sorting out the simple recurrence formula for V_3. The first part of (c) was usually pushed through to at least giving a multiple of S (and usually the right one). The second part elicited quite a range of responses, showing some candidates did not understand this part, but often the alpha was already achieved (and not lightly: often after many previous pages of work).

Course: MATHEMATICAL BIOLOGY

Paper no. 1 Question no. 6C

Short paper 1 (discrete age structure question): Generally, adapting the ideas from lectures to discrete time was done very well. There was some too short explaining of the equations (e.g. simply saying the first equation is 'births' and nothing else). Subbing in the similarity solution usually worked out, though sometimes by contorted routes, or fudges made to repair slips. Not many got full marks for the last part, e.g. just claiming that ϕ strictly decreasing is sufficient for a unique solution, but this did not cost the beta if the rest was done competently.

Paper no. 2 Question no. 6C

Short paper 2 (Turing instability question): Very well done, there will be a high rate of betas for this one but this is not inappropriate given that making progress with this question requires sound basic knowledge of Turing instabilities. The mark scheme was somewhat front-loaded, so the beta was usually achieved if there was at least some engagement with trying to determine a condition for $det(J_{mod}) < 0$ for some k. Quite a few students made some error or forgot to combine the conditions at the end, but many fully completed the question.

Paper no. 3 Question no. 6C

Short paper 3 (Wound healing question): there was an error in the question: the leading edge of the wave is at f = 0 not f = 1. In the event, only a very small number of candidates seem to have affected by this in practice (if at all) and credit was given for the right working in this case. There were many very short attempts (a line or two, stopping well before linearising), so question statistics will be odd. Most attempts that got through the first part wrote something sensible for the wound healing part, though there were some interesting sketches of wounds healing from one side only, or not from the edges but growing uniformly upwards (the beta was given in these cases as the rest of the working and ideas were correct).

Paper no. 3 Question no. 13C

Long paper 3 (Stochastic population): this has proved a useful question for discriminating between students who only able to reproduce bookwork as they have seen before and those who understand it securely enough to adapt it slightly. The first part was routine and the question guided students through it. Some candidates finished this succinctly, while others made a meal of a linear first order constant coefficient ODE (but generally got there in the end). Writing down the new master equation was done correctly in about half of the attempts. The most common type of error (putting k into the rate rather than the number added) resulted in the right equation for the evolution for the mean so generally these were able to reach a beta even after a few minor gaps. The point about the previous method failing was not often well answered, but a few students showed they had a complete grasp of why the previous method had worked.

Paper no. 4 Question no. 6C

Short paper 4 (delay logistic): this was not at all well done, with few attempts showing competency at a method which on the examples sheet (and in a past exam paper). Many attempts were abandoned early so the beta rate will be low. This might be as stronger students suspect the question was overly long as it had two parts (whereas the second part was very short if the methods were used correctly). A common error for those persisting was to Taylor expand in T, and then using this for non-small values of T.

Paper no. 4 Question no. 14C

Long paper 4 (plant disease): a very wide spread of performance here. The intersection of candidates who had a sense of what was going on in the model and those candidates who could cut through the algebra was small so the alpha rate will be low (but high beta rate). Many got that the disease neither caused death not affected reproduction, nor was there any recovery. Generally the phase plane analysis in (b) was done well, but often with excessive algebra (e.g. > 1 page to just find the fixed points). Many candidates left themselves stuck in (c) and (d) with poor algebra (including surprisingly many incorrect applications of quotient rule). Some had the key idea, that N tends to its stable equilibrium, which then dictates the θ dynamics. There were a few superb answers to the whole question.

Course: NUMERICAL ANALYSIS

Paper no. 1 Question no. 40C

Long paper 1 (Jacobi): All who attempted this question knew what the Jacobi method was and had a good idea about convergence. Many attempts were rather lengthy in sorting out the eigenvalues for (b). For part (c), a wide range of approaches were used, but most commonly as anticipated in the crib (alternate signs of a vector either to show $xA'x$ always positive, or same eigenvalues). Some made the mistake of assuming the same elements down each diagonal (so Toeplitz). If the method to complete this special case would have worked more generally, enough credit was given for the alpha.

Paper no. 2 Question no. 39C

Long paper 2 (9 point method and TST matrices): This question was too easy: essentially the candidates were led through every step. Some candidates were careful and gave clear explanations showing they really had understood what they were doing. Others were clearly reproducing standard notes without demonstrating understanding, missing the alpha if they showed gaps in multiple parts of the question. This type of question should be avoided in future as a section 2 question: there should be some more testing problem element and much less guidance given.

Paper no. 3 Question no. 40C

Long paper 3 (diffusion equation): The first part of this question was exactly from notes, but the second part was a new variant. There was considerable confusion over convergence vs stability, and many candidates missed out key points of argument for (a). Despite the question saying to prove directly, some took the route of showing stability and then invoking the Lax equivalence theorem, though partial credit was given if it was done accurately. Most candidates had no difficulty in getting from the new method in (b) to writing down the eigenvalues (given familiarity with TST matrices which were on the previous paper), though progressing from there to conditions on the parameters for stability was highly error-ridden (handling the trig over the range of k, handling the modulus signs, care with direction of inequality when multiplying up by something which could be negative, etc). Given that this was the original part of the problem, there was substantial credit available for doing this part correctly.

Paper no. 4 Question no. 39C

Long paper 4 (2-periodic functions): this was a very straightforward question, and it was very surprising that there weren't more students choosing to do this question. This was essentially two short questions (first is special case of something in notes, second verbatim from notes). Given that there was not too much problem to do, any non-trivial errors or key gaps in explanation were marked down.

Course: PRINCIPLES OF QUANTUM MECHANICS

Paper no. 1 Question no. 33B
A very popular question. Many candidates were no doubt attracted by the easy bookwork in the first section however the latter parts on the 3d oscillator were more challenging. Surprisingly many candidates were unable to reproduce the correct normalisation of the eigenstates or even the defining condition of the ground state (the fact that it is a zero eigenvector of all the annihilation operators). Constructing the required linear combination for the final part of the question also proved challenging to many although one did not need to complete this perfectly to obtain an α.

Paper no. 2 Question no. 33B
This question on time-dependent perturbation theory attracted signifcantly fewer attempts than the others on this course. Part b) required a careful application of the bookwork asked for part a). The final part requiring the exact diagonalisation of the Hamiltonian required some familiarity with the properties of the Pauli matrices and the correct use of the identity given in the hint. Several attempts took a wrong turning at this point leading to incorrect expressions for the eigenvalues.

Paper no. 3 Question no. 33B
A popular question on perturbation theory which most found relatively easy. The question tested the understanding of both the degenerate and non-degenerate cases which was demonstrated in most attempts. The final part of the question simply required finding the eigenvalues of a 3×3 matrix and candidates who got that far were rewarded with some easy marks.

Paper no. 4 Question no. 32B
Another very popular question. Most candidates were able to reproduce the bookwork for the first half of the question and this was sufficient to achieve at least a β in most cases. The middle section of the question, in which an explicit diagonalisation of the given Hamiltonian was required was more demanding but was completed in almost half of the attempts almost always resulting in an α. The best answers correctly described the role of rotational symmetry in the $B \to 0$ limit as an explanation of the resulting degeneracies.

Course: QUANTUM INFORMATION AND COMPUTATION

Paper no. 1 Question no. 10
This Q quantum teleportation worked well. Students' answers were a bit vague on how to invoke the no-signaling theorem.

Paper no. 2 Question no. 10
This Q on the BB84 algorithm worked well. Pretty much everyone got through parts a and b. The real distinguisher was the bit error rate. Quite a few students had trouble setting up that computation in an efficient and precise manner (e.g. many analyzed all possible cases rather than just a minimal subset).

Paper no. 2 Question no. 15
This Q on state discrimination (distinguishability) worked well. The hardest part was defining mathematically the right projector for part iv.

Paper no. 3 Question no. 10
This Q on a variant of Grover's algorithm worked well. Very few people showed explicitly that I_G requires indeed only one call of U_f.

Paper no. 3 Question no. 15
This Q on quantum oracle promise problem worked well. People struggled finding the right quantum algorithm to solve quantum oracle problem in part c as requires guessing the right state to begin with (all students knew it was a matter of QFT-U_f-QFT, but many choose "wrong" state).

Paper no. 4 Question no. 10
This Q on Shor's algorithm worked well. Very few people were careful in proving that the assumptions of theorem on Continuous Fraction were actually satisfied ($1/N^2 < 1/r^2$).

Course: STATISTICAL PHYSICS

Paper no. 1 Question no. 35
This Q on entropy and heat capacity worked well. Only few realized to include latent heat in calculation of S.

Paper no. 2 Question no. 35
This Q on ideal gas worked well. Many students forgot the $1/N!$ in the free energy, but that did not matter in any of the following calculations.

Paper no. 3 Question no. 35
Many students got the wrong C_V because of a confusion on the order of limits. As $T \ll \epsilon_F$ they neglected the T term in the integral. Also, I didn't give the full mark of the first question (what is μ) to almost anyone as most people gave very short answers such as $\mu = dE/dN$.

Paper no. 4 Question no. 34
This Q on chemical potential and fermions at finite density worked well. The first question "what is μ?" was hard to grade as students gave a mix of very long and precise as well as ultra concise answers (e.g. $\mu = dE/dN$). The limit $T \ll \epsilon_F$ in the final integral was confusing and many students got the wrong result by taking that limit at the wrong moment. Also, the constant contribution to the energy was a bit unclear.

Course: WAVES

Paper no. 1 Question no. 39
This question was quite well-answered. Most students appreciated the concept of a vacuum developing, though most did not understand how to apply the (well-covered) properties of a classic expansion fan to construct the required leading order approximation in the last part of the question.

Paper no. 2 Question no. 38
This question was very well-answered. Most students understood how to set up the problem and boundary conditions correctly, and also how to manipulate the equations to construct the required quantities.

Paper no. 3 Question no. 39
This question was well-answered. Most students applied the boundary conditions correctly, and understood the correct form for the various reflected and transmitted waves. An unfortunate typographical error in the given form for the amplitude of the transmitted wave (the bracket $(\lambda + \lambda^{-1})$ should have had a superscript 2) emerged and was corrected regrettably late in the examination. The relevant parts of the question were marked generously, although the (positive) power of this bracket does not modify qualitatively the requested discussion of two asymptotic limits.

Paper no. 4 Question no. 38
This question was exceptionally well-answered. Most students who attempted this question demonstrated real facility with the required manipulations both to derive the ray tracing equations and to describe the waves behind a stationary object, closely related to the canonical Tripos problem concerning the flow behind a duck.

COMMENTS ON QUESTIONS

MATHEMATICAL TRIPOS PART IA, 2018

Course: ANALYSIS I

Paper no. 1 Question no. 3E

The bookwork part was well done. In the second part, many candidates considered what happens if (x_n) is bounded above by some c, but very few noticed that in that case $f(x_n) \geq f(c)$ for all n, and hence (x_n) is in fact not bounded above.

Paper no. 1 Question no. 4D

This was a poorly done question. The bookwork for radius of convergence resulted in many mangled proofs, with candidates confusing convergence with absolute convergence or making random jumps in the logic. Perhaps part of the problem is that, because it is a short proof, students do not realise that it is not trivial at all.

Paper no. 1 Question no. 9F

Part (a) was easy bookwork and almost all students that attempted it got it correctly. The idea for part (b) seemed to have been understood by most students, though many arguments were messy. Perhaps more structure in the phrasing of the question would have produced more consistent results. Part (c) had a good level of difficulty.

Paper no. 1 Question no. 10F

Part (a) was bookwork and done well. The students had most difficulties with the second part of (b), which required a little bit of thinking. Overall the level of difficulty seemed to have been about right.

Paper no. 1 Question no. 11E

Most candidates could state and prove the Comparison Test only losing the occasional mark for not carefully justifying each step. A very few candidates confused the Comparison Test with other results in the course but, strangely, they applied the correct result in later parts of the question.
The next part concerning $\sum x_n$, $\sum x_n^2$ and $\sum \frac{x_n}{1-x_n}$ was generally well done. The one part that caused some problems was showing the convergence of $\sum \frac{x_n}{1-x_n}$ assuming the convergence of $\sum x_n$. Proving that the convergence of $\sum x_n$ implies $nx_n \to 0$ proved surprisingly problematic for the majority of candidates. Showing that the converse is false was somewhat more successful.
The last part was more difficult but there were several successful attempts. It was possible to achieve an alpha without this part and there were a number of candidates in that category.

Paper no. 1 Question no. 12D

The integrability part produced many attempted proofs of the (false) fact that the function cannot be integrable if the weights on the rationals do not tend to zero. The fundamental theorem of calculus bookwork was extremely well done, with pretty much everyone who did this question supplying accurate proofs. The problem at the end was not so well done.

Course: GROUPS

Paper no. 3 Question no. 1D

A well done question. Candidates had a variety of methods for counting the various cycle types.

Paper no. 3 Question no. 2D

The bookwork on products of three rotations was on the whole well done. However, the last part of the question produced a lot of incorrect answers, often of the form 'the proof above does not give that every element is a product of at most two reflections, therefore it cannot be true that every element is a product of two reflections'.

Paper no. 3 Question no. 5D

Candidates showed great ingenuity in proving that the signature is the only homomorphism. Several different proofs were given, which made this question that great rarity, a fun question to mark.

Paper no. 3 Question no. 6D

I had expected candidates to struggle with the very abstract nature of this question, but actually it was very well done. Most people had no problem at all considering homomorphisms to the group of automorphisms.

Paper no. 3 Question no. 7D

Usually quotient groups are found difficult, and questions on them have a low takeup rate. So it was nice to see a good number of attempts on this question. They were generally pretty good. Candidates managed to get to grips with the notion of metacyclic groups, and on the whole produced coherent and sensible arguments.

Paper no. 3 Question no. 8D

This was a shockingly badly done question, but not for the reasons I had expected. I had expected candidates to mis-state the direct product theorem (perhaps leaving out the commutativity), but they nearly all got this right. I had also expected lack of clarity with the proof, but again candidates were generally accurate on this. However, when it came to applying the theorem there were all sorts of errors. Perhaps the most depressing was the notion, common to well over half of all attempts, that if one has a theorem stating 'If X holds then G and H are isomorphic' then to prove that G and G are not isomorphic it suffices to show that X does not hold. This shows a very worrying lack of actual thinking.

Course: NUMBERS AND SETS

Paper no. 4 Question no. 1E

The majority of candidates had no problem with this fairly straighforward bookwork question. The following were typical reasons for losing marks: not saying that p is prime or that $(x,p) = 1$ in Fermat's theorem; not checking the condition $(x,p) = 1$ in applying Fermat; forgetting that 2 is a prime in the last part.

Paper no. 4 Question no. 2E

The first part was straightforward bookwork, yet most attempts were disappointingly poor. Some vague reference to unique prime factorization, or saying that $(p, q) = 1$ implies $(p^2, q^2) = 1$ without proper justification was not sufficient. The problem element that made up the second part of the question was generally well done.

Paper no. 4 Question no. 5E
The first part was generally well done, however instead of using Bezout, some candidates used more advanced results that follow from Bezout without justification. Wilson's theorem was fine. The difficulty with finding the exponent of a prime in the factorization of $n!$ was to pin down a convincing argument which was not always successful. The question finished with some applications of the earlier parts which was again reasonably well done.

Paper no. 4 Question no. 6E
The main part of the question was to find upper and lower bounds on the size of a largest k-code. Candidates attempting this question either realized how to do this and most of those ended up with an alpha, or they could not in which case they might have achieved a beta if they correctly solved the other, fairly straightforward parts.

Paper no. 4 Question no. 7E

Candidates did generally well on the first part and were able to deduce the next two formulae correctly. Surprisingly, many candidates deduced the inclusion-exclusion formula by summing the previous formula over $t = 1, 2, \ldots, n$ instead of simply putting $t = 0$. Deducing the formula for Euler's totient function was fine, although sometimes careful justification was missing. Less than half the candidates attempting this question could do the last part on Carmichael numbers.

Paper no. 4E Question no. 8
A very large number of the candidates only did the first bookwork part which was generally fine. Very few had a decent attemt on the substantial problem part of the question. Those who had some good, relevant ideas here earned generous marks.

Course: PROBABILITY

Paper no. 2 Question no. 3F
This question was done almost perfectly by most students. The bookwork component was too large and also otherwise the question seemed to have been too easy.

Paper no. 2 Question no. 4F
This question was not done as well as 3F, but still quite well by most students. Again it was probably a bit too easy.

Paper no. 2 Question no. 9F
The question was fairly technical and did not require very clever insight. Part (a) and the first step of part (b) were done reasonably well. The induction in part (b) was only carried out really well by few students. Overall the level of difficulty seemed to have been okay.

Paper no. 2 Question no. 10F
Part (a) was done well by the majority of students. Part (b) was somewhat tricky and few students

got it correctly. Part (c) was done a bit better, still not many students got it completely right. This question did not have a significiant bookwork part, explaining the lower results compared to other questions. Overall I think it was an okay question.

Paper no. 2 Question no. 11F

This question had too much bookwork. In particular, part (a) had a lot of bookwork resulting in high marks. Part (b) was done well by most students. Overall this question was too easy.

Paper no. 2 Question no. 12F

Part (i) was a bit technical. Many students got the idea correct, but much fewer produced a rigorous argument. I was perhaps a bit too lenient in deducting marks for this. Part (ii) was very easy and most students that attempted it got it correctly. Part (iii) had a better level of difficulty. Overall the level of difficulty seemed to have been about right. There was almost no bookwork component, explaning the low take up.

Course: DIFFERENTIAL EQUATIONS

Paper no. 2 Question no. 1B
Answers were very variable in quality. Some ignored the possibility $a < 0$, despite the fact that this was signposted in the question. Others invoked the wrong criterion for stability, forgetting that this was a difference equation and not a differential equation. Algebraic mistakes often made life complicated. Some candidates used, initially at least, ?stable? to mean ?steady?. A few good answers sailed through the question, setting out the key facts and describing the behaviour, in less than a page.

Paper no. 2 Question no. 2B
The demonstration that $\partial P/\partial y = \partial Q/\partial x$ implies the existence of an F was done by very few. Some simply stated that the result was true (provided that the domain was simply connected) which did not meet the requirement of ?show that?. But the other parts of the question were completed by most candidates (and were sufficient for a β if done well).

Paper no. 2 Question no. 5B
Good answers to this question were very rare. Most solved a relevant eigenvalue problem and wrote down a complementary function, then proceeded to finding a particular integral. This could be done by brute force calculation, and some did it that way, but the calculation could also be simplified by spotting the structure in the problem ? e.g. a $t\times$ exponential term. A very small number of candidates took the approach that I had intended, by exploiting a basis of eigenvectors to derive two uncoupled first-order equations. The ?special? values of b and c were correctly explained by a small number of candidates. Several candidates guessed wrongly that they were associated with resonance, when in fact they were associated with absence of resonance.

Paper no. 2 Question no. 6B
There were many good answers to this question, which candidates found more straightforward than I had expected. Most completed the chain rule part, though some unwisely chose to derive $\partial/\partial\xi$ and $\partial/\partial\eta$ in terms of $\partial/\partial x$ and $\partial/\partial y$ rather than vice versa. Some of the reasoning was shaky in the second part of the question and there were some mistakes in the final part of the question, e.g. in insisting that the functions appearing in the solution were linear.

Paper no. 2 Question no. 7B
This question was specifically designed not to require advance knowledge of the non-power-series (logarithmic) solution, but to lead the candidate to deduce the need for it at the end of a number of clearly specified steps. But relatively few candidates actually followed these steps, e.g. several did not actually write out the requested explicit terms in the series expression. Some derived the formula for the second solution, although that was not requested. Surprisingly few made the step from substituting a power series into the integral to deducing the need for a logarithmic term, perhaps because this seemed too ?informal?.

Paper no. 2 Question no. 8B
Most candidates provided the explanations requested for the first part of the question, though several candidates simply failed to provide what was requested and therefore lost marks. There were some errors in solving for T_1 and U_1, mainly caused by confusion about how to take account of the delta function. Solving for T_2 and U_2 required more attention to detail in the algebra and many were defeated by this. Most candidates seemed to realise that the second heating protocol would tend to a delta function in the limit $\tau \to 0$ and got credit for mentioning that.

Course: DYNAMICS AND RELATIVITY

Paper no. 4 Question no. 3A
Straightforward question on Galilean relativity and equilibria. Reasonably popular and well done on the whole.

Paper no. 4 Question no. 4A
A bookwork question deriving Kepler's law. Quite popular and well done.

Paper no. 4 Question no. 9A
A popular, hard question on bodies rolling down hills. Candidates met with variable success. Only a handful got full marks. Nearly everyone got 5 marks for the bookwork rider.

Paper no. 4 Question no. 10A
A hard question on 4-momenta, quite popular. Many forgot to conserve momentum in anti-proton production (and thus didn't realise that the final state particles must all be moving in the lab frame). Candidates found the last part especially hard: perhaps a hint suggesting to work in the rest frame of particle B would have been appropriate.

Paper no. 4 Question no. 11A
I was impressed with the quality of answers on this medium popularity question on mechanics and the Lorentz force law. I forgot to define the mass m of the ball, but all candidates either set it to 1 or m in their responses (either of which was accepted while marking).

Paper no. 4 Question no. 12A
Not stupendously popular, but many students did very well and achieved full marks on this question about rotating frames.

Course: VECTOR CALCULUS

Paper no. 3 Question no. 3C
This question starts off with a little bit of tedious algebra, but ends with a familiar result. Many students messed up the algebra in the first part, but if they knew where they were heading then they could still get the beta mark. Unfortunately, a large number that got the wrong answer did not state that the correct result should be $1/a$, and were content with completely incorrect answers, or expressions that were $1/a$ only at one point, or constant but incorrect values.

Paper no. 3 Question no. 4C
After some simple but obscure bookwork, the students are asked to solve a straightforward Poisson equation, where the small trap is that, due to the Neumann boundary condition, the general solution includes an arbitrary additive constant. The last part is hinting towards it, but surprisingly most students failed to include the constant, with some just blindly stating uniqueness and others blindly stating "unique up to a constant" without realising that it directly implies that their answer is wrong.

Paper no. 3 Question no. 9C
This is a standard change-of-variables integral question with simple algebra but a small twist in that the map is not bijective. Although only a few marks would be deducted for not noticing this, most students lost many more marks. A common error, which was much more prevalent than I had expected, was incorrectly assuming that D' would be a rectangle. Also, many did not pay attention to the implication that the Jacobian is best expressed in terms of $\partial(u, v)/\partial(x, y)$ instead of $\partial(x, y)/\partial(u, v)$ and had to do treacherous extra algebra to make up for it. Many students did not make use of the parallel calculations to check their work, but instead tried to fudge their answers to agree.

Paper no. 3 Question no. 10C
This is a standard Stokes question (albeit with two surface parts and two boundary parts), which was typically done well. The calculations are quite simple, unless one tries to do the radial integrals without eliminating most of the terms first by doing the azimuthal integrals or appealing to symmetry. Many lost a mark on stating the orientation incorrectly - e.g. by saying that the normal should be "outward", "on the left", or "pointing out of the surface". Some lost marks by trying to include the seam between the two surface parts as a boundary and adding a contribution from it. Like in question 9, some students tried to fudge their calculations to match, and sometimes after incorrectly getting one answer to be 0 they would quickly find a way to make the other result be 0 too!

Paper no. 3 Question no. 11C
Few students attempted this question. Some only did one of the two bookwork parts and stopped there. Some lost marks on sign errors in $\mathbf{E}$ or ϕ, or getting the wrong additive constant for ϕ. A decent number, but fewer than I expected, made sufficient progress on the last part to earn the alpha. Some tried incorrectly to add and subtract charges from different parts before plugging into a spherically symmetric formula, rather than correctly using the formula on each part and then adding or subtracting the resulting field and potential.

Paper no. 3 Question no. 12C
Few students attempted this question. The bookwork in (a) and (b) was generally done well, although some lost a mark for stating rather than deriving the expressions in (a), and for forgetting that the zero tensor is an isotropic tensor of rank 1. A decent number managed to do part (c),

which wasn't too tricky. I knew they would find part (d) difficult, but in the end only one person did it, which was a bit below my expectations.

Course: VECTORS AND MATRICES

Paper no. 1 Question no. 1C
Disappointingly few students managed to state the basic bookwork correctly. Many failed to take the principal value in (iv). Some incorrectly excluded solutions in (i). Recurring algebraic mistakes include: getting the wrong argument of $\sqrt{3}+i$, writing $\log(i) = \pi/2$ or $\log(i) = 1 + i\pi/2$, and doing z^i instead of i^z in (iii).

Paper no. 1 Question no. 2A
Most students did not even tender an attempt at this question, surprisingly. I can only assume that they did not know how to find the inverse map.

Paper no. 1 Question no. 5C
As intended, the main difficulties were figuring out to calculate $\mathbf{m} \cdot \mathbf{m}$ and to make an appropriate ansatz in the last part (or invert a matrix). Surprisingly many minor marks were lost on missing out answering small easy parts of the question, but overall the students performed well as expected.

Paper no. 1 Question no. 6B
Intended as a straightforward question. (a) Candidates lost marks for not providing the information requested in the question. A surprising minority chose not to use eigenvalue/eigenvector properties to determine the properties of the rotation in the second part of the question. (b) Many candidates provided full details of a diagonalisation explicitly when they didn?t actually have to do this. Some lost marks by selecting the wrong points on the ellipsoid or by failing to check that the points they provided, whilst in the correct directions, were actually at the required distance from the origin.

Paper no. 1 Question no. 7B
(a) was very standard and most did it correctly, though the clarity of the brief explanation provided for maximisation of $v^T Av/v^T v$ varied a lot. The part answered least consistently well was the demonstration of a real eigenvector, e.g. many arguments were equivalent to showing that all choices of eigenvector were real, which clearly is not true, since a real eigenvector can be multiplied by an arbitrary complex number to give another (complex) eigenvector. (b) was non-standard but much of it followed from (a). Again the demonstrations of a real eigenvector were variable in quality. Some candidates (but not many) solved for the eigenvalues of A and for the eigenvalues of B and then attempted to argue that some combination of these was relevant to maximising the function.

Paper no. 1 Question no. 8A
A popular and fairly straightforward question on diagonalising 2 by 2 symmetric matrices, well done on the whole with many students attaining full marks. Students often forgot to express the eigenvalues in terms of the determinant and trace.

COMMENTS ON QUESTIONS

MATHEMATICAL TRIPOS PART IB, 2018

Course: ANALYSIS II
No comments received.

Course: COMPLEX ANALYSIS
No comments received.

Course: GEOMETRY

Paper no. 1 Question no. 3G
This question was generally well done by those that attempted it. A few students tried to prove $e(S^2) = 2$ by induction on the number of triangles, which isn't a good idea – you need to work with more general polyhedral decompositions, and to be careful about the base case. But they nevertheless usually got a β when the rest of the question was done correctly.

Paper no. 2 Question no. 14G
Not a popular question, but well done by those that made a serious attempt at it. I was particularly impressed by how well the slightly subtle part (b) was handled in many attempts.

Paper no. 3 Question no. 5G
There were only four attempts, of which only one earned a β. Evidently this question was very off putting. I still feel that the difficulty level is, in an absolute sense, appropriate – with the given formula for hyperbolic distance, the question is a very easy computation – but perhaps most students don't feel confident enough in hyperbolic geometry to attempt it on a Section I question.

Paper no. 3 Question no. 14G
This question has quite a few good attempts. Most students did enough to see why the claimed fact is true, but some αs were then lost for insufficient rigour.

Paper no. 4 Question no. 15G
Despite being on advanced material, this calculational question attracted the most attempts of the Section II Geometry questions. There was a good spread of answers, from completely successful to poor. The most efficient method, only spotted by a few, was to notice that the first fundamental form is nice enough that one can apply the curvature formula for geodesic polar coordinates.

Course: GROUPS, RINGS AND MODULES

Paper no. 1 Question no. 10G
Parts (a) and (b), both bookwork, were generally very well done, of course. (I was surprised to see a few non-standard proofs of Sylow's first theorem in part (b)!) In the unseen part (c), a common mistake was for students to compute the orders of the elements of G, notice that they are the same as for A_4, and conclude that G must be isomorphic to A_4. Most students who realised that they should use the conjugation action of G on its four 3-Sylow subgroups did enough to get an α.

Paper no. 2 Question no. 2G
It is perhaps not surprising that this question was not very popular, since it is long for a Section I question. Part (b) was very poorly done, and as a result there were few βs.

Paper no. 2 Question no. 11
To my surprise, the unseen part (b) was found very easy, and as a result many αs were awarded.

Paper no. 3 Question no. 1G
An easy question, with many correct attempts, as expected. Some students did fall down on the more conceptual part (b), though, so I feel the question provided a useful test of understanding.

Paper no. 3 Question no. 11G
A routine question with a large number of attempts, mostly correct and deserving of an α. Students found a variety of ways to complete the unseen part (c). Eisenstein's criterion (substituting $Y = X + 1$) is the easiest, but many of those who missed it still got their α by checking a finite number of cases coming from Gauss' lemma. The most common mistake was to claim that Gauss's lemma implies that a reducible polynomial over the rationals must have an integer solution. (This is true for monic polynomials, but the polynomial in the question is not monic.)

Paper no. 4 Question no. 2G
Not a tremendously popular question (like most Section I questions with a genuinely unseen component), but there was a nice variety of answers to part (b).

Paper no. 4 Question no. 11G
The question garnered a nice spread of answers. There were plenty of correct solutions, but also many unsuccessful attempts too. The most common mistake was to confuse the rational canonical form with the Smith or Jordan normal form. Most successful answers spotted a shortcut: it was enough, in this case, to compute the minimal and characteristic polynomials.

Course: LINEAR ALGEBRA

Paper no. 1 Question no. 1E
This question was too hard. Most candidates did the first part (which has appeared on problem sheets in the past) well, but it was common to make no progress with the last part.

Paper no. 1 Question no. 9E
Most parts of this questions were done well, though the last part was usually done by brute force and not how the examiner had hoped. Making the matrix be $2n \times 2n$ instead of 4×4 might have caused candidates to try a more conceptual approach. The JNF for $J_n(\lambda)^2$ was done quite poorly, and very few candidates got the JNF for $J_n(0)^2$ right. (Most observed that $J_n(0)^2$ has 1's on the 2nd superdiagonal, then asserted that $(J_n(0)^2)^r$ must have 1's on the $(r+1)$st superdiagonal.) Unfortunately this point, along with minor lapses in other parts, meant that relatively few attempts got α's.

Paper no. 2 Question no. 1E
The bookwork part was very well done, as was the calculation, with candidates using varied methods and getting varied (correct) answers.

Paper no. 2 Question no. 10E
The previously seen parts of this question were well done, with candidates doing the first part either using row and column operations (though a mark was often lost for not making explicit the connection between such operations and left- and right-multiplication by certain matrices) or more usually by translating it to a coordinate-free statement which is easy to prove. The determinant calculation was almost universally done well. The unseen part was done very poorly, and the large number of marks it carried lead to the very low α-rate. Despite the hint of calling the matrix X, most candidates did not try to apply the observation in the first part to X.

Paper no. 3 Question no. 10E
Although it attracted relatively few attempts, a number of candidates did this question very well; the average mark is quite low due to the many candidates to did not make much progress with the last part, and with the many candidates who simply asserted that complex matrices are upper-triangulable (there were 4 marks available for proving this as a step to the Cayley–Hamilton Theorem). A large number of candidates gave a proof of the Cayley–Hamilton Theorem using the formula $\mathrm{adj}(M) \cdot M = \det(M) \cdot I$, and lost marks for not saying anything about what $\mathrm{adj}(M)$ is or why this formula might be true.

Paper no. 4 Question no. 1E

This question was very popular, though without good reason: it does not seem to have been especially easy. While some candidates did it quickly and well, many candidates have not understood the relation between expressions such as $x^2 + 2xy + 2y^2 + 2yz + 3z^2$ and quadratic forms given in terms of a bilinear map, and in particular what the symbols x, y and z denote. It was common to refer to a new basis with $x' = x + y$ and so on.

Paper no. 4 Question no. 10E

This question was very well done. The part “Characterise the...” was too vague, and candidates gave wildly different volumes of writing as solutions: as I had intended this point to be very brief I did not feel I could deduct marks for being brief, but this no doubt disadvantaged candidates who spent too much time on it. (I would support the format of examinations to be changed to specify the number of marks available for each part of the question: it does not seem useful to test how well candidates can guess the examiners’ intentions.) The question would have been better without this part.

Course: METRIC AND TOPOLOGICAL SPACES

Paper no. 1 Question no. 12E
This question was too easy. There were too many marks available for deducing that S^1 is compact, which almost all candidates did, and too many for the last part, which turned out not to be as difficult as the examiner had anticipated.

Paper no. 2 Question no. 4E
I had thought that this question was quite standard, so was surprised that it attracted so few attempts and was not so well done. Even candidates who could pick a sequence of functions f_n with the sequences $d_1(f_n, 0)$ and $d_2(f_n, 0)$ showing different behaviour often had trouble translating this into a topological conclusion. Quite a few candidates implicitly used, or even claimed outright, that metrics induce the same topology if *and only if* they are Lipschitz equivalent.

Paper no. 3 Question no. 3E
Candidates find it difficult to write careful proofs under timed exam conditions. This observation is presumably not new, but solutions to this question provide evidence for it. For example, rarely did candidates try to show that K_x was not empty.

Paper no. 4 Question no. 13E
A surprisingly large number of candidates misread "d divides n" as "n divides d"; I tried to give as much credit as I could for arguments made using this definition which would have worked with the correct definition. All candidates could show that X is neither Hausdorff nor compact, seemingly without much effort: this was clearly too easy. I was very surprised that so few candidates could do the penultimate part; this is such a standard result in topology that it has a name: the Gluing Lemma.

Course: MARKOV CHAINS

Paper no. 1 Question no. 20H

This question, on calculating return probabilities in a coin-tossing game, was not answered well. It was probably too challenging. Several candidates failed even to identify a Markov chain describing the state of the game.

Paper no. 2 Question no. 20H

First part was very straightforward: the relationship between mean recurrence time and invariant distribution. But the problem part was more of a challenge. Candidates who noticed the symmetries of the Markov chain found the question relatively easy; others could spend a lot of time with recurrence relations.

Paper no. 3 Question no. 9H

Candidates found this question straightforward: classify states of a simple 5 state chain, and calculate a hitting probability of an absorbing state.

Paper no. 4 Question no. 9H

A very straightforward question on detailed balance: it is taught at the end of an Easter term course, which perhaps explains the relatively low number of attempts.

Course: OPTIMISATION

Paper no. 1 Question no. 8H

The transportation problem, with an example candidates were asked to solve. A straightforward question, answered reasonably well in the moderate number of attempts.

Paper no. 2 Question no. 9H

A question which was found not easy. There was some conceptual challenge, but close to a question on the example sheets. Not a popular question.

Paper no. 3 Question no. 21H

Proving the Lagrange Sufficiency Theorem got a few marks for many candidates. The non-negativity constraint on the variables in the optimization example needed to be treated rather than ignored for a good mark.

Paper no. 4 Question no. 20H

The max-flow min-cut theorem was generally well stated, but not many candidates made the comment that if the capacities of edges are integral than a maximum flow can be found where the flow along each edge is integral. This comment was needed (and asking for it was a hint) for the problem in the main part of the question.

Course: STATISTICS

Paper no. 1 Question no. 7H

The most common difficulty was not realising the relationship between the maximum likelihood estimator of the parameter and the maximum likelihood estimator of the median.

Paper no. 1 Question no. 19H

This was a question on the general linear model. Part (a) was book work, and done well - just a few mistakes over the degrees of freedom of the chi-squared distribution. Part (b), on estimating the angles of a plane quadrilateral, served to separate the candidates.

Paper no. 2 Question no. 8H

A question on the Neyman-Pearson lemma. Some candidates paid too much attention the precise threshold for the likelihood ratio, rather than realising the lemma gives the form of the critical region and the size of the test gives the precise boundaries of the critical region. Many candidates picked up a few marks by attempting just the first paragraph.

Paper no. 3 Question no. 20H

A question on contingency tables. Some candidates failed to outline a justification for Pearson's chi-squared statistic from the generalised likelihood ratio statistic. A pleasingly high number of candidates were able to comment sensibly on the use of the statistical technique in the numerical example.

Paper no. 4 Question no. 19H

A question on Bayesian inference for a binomial model, with two priors and two loss functions. Candidates handled the problem well.

Course: COMPLEX ANALYSIS OR COMPLEX METHODS

Paper no. 1 Question no. 2A

This question was done very well by nearly all candidates. The common approach to (b) was to consider just the mapping of three points. Candidates who did not explain why this was sufficient (e.g. Mobius maps send circlines to circlines) lost marks, but these were typically not sufficient to lose out on the beta.

Paper no. 1 Question no. 13A

Candidates struggled with justifying why the integration along the vertical contours was negligible, but otherwise tackled this question as expected. There was abundant but inappropriate quoting of Jordan's Lemma in section (b) if candidates considered both $e^{i\pi z}$ and $e^{-i\pi z}$ terms.

Paper no. 2 Question no. 13A

I expected this question to be answered better - a significant number of candidates could not answer the bookwork part (a). Determining that all singularities of F were removable was also found tricky, which made completing the rest of the question hard going.

Course: COMPLEX METHODS

Paper no. 3 Question no. 4A

This question was clearly avoided by most candidates. Of those who attempted it, I was surprised that none managed to obtain full marks for section (a) despite it being identical to a question on the example sheets.

Paper no. 4 Question no. 14A

Candidates generally understood the relationship between parts (a) and (b) of the question, although the homogeneous solution to the (transformed) equation in (b) was often overlooked, or set to zero. This meant while K was found correctly, the limits of the final integral were often wrong, or a Heaviside function had been introduced. Some confusion also arose due to the term $x - \xi$ in the integral, leading candidates to (incorrectly) believe the convolution theorem should be used.

Course: ELECTROMAGNETISM

Paper no. 1 Question no. 16C

A disappointing take-up for this question, probably because candidates were put off by the rather long (bookwork) derivation called for at the beginning. There was one perfect solution, whereas all other attempts were mostly just partial.

Paper no. 2 Question no. 6C

Mostly done well, though candidates often simply asserted $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$ rather than deducing this as a consequence of the Maxwell equation $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$. There was some minor confusion with signs, both the identity $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{A}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{A}) - \nabla^2\mathbf{A}$ and (often compensating!) the fact that $\nabla^2(1/r) = -4\pi\,\delta^3(\mathbf{r})$.

Paper no. 2 Question no. 18C

I was disappointed that even the bookwork part of this question caused problems for many candidates, who leapt to use Gauss' Law to find a discontinuity in the *normal* component of $\mathbf{E}$ and Ampere's Law to find a discontinuity in the *tangential* components of $\mathbf{B}$, whereas the question asks about the *tangential* components of $\mathbf{E}$ and the *normal* components of $\mathbf{B}$. Those who avoided this mistake typically went on to do the Lorentz transformation part well, though the final result was not always expressed in terms of quantities in Albert's frame.

Paper no. 3 Question no. 17C

The first part of the question – deriving energy conservation from Maxwell's equations – was usually done well, though several candidates either gave incorrect interpretations of the various terms in this equation, often not specifying that $\frac{1}{2}\int\left(\epsilon_0\mathbf{E}\cdot\mathbf{E} + \frac{1}{\mu_0}\mathbf{B}\cdot\mathbf{B}\right)d^3x$ was the energy *of the electromagnetic field*. (Several candidates omitted this part of the question altogether. The second part of the question was poorly done. While most candidates realised they needed to use Ampere's Law to find the magnetic field, few realised that the field in the region between the capacitor's plates was due to the changing electric field (as the plates discharge) rather than due to any current.

Paper no. 4 Question no. 7C

This question was done well by almost all candidates who attempted it. There were occasional attempts to justify the conservation equation using something other than Maxwell's equations, but essentially all the candidates who obtained the correct conservation equation then successfully completed the final part of the question.

Course: FLUID DYNAMICS

Paper no. 1 Question no. 5D
This was a straightforward question done straightforwardly.

Paper no. 1 Question no. 17D
This question was very well done by most people. Though there were a large number of alphas, I don't consider that the question was too easy but rather that the students had learned the fluid-mechanical principles associated with parallel flow very well and could apply them.

Paper no. 2 Question no. 7D
This question was all bookwork, albeit from the end of the course, and I consider it to be very straightforward. It was disappointing, therefore, that there were so few attempts and even fewer good ones.

Paper no. 3 Question no. 18D
The first half involved using fundamental fluid-mechanical principles (conservation of mass, use of Bernoulli's equation for potential flow). A good number of students who tackled this question stumbled their way to the marks but were somewhat disorganised piecing things together. The second half was straightforwardly mathematical and exposed students' lack of fluency with integration.

Paper no. 4 Question no. 18D
This question was mostly well done but I was surprised that it was not done even better. It is a straightforward example of linear stability analysis. Most marks were lost for want of a clear setting up of the fluid-mechanical problem – Laplace's equation for potential flow with boundary conditions appropriate to the problem set.

Course: METHODS

Paper no. 1 Question no. 14C
The bookwork on defining a convolution and showing the Fourier transform of $f * g$ is $\tilde{f} \times \tilde{g}$ was done well by most candidates. There were various approaches to finding the Fourier transform of $(\sin x)/x^2$, often very inventive, and correspondingly the final result could take many (equivalent) forms. Full credit was awarded to any correct method. Some candidates were confused over whether to use the convolution theorem or its inverse.

Paper no. 2 Question no. 5C
This seemed to be an unpopular question. Again there were various approaches to proving the desired equality, and anything accurate was awarded credit.

Paper no. 2 Question no. 16A
Overall this question was answered well, although a number of candidates did not attempt the express the solution in (b) as a Fourier Series. Some tried Green's functions, whilst others wrongly considered $L(y) = \lambda y$. Algebra errors in the Fourier Series solution were common but marked forgivingly.

Paper no. 3 Question no. 7A
A standard Green's function question answered well, although algebra was overall poor, so despite the high beta count, relatively few candidates actually got the correct final solution.

Paper no. 3 Question no. 15A
Most candidates approached this question as expected through repeated use of the sampling property. Some gave additional definitions (correctly or incorrectly) for the delta function, but these often caused problems when trying to solve (e)(ii).

Paper no. 4 Question no. 5A
This question was answered well overall and nearly all candidates followed the expected method.

Paper no. 4 Question no. 17C
The bookwork part on Green's theorem was generally well done, though many candidates ignored the instruction to treat the singular point carefully and just asserted that $\nabla^2 G_0(\mathbf{x}, \mathbf{x}_0) = \delta^3(\mathbf{x} - \mathbf{x}_0)$. Full credit was awarded here only if the candidate had provided some justification of the δ-function, or else had applied the given equation to a region with a neighbourhood of $\mathbf{x}_0$ removed. The second part of the question was found challenging. While most candidates realised they needed to find a Green's function obeying $G(\mathbf{x}, \mathbf{x}_0) = 0$ for $\mathbf{x} \in \partial\Omega$, not everyone saw how to use the hint to find this.

Course: NUMERICAL ANALYSIS

Paper no. 1 Question no. 6D
The first part (bookwork) was done well by most people, many doing things slightly more efficiently than in my model solution. I would have liked to see people explaining the relationship between the local error they found and the global error of the scheme but this had not been explicitly asked for and I didn't penalise the omission.

Paper no. 1 Question no. 18D
Statistically, this hit the right level of difficulty. The greatest loss of marks was associated with the

level of care taken to explain explicitly how the Householder transformations are used in creating an algorithm for QR decomposition.

Paper no. 2 Question no. 19D
There was a minor typographical error, spotted early in the examination, that did not seem to have affected anyone's analysis. Most students found this question very straightforward. Most marks were lost for want of *careful* explanations justifying the main result.

Paper no. 3 Question no. 19D
This turned out to be entirely straightforward for most people, with 84% obtaining an alpha. The most common source of error was a lack of care regarding when the functional commutes with the integration (only on a *fixed* interval).

Paper no. 4 Question no. 8D
An appropriately straightforward short question with 79% betas.

Course: QUANTUM MECHANICS

Paper no. 1 Question no. 15B
This question was disappointing. Many attempts failed to use the hint and attempted to solve the Schrodinger equation by series solution, getting bogged down in algebra. Some attempts did not know what a bound state was. Candidates who used the hint, and who knew a bound state had negative energy, tackled the question well.

Paper no. 2 Question no. 17B
Surprisingly the very first part of this question was not well done with some candidates unable to identify correctly the terms in the radial equation for the hydrogen atom. The next part of this standard hydrogen atom question was well done with most candidates able to correctly identify the radial wave function. Finding the expectation value of the radius was not well done with many candidates not realising the need for the wave function to be normalised.

Paper no. 3 Question no. 8B
The first part of this question was well done on the whole. The second part on Ehrenfest theorem was less well done and many candidates failed to recognise that they should first prove Ehrenfest theorem and then apply it. Those that did, or essentially proved it for the special cases, did well.

Paper no. 3 Question no. 16B
The attempts on this question on the generalised uncertainty principle were mixed. Not all attempts were able to show the first part satisfactorily. Some attempts failed to prove the generalised uncertainty principle satisfactorily. The final part of this question was mixed with some candidates unable to apply the first part to the expectation value of the Hamiltonian to show that every energy eigenvalue satisfied the inequality given.

Paper no. 4 Question no. 6B
This question on the conservation equation was very well done. The first part was standard book-work and a simple application which most attempts able to complete.

Course: VARIATIONAL PRINCIPLES

Paper no. 1 Question no. 4B
This question was well done. Most attempts tackled the first part well but some were confused by the second part.

Paper no. 2 Question no. 15B
This standard question containing an extension of bookwork was done well with most candidates able to derive the Euler-Langrange equations for this functional. The application was mainly done well though some attempts were sloppy and imprecise.

Paper no. 3 Question no. 6B
Most candidates did this question well with most able to derive the equations of motion and show the angular momentum was a conserved quantity. Not all were able to use the information to simplify to deduce h was a conserved quantity and the attempts to identify h were disappointing.

Paper no. 4 Question no. 16B
On the whole this question was very well done. Most candidates were able to complete the first part and apply this to the second part. Not all candidates were able to show that the special condition reduced the anharmonic oscillator to the harmonic oscillator and that J was the angular momentum, conserved by Noether's theorem.

MATHEMATICAL TRIPOS PART IA, 2017

COMMENTS ON QUESTIONS

Course: ANALYSIS I

Paper no. 1 Question no. 3F

Surprisingly, many more students than expected had difficulties with this question. In particular, there were a lot of answers that made no sense at all, arguing by applying the triangle inequality in the wrong direction, replacing $n\varepsilon$ by ε, and so on. The students that argued in this way did not use the assumption that the given sequence is monotone (and seemed to not have been surprised not to use an assumption) and generally received no points.

The low outcome seems to be explained by the following: (1) Many students attempted the question, but did not produce anything reasonable and thus received 0 points. Not including these students in the statistics, the average mark is between 6 and 7, which is more reasonable. (2) The question did not have multiple parts, which would have made it possible to award marks for partially correct answers.

Paper no. 1 Question no. 4E

This was a straightforward bookwork question. Those who missed out on a β had trouble with the first part. Some seem to have misread the question and assumed that $\sum a_n z_0^n$ was absolutely convergent. Others tried to use the Ratio Test which does not help here.

Paper no. 1 Question no. 9D

Bookwork and the first part of (c), which was in an examples sheet, were done very well. The very last part of the question was new and unfamiliar; though not hard, most found it challenging, and it was the reason for the relatively low alpha count.

Paper no. 1 Question no. 10D

The number of attempts on this question was surprisingly low. All of the concepts covered in the question were extremely elementary, and even "convexity implies continuity" had been an exercise on an examples sheet. The inequalities the question asks to establish however were new, and these may have looked too daunting to many. On the plus side, those who attempted the question generally did it well. The last part asking to show a local minimum is a global minimum for a convex functions was meant to provide some challenge, but I was pleased to see many complete proofs of this.

Paper no. 1 Question no. 11F

The first part of this question was bookwork, and so as a consequence most answers were memorised versions of the proof given in the lectures. In writing down these proofs quickly, the students often forgot to use or mention the assumptions needed for the steps. The other parts were generally done satisfactorily.

Paper no. 1 Question no. 12E

Most candidates had a decent attempt at this question. There were a number of reasons for losing marks. Almost all candidates forgot to say that a continuous function on closed bounded interval is bounded, which is necessary before one can even contemplate integrability. Just quoting without proof that a continuous function on a closed bounded interval is uniformly continuous was not sufficient. When showing that any extension to $[0,1]$ of a bounded continuous function on $(0,1]$ is integrable, many treated the first point of the dissection as both a fixed point and as a variable at the same time. Finally, neither the Fundamental Theorem of Calculus, nor swapping a limit with integration was of any help in integrating the derivative in the last part of the question.

Course: GROUPS

Paper no. 3 Question no. 1E
Despite the hint, many candidates failed to deal with the three cases when one of w_1, w_2 and w_3 is ∞. The attempts on the second part produced lots of good answers, but also many sloppy arguments.

Paper no. 3 Question no. 2E
Many candidates lost marks on the easy bookwork questions by not being precise enough. The definition of normal subgroup must contain the requirement of being a subgroup. The definition of quotient group, as the definition of any group, must specify a set *and* a binary operation. The Isomorphism Theorem includes the statement that the kernel is a normal subgroup. That said, those who could do the familiar problem question at the end, generally earned enough marks for a beta.

Paper no. 3 Question no. 5E
Most who tried this question could do the first part well, but only about half the candidates had a serious go at the second part. The special case p=2 of the second part is bookwork (splitting of conjugacy classes of even permutations), and the general case is not that different. So having so few attempts is somewhat surprising.

Paper no. 3 Question no. 6E
This question was entirely bookwork. However, the last part, classifying groups of order 8, is not an easy bookwork. It requires one to gather their thoughts carefully and consider several cases. This was not done very successfully. Even the first half of the question caused some difficulties. For example, despite being asked specifically, very few candidates proved that the order of an element in a finite group is finite, and many simply stated without proof that the order of the subgroup generated by an element is the order of the element.

Paper no. 3 Question no. 7E
This question was reasonably well done. Most people could do part (a) and the first half of part (b), although noone could really provide a completely rigorous proof for the size of the stabiliser. A good number undestood how to use Burnside's lemma to compute the number of different colourings of the cube with only occasional computational error.

Paper no. 3 Question no. 8E
I expected almost everyone to be able to produce the standard proof that sign is well defined, but this was not the case. It was clear from the attempts on part (b) that many don't quite get the power of conjugation. Hence there were some laboured arguments for showing that (1 2) and $(1\ 2\ \ldots\ n)$ generate S_n. Few could prove that $(1\ k+1)$ and $(1\ 2\ \ldots\ n)$ generate S_n if k and n are coprime, and even fewer could show the converse.

Course: NUMBERS AND SETS

Paper no. 4 Question no. 1D
Easy question done extremely well.

Paper no. 4 Question no. 2D
Straightforward question requiring knowing defintions and elementary set identities. The only slightly non-trivial part was establishing transitivity of the relation which a surprising number of candidates managed to muddle up!

Paper no. 4 Question no. 5D
This was found to be much easier than I thought. In hindsight, there should have been a fourth, challenging part.

Paper no. 4 Question no. 6D
We hit the mark with this one I thought, in terms of the level of difficulty and the number of attempts attracted.

Paper no. 4 Question no. 7D
This proved to be surprisingly unpopular, although the binomial theorem and manipulation of binomial coefficients are all that was needed. Given the unfamiliar nature of the question, the candidates would have had to think a little to understand what's being asked, and to connect the first part to the second. Beyond that, no particular cleverness was required.

Paper no. 4 Question no. 8D
Popular, easy question done very well. Some failed attempts at finding an example as required.

Course: PROBABILITY

Paper no. 2 Question no. 3F
Most students were able to prove the easier upper bound. The lower bound by Cauchy-Schwarz caused more difficulties. The overall outcome was roughly as expected.

Paper no. 2 Question no. 4F
For many students, this problem was straightforward. However, some students were confused about the definition of conditional probability density function, resulting in a number of answers that started from wrong assumptions.

Paper no. 2 Question no. 9F
This question was elementary bookwork, except for the last part. As a consequence, most students did very well in all but the last part (which was however also done okay by many).

Paper no. 2 Question no. 10F
This question was also mostly bookwork, and again most students did very well. It seems that the first part was a bit too easy as it seems to have been covered essentially literally in class. It would have been a better problem if part of the problem had not been seen before, e.g., by asking for the large deviations of a different distribution than the Gaussian one.

Paper no. 2 Question no. 11F
Very few students attempted this questions. However, most of those who did attempt it did well. It seems that the less familiar look of the question discouraged students to attempt it, even though the actual work was not difficult.

Paper no. 2 Question no. 12F
This question seemed to have been at a good level. It definitely was more challenging than 9F and 10F, but at the same time many students attempted it because it started with a familiar bookwork component. It also helped that the intermediate answers were given in the question; knowing these, it seems that the students are much better at finding correct solutions.

Course: DIFFERENTIAL EQUATIONS

Paper no. 2 Question no. 1C
Fine.

Paper no. 2 Question no. 2C
Fine.

Paper no. 2 Question no. 5C
Mostly calculational. (a) Mostly OK. (b) Many forgot to differentiate V. (c) students who simplified the form for the time periodic solution in (c) using expression for ω_0 mostly got the answer, but if left to the end it became a struggle to do the integral and calculate the quality factor Q correctly. c(ii) bookwork, mostly well done.

Paper no. 2 Question no. 6C
(a) quite well done, straight from example sheets, but (b), which was unseen, was less well done than expected.

Paper no. 2 Question no. 7C
Many students think of the Wronskian more as a trick device to get a second solution than as a test for linear independence. First theory part of the question not all that well done , but the second part (finding the power series solutions explicitly) produced more successful attempts.

Paper no. 2 Question no. 8C
(a) Not very well done - many tended to give "word solutions" to the first homogeneous equation, which they could not generalize to the inhomogeneous case even with the correct solution to the ode available. If they were taught to solve homogeneous problems either by introducing characteristic curves explicitly or by using a complete change of variables they would do better on this topic in future. (b) was generally better done.

Course: DYNAMICS AND RELATIVITY

Paper no. 4 Question no. 3A
This question was done well by most candidates, but it was disappointing that many struggled to handle the (rather standard) expansion of $\dot{\mathbf{y}}_i \cdot \dot{\mathbf{y}}_i$ and to write the correct definition of the moment of intertia of a particle in terms of its mass and perpendicular distance to the axis of rotation.

Paper no. 4 Question no. 4A
This was a nice elegant question, but it posed problems to quite a few candidates. Many struggled with the last part, where the candidates were supposed to write a differential equation for v and perform a routine integration after separating the variables.

Paper no. 4 Question no. 9A
From my past experience, many students do not find questions of this type trivial. Hence, I was pleasantly surprized that this question was done well by almost every candidate who attempted it. With the hindsight it should have been made more challenging.

Paper no. 4 Question no. 10A
Many candidates struggled with this seemingly accessible question. As a minor issue, in part (a) many misplabled the sign of k, implicitely assuming potenial $-k/r$ as opposed to k/r stated in the question. Many bogged down in over-complicated calculations in parts (b), especially in (b)(iii). In addition, quite a few candidates demonstrated the misconception that p corresponds to the minimum of the effective potential.

Paper no. 4 Question no. 11A
This was a question on rotating frames. It attracted fewer attempts than other questions and was poorly done. The set-up was very simple, yet the solution required solid understanding of the formalism and clear physical intuition. Despite the clear verbal explanation supported by the diagram, some candidates misinterpreted the configuration of the hoop, assuming that y'-axis is vertical. Many candidates calculated the ficticious forces successfuly, but struggled to use their results in order to extract the force excerted by the hoop on the bead. Understanding of the nature of the force of normal reaction was poor.

Paper no. 4 Question no. 12A
This was an extremley popular question, which has been done successfully by the majority of candidates. I thought that the two-stage rocket would pose a considerable challenge, but with the hindsight this question could have been made more difficult. Many candidates did struggle with the very last part, where they were expected to analyze the case of $k = 1$. Many arrived at apparent contradiction $v_2 = 2v_1$ (here indices refer to the final speeds of the two stages), but did not resolve it (e.g. with the help of the l'Hopitale's rule).

Course: VECTOR CALCULUS

Paper no. 3 Question no. 3B

This was a very routine question (an area integral), and it was done well.

Paper no. 3 Question no. 4B

A much better response than I had expected for the quotient theorem. Some just did the first part for 3 marks, but most realised what was required for the second part (i.e. to show that T transforms according to the tensor transformation law). Only about 5 wrote an essay on linear maps, getting no marks because it wasn't an 'or otherwise' question and also because they mostly showed little understanding of what they were trying to write. An accurate statement of the quotient theorem was required. A surprising number thought that you have to conisider $T_{ij}v_{ij}$, where v_{ij} is the arbitrary tensor as well at $T_{ij}u_j$ (which made for extra work but didn't lose any marks). I was also generous to those who thought that a 3×3 array corresponds to a rank three tensor.

Paper no. 3 Question no. 9B

This was quite technical and there was an encouraging number of very competent answers. The very first part, however, requiring two applications of the chain rule, was extremely poorly done. Given $\mathbf{z}(\mathbf{x}, t) = \mathbf{x}t$, quite a few started with $\frac{\partial t}{\partial z_i} = \frac{1}{x_i}$ and it got worse from there. The average mark was quite low (in comparison with the quality of the response) because a lot of candidates just proved that $\mathbf{B} = \nabla \times \mathbf{A} \Rightarrow \nabla \cdot \mathbf{B} = 0$ for two marks, then moved on.

Paper no. 3 Question no. 10B

Too easy, it seems; though I am amazed that so many people knew the formula (involving cross products) for the surface area element).

Those who forgot the 'cap' in the surface integral could still get their alpha. However, quite a few then went back and carefully fudged their (possibly) correct answer to the curved surface integral to give an incorrect answer equal to the volume integral, with the intention of deceiving me. They succeeded, and thereby lost two further accuracy marks and their alpha, leaving me with a sense of just deserts having been served and a little warm glow inside.

Paper no. 3 Question no. 11B

The response was disappointing in both quantity and quality. Part (a) was about parameterised curves, tangent and normals, curvature and torsion which is on the first line of the schedule. Part (b) followed on, and was a slightly easier version of Q7 on Examples Sheet 2. Part (c) was a straightforward application of the formula in Part (b).

Paper no. 3 Question no. 12B

Almost everyone could do the opening uniqueness bookwork. Many candidates took the next two parts, which I had thought brief and rather elegant, as an invitation to write pages and pages of gibberish.

A very frequent mistake was to state that if $\phi = \psi$ on the surface S then $\nabla\phi = \nabla\psi$ on S.

Course: VECTORS AND MATRICES

Paper no. 1 Question no. 1A
The majority of candidates answered part (a) well, but many struggled with part (b), where getting the sign of $\arg(1-z)$ was a typical difficulty.

Paper no. 1 Question no. 2C
Generally well done. Some confusion/errors over commutativity of matrix multiplication.

Paper no. 1 Question no. 5A
Generally this was a well done question. Few candidates did not establish the expression $\mathbf{x}_{n+2} = -\lambda^2\mathbf{x}_n$ in part (b)(i), which would have simplified subsequent calculations. Many candidates struggled to find $\cos\theta$ in the last part.

Paper no. 1 Question no. 6B

Disappointing. I expected a much better response to this rather easy question testing basic understanding of eigenvectors and eigenvalues. Part (b) could have been done in a very low-tech way by choosing a basis ($\mathbf{m}$, $\mathbf{n}$ and $\mathbf{m}\times\mathbf{n}$) or, more high tech, by using standard results such as the sum of the eigenvalues being the trace of the matrix and B^TB, being a symmetric matrix, having orthogonal eigenvectors. Quite a number of candidates instead babbled about kernels and image spaces, etc, and ended up not proving very much.

Paper no. 1 Question no. 7B
The great majority of candidates who proved that $x^TAx = 0$ if A is antisymmetric ($x^TAx = (x^TAx)^T = -x^TAx = 0$) thought it was OK just to write down the same argument backwards to prove the converse, not realising that if $x^T(A)x = 0$ for all x, you can't trivially knock off the x's and state that $A^T + A = 0$. Some candidates went through a long argument involving diagonalising the symmetric part of A, perhaps remembering something similar that came up last year.
Almost all candidates who attempted the question could do the semi-bookwork in part (b).
Almost all candidates who attempted part (c) forgot to show that $\mathbf{a}$ can be chosen to be real, or just assumed that it was real. Annoyingly, candidates who used a rather infantile approach, just writing out long hand the elements of a general anti-symmetric matrix, obtained the whole of part (c) with little understanding of what is going on.

Paper no. 1 Question no. 8C
Well done ; turned out to be on the easy side for a long question .

MATHEMATICAL TRIPOS PART IB, 2016

COMMENTS ON QUESTIONS

Course: ANALYSIS II

Paper no. 1 Question no. 11
Generally well done. A couple of candidates observed that the interval in the first piece of bookwork needed to be bounded (as the examiner had intended), and everyone else just tacitly assumed it.

Paper no. 2 Question no. 3
A very easy question on uniform convergence, mostly well done, although strangely a significant number of candidates gave the definition of uniform continuity instead of that of uniform convergence (but then went on to answer the rest of the question perfectly).

Paper no. 2 Question no. 12
Fears that this question might prove difficult to understand were unfounded.

Paper no. 3 Question no. 2
Surprisingly few candidates gave a satisfactory answer to the last part (give a metric on $\mathbf{R}$ with respect to which it is not complete). As least as many candidates proposed the p-adic metric on $\mathbf{R}$ (!) as a counterexample.

Paper no. 3 Question no. 12
Unfortunately the final part of the question turned out to be easier than anticipated. Originally intended to be a function which is not a contraction (but possessing an iterate which is), after revisions to the question it unintentionally ended up as a contraction mapping (and worse, one for which the existence and uniqueness of the fixed point can be proved by simple calculus). However together with the bookwork the question was sufficiently challenging to give a reasonable spread of marks.

Paper no. 4 Question no. 3
Definitely on the easy side, but some surprisingly incoherent answers.

Paper no. 4 Question no. 12
This looked too easy, but the bookwork was not well done, and many candidates were confused about how to show that a function was not differentiable at a point.

Course: COMPLEX ANALYSIS

Paper no. 3 **Question no. 13F**
The bookwork on Taylor's theorem was well done, as was the inequality coming from Cauchy's formula. But the case of equality was fudged by most candidates, who generally applied 'wishful thinking' to the integrals.

Paper no. 4 **Question no. 4F**
The first part of this question, which was bookwork about antiderivatives, was well done, but the second part was very poorly done. Most students quoted false statements about antiderivatives, with no regard to whether the domain was a star domain or not – despite the fact that the question clearly suggests that this distinction is key.

Course: GEOMETRY

Paper no. 1 **Question no. 3**
An easy question on hyperbolic polygons, many candidates achieving full marks.

Paper no. 2 **Question no. 14**
A routine question on Euclidean geometry, though the simple rider (give an orthogonal transformation of the plane which cannot be written as a product of fewer than 3 reflections) stumped most of the candidates.

Paper no. 3 **Question no. 5**
Strangely unpopular, with only 3 serious attempts.

Paper no. 3 **Question no. 14**
This proved to be far too easy. Despite a draconian mark scheme, most candidates obtained an alpha with little effort.

Paper no. 4 **Question no. 15**
Some excellent answers to a question which tested the candidates' ability to think geometrically and to articulate their thoughts on paper.

Course: GROUPS RINGS AND MODULES

Paper no. 1 **Question no. 10E**
This question covered Sylow's theorem and its applications in group theory.

Paper no. 2 **Question no. 2E**
This question covered elementary aspects of ring theory with a mixture of bookwork and unseen elements.

Paper no. 2 **Question no. 11E**
This question examined more advanced aspects of rings and ideals. Average mark is perhaps surprisingly relatively low.

Paper no. 3 **Question no. 1E**
This question examined simple properties of modules.

Paper no. 3 Question no. 11E
A question about Euclidean domains and rings related to the integers. Again average is unexpectedly low.

Paper no. 4 Question no. 2E
A question examining knowledge of the more elementary properties of groups.

Paper no. 4 Question no. 11E
This question is about modules over Euclidean domains and PID's. Average is relatively high.

Course: LINEAR ALGEBRA

Paper no. 1 Question no. 1F
This was well done: students were generally able to prove Steinitz Exchange correctly and with the detail correct, which was impressive.

Paper no. 1 Question no. 9F
An impressively-answered question. Candidates were good at the simple manipulations of inverses and restrictions, and were also particularly good at finding the counterexamples. With hindsight, perhaps this question was too easy.

Paper no. 2 Question no. 1F
Candidates could prove Rank-Nullity and generally had little problem constructing simple examples.

Paper no. 2 Question no. 10F
Very good answers on the rank inequalities and also on the problem part. Students seemed genuinely good at seeing how the rank is behaving.

Paper no. 3 Question no. 10F
A poorly answered question. The diagonalisation of a quadratic form was well done, but the proofs of the Law of Inertia were confused and usually incorrect. And the problem part showed that students were very uneasy in any actual manipulation of an explicit quadratic form.

Paper no. 4 Question no. 1F
Generally well done. But a lot of verbiage and imprecise argument in showing that an inner product space is the direct sum of a subspace and its perp.

Paper no. 4 Question no. 10F
Candidates were able to reason extremely well about dual spaces. Even arguments that boil down to 'these two abstract spaces are equal because one is a subspace of the other and by an earlier calculation their dimensions are the same' were well done. The command shown was impressive, particularly as dual spaces are notorious as being one of the hardest parts of the course.

Course: METRIC AND TOPOLOGICAL SPACES

Paper no. 1 Question no. 12E
This question examined connectedness and path-connectedness properties of subsets of the line and the plane.

Paper no. 2 Question no. 4E
A question on easy properties of metric spaces.

Paper no. 3 Question no. 3E
A question on topology on product spaces.

Paper no. 4 Question no. 13E
A popular question with relatively higher average, on compact spaces. The hint included in the question explains the high average.

Course: MARKOV CHAINS

Paper no. 1 Question no. 20H
Showing that X is a Markov chain and writing the transition matrix was done very well by everyone. For the last part of the question most students used induction, since the answer was given. This meant that they were able to show the expression on the expectation but did not explain why this solution was the minimal one.

Paper no. 2 Question no. 20H
The first part of the question was done well and so was the second part. Regarding the final part, many students attempted to find the invariant distribution instead of using the result of part (b).

Paper no. 3 Question no. 9H
Surprisingly this question was done very poorly despite the fact that all parts of this question were bookwork. Students seemed not having understood well the notion of reversibility.

Paper no. 4 Question no. 9H
This question was bookwork but many students struggled with it. It seemed that most students did not know let alone prove the order of the probability of being back at 0 in n steps.

Course: OPTIMIZATION

Paper no. 1 Question no. 8H
This was a standard simplex algorithm question and the students who attempted it had no problem at all.

Paper no. 2 Question no. 9H
The first part of this question was bookwork and students did it very well. Even though the second part was also bookwork, some students struggled with it, in particular proving the weak duality property.

Paper no. 3 Question no. 21H
The first two parts of this question were bookwork and were done generally very well. Many students had no trouble with the game when $n = 3$. The last part for general n was the hardest. Some students noticed the domination by the first three rows, but others attempted induction or trying to find a pattern unsuccessfully.

Paper no. 4 Question no. 20H
Surprisingly many students had trouble with the first part of this question even though it was bookwork. Despite this, they were able to do the other parts of the question quite well.

Course: STATISTICS

Paper no. 1 Question no. 7H
The first part of the question was bookwork so students had no trouble with it. The first part of the second question was done very well, despite having a typo in the question. Some people struggled with the second part involving the conditional expectation.

Paper no. 1 Question no. 19H
The first two parts of this question were either bookwork or direct applications of material done in lectures, so students did very well. Some people struggled with the last part.

Paper no. 2 Question no. 8H
Students did generally well in this question. It was mostly bookwork or similar to examples they had seen in lectures.

Paper no. 3 Question no. 20H
The first part of the question was bookwork and it was done well. Some students also provided the proof of the Gauss-Markov theorem in order to prove the inequality on the variances without appealing to the theorem. The last part was done generally well, but some students did not fully justify the independence between $\hat{\beta}$ and $\hat{\sigma}^2$.

Paper no. 4 Question no. 19H
This question was done very well. The first part was bookwork and the second part was an application of the Neyman-Pearson lemma.

Course: COMPLEX ANALYSIS OR COMPLEX METHODS

Paper no. 1 Question no. 2A
This seemed to be a straight-forward question, with most attempts being successful. Some candidates "guessed" a solution but failed to show it then satisfied the C-R equations, resulting in a loss of 2 marks.

Paper no. 1 Question no. 13A
This was not a popular question. Of those attempts that got through to the end it was unfortunate to see a majority of them closing in the LHP for part b)ii) which is incorrect. Many solutions also lacked proper justification of why b)i) was analytic specifically at $\lambda = \alpha_+$, despite the note in brackets at the bottom of the question. Part b)iii) was answered well, with candidates either summing their two previous solutions or closing in a rectangular contour.

Paper no. 2 Question no. 13A
This proved to be a bit too simple for a long question but there were a remarkable number of algebraic mistakes - these were penalised harshly given the question was relatively easy.

Course: COMPLEX METHODS

Paper no. 3 Question no. 4A
Most candidates, in the spirit of a "show that" question, avoided any messy algebra and quoted the required fractions for a partial fraction separation of $\hat{y}(s)$ (or the residues from the poles). Candidates who attempted to calculate these themselves were therefore marked very kindly for any algebraic mistakes. The majority of candidates who did not get a beta did not attempt any sort of inversion/separation once they had transformed the equation.

Paper no. 4 Question no. 14A
This question seemed to be pitched at the right level, although there were a lot of low-mark attempts that could not make progress because they could not conformally map the starting domain to the UHP. The bounded condition (decaying in the upper half η-plane) led to some confusion over the general solution, with many using $\phi = e^{-k\eta}$ rather than $\phi = e^{-|k|\eta}$. Many attempts also did not explicitly obtain a general formula for $F(\xi)$ in terms of f and x, however it was more likely in the specific case $f = \sin x/4$ they did - $\sin \xi/4$ was a popluar (but wrong) answer.

Course: ELECTROMAGNETISM

Paper no. 1 Question no. 16C
A question on electromagnetic waves and their reflection from a plane surface. It was generally well done. The first part was bookwork and was well reproduced by most candidates. The second part used an unfamiliar notation but was well answered by many. Algebraic arguments were used rather than geometrical arguments involving the reflection of the vectors in the boundary. Marks were often lost for not showing that the reflected wave satisfies Maxwell's equations.

Paper no. 2 Question no. 6C
An easy short question about a spherical capacitor and Gauss' Law. This was quite poorly done in general. Many had errors in the electric field, or wrote down potentials that were discontinuous. Relatively few correctly integrated the electrostatic energy and related it to the capacitance.

Paper no. 2 Question no. 18C
Mainly bookwork on the electromagnetic tensor and Lorentz transformation of the field components in special relativity. Perhaps unsurprisingly, many candidates had various sign errors in their expressions. The last part of the question involved an application to charges moving along a wire. While some candidates correctly obtained and transformed the field components, remarkably few gave a convincing physical explanation for the appearance of an electric field in the moving frame.

Paper no. 3 Question no. 17C
This question was about the method of images in a sphere, approached in a novel way. The first part was attempted by a variety of methods. Some candidates referred to the circle (or sphere) of Apollonius, which was fine if the quoted results were adequately stated. The translation from the result of the first part to the second part (with the sphere centred at the origin) caused difficulty for many. Some quoted or derived the result for the image in a sphere separately. Very few candidates obtained a correct expression for the force on the particle.

Paper no. 4 Question no. 7C
A short bookwork question on the magnetic field of a wire loop in the dipole approximation. This was very badly done, with only two candidates obtaining more than half marks and no one obtaining a fully correct solution.

Course: FLUID DYNAMICS

Paper no. 1 Question no. 5

This should have been an entirely straightforward question involving elementary kinematics. I was surprised by the number of students who confused incompressibility (div u = 0) with irrotationality (curl u = 0). Although the average mark and number of betas is high, I was disappointed that these values were not even higher.

Paper no. 1 Question no. 17

This question is about the hardest that could be set at this level on parallel, viscous flows, and I demanded a lot in terms of clear setting up of the mathematical problem from its physical description. It was done very well by most students.

Paper no. 2 Question no. 7

Relatively few attempts at this question on Bernouilli's equation and fewer betas than one might have hoped for. Students had difficulty both the bookwork of deriving Bernoulli's equation and with the application.

Paper no. 3 Question no. 18

Successful students, by and large, drew pictures and considered mass conservation from first principles. Many students did not remember the expression for divergence in cylindrical polar coordinates. Disappointingly few students recognised the problem as an example of vortex stretching and conservation of angular momentum.

Paper no. 4 Question no. 18

This question on rotating flows comes from the very last part of the course and I had anticipated therefore that it might not be popular. I was pleased both with the number and the quality of the attempts. Most marks were lost in the final part of the question, which required an understanding of the properties of waves.

Course: METHODS

Paper no. 1 Question no. 14

This question was very well done, especially considering that the final part of (b) (deriving the d'Alembert solution using Fourier transforms) was an unseen application of the FT formalism and its properties. (a)(i) was straightforward and correspondingly very well done. For (a)(ii) many (but not most) students realised that to invert the FT expression there, it is much easier to first use partial fractions (to get simpler additive terms), but many more students instead just used the convolution property for products to get the answer in an algebraically more complicated way. The most common error was to not properly treat negative values of w (i.e. needing to use $|w|$ rather than just w in the final inverse FT).

Paper no. 2 Question no. 5

This straightforward question on Fourier series was generally well done. A number of students (still correctly) used the formula for full complex Fourier series coefficients for the function $f(x) = x$ here, rather than more simply *ab initio* using just the Fourier sine series (as f is odd). There were various (correct) approaches given for the last part ("calculate a_0 another way").

Paper no. 2 Question no. 16A

Whilst seemingly a standard separation of variables problem, candidates quickly got into trouble by not recognising Bessel's equation. There was also a tendency to give only a bounded general solution (excluding the Y_n and e^{mz} terms) for the first part which lost marks. Overall those who had found the correct general solution were slightly put off by having an extra degree of freedom in the z terms, but nevertheless proceeded to find an appropriate solution to the given problem.

Paper no. 3 Question no. 7A

Overall this was answered well - the most difficult step seemed to be obtaining an expression for v in terms of the parameterisation variable, leading to the common and costly error of excluding the xe^{-x^2} term from the solution.

Paper no. 3 Question no. 15A

Dealing with the discontinuity condition was handled much better than I expected, although dealing with inhomogeneous boundary conditions was not - a lot of candidates were eager to find a second Green's function for the new boundary conditions. This was penalised harshly as the question specifically requested you use the function already determined. Of those who appropriately found a particular and homogeneous solution, there were varying degrees of success in accurately integrating the Green's function although this was usually not detrimental to obtaining the alpha.

Paper no. 4 Question no. 5A

The first part was answered very well by almost all, using standard induction on the recursion formula. The second part had more mixed results - some wished to use the fact that P_n is a polynomial of degree n that satisfied the recursion relation to induct on the orthogonality relations, leading to rather obscure (and incorrect) arguments as to why $\int_{-1}^{1} xP_n(x)P_m(x)dx$ could be zero. Others merely stated the differential equation is Sturm-Liouville therefore the solutions must be orthogonal, completely foregoing the required proof of this. The third part was generally attempted well either by finding $R_n(1)$ or the coefficient of x^n.

Paper no. 4 Question no. 17

This question (on Green's function and fundamental solution of heat equation) was very poorly done. Many of the attempts did not even get past (a)(i) and even there, very few could give the definition of the fundamental solution. (a)(ii) (Duhamel's principle) was better, when it was attempted. In (b) many attempts did not incorporate the insulated BC (although then one still, albeit incorrectly, gets the stated final answer) and thus omitted the needed image of the initial condition in the left half plane, to get an even extended solution.

Course: NUMERICAL ANALYSIS

Paper no. 1 Question no. 6C
A short bookwork question on Lagrange cardinal polynomials, polynomial interpolation and divided differences. This was fairly well done. Some candidates lost marks for not giving an explicit expression for the divided difference, or for gaps in the proof for the last part of the question.

Paper no. 1 Question no. 18C
A relatively difficult question about third-order explicit Runge–Kutta methods. This was quite algebraically intensive and very few candidates obtained the complete set of conditions on the coefficients. The last part of the question, about the linear stability domain of the method, was well done and many obtained the correct answer from a subset of the conditions from the previous part.

Paper no. 2 Question no. 19C
This popular question involved bookwork about the linear least-squares problem, followed by an explicit calculation involving QR factorization via a single Householder reflection. It was well done and the majority of candidates obtained the correct answer. Several marks were deducted if numerical errors were introduced that led to the wrong answer.

Paper no. 3 Question no. 19C
This question involved bookwork on the zeros of orthogonal polynomials and Gaussian quadrature, followed by an application of the Peano kernel theorem. The bookwork was generally very well reproduced. Candidates were much better on quoting the Peano kernel theorem than on applying it specifically to the case of Gaussian quadrature.

Paper no. 4 Question no. 8C
This short question asked for an LDL^{T} factorization of a symmetric matrix, followed by a Cholesky factorization in a special case. It was quite easy and was very well answered in general. Almost all candidates solved the first part correctly.

Course: QUANTUM MECHANICS

Paper no. 1 Question no. 15
This question (on existence of bound states) had a large take-up and was generally well done. Many student omitted to justify continuity of the wavefunction at the delta function points $x = \pm a$ (but most students correctly derived the discontinuity condition for the derivative). The final (most substantial part) was very well done.

Paper no. 2 Question no. 17
Part (a) was just standard bookwork (deriving energy levels of harmonic oscillator) yet it was not especially well done. Part (b) had the ‘unseen’ element of needing to complete the square in the hamiltonian, which was done very well by many students (including some who did not complete (a)).

Paper no. 3 Question no. 8
This question had a surprisingly small take-up considering that it was very straightforward and essentially bookwork (and considering that many students attempted the other (long) QM question on this paper). Amongst the actual attempts there were many good solutions showing good understanding of the principles here.

Paper no. 3 Question no. 16
This question (on quantum angular momentum) was not well done despite a good number of attempts. Part (a) (very easy) was fine but (b) (deriving a generalised uncertainty relation) was not so well done, unexpectedly so, considering similarity to an exercise sheet question. Part (c) (involving application of the angular momentum operators to a given wavefunction) was messily done with many students generating pages of calculations that could have been avoided with a little prior thought and systematisation.

Paper no. 4 Question no. 6
Quite poorly done considering the very standard nature of the question (reflection from a step potential). In (a) very few students were able to give any kind of adequate physical interpretation of the wavefunction. (b) was better done. Many students didn't properly cover both cases viz. $E < V_0$ and $E > V_0$, with their different behaviours.

Course: VARIATIONAL PRINCIPLES

Paper no. 1 Question no. 4
This question was appropriately straightforward. Quite a number of students wrote down separate equations for x and y variations, not recognising the partial-differential character of the problem.

Paper no. 2 Question no. 15
A lot of rather weak attempts really brought the average mark down. This may be due to the fact that the problem is seemingly more 'applied' than others asked of this course and uses terminology from physics.

Paper no. 3 Question no. 6
The average mark and the number of betas is in line with expectations but lower than might be hoped for given the entirely routine nature of the problem. A lot of students 'accountancy' was poor owing to general disorderliness and poor writing.

Paper no. 4 Question no. 16
This question was from an obscure part of the Schedules that hadn't been much examined previously. The question was nevertheless straightforward in the sense of requiring bookwork and an example of a fairly standard type. The large number of alphas awarded was appropriate given the thorough and detailed answers given by most students.

MATHEMATICAL TRIPOS PART II, 2017

COMMENTS ON QUESTIONS

Course: NUMBER THEORY

Paper no. 1 Question no. 1
There were some very good, competent answers. Candidates seriously attempting this question on quadratic residues usually made progress though not always enough to earn a merit mark. Typical misconceptions included considering $p-1$ instead of $(p-1)/2$ integers in Gauss' lemma and/or, in the last part, some candidates showing -1 is a quadratic residue mod p and getting stuck after that.

Paper no. 2 Question no. 1
A somewhat mixed set of answers, ranging from very good to partial or scratches. Marking scheme was slightly adjusted as most candidates did not realize they were expected to use inclusion-exclusion in the proof of Legendre's formula and only used it in the example.

Paper no. 3 Question no. 1
Disappointingly small number of serious attempts, though this question on pseudoprimes was not really difficult. Possibly the candidates were not quite prepared to use certain skills from Part I Number and Sets. Many just stated the initial definition(s) and then apparently "randomly" tried to manipulate the given conditions.

Paper no. 3 Question no. 10
There was some variations among the serious attempts in how carefully the candidates reproduced the required bookwork on continued fractions and dealt with the example, but many were able to produce answers worth of α. It was perhaps a bit disappointing to see that too many did not demonstrate enough confidence about applying the theory in the last part and fell back to using 'brute force' calculations instead.

Paper no. 4 Question no. 1
This question on the distribution of the primes was rather well done. Most candidates were able to reproduce the required analytic number theory bookwork, or at least the argument for the first to two estimates, fairly accurately. Making some progress into the second part usually earned them a merit mark.

Paper no. 4 Question no. 10
The question attracted many attempts, including many good ones though occasional candidates, while being persistent and apparently having some idea, were a bit too vague in their answers. Typical points of confusion included arguing for mod $4p$ rather than mod p in the criterion of representing p by binary quadratic forms and/or not checking whether there are other reduced binary quadratic forms with the same discriminant -60. There were confused attempts of expressing solubility of $x^2 \equiv -60 \bmod p$ via appropriate quadratic residues mod 15.

Course: TOPICS IN ANALYSIS

Paper no. 1 Question no. 2F

The expected errors indeed denied several their β. For instance, forgetting to exclude rationals in the statement of Liouville's theorem and forgetting to argue that if ζ_n is 1 infinitely often then the required number is indeed irrational, so Liouville can be applied.

Paper no. 2 Question no. 2F

Not much to say. A true of false question that most who attempted got right.

Paper no. 2 Question no. 10F

Well answered in general. For the second part concerning the sequence f_n, quite a few attempted to apply Baire's category theory without using the fundamental assumption of continuity off f_n. This is where most points were lost on this question.

Paper no. 3 Question no. 2F

Well answered for the most part with a relatively large take-up.

Paper no. 4 Question no. 2F

Comparatively low take-up; many only did the first part.

Paper no. 4 Question no. 11F

Relatively large take-up. Most indeed understood the importance of compactness and uniform continuity and answered reasonably well, though the precision varied considerably.

Course: CODING AND CRYPTOGRAPHY

Paper no. 1 Question no. 3

A well done question. Most candidates were able to reproduce the bookwork on decoding rules accurately enough and do the right computation in the example.

Paper no. 1 Question no. 10

The question on binary linear codes was very well done. Almost every candidate who seriously attempted it and was not running out of time made good enough progress to get an α.

Paper no. 2 Question no. 3

This question on decipherability had a fair amount of good, sometimes elaborate answers. Some candidates only did about a half of the argument, apparently assuming the rest without proof. Some attempts were scratches, e.g. just stating Kraft's inequality.

Paper no. 2 Question no. 11

This question on entropy and channel capacity was rather well-done by many candidates. Some were confused in the last part e.g. confusing input and output random variables in the argument. Some answers had a minor inaccuracy concerning the use of logarithm to base 2 or e.

There was unfortunately a misprint in the inequality to be proved which was missing a term $+(1-p_1)$. This was pointed out by two candidates and a correction was issued approximately half-way through the exam. Many candidates seemed to figure out the correct version remembering a similar inequality in an example sheet. This part (and any direct dependencies) was marked sympathetically, also noting any candidate who appeared to have lost significant time due to this mistake.

Paper no. 3 Question no. 3
Attempts to this question on binary cyclic codes varied from some good answers to some which looked a bit superficial ones which could do better with a bit more effort. Some β marks were really borderline.

Paper no. 4 Question no. 3
The success rate on this question about the mathematics of RSA encryption varied between candidates. Most remembered (possibly from Part I Numbers and Sets) and were able to write out the classical RSA procedure and explain the issue of homomorphism attack. Some candidates went on to reproduce the notes on El-Gamal signature, though some were more vague or even stopped at that point.

Course: AUTOMATA AND FORMAL LANGUAGES

Paper no. 1 Question no. 4
Both parts were taken from an example sheet. Alas, probably the worst done amongst the four short questions in A&FL, so this did not get things off to a flying start in this option.

Paper no. 1 Question no. 11
Mostly unseen material. Reasonable efforts at the first part. However part (b), which really just calls for repeated use of the pumping lemma, was poorly tackled.

Paper no. 2 Question no. 4
Plenty of convincing attempts at providing examples of various languages and of subsets of $\mathbf{N}$ with certain properties. All material was either bookwork or from an example sheet.

Paper no. 3 Question no. 4
Quite a few decent attempts. Part (a) on the CNF was from the lectures; the other two parts were adapted from example sheets. Almost everyone gave the same examples. The least familiar part was (b) in which there was a lot of handwaving.

Paper no. 3 Question no. 11
The first two parts were bookwork and done well. The specific example of one set being a a many-one reduction of another proved difficult. Those attempting a solution produced rather loose arguments.

Paper no. 4 Question no. 4
The bookwork part (a) was done mostly in too cavalier a manner. Part (b) was (easy) unseen and just needed several applications of the pumping lemma - it was done far better than first part though.

Course: STATISTICAL MODELLING
No comments were submitted

Course: MATHEMATICAL BIOLOGY

Paper no. 1 Question no. 6B
This was found to be quite difficult even though it was simplified from an earlier version. Despite being given the solution to the differential equation it was long, with two changes of variables.

Paper no. 2 Question no. 6B
Many candidates were unable to write down the corresponding differential equations even though it was only a small variation on lectured material.

Paper no. 3 Question no. 6B
This was almost standard bookwork and yet a large number were not able to go from the general case to the specific example.

Paper no. 3 Question no. 12B
A pretty standard question and most candidates knew what to do. It was long and the exploration of the full parameter space was quite difficult, and the inability to describe it all was the reason for most of the β's.

Paper no. 4 Question no. 6B
This appeared to be a standard question, yet most candidates were unable to make progress beyond the identification of the fixed points.

Paper no. 4 Question no. 13B
This question was well done, with candidates able to deal with the homogeneous system and then to include the effect of a spatial perturbation.

Course: FURTHER COMPLEX METHODS

Paper no. 1 Question no. 7E
A good question. Some students struggled to find the residue, making trivial arithmetic mistakes.

Paper no. 1 Question no. 13E
First part presented no difficulty to most students. In the second part the main issue was the determination of the range of s — very few students had done it correctly. The last evaluation cost many students lost points because of the arithmetic errors, but was quite doable.

Paper no. 2 Question no. 7E
This questions was pretty hard. Students either knew the trick (likely mentioned in lectures) or did not do it well. Many skipped the proof and simply found the numeric value of the constant.

Paper no. 2 Question no. 12E
A scary looking question. First part was rather easy. In the second part there was a lot of arithmetic issues, although many students still got A correctly. For the B only a few found the compact expression.

Paper no. 3 Question no. 7E
A relatively easy question, most students did well.

Paper no. 4 Question no. 7E
A good question. Although maybe slightly too hard for Section I. MOst students got the right form of f, although a surprisingly large number of them struggled to solve the first order ODE for f correctly. Then about half of the students did not even attempt to take the contour integral, leaving the result in the explicit form. Out of thos who took the integral some got the residues wrong. But the main issue was that even when they would get the correct answer they did not try to give the real solution (as they were asked) leaving it in the complex form. I believe only two students did it fully.

Course: CLASSICAL DYNAMICS

Paper no. 1 Question no. 8E
Many students found the formulation of the first half of the question very vague and simply did not understand what is being asked. Only very few were able to do correctly the second part. This question seems too hard for Section I.

Paper no. 2 Question no. 8E
Good Section I question.

Paper no. 2 Question no. 13E
Good question with a right balance of physical background, technical calculations, and arithmetic.

Paper no. 3 Question no. 8E
Good question. Some were lost in arithmetics when finding the constant c.

Paper no. 4 Question no. 8E
A good question, which tested students' knowledge in several directions. Clearly formulated, most students understood well enough what is being asked.

Paper no. 4 Question no. 14E
A good question. Some students did not understand that they need to rigorously prove the first part. But many did it very well. In the second part the main issue was solving the Lagrange equations correctly — many students struggled with getting the right solution because they complicated things too much and could not selve the resultant equations. Some simply gave up and guessed (I don't know how) that the geodesics are circles.

Course: COSMOLOGY
No comments were submitted

Course: LOGIC AND SET THEORY

Paper no. 1 Question no. 15
The bookwork material on Completeness, Compactness and Decidability was done well. The attempts at the rider to produce examples of finitary sets weren't too bad, though several people took an overly complicated route on the very last part.

Paper no. 2 Question no. 14
Fairly straightforward question on ordinal calculus. Parts (iv) and (v) were unseen and attracted some innovative attempts.

Paper no. 3 Question no. 14
Quite a easy question on Zorn's lemma with applications to the existence of vector space bases. The final part on the common cardinality of the bases was the downfall of some.

Paper no. 4 Question no. 15
Probably the hardest of the four questions asked with fewer takers. The definition (and use) of *attempt* functions being particularly problematic for some. The final part on a family of sets defined for each ordinal by recursion also proved tricky.

Course: GRAPH THEORY

Paper no. 1 Question no. 16
The first half was bookwork on Hamiltonian graphs, and done well. The second part asking for examples was unseen, but also done well.

Paper no. 2 Question no. 15
Almost everyone did well on this question about the graph theoretic statement of Hall's marriage theorem together with a straightforward application to stochastic matrices.

Paper no. 3 Question no. 15
Basic properties of the Ramsey numbers were well understood and derived. The unseen applications to various colourings of K_n were a little less well done.

Paper no. 4 Question no. 16
Bookwork properties of regular graphs done very well, as was the straightforward application to finding the eigenvalues of the Petersen graph.

Course: GALOIS THEORY

Paper no. 1 Question no. 17I
This was a reasonable question done generally well. The only point where some candidates struggled was in showing that the second given polynomial was irreducible (a few gave conditional answers, some tried to solve the algebra by hand, and very few used characteristic p reduction as intended). Some people struggled to remember and hence use the formula for the discriminant. [Mark scheme: 2+7, 5, 6.]

Paper no. 2 Question no. 16I
This question was generally done well, with fluent proofs of the primitive element theorem. Some candidates forgot to treat the case of finite fields separately. In the final part, those candidates

who forgot the canonical example (seen on question sheets) struggled, since no example not both in finite characteristic and of transcendence degree at least two can work for general reasons, so their attempts to come up with an ad hoc example were basically doomed. [Mark scheme: 2+8, 5, 5.]

Paper no. 3 Question no. 16I

In response to a query from a candidate, a correction to this question was announced one hour into the examination: it's not clear that was the right decision. The correction changed the polymomial in (b)(i) from t^4+t to t^4+1 (as had been intended). The former is not separable, so it is debatable whether the Galois group has actually been defined for such a polynomial. On the other hand, most students had proceeded to just declare that the Galois group was trivial, and were irritated to have to re-visit. (A few candidates incorrect thought that t^3+1 was irreducible over F_3 and insisted on sticking to their solution to the question as originally given. Any candidate who wrote something sensible for either version of the question got full credit.)
In any case, the question was absurdly easy: the bookwork part was shortcircuited by many candidates by their using the classification of Galois extensions as ones for which $\mathrm{Aut}_K(L) = [L : K]$ rather than as normal separable extensions, which made the easy bookwork even easier. The examples in (b) were simply too simple in either version of the question. I am confident no candidate was disadvantaged by the error: there were few attempts not gaining an α, most of which simply omitted some large part of the question. [Mark scheme: 10, 2, 4, 4.]

Paper no. 4 Question no. 17I

This question was much too easy, and got a large number of high scores even with an attempt to mark it stringently in order to differentiate. Some candidates stated the fundamental theorem too loosely (e.g. not explaining what the bijection is or that it is order-reversing); most marks were lost by candidates who simply stated the answers in the last part (or said "by inspection") despite the clear instruction to justify those answers. [Mark scheme: 4, 3+4, 1+8.]

Course: REPRESENTATION THEORY

Paper no. 1 Question no. 18

The bookwork and example on faithful and irreducible representations were well done by most candidates attempting this question. Some struggled, with mixed success rate, with getting the character table right in (c).

Paper no. 2 Question no. 17

A popular and often well done question on characters of finite group representations. In (i), some candidates did not find the right method (albeit a simple one) and tried to appeal to Galois theory instead. They typically received half of the maximum marks allocated to the required argument. In (ii), a few candidates did not spot the right method and tried (unsuccessfully) to proceed by proving that $g \mapsto g^2$ is a bijection of G onto itself.

Paper no. 3 Question no. 17

This question based on Burnside theorem had a somewhat disappointing outcome. The students did not seem to remember the IA Groups well enough (or to realize those IA skills would be needed here). Some struggled finding the appropriate conjugacy classes argument in (b) part, though the (c) part was generally more successfully done. Almost no one spotted (in (b)) that there is a conjugacy class of size a power of p, so one can appeal to the respective part of argument of Burnside theorem.

Paper no. 4 Question no. 18
A popular and generally well done question on the representations of $SU(2)$. Most candidates did not have any problem reproducing the required material from lectures. Occasionally a treatment of the unseen example did not contain perfectly complete details, though it usually was essentially correct and sufficient for some kind of α mark.

Course: NUMBER FIELDS

Paper no. 1 Question no. 19
This dealt with properties the ring of integers in a number field and attracted a lot of unsuccessful *ad hoc* attempts. The given existence of an integral basis did not help to suggest a method of attack (some ignored its existence completely).

Paper no. 2 Question no. 18
part (a) of this question dissected, with hints, sections of the proof of Dirichlet's unit theorem. It was unpopular and very poorly done by those brave enough to have a go. Perhaps this was because the full proof of the unit theorem is starred on the Schedules. The routine calculation of the norms of units in (b) was very well done, however.

Paper no. 4 Question no. 19
The question, which invited a calculation of the integral basis of a slightly unpleasant number field, was the most popular of the three questions. Most people only got half way through the calculation then simply wrote down their purported basis without further comment. Often their guess was wrong. Many ignored the hint. The second part which was a calcuation of an ideal class group attracted a variety of methods; those following the standard route invariably got the right answer, while those trying something more innovative usually began by overestimating the Minkowski bound, which then led them down an overly complicated path

Course: ANALYSIS OF FUNCTIONS

Paper no. 1 Question no. 22F
Good number of decent attempts, though few were able to draw points from (d).

Paper no. 3 Question no. 20F
Those that attempted this problem did for the most part reasonably well. Few however were able to determine the function h_n in part (iii), though one student was indeed able to get full marks.

Paper no. 4 Question no. 22F
Take-up was extremely low relative to the questions from this course on previous papers, and essentially none of the question was answered beyond part (i).

Course: ALGEBRAIC TOPOLOGY

Paper no. 1 Question no. 20I

This question was done in a surprisingly sloppy fashion. Very few people explained in (a) why concatenation defines a map at the level of the fundamental group and not on the set of based paths; many candidates consistently (or when convenient) pretended that the set of based paths itself forms a group and cancelling elements without arguing via constructions of nullhomotopies; there was in general some confusion between homotopy and based homotopy. In particular, the construction of the explicit based homotopy in (d) was often incomplete, though most candidates understood that the key point was that the fundamental group is abelian. [Mark scheme: 2,4,5, 5+2, 2.]

Paper no. 2 Question no. 19I

Very few candidates could properly define a push-out. Most of the rest of the question attracted quite a few good answers; the point people found hardest was the drawing of the covering space in (c)(i), which was usually slapdash (in the end few marks were lost for a convincing diagram with even the briefest explanation of where it had come from). Although the constant map provides a retraction to a graph for (c)(ii), only one candidate realised this; those that tried to give retractions to "more interesting graphs" were successful about half the time. Quite a few candidates incorrectly thought that (c)(iii) was impossible because the fundamental group of $\widehat{X}$ was not free (which it is) rather than realising that they should argue by consideration of homology. [Mark scheme: 2+1+2, 3+3, 4+2+3.]

Paper no. 3 Question no. 18I

The computation of the homology of the torus worked pretty well as a question. Everyone who made a serious attempt knew that the crux was to use Mayer-Vietoris and cut up the final circle factor into two intervals. From that point on the level of care varied considerably. Only one or two candidates both handled the algebra systematically, explained why short exact sequences of free abelian groups split, and derived the inductive formulae for the ranks of those groups in a complete fashion, but the partial answers showed good general understanding. [Mark scheme: 20.]

Paper no. 4 Question no. 20I

This was a poorly done question. There were many sloppy answers for the third part involving carelessly formulated and unproven statements about cycle representatives for the fundamental class and orientations. A couple of people proceeded via the Lefschetz fixed point theorem. Only one candidate gave a proof by Mayer-Vietoris which inter alia established the required claim that $H_n(S^n) = \mathbb{Z}$. The final part was also found surprisingly difficult. [Mark scheme: 2,5,7,6.]

Course: LINEAR ANALYSIS

Paper no. 1 Question no. 21F
High take-up and well answered. For (c), some tried to sketch the construction of Y in terms of Cauchy sequences instead of the intended answer via X^{**}. With that approach, it was hard to give enough detail so as to merit full marks.

Paper no. 2 Question no. 20F
Again very high take-up and well answered. In part (c), some gave no justification of the fact that finite dimensional subspaces are closed.

Paper no. 3 Question no. 19F
Very high take-up with large number of α's. Points were mostly lost for trivialities, for instance, in incorrectly representing the maximum in terms of $|\cdot|$. Most got (iv) even though unseen.

Paper no. 4 Question no. 21F
By far the hardest of the Linear Analysis questions. Very few got full marks on part (iii).

Course: RIEMANN SURFACES

Paper no. 1 Question no. 23F
In general, well answered. The problem turned out to be slightly easier than the model solution would indicate, as the fact that the degree of f was two could be used to infer more easily to exclude the case $e_p = 3$. It was only really the last part that somewhat separated the top scripts from the rest.

Paper no. 2 Question no. 21F
The middle part was the most difficult. The last part had a simpler answer than that of the model solution—and many found this.

Paper no. 3 Question no. 21F
Well answered in general, though the level of detail in justifying the first part varied. A few students mixed up in the ramification indices of π and τ. Several students seem to have forgotten the assumption that n is even in answering the last part.

Course: ALGEBRAIC GEOMETRY

Paper no. 1 Question no. 24I
This question was generally well done. Most candidates solved the final part by showing that one of the varieties was singular and the other smooth, rather than using properties of the rings of regular functions; those who tried to use algebraic properties of the rings of regular functions often struggled. [Mark scheme: 5,5,5,5.]

Paper no. 2 Question no. 22I
This question unfortunately contained an error, which was not noticed in the exam, nor by any candidate who attempted the question (in fact the error occurs in the classic textbook on the subject by Hartshorne, from which the question was taken). Namely, in part (b), one should add the hypothesis that the degree of the polynomial is not divisible by the characteristic of the field k. No candidate seems to have been disadvantaged by the error, since none noticed the need for the

hypothesis. In fact, a really proper answer to the final part of the question would involve showing that the given equation is irreducible over fields not of characteristic 2. Only one candidate even attempted to show that, and the others were not penalised for not trying since it seems (from later discussions with the lecturer) that this was swept under the rug in the corresponding question from the example sheets. In retrospect, it would have been better to simply give that hypothesis; however, the few candidates who noticed they were making an assumption don't seem to have lost time, and got very high marks.
That aside, the question was generally done well. Only a few candidates realised for the last part that since the singular set cannot be everything, by general theory, the equation must a posteriori be reducible in characteristic 2 (something which is actually evident from the formula). [Mark scheme: 4,7,5,4.]

Paper no. 3 Question no. 22I
This question was well done, but perhaps a bit too straightforward. Identifying the preimage of the blow-up point in the proper transform was the only point at which some candidates came unstuck. [Mark scheme: 2+2,2+6,8.]

Paper no. 4 Question no. 23I
This question was a bit too easy. Several candidates missed that the point $(0:0:1)$ of ill-definition of the map becomes a ramification point, and no-one pointed out where their solution to the second part used the characteristic-not-equal-to-two hypothesis. The final part on Riemann-Roch was done essentially perfectly by all who attempted it, and was probably too close to example sheet material. [Mark scheme: 4,6,4,6.]

Course: DIFFERENTIAL GEOMETRY

Paper no. 1 Question no. 25I
This was a very easy bookwork question which (outside the list of definitions requested) was done surprisingly poorly. In particular, many candidates in the final part just showed that 0 is not a regular value of the determinant map, rather than proving that the zero-set is not smooth. [Mark scheme: 2, 2, 1+1, 6+2+2, 4.]

Paper no. 2 Question no. 23I
This question unfortunately contained an error which a candidate pointed out 5 minutes before the end of the examination: in the second part of the question, the hypotheses should assert that the curves α and $\tilde{\alpha}$ have both curvature and torsion agreeing on the subinterval J, not merely curvature. Of the fifteen or so candidates who attempted the question, most simply asserted (incorrectly) that equality of the torsions followed and proceeded to argue via the fundamental theorem of curves as intended; they were given full credit for that part. A few candidates omitted that part; in all cases, the rest of the question was marked generously, so that they obtained the same merit mark as if they had been given full credit on that part of the question. Two candidates simply stopped at that point of the question, and may have lost time due to the error, but it was hard to deal with this in the context of this question alone.
The error aside, the question worked quite well: most candidates who attempted the last part understood the key fact that the principal curvatures had the same sign, and were able to give examples via cylinders and planes of all the desired phenomena. A couple of candidates were confused by what is involved in showing an inequality is "sharp" (they were not penalised if they

gave valid examples for the case of inequality rather than equality). [Mark scheme: 2+2+2, 4, 2+4, 4.]

Paper no. 3 Question no. 23I
This was an easy question done rather averagely. Several people showed that energy minimisers were geodesics without remembering to relate length and energy. A few candidates in their discussion of geodesics on the punctured sphere were not careful about when they were working on the closed sphere and when on the punctured sphere, and two didn't seem to notice that the relevant great circle might pass through the puncture. However, there were a few very careful and thorough answers. (No marks were withheld for candidates who just asserted that geodesics on the sphere were great circles, although a couple of candidates did go to some length to demonstrate that.) [Mark scheme: 4, 10, 6.]

Paper no. 4 Question no. 24I
This rather easy question received few attempts, but of those who did make a serious attempt, there were some good answers. In particular, several candidates both understood how to address the second part using a subsurface of a cone or other singular surface, and some candidates gave good justifications for the impossibility of the construction outlined in the last part. [Mark scheme: 4,6,4,6.]

Course: PROBABILITY AND MEASURE
No comments were submitted

Course: APPLIED PROBABILITY

Paper no. 1 Question no. 27K
This was a fair question and I in fact expected the students to have done better on it. The first part of the question is bookwork and the second part of the question followed from an example sheet question (sum of exponentials up to a geometric time is exponential). The third part of the question is similar.

Paper no. 2 Question no. 25K
This was a fair question. Students did not have much difficulty with part (a) because this was bookwork. Part (b) is a standard sort of calculation involving the Poisson distribution that students did several times on example sheets and had seen in class.

Paper no. 3 Question no. 25K
This was a fair question and students performed as expected. The first part of the question is bookwork and the first part of part (b) is a standard fact about renewal processes. Many students struggled with the last part of part (b) as it was unseen. I was impressed that some students were able to do well, though, without using the hint.

Paper no. 4 Question no. 26K
This was a fair question and perhaps on the easy side. I expected students to have done better, but some were confused the wording in part (b) (although it is correct and clear as written) regarding the meaning of "total time being served".

Course: PRINCIPLES OF STATISTICS

Paper no. 1 Question no. 28K
Students did this question well. Part (a) was a straightforward computation, which did not cause difficulty. Part (b), (i) was bookwork. Some students lost points on part (b), (ii) for making computational mistakes.

Paper no. 2 Question no. 26K
This was a fair question. Students lost typically lost points for not correctly calculating the deviations of the MLE in part (c) from θ which in turn caused problems in part (d).

Paper no. 3 Question no. 26K
This question turned out to be quite easy for those who attempted it. Part (a), (b), and (c) were mostly bookwork.

Paper no. 4 Question no. 27K
This was a fair question. Most students had no difficulty with part (a), but a number of students had a little bit of difficulty correctly finding the confidence interval in (b). Some had difficulty setting up the optimization problem necessary to solve (c). Those who made it to (d) did not have difficulty making the connection to Wilks' theorem.

Course: STOCHASTIC FINANCIAL MODELS
No comments were submitted

Course: OPTIMIZATION AND CONTROL

Paper no. 2 Question no. 28K
This was a fair question and students performed quite well on it. Those who attempted it mainly lost points in part (c).

Paper no. 3 Question no. 28K
This was a fair question and students performed quite well on it. The main difficulty that students had was finding the limiting form of the optimal control and the minimal average cost per unit of time.

Paper no. 4 Question no. 29K
This was a fair question and students performed quite well on it. Those who attempted it only struggled with the final part of the question.

Course: ASYMPTOTIC METHODS

Paper no. 2 Question no. 29E
A lengthy question with many places in which points can be lost. Most students were not able to identify all sources of approximation needed for getting the remainder term right. Very few students got a full mark.

Paper no. 3 Question no. 29E
A question involving a lot of tedious calculations. One has to keep concentration to be able to solve it without a loss of points. Maybe one or two students only found the correct form of the full asymptotic expansion.

Paper no. 4 Question no. 30E
A more straightforward question. The main difficulty seemed to be the derivation of the recursive relations. A lot of students also struggled to derive the correct form of equations for S_j, especially for S_0. Getting the arithmetic right was also one of the main issues (a lot of tedious calculations).

Course: DYNAMICAL SYSTEMS

Paper no. 1 Question no. 30A
This had seemed like a reasonable question, given that it was a slightly easier version of a 2004 question, but it caused surprisingly much diffculty this year. In part (a) [find fixed point, analyse them, sketch] despite all some Jacobians being diagonal, many candidates spent much time finding the nature of the eigenvectors using trace and det, and doing this repeatedly for identical fixed points. The sketches were a mixed bag, with some being inconsistent with the previous results. On the other hand, some were rather good, showing use of the symmetries (and invariant lines). In part (b) [find something about domain of stability] only a few actually reached the result for the domain, but many had the right idea of using the correct Lyapunov function, and their working showed attempts to find a contour of V that guaranteed negative V'. The missing idea for many was the idea of parameterising a curve of constant V to find max V' for a given V=C (several ways to do this, and all that were done correctly ended up being able to use the hint). Again, credit was given for clear arguments and the right approaches.

Paper no. 2 Question no. 30A
This will have low alpha/beta rate as many attempts were simply dumping the ap- propriate theory and not engaging with the given problem. Generally students knew the theorems well, but had trouble creating a suitable Lyapunov function for (a), which was only a minor modification of a standard example. Some tried to use approximation of lower order terms without spotting those must be zero on a line and hence higher order terms matter. For (b), there are many variants of suitable regions for P-B theorem, but some were stuck by reading theta and p as polar coordinates (though damped pendulum is a standard example and similar). For (c) this is different to example sheet and past papers in that it is not a special value of H that is sought where a periodic orbit exists, but rather a special value of a parameter that ensures a particular orbit persists (given H find k, as opposed to find H(k)). This threw some students who launched into the usual integral with a general value of H.

Paper no. 3 Question no. 30A
This was really not well done. Many students just had disorganised algebra. Others were reasonably organised, but had not applied any thought to the form of the centre manifold so had a large number

of coefficients to resolve, and hence the question must have seemed intractable. With either a more intelligent approach than crunching coefficients, or minimising the number of coefficients needed before subbing in, this was an reasonable question in length, and a few students cleared it quickly. The wording of answers for the Centre Manifold Theorem ranged very widely, but marks were given so long as the ideas were used correctly later. Some algebraic slips were costly (not changing coordinates correctly in some unfortunate way) but others had no consequences (incorrect V didn't change dynamics on manifold).

Paper no. 4 Question no. 31A
For those comfortable with bifurcations of 1D maps, this should be a routine question, and indeed many managed without major hitch. The bifurcation diagrams were generally well done, many using the hint of "no further bifurcations" to put together the only likely picture without excessive calculation. There was some excessive algebra for (b), including calculation of F2 even though it was given. A few attempts calculated F(x) for the period 2 points, without connecting up already which must be in the same cycle, but this merely wasted time rather than sinking the question. The last diagram was generally well done, including expectation of period doubling and chaos. This question also rewarded those with insight and clean algebra, with a few 2-page solutions gaining an alpha.

Course: INTEGRABLE SYSTEMS

Paper no. 1 Question no. 31A
This was an interesting question drawing together different parts of the course, but caused considerable difficulties: many candidates abandoned after either just a few definitions or part way through, without engaging with the main problem at all. I suspect the more applied students were just not comfortable with the Lie point symmetries part of the course. Like other Integrable Systems questions this year, there was uncertainty about how much formality was re- quired, with some attempts going into too much detail and others asserting too much without any clear arguments.

Paper no. 2 Question no. 31A
The alpha/beta ratio here will be low as quite a lot of attempts were just the first bit (ZCE) or that and a brief stab at the next bit then question abandoned. Very few attempts managed the middle bit: most used uniqueness without matching the x=0 condition also, hence something was skipped or fudged. The later parts did not depend on this though, and candidates that continued on put themselves in range for an alpha if these were done well.

Paper no. 3 Question no. 31A
This was the best done of all the Integrable Systems questions this year, and most who started it got most of the way through at least. There was confusion and much variation about how much formality was required and how much could be assumed. Marks were awarded for approaches which showed competence with the material, and solutions which had clarity on what was being done at each stage were given more benefit of the doubt if minor things went wrong (as opposed to solutions which reproduced the algebra from bookwork with little explanation or justification at any point, then made a slip). The last bit did not go well for almost everyone, so credit was given for sensible ideas and approaches.

Course: PRINCIPLES OF QUANTUM MECHANICS
No comments were submitted

Course: APPLICATIONS OF QUANTUM MECHANICS
No comments were submitted

Course: STATISTICAL PHYSICS

Paper no. 1 Question no. 34D
A fair question, but one that was done rather poorly. The stumbling block for many was plotting a function that involved the difference of two coth terms. Despite the fact that only the asymptotics were needed, this was enough to trip up most and lead to many wrong results for the final part of the question.

Paper no. 2 Question no. 34D
Like many manipulations in thermodynamics, it's very easy to go around in circles if you don't know where youre heading. While many students did a good job of this, only a handful managed to get all answers correct. I was fairly lenient in penalising algebraic slips.

Paper no. 3 Question no. 34D
This was a challenging question, but I was pleasantly surprised at the quality of the answers. The structure of the question held the students by the hand as they tiptoed their way around the triangular carnot cycle, pointing out the subtlety that, as far as heat goes, the hypotenuse both giveth and taketh away. Nonetheless, it was non-trivial to find the correct value of the efficiency, $\eta = 16/97$. A number of students found $\eta = 1$; when the exams are over I will attempt to sell them the new perpetual motion machine that I've developed.

Paper no. 4 Question no. 34D
A long question which was mostly bookwork. The students exhibited a good understanding of the Maxwell construction and the meaning of the critical point.

Course: ELECTRODYNAMICS

Paper no. 1 Question no. 35D
This was a straightforward question which required the candidate to perform a Lorentz transformation in order to determine the motion of a particle in an E and B field. It was slightly fiddly in places, but with a very low number of attempts its difficult to judge its difficulty.

Paper no. 3 Question no. 35D
Quite a tricky question which required the students to compute the force between two plates using the stress-tensor. Although there were only a handful of attempts, they included a couple of perfect answers.

Paper no. 4 Question no. 35D
This question cleverly combined two of the hardest parts of the course; radiation and electromagnetism in media. The students were clearly unimpressed by this combination, given the extremely low take up.

Course: GENERAL RELATIVITY

Paper no. 1 Question no. 36D
This question differed from bookwork only in using a 5d black hole rather than the more familiar 4d black hole. This made it too easy, and there was an abundance of alphas. Those that tripped up typically didnt do the stability analysis correctly, and so failed to notice that life in five dimensions is rather harder to maintain than in four.

Paper no. 2 Question no. 35D
A nicely balanced question, that involved a good mix of bookwork and calculation. A number of students would have done much better had they realised that the Einstein static Universe was static. Some students polished off the stability analysis in just a few lines and gained full credit; others took several pages.

Paper no. 3 Question no. 36D
A slightly left-field question, whose novelty is likely responsible for the low-take up. Nonetheless, it was a very straightforward – perhaps overly so – and the majority of those students who were brave enough to attempt it did very well.

Paper no. 4 Question no. 36D
The first part of the question was straightforward application of what was done in lectures; around half the students who attempted the question did a good job. The second part of the question was arguably off-schedule: even though the main formula was given, they still needed to recall what a quadripole moment is and this is starred material. Only two people attempted this the second part of the question, one of whom got the right answer.

Course: FLUID DYNAMICS II

Paper no. 1 Question no. 37B
This was generally well done, with the principle of a similarity solution to the boundary layer equations well understood. Some found the algebra taxing.

Paper no. 2 Question no. 36B
About half the candidates knew the basis of lubrication theory and could set up the geometry correctly. Others found the geometry challenging.

Paper no. 3 Question no. 37B
This question was more a question in the use of suffix notation and was generally well done.

Paper no. 4 Question no. 37B
A standard question on shear flow instability, which the majority did well. Some had difficulty with the boundary conditions and in interpreting the final result.

Course: WAVES

Paper no. 1 Question no. 38B
There were few attempts at the Waves questions. In this case almost all attempts were good.

Paper no. 2 Question no. 37B
Almost all attempts were of a very high standard. The question was a repeat of a question set in 2007.

Paper no. 3 Question no. 38B
This question on ray theory was the weakest of the Waves questions. Although a lot of it was bookwork, this was not well done and the application to the specific problem was not done well.

Paper no. 4 Question no. 38B
The Rossby wave question and the use of stationary phase was done pretty well on the whole. Some had trouble with the various parameter ranges.

Course: NUMERICAL ANALYSIS

Paper no. 1 Question no. 39A
Any student who could not state the H-J theorem would not start this question, and any who could would be very likely to be able to tackle the rest of the question, so there will be a high beta rate here. However, some got into a tangle with the extra omega in front of the b, and hence may have not reached an alpha. Generally well done though.

Paper no. 2 Question no. 38A
This was a completely standard question, but the length and fear of the amount of algebra will have put many off. Many students ignored the nice notations offered in the hint and charged on with pages of calculations. Some solutions were just the algebra and little or no argument presented at any point. Many students guessed what they were aiming for, and some claimed it even when it contradicted their working.

Paper no. 3 Question no. 39A
This was actually a rather straightforward problem which did not require much knowledge of theory or specific examples beforehand, so the very small number of attempts was a surprise. The majority of the serious attempts managed it without difficulty, though some wrote out the same working repeatedly for each part of the question. Given that the substance (indeed entirety) of the question was working out the convergence rates of three methods, solutions that had chaotic algebra that led to errors was penalised.

Paper no. 4 Question no. 39A
Most who knew enough to start the question got at least part (a) out without major problems, the only issue was students who tried to apply the same method to (b) without noting that it is now second order in time so a slightly different approach was needed. There will be a high alpha rate for this question, but it should be borne in mind that (i) this type of question reveals enough of its contents that only those secure in the area will attempt it (no scavenging going on) and (ii) the majority of those getting the alpha had to do quite a bit of work.

MATHEMATICAL TRIPOS PART II, 2018

COMMENTS ON QUESTIONS

Course: ALGEBRAIC GEOMETRY

Paper no. 1 Question no. 25I
This question could have used a more interesting problem part. The bookwork was generally done well, although some people forgot how to do the first part. Solutions to the problem part were not very clear, but rarely so muddled as to lose more than a few points.

Paper no. 2 Question no. 24I
Most candidates did well on part a), but very few know what they were doing in part b). I marked this part fairly generously.

Paper no. 3 Question no. 24I
This question was nearly all bookwork, but was a pretty good test of candidates' understanding. Relatively few people understood how to complete the square to put the cubic in Weirstrass form. The other difficult bit was surjectivity of the map onto the divisor class group.

Paper no. 4 Question no. 24I
n contrast to paper 3, this question was nearly all problem. Almost all candidates were able to check the curve was smooth, but few managed to apply the Riemann-Hurwitz formula without making a mistake somewhere. The most common error was to somehow miscompute the discriminant.

Course: ALGEBRAIC TOPOLOGY

Paper no. 1 Question no. 21H
Not a very well done question, with relatively few attempts. The students struggled with practically all parts of the question. In particular, constructing the homomorphism in part c) seemed completely beyond most students. A very hard question, reflected in both the number of attempts and average mark.

Paper no. 2 Question no. 21H
Bookwork formed the main part of this question (15 marks), thus it is unsurprising that so many students scored an alpha. Only one poor attempt, the rest were solid to good. The final unseen question (though similar to an example sheet question) was found challenging by some students, but then again others used it to gain marks even though they had not been able to do parts of the bookwork.

Paper no. 3 Question no. 20H
The hardest part of this question was finding an actual 4-sheeted cover of the space given. Many students simply guessed, and their covering space was quite flawed. There were many rushed attempts at this question scoring few marks; done clearly as a last-ditch effort. Those who knew how to approach the final part and had time to do so were generally quite successful, but many simply had no idea. It may have helped to give the students some sort of hint at how to find the covering space, or a vague description of it, so that they could get on with the rest of the question.

Paper no. 4 Question no. 21H
By far the most popular question of the course. The students seemed to really like this question, demonstrated by the fact that there were practically no rushed attempts, and very few betas. They all managed the bookwork in the first two parts well; a few failed to show that their representative element was a cycle, that was the only hiccup. Almost all students made the right sort of Mayer-Vietoris argument in the last part. However, several struggled with the final parts of the argument (rigorously showing that the homology group was indeed $\mathbb{Z}^2$). There were a lot of hand-waving half-arguments. Hence the large number of low alphas; most marks were lost in this final part.

Course: ANALYSIS OF FUNCTIONS

Paper no. 1 Question no. 23F
No attempts.

Paper no. 3 Question no. 22F
This was by far the most popular of the Analysis of Function questions, with several very good attempts, even of part (c). Several students' solutions did not make use of the hint.

Paper no. 4 Question no. 23F Question no. 23
Essentially no attempt of note.

Course: AUTOMATA AND FORMAL LANGUAGES

Paper no. 1 Question no. 4G

Most made a fair attempt at this question. There were a few inaccurate statements of the pumping lemma for CFLs; the riders were mostly adapted from the example sheets, though (b)(iii) caused most of the problems in showing that the given language was not a CFL.

Paper no. 1 Question no. 12G

Standard question on many-one reductions together with some properties of the halting set. The riders in (c) and (d) were unseen, but nevertheless were handled well.

Paper no. 2 Question no. 4G

Oddly this was the least popular of the four short questions. Almost all of this question was bookwork relating to the DFA obtained from a given ϵ-NFA via the subset construction with ϵ-transitions. The inductive proof in (b) was particularly well done.

Paper no. 3 Question no. 4G

Mostly bookwork material relating to the Chomsky Normal Form. The bulk of the marks was for a description of the algorithm converting a given CFG into one which was in CNF. While only the steps were to be outlined quite a few of the descriptions offered were far too sketchy, missing out crucial steps or failing to keep track of the change in the language from step to step. The (unseen) explicit example at the end seemed either to produce a one-line conversion (without comment) or many pages of descriptions simply repeating part (b).

Paper no. 3 Question no. 12G

Another question on the pumping lemma, this time for regular languages. Proofs of this result were mostly sound. The unseen part (b) concerning the non-existence of certain paths in transition diagrams for a minimal DFA with
finite langauge produced some handwaving arguments, but overall it was actually done better than I'd expected.

Paper no. 4 Question no. 4G

The statements of the various results to be used later were mostly sound, though a few blurred the distiction between partial recursive and total recursive functions. Apart from (d), all of the remaining questions were from example sheets and done competently. Once they realised it was really an application of Rice's theorem, the unseen part (d) wasn't too badly done either.

Course: CODING AND CRYPTOGRAPHY

Paper no. 1 Question no. 3H
A straightforward bookwork question, and a good choice of one. Most of those who attempted the question seriously were able to achieve a beta, with several just missing out for silly mistakes or oversights. Many students made a rushed attempt at the question, scoring only a couple of marks.

Paper no. 1 Question no. 11H
The bookwork and example sheet parts were generally done quite well. Most students struggled with the unseen content (generator matrix for the dual of a bar product, and elements of even weight in $RM(d, r)$); this was mostly what distinguished between alpha and beta marks. Only a handful of poor or rushed attempts.

Paper no. 2 Question no. 3H
Full, or mostly full, marks for most students. A very short question. Almost all marks lost for not discussing the symmetry between 000 and 111, and not mentioning that one must be sent (so no need to consider the probability of which is sent). The last part should have been made slightly more difficult by not stating the explicit probability, and instead asking the students to compute it themselves.

Paper no. 2 Question no. 12H
A very well done question. The unseen parts were done particularly well, with many students scoring most of the marks available there. The biggest hurdle was the long and hard technical bookwork in the middle; many students lost most of their marks there.

Paper no. 3 Question no. 3H
Surprisingly few attempts. Most students who knew what to do gained near-perfect marks. The others simply took guesses at parts of the question. It seems that half the students struggled to perform the correct decoding of $r(X)$; many didn't even attempt it.

Paper no. 4 Question no. 3H
The students seemed fairly comfortable with the latter part of the question (unseen standard problem). In fact, most marks were lost in the earlier bookwork parts of the question; though the students could usually implement the Berlekamp-Massey method, they struggled to describe it, or even formally define a linear feedback shift register.

Course: DIFFERENTIAL GEOMETRY

Paper no. 1 Question no. 26I
Relatively few attempts for what I was expecting would be an easy question. Parts c) and d) were done well, but part b) was considerably harder.

Paper no. 2 Question no. 25I
Again, relatively few attempts. There were some good solutions to the last part. On part b), a number of candidates made the mistake of saying that a helix has the same curvature as the circle it projects to.

Paper no. 3 Question no. 25I
This question attracted more attempts, presumably because it has a large bookwork component on a fairly obvious topic. Proofs of the Theorema Egregium varied widely in quality, with some people losing the thread after showing that Christoffel symbols are determined by the first fundamental form, and others carrying through to the end. The problem part was very difficult, and I marked so that it was possible to get an α without it.

Paper no. 4 Question no. 25I
Poorly done, although it was possible to get quite a few points just by thinking about the cone and the cylinder. Most candidates had a general idea as to why i) should be true, but none were able to explain it with any degree of rigor. Only one candidate managed to give an example of an incomplete manifold where the curvature blows up near a point.

Course: GALOIS THEORY

Paper no. 1 Question no. 18I

This was a good question. Responses to the question about the resolvent cubic were very good overall. The problem part at the end was more difficult, and the quality of answers varied widely. A surprising number of people failed to compute the discriminant correctly.

Paper no. 2 Question no. 18I

This question was relatively easy. The bookwork was generally done well. The majority of responses to the question about the least common multiple were fuzzy, but good enough to get credit. The rider at the end was more discriminating, and in retrospect it might have been better to have it worth more points.

Paper no. 3 Question no. 18I

This question was on the easy side. Some people had difficulties with the latter parts of the proof of Artin's theorem, but on the whole it was done well. The problem parts at the end are too easy; in retrospect, the hint at the end of (ii) should not have been included.

Paper no. 4 Question no. 18I

This question was easy for those candidates who spotted the fact that $\alpha + 1$ is a root if α is, and virtually impossible for those who did not.

Course: GRAPH THEORY

Paper no. 1 Question no. 17I

This was a good question with a wide range of scores. The first 10 points of bookwork were pretty routine but the bookwork in part c) was more challenging. The problem at the end could be solved directly without using the rest of the question. The vast majority of correct solutions took this route.

Paper no. 2 Question no. 17I

This was another good question. Most attempts could handle the bookwork and the cycle of length k, but the question about the cycle of length $2k$ was more difficult. The rider at the end was relatively easy.

Paper no. 3 Question no. 17I

This question was hard. The question about graphs with chromatic number $> r$ was more difficult than intended, and the bookwork was also quite tricky. Very few people solved the problem part at the end.

Paper no. 4 Question no. 17I

This question was easier than the other three, and was done well by the vast majority of candidates. Solutions ranged from short and easy to long (but generally correct) case-by-case analysis.

Course: LINEAR ANALYSIS

Paper no. 1 Question no. 22F
Many attempts. Part (b) gave most of the difficulty, with many students not understanding what is the non-trivial aspect of the question.

Paper no. 2 Question no. 22F
Like on paper 1, it was part (b) that gave the most difficulty. Many students applied Arzelà–Ascoli to something which does not satisfy its assumptions.

Paper no. 3 Question no. 21F
A large number of attempts with a good breakdown in scores. Successful solution to part (c) distinguished the better answers. Note that various distinct arguments based on uniform boundedness or the closed graph theorem were employed to yield correct solutions to (c).

Paper no. 4 Question no. 22F
A surprisingly small number of attempts relative to the other days. Part (b) already caused difficulty for many. A small number of students seem to have wasted time on part (c) prior to the announced correction, though those students seemed to finally have done well on the question.

Course: LOGIC AND SET THEORY

Paper no. 1 Question no. 16G
The bookwork material on ordinal exponentiation was well done by almost everyone. The two riders at the end, especially the final one, were much less convincing.

Paper no. 2 Question no. 16G
Most people successfully negotiated this question on properties of order-preserving maps of posets. Quite a few found the counterexample to the Knaster-Tarski theorem (when completeness of the poset is relaxed) somewhat challenging; most of the marks were lost here.

Paper no. 3 Question no. 16G
The bookwork (Compactness and the Upper Lowenheim-Skolem theorem) was done well. The final two riders (part (iv) *et seq.*) done rather less well, even though the final bit really just boiled down to two applications of Completeness.

Paper no. 4 Question no. 16G
All of the question had been seen before except the very last paragraph. A few struggled to recall fully Mostowski's Collapsing Theorem, but on the whole the bookwork was done better than expected. One or two blurred the distinction between function and function-class, and there were a few creative ways to define what f 'is an attempt' means. In the final unseen part, quite a few missed the distinction between finite and infinite sets, rather fixating on the half-way house of countable sets.

Course: NUMBER FIELDS

Paper no. 1 Question no. 20G
This was the most popular of this topic's questions, but (like the other two questions) done very badly. In part (a) people always defaulted to trying to shoehorn the question to something approachable via Dirichlet's Unit Theorem, often going nowhere, whereas it was really an easy application of Dedekind's criterion (I think only two or three people realised this). The fairly standard part (b) was done better, but quite a few even bungled the calculation of the Minkowski constant.

Paper no. 2 Question no. 20G
Another question where people automatically reached for Dirichlet as the 'general result' to be used, whereas it was much easier to apply Kummer's lemma.

Paper no. 4 Question no. 20G
A lot of this was unseen or non-standard variations of material given in lectures and it was catastrophically done by the few brave souls who had a go. Most simply quoted Eisenstein at the beginning and ignored the very helpful hint to get the degree of the field extension. Almost no-one had the first notion about how to find the dual basis, and fewer still (I think just one person) made it to part (c) where one had to exhibit an explicit form for the ring of integers in a special case.

Course: NUMBER THEORY

Paper no. 1 Question no. 1G
A straightforward question on the Chinese remainder theorem with a simple application to systems of squarefull integers. Many stated and proved the CRT only for a system of two equations but then applied it to a system of a thousand (!) equations without any comment. On the whole well done.

Paper no. 2 Question no. 1G
An easy question on the definition and use of the Legendre symbol. Generally well done by those who attempted it.

Paper no. 3 Question no. 1G
A popular question on multiplicatve functions and in particular the Mobius inversion formula. The statement of that formula contained a typo, but it was quickly spotted and corrected. All parts were well done, though I do wish people would remember that in defining a function one is supposed to specify both a domain and a codomain.

Paper no. 3 Question no. 11G
A very popular question on reduced positive definite BQFs. It was well done down to the very last part (which was unseen) on representations of primes by such forms. This latter part did cause a few problems.

Paper no. 4 Question no. 1G
I thought this looked quite an easy question on representations of continued fractions, however many people got stuck or made inexplicable errors. For the bookwork, the majority didn't think to consider first a purely periodic continued fraction and then tackle the general case separately. An alarming number of people could not correctly find a positive root of $\theta^2 - 7\theta - 1 = 0$.

Paper no. 4 Question no. 11G
Alas there was a typo in the second sentence of part (i), but it was spotted very quickly. There were some inaccurate statements of the Fermat-Euler theorem, with some not bothering to use Euler's function in the statement. The standard material on pseudoprimes and Carmichael numbers was competently done, though often by laborious methods. No one got a hold of the nice but rather tricky final part about there being a finite number of Carmichael numbers of a given type. It required solution of a certain pair of simple linear equations, but few got close even to writing these equations down.

Course: REPRESENTATION THEORY

Paper no. 1 Question no. 19I
A good question which produced answers of varying quality. The bookwork was generally well done, and there were many good solutions to the problem part.

Paper no. 2 Question no. 19I
This was a relatively easy question, but attracted few attempts, probably because it looks so long. Candidates gave many good solutions to the problem parts, by a variety of routes. The only part which was really difficult was finding an example in part c) for which χ splits into 3 irreducible characters.

Paper no. 3 Question no. 19I
Candidates were unsure just how much detail they were supposed to provide in the proof that $\chi(1)$ divides G, with some simply asserting that $|\chi(g)|\mathcal{C}_g|/\chi(1)$ was an algebraic integer, and others giving the proof. Most candidates could do the three easy problem parts, but very few were able to make progress on c).

Paper no. 4 Question no. 19I
This question was done well by almost all who attempted it. Question a ii) was the most difficult part.

Course: RIEMANN SURFACES

Paper no. 1 Question no. 24F
Well answered in general. Many students did not, however, correctly identify the group H.

Paper no. 2 Question no. 23F
Well answered in general. As expected, there were several different arguments given for the last two parts.

Paper no. 3 Question no. 23F
Very well answered by those who attempted it. No serious difficulties with any part of the question.

Course: TOPICS IN ANALYSIS

Paper no. 1 Question no. 2F
For the most part well answered by those who attempted it, but varying levels of formality in the proofs provided.

Paper no. 2 Question no. 2F
Very few attempts, but those well answered.

Paper no. 2 Question no. 11F
Reasonably well answered. Several only attempted the true and false part (b). No marks were given for answers without any justification.

Paper no. 3 Question no. 2F
Generally well answered. Many more attempts than the Section 1 questions from previous days.

Paper no. 4 Question no. 2F
In general, well answered, with a reasonable number of attempts. Students did well even with the last part.

Paper no. 4 Question no. 12F
Many students simply quoted Runge's theorem, even though they were meant to prove this in the case of the domains in question. The mark scheme was adjusted so that these students could still receive alphas under appropriate circumstances.

Course: APPLIED PROBABILITY

Paper no. 1 Question no. 28J
There was a lot of variety in the given definitions of an inhomogeneous Poisson process. The part with the time change went quite well, the unseen part turned out to be challenging for most of the students.

Paper no. 2 Question no. 27J
In the bookwork part a number of students came up with the definitions for a discrete time Markov chain. When justifying the Markov property for Y_n, most students missed the fact that the strong Markov property of X needs to used. The transition probabilities for Y_n have been derived quite often, but a number of students decided to express it in term of the generator of X, which lead to some confusing formulas.

Paper no. 3 Question no. 27J
As expected most of the students solved the relatively easy bookwork part with confidence. In the unseen part there were a good number of slight computational errors when deriving a formula for $E[Q_n]$. Overall the question worked quite well.

Paper no. 4 Question no. 27J
In the proof of Wald's identity many students only used the tower property and concluded the statement from the i.i.d. property of the X_i's without noticing or arguing that the stopping time property of M is essential. In the proof of the renewal theorem many students applied the law of large numbers along the random times $N(t)$. Another common mistake was to claim that the statement would follow directly from the almost sure convergence of $N(t)/t$ without further explanations. In the somewhat challeging last part many students tried to solved it by using a discrete time Markov chain rather than renewal theory.

Course: OPTIMISATION AND CONTROL

Paper no. 2 Question no. 30K

This question was of an appropriate level of difficulty. Many students solved it perfectly, but the marks show a good spread. It is possible to answer the final question by remarking that, if we never stop searching, the final reward is fixed and does not depend on the policy, so we should aim to minimise the searching cost as in part (a). This type of answer received full marks.

Paper no. 3 Question no. 30K

The average score in this question was low, because almost no one proved the normality required in part (a) rigorously. The problem requires proving that x_t is $\mathcal{N}(\hat{x}_t, V_t)$ *conditional on* W_t, which is due to the fact that a product of normal densities is normal, but most people proved it marginally. Parts (b) and (c) were more attainable.

Paper no. 4 Question no. 30K

The level of this question was appropriate. About half of the students used the approach suggested in the solutions, positing a separable form for the value function $F(x, t)$ as a solution of the differential equation in part (a). The other half used Pontryagin's maximum principle.

Course: PRINCIPLES OF STATISTICS

Paper no. 1 Question no. 29K

We expected this question to be on the easier side and the results show that. Many students permuted θ and $1-\theta$, which was not heavily penalised. The application of Slutsky's lemma was examined strictly, as several students suggested the implication was almost sure convergence. The most common mistakes were computational.

Paper no. 2 Question no. 28K

This difficulty of this question was appropriate. Computational mistakes were common, especially in deriving the Bayes risk. Very few students were able to answer part (d). Many students pointed out that the MLE is a limit of Bayes rules and has constant risk, then cited a result from lecture stating that Bayes rules with constant risk are minimax. However, the limit of Bayes estimators is not a Bayes estimator and Bayes risk is ill-defined with improper priors, therefore only partial credit was awarded for this answer.

Paper no. 3 Question no. 28K

This question could have been improved by removing or restricting the condition in italics at the bottom. In fact, most candidates attempted to prove directly that $\hat{i}_n \to I(\theta_0)$ in $\mathbb{P}_{\theta_0}$, in part (b), and that the limiting distribution of the Wald statistic is χ^2_p, in part (c), even though they could technically just cite the result from the course. Because of this, proofs that were not entirely rigorous had to be given credit, which inflated the average.

Paper no. 4 Question no. 28K

The level of difficulty was appropriate. In (b), many candidates failed to explain why the ξ_i are independent. A different Greek letter might have avoided confusion with ε_i. Very few people recognised the fact that $\hat{g}_N$ is a least-squares estimator, and many simply argued that it is an average for something with expectation close to $g'(x_0)$, and averages minimise the square loss. A few candidates were vexed by the mention of "intuition". A common mistake was to assume $\hat{g}_N$ is unbiased, and to apply a bias variance decomposition.

Course: PROBABILITY AND MEASURE

Paper no. 1 Question no. 27J
For part (a) all of the students solved it by differentiating to find that the minimum is attained for $a = E[X]$. Parts (b) and (c) worked well and parts (d) and (e) turned out to be as challenging as anticipated, but a good number of students managed to solve it.

Paper no. 2 Question no. 26J
This question turned out to be not as easy as expected, in particular there were not many attempts for part (b). But it produced a good range of marks.

Paper no. 3 Question no. 26J
The question worked quite well, although many student got easily 10 marks just for completing the relatively easy bookwork part. However, only a few student managed to solve the quite challenging unseen part of the question.

Paper no. 4 Question no. 26J
Given that ergodic theory has been only lectured in the final week of term, the question turned out to be surprisingly easy, which is mainly due to the large bookwork part. Many students earned an α by completing the bookwork and the easy unseen part in d). For c) the most common solution was the Baker map rather than the rotation map in the model solution.

Course: STATISTICAL MODELLING

Paper no. 1 Question no. 5J
This question went well and produced a good range of marks and lots of β's. Many student did not state Wilks' theorem explicitly as requested, although they managed to apply it correctly.

Paper no. 1 Question no. 13J
This question was more on the easy side. Many students had no problems to derive the relation for the hazard function. The 6 marks for the log Likelihood function in the second part (according to the marking scheme) was maybe a bit too generous.

Paper no. 2 Question no. 5J
This question worked quite well, although I expected more attempts, and a good number of students only wrote down the F-statistics (often incorrectly).

Paper no. 3 Question no. 5J
This question turned out to be harder than expected, only 11 attempts. One common mistake was to identify the correct parameters that determine the degrees of freedom.

Paper no. 4 Question no. 5J
Not many students spotted the correlation of the residuals in the plots. Partial credit has been given for somewhat reasonable explanations in order to create more β's.

Paper no. 4 Question no. 13J
As expected, in a) and b) a number of different explanations came up. They have been generously marked as long as they were somewhat reasonable. Part c) went overall well, some students tried a Likelihood approach in the regression model rather than the delta method.

Course: STOCHASTIC FINANCIAL MODELS

Paper no. 1 Question no. 30K
This question was on the easier side, but many candidates struggled to prove the uniqueness of Doob's decomposition. Many simply assumed the construction and cited the uniqueness of conditional expectation up to null sets. The implication (ii) $\Rightarrow$ (i) in part (d) also proved difficult for many candidates.

Paper no. 2 Question no. 29K
This question was of an appropriate level of difficulty. It was clear that many students had memorised the proof required in part (a), but the answers not always made sense, so they were carefully scrutinised. There was a correction to this question early in the examination, which eliminated the term $-\sigma^2/2$ in the definition of the Black–Scholes process S_t. This correction only redefines the drift constant μ, and the answer does not depend on this constant, so it was possible to solve the problem with the same method and obtain full credit even if the candidate missed the correction on the blackboard.

Paper no. 3 Question no. 29K
The level of difficulty was adequate. Fewer than half of the candidates managed to prove the implication (iii) $\Rightarrow$ (i) in part (c). Part (d) was very similar to a part in Paper 1, so we required a careful argument for full marks.

Paper no. 4 Question no. 29K
The difficulty was adequate. Few candidates properly justified taking derivatives inside the integral, and this was not penalised. When doing this, many failed to recognise that it is much easier to apply the change of variables $X = Y\sigma + \mu$ with Y standard normal. This led to many computational mistakes, unfortunately.

Course: APPLICATIONS OF QUANTUM MECHANICS

Paper no. 1 Question no. 34A
This was a straightforward question which rewarded candidates who understood the bookwork on one of the more difficult topics in the course.

Paper no. 2 Question no. 35A
This question on a topic at the end of the course had a low take up (12 attempts). Those who attempted the question generally did well (7 alphas) and the question itself seemed to be at the right level.

Paper no. 3 Question no. 35A
The section of the course on scattering in three dimensions is usually perceived as hard by the students and questions typically appear long due to the need to give some of the more complicated formulae in the text of the question. However, this question was relatively popular and students who had learnt the material in the notes and problem sets were appropriately rewarded.

Paper no. 4 Question no. 34A
A good question which combined material from different sections of the course (Born approximation and 3d lattices). The question was well structured and straightforward except for the final unseen part. The high alpha rate (14 out of 21 attempts) possible reflects a selection effect.

Course: ASYMPTOTIC METHODS

Paper no. 2 Question no. 31

Good question, not very difficult. In the first part a number of students did a direct change of variables obtaining a correct expression but without motivation (e.g. using Jordan's lemma) thus losing marks. Many derived their own versions of the Jordan's lemma, which was fully acceptable. The last two parts were straightforward for the majority of students as they involved direct application of the stationary phase methods, which they must have mastered in the course.

Paper no. 3 Question no. 31

A very nice question, consisting of two separate parts. In part (a) most students correctly identified the condition for finding the paths of steepest descent (from vanishing of the imaginary part of the argument of the exponential). However, subsequent analysis of this condition led to very mixed results. In many cases the extremal paths were identified but paths of steepest ascent (and not descent) were given as an answer. Asymptotics of these paths also caused a lot of trouble. But a small fraction of students ($10-20\%$) still got this part fully correctly. Most of the students probably were too exhaused when they got to the second part and only a few of them did it correctly. Some of those who started had difficulties getting the desired final form as they did not realize they needed to apply the steepest descent method here.

Paper no. 4 Question no. 31

This question ended up being surprisingly easy. It was attempted by a large number of students and many had no issues with it. In the first part only a handful of students could not show that the provided integral is a solution of a given equation; most have no difficulty demonstrating this result via integration by parts. Asymptotic expansion of the integral caused no problem at all, very few students lost marks there. The same is true about the last part of the problem. A number of students did not go through the direct derivation of the Liouville-Green series and used the formula they derived in lectures, which was fine. The exact form of the equation was such that it was difficult to make major mistakes on the way even if the simplifying assumptions were introduced from the start. Some parts of the problem (e.g. equation transformation) were very trivial and gave all students several extra marks. I suggest making this problem harder in the future.

Course: CLASSICAL DYNAMICS

Paper no. 1 Question no. 8B
Good question. Some students failed to derive Hamilton's equations from the action principle. Demonstration of the conservation of F did not cause major problems generally, having been just reminded of the Hamilton equations. However, deriving the solution ended up a bit tricky for some students: some did not understand they could just use integrals of motion for that and instead went on to come up with some differential equations based on Hamilton equations (they were explicitly asked to "solve Hamilton equations" and this might have confused some).

Paper no. 2 Question no. 8B
Good question, student performance consistent with expectations. Some students failed to derive the stated result using Lagrange multipliers. However, starting with the equation of motion the majority were able to obtain the resultant motion.

Paper no. 2 Question no. 14B
A very good problem. First part had mixed results: when deriving the appropriate equation some students failed to show that the matrix they use to define the angular frequency vector is anti-symmetric. Some failed to provide any derivation at all. But many did deliver a clean and logical derivation of the result. Derivation of the Euler equations was straightforward for everyone. When showing the conservation of energy some erroneously assumed that energy depends on $\dot{\omega}_i^2$, not ω_i^2, which is a pretty serious issue. For the conservation of L^2 some simply used the fact that Euler equations are a direct consequence of the conservation of $\mathbf{L}$ — this part of the problem text can be clarified better to make sure students understand what exactly they are asked to do. The last part of the problem was relatively easy, given the previously identified two integrals of motion, but some students got lost in algebra and could not eliminate both ω_1 and ω_2 in favor of ω_3.

Paper no. 3 Question no. 8B
Good question. Looks a bit academic at first, but was well understood by the students. Most of them were able to write down the matrices correctly, although there were cases when the arithmetic mistakes led to the wrong results. In these cases the eigenvalues and eigenvectors were incorretly computed. But most students did well starting with the appearance of the matrices.

Paper no. 4 Question no. 8B
A good question testing the knowledge of Noether's theorem. Some students did not provide a detailed proof of the theorem, losing marks. In the second part the majority of students did come up with one of the continuous transformations to obtain the integral of motion using Noether's theorem without much difficulty. Almost all have also correctly identified the energy as the second required integral (reference to the number of integrals of motion has likely served as a hint).

Paper no. 4 Question no. 15B
A good question testing important concepts. A number of students provided improper derivation of the Hamilton equations (they were asked to use Lagrange equations and the Legendre transform) or no derivation at all. But a good fraction did well on this part. In the second part the main issue was the failure to express Hamiltonian in canonical variables. However, once this was done the third part was straightforward.

Course: COSMOLOGY

Paper no. 1 Question no. 9B
A good short question. It was made a bit simpler by stating the desired results to which the students were supposed to arrive. The only issues were arithmetic mistakes.

Paper no. 1 Question no. 15B
Conceptually pretty straightforward question and students knew very well what they needed to do. The only issue was the lengthy and cumbersome calculations, but they were made easier by providing the explicit forms of the final results. In the end the majority of students were able to reproduce these expressions. The most difficult part was the the last one — derivation of the time at which solution changes from deceleration to acceleration. Only a small number of students got that part right.

Paper no. 2 Question no. 9B
First part was easy, almost everyone was able to provide a desired argument. The second part was a bit tricky. In deriving the horizon size many students integrated $c/a(t)$ over time, rather than $ca(t_0)/a(t)$. As a result, many got incorrect result, even with wrong dimensions. The horizon problem was described with sufficient level of understanding by virtually everyone.

Paper no. 3 Question no. 9B
Most students had no issue taking the high energy limit and getting the first part of the problem right. This part was not supposed to cause problems from the start. The second part is a marginal test their knowledge of cosmology — it focuses on the manipulation of a particular integral, which is pretty much an arithmetic exercise. A fraction of students failed to cope with it.

Paper no. 3 Question no. 14B
A pretty straightforward exercise heavily based on lecture material. It was made even easier by giving the result from the start in most cases and asking them to prove it. A few students failed to specify the appropriate boundary condition on pressure derivative in the centre of the star. The rest of the problem was relatively easy, with just some arithmetic mistakes in powers, etc. in cumbersome expressions (such as in part d).

Paper no. 4 Question no. 9B
This question probably required good familiarity with the topic. Its formulation is a bit oibscure. Only a handful of students attempted it and only one or two got all of it right. The main trick was getting the coefficient in the first orde expansion right, which required high order expansion of time in terms of a parameter. Most students did not go to high enough order and lost corresponding terms, costing them marks. In the second part many students confused "linear perturbation" with the size of the perturbed region. In the future I would suggest providing more clarifications in the text of the problem.

Course: DYNAMICAL SYSTEMS

Paper no. 1 Question no. 31E
This question seemed to split the candidates somewhat, with many either getting an alpha or failing to get a beta. The fixed point analysis at the beginning was often done carelessly, which resulted in errors carried on to the rest of the question. A surprising number of students, faced with a 2×2 diagonal matrix, decided to compute the trace and determinant in order to find the eigenvalues. Some of them even did this correctly. The sketching was often not completed with enough care, and resulted in marks lost. Many failed to realise that outside the unit circle the trajectories must move monotonically downwards. The final part, analysing the bifurcation was mostly done well.

Paper no. 2 Question no. 32E
This was fairly well answered on the whole. Some students lost marks by not reading the question carefully and missing parts out. Many failed to realise that the homoclinic orbit they were seeking would pass through $(0, 0)$ and so they should set $H_0 = 0$ when using the energy-balance method. This led to intractable algebra. The final part, where the candidates were asked about what parameter range they would expect to see a periodic orbit was correctly answered by very few students.

Paper no. 3 Question no. 32E
The first part was mostly well answered, but many students failed to completely nail down the justification of asymptotic stability as many failed to recognise their Lyapunov function was not strictly monotone away from the origin. The section on finding stationary / Hopf bifurcations was on the whole poorly answered, with many seeming not to know the criteria for either type of bifurcation. As on paper one, the part that involved finding a centre manifold was generally well answered.

Paper no. 4 Question no. 32E
This question was well answered by the majority of students who attempted it. The first part, showing that if F has a 3-cycle, then F^2 has a horseshoe seemed to cause little problem to most of the candidates. A few lost marks for carelessness in stating Sharkovsky, and a few failed to realise that they needed to use the result to then show that chaos is a consequence of having an $N-$cycle for $N \neq 2^p$.

Course: ELECTRODYNAMICS

Paper no. 1 Question no. 36D
The first part of this question was fairly standard bookwork. Nearly all students did well on this. The second part involved a detailed calculation of the relativistic motion in background electric and magnetic fields. This too was done well by a good fraction of students.

Paper no. 3 Question no. 37D
The first part of this question was standard, but involved bookwork. Nearly all students did this without difficult. I thought that the second part would be challenging, but most students got a perfect answer or close to it. This showed a genuine understanding of a tricky part of the course.

Paper no. 4 Question no. 36D
This was a fairly straightforward question on electromagnetism in media, a part of the course which scares many students. This was reflected in the low take up. Students tended to do very well, or barely get off the ground.

Course: FLUID DYNAMICS II

Paper no. 1 Question no. 38C
Most students answered this competently. The main errors were to equate pressures at the free boundary (rather than normal stresses) and to ignore completely the requirement of zero shear stress at the free boundary.

Paper no. 2 Question no. 38C
There were very few decent attempts at this Kelvin-Helmholtz-like instability problem. Weaker candidates did not realise that the kinematic boundary condition at an interface should be applied to the velocity fields on both sides in order to connect the two. Some candidates did not think about the symmetry of the velocity potential in the central region in order to simplify the algebra. Others thought about this and made an incorrect choice.

Paper no. 3 Question no. 39C
The bookwork (reciprocal theorem; resistance tensor) was well answered, and only a few candidates failed to notice that they were expected to show that the resistance tensor $\mathbf{A}$ is symmetric. But very few candidates were able to find the angle of sedimentation of the inclined rod. This requires little more than A level mechanics, with resolution of forces and velocities along the principal axes of the rod.

Paper no. 4 Question no. 38C
Many answers to this lubrication problem were close to perfect. Various standard algebraic errors (e.g. errors of sign) do not change the predicted rate of volumetric pumping through the gap. However, they do indeed affect the wall shear stress and the torque computed in the second part of the question. This led to an increased spread of marks.

Course: FURTHER COMPLEX METHODS

Paper no. 1 Question no. 7B
This question generally caused little problem. Most students were able to find one of the suitable substitutions and demonstrate the stated results. Some lost points when specifying the values of z for which the first relation holds.

Paper no. 1 Question no. 14B
Standard application of the Laplace's method. Most students were able to derive the form of equation for function f, and majority have correctly solved it as well although some did lose points because of wrong solution. Identification of two finite contours ended up a bit tricky, although the one along the y-axis was identified by almost everyone correctly. Infinite contours and derivation of a second real solution also caused issues for some students.

Paper no. 2 Question no. 7B
Standard application of the Cauchy pripcipal value method. Having the answer provided to students certainly simplified things a lot. Majority had no issues identifying the appropriate integration contour and taking the integrals.

Paper no. 2 Question no. 13B
The part with the definitions was very straightforward, as well as showing that zero and infinity are branch points. Cauchy-Riemann conditions were easy as well. The tricky part was in the end, when the students needed to fix the branch of v — only a few did this appropriately for the whole complex plane. Some students did not use the Cauchy-Riemann results for showing that $dw/dz = 1/z$, instead just directly differentiating the $z = e^w$ relation.

Paper no. 3 Question no. 7B
A nice short question testing major concepts. Many students did well on it, with a good share getting the full grade. Things were made a bit simpler for them by giving the final result. Some have struggled with the Laurent expansion. A small number of students did not know how to use the branch cut and took the integral using other means.

Paper no. 4 Question no. 7B
Easy question. In the first part the only issues with the singular point classification were at infinity due to errors in making the coordinate change in the equation. With the provided substitution the last part of the problem was easy. Looks like a direct application of the concepts learned in class.

Course: GENERAL RELATIVITY

Paper no. 1 Question no. 37E
This was a relatively straightforward question involving geodesics in de Sitter. On the whole it was well answered. Where candidates fell down it was usually because they didn't use all the symmetries of the Lagrangian, or were careless extracting Christoffel symbols. The final part concerning the interpretation was not well answered, with many assuming the geodesic incompleteness implied a big bang singularity.

Paper no. 2 Question no. 37E
This question was mathematically fairly simple, but required some care in interpretation of the results: the majority of the marks were for the descriptive parts. Where students lost marks it tended to be through not taking enough care to fully describe the dynamics and consider all possible cases.

Paper no. 3 Question no. 38E
This question contained some challenging algebra, but those students who persisted mostly got through to the end. Some worked with forms rather than transformation matrices, which made the computation slightly cleaner. The limits in the last part were slightly subtle, with many failing to notice that $\sqrt{(x-t)^2} = |x-t| \neq x-t$ in general.

Paper no. 4 Question no. 37E
This question had few attempts, and these were generally not good. No student completely convincingly extracted the Newtonian limit, and most struggled with the later parts. A common mistake was to neglect the effect of the gravitational field on the rate of passage of time according to the observer. As a result, most students found a local speed of light which differed from 1. None seemed concerned by this.

Course: INTEGRABLE SYSTEMS

Paper no. 1 Question no. 32A
A very popular question with 50 attempts. The question was well structured with five subsections of ascending difficulty leading to a good distribution of marks.

Paper no. 2 Question no. 33A
Very popular with 55 attempts. The early parts of the question were standard bookwork. The unseen part of the question ended up being too easy and did not adequately seperate out the more able candidates (hence 31 candidates achieved alphas).

Paper no. 3 Question no. 33A
Very popular 54 attempts/30 alphas. As for the corresponding question in Paper 2, the unseen part ended up being too easy. The first draft of this question (which we subsequently decided to shorten) was probably about right.

Course: MATHEMATICAL BIOLOGY

Paper no. 1 Question no. 6C
A straightforward question, usually well answered. Yet some candidates were unable to write down the correct master equation for the evolution of the probability of the population size.

Paper no. 2 Question no. 6C
A straightforward question. One candidate asked during the exam whether the constants a, b and c are positive: the examiners had already discussed this and had decided that applied mathematicians should have learned to think about sensible choices of parameters in their models. Mark losses were mainly due to simple algebraic errors when finding the endemic state.

Paper no. 3 Question no. 6C
A straightforward question that most people answered well.

Paper no. 3 Question no. 13C
(a)–(c) were bookwork and were well answered, though some candidates made errors of sign when evaluating the probability flux in (a). Marks were most often lost in (e) or (f), either because "stationary state" was ignored, or because no route was found to show that D_{ij} is the inverse of C_{ij}.

Paper no. 4 Question no. 6C
A straightforward question. Some candidates had trouble showing that the fixed point is stable, and others lost marks on poor (or non-existent) spider diagrams.

Paper no. 4 Question no. 14C
This question (Turing instability, modified from an earlier year) was far too close to material given in lectures, and consequently attracted a large number of candidates. The best gave a clear description of what they were doing, and deservedly earned full marks. Others appealed directly to results given in lectures, and one even explained the difference between his/her sign convention and that of the lecturer. Most candidates scored high marks, but sometimes the final sketch of regions of instability gave reason to suspect that the material was being reproduced from memory without full understanding.

Course: NUMERICAL ANALYSIS

Paper no. 1 Question no. 40E
Considering that this question was, in principle, accessible to any student with a decent understanding of first year linear maths, I was surprised how poorly it was answered. Very few could concisely argue that $\hat{A}$ must have a special structure and deduce the implications for A.

Paper no. 2 Question no. 40E
This question was well answered on the whole. The descriptions of the relaxed Jacobi method were often cursory at best, with some losing marks for not clearly explaining terms introduced. Some were careless in computing eigenvalues of A which resulted in errors carried through the rest of the question, but a respectable number made it to the end.

Paper no. 3 Question no. 41E
This question was mostly well answered. Some lost one or two marks on the first part for diving in to algebra without explaining what they were doing, or for meandering computations. The stability analysis was well done on the whole, with most students immediately applying the Fourier method. The hint in the question was slightly misstated but no student appeared to notice, and full marks were awarded for applying the hint as stated.

Paper no. 4 Question no. 40E
This question was fairly well answered by the students who attempted it. Some lost marks for being too vague in describing the FFT method. A few dropped marks by not applying the algorithm carefully enough in the examples at the end. On the whole, the students who made it more than 25% of the way through got to the end.

Course: PRINCIPLES OF QUANTUM MECHANICS

Paper no. 1 Question no. 33D
This question required some simple bookwork, followed by a calculation in perturbation that both lecturer and examiner thought some would find tricky. Both lecturer and examiner were completely wrong. The question was way too straightforward and the vast majority of students got full marks.

Paper no. 2 Question no. 34D
This was a fairly tricky question on the parity of particles in various decay processes. It is close to questions on the examples sheet, but is a topic that students often have trouble with. This was the case here. A couple of students got full marks with beautifully succint answers of around 1/2 a page, and a few others did well. But most struggled to appreciate the subtle correlation between the spin and angular momentum due to the anti-symmetric nature of the wavefunction. Given the large number of poor attempts, I was fairly generous in marking

Paper no. 3 Question no. 34D
This question, on the interaction picture, got a range of answers. Most students could do the bookwork at the beginning. But you had to think to figure out what matrix element to compute, and then keep your head when doing it. I thought it was a good question which separated the students.

Paper no. 4 Question no. 33D
A straightforward question on the algebra of spin. Students could answer this very well.

Course: STATISTICAL PHYSICS

Paper no. 1 Question no. 35A
A good question which combined standard book work with a nice unseen part. Credit was fairly evenly distributed through the question and this was refelcted in the distribution of marks. Ten alphas out of twenty attempts is good however there were also several incomplete attempts/fragments which scored poorly.

Paper no. 2 Question no. 36A
This question proved to be too hard with no candidates gaining an alpha. Candidates often stumbled in the bookwork in part (a) and on the first part of (b) and struggled thereafter. The problem might have been improved with suitable hints.

Paper no. 3 Question no. 36A
Like other questions on this course this proved hard with a very low average mark of 7.81 over 27 attempts. The unseen part in particular seemed to throw many candidates. Answers typically ended up being quite long.

Paper no. 4 Question no. 35A
A reasonable number of attempts (18) but the alpha rate was low (5/18). Many candidates struggled with Part (b). This question was undoubtedly too hard (and pssibly too long).

Course: WAVES

Paper no. 1 Question no. 39C
(Linear sound waves) Most candidates did fairly well. Some ignored the inertia of the spherical metal shell; one candidate ignored the hint and quickly went wrong. Surprisingly few candidates were able to show that there are infinitely many eigenfrequencies, and some claimed that there are none.

Paper no. 2 Question no. 39C
This shock wave question is a classic question that requires pre-exam preparation if an efficient solution is to be presented in the exam. A large number of candidates took the upstream (undisturbed) pressure and density to be p_2 and ρ_2, repectively, instead of p_1 and ρ_1. They subsequently became bogged down in algebra which led nowhere.

Paper no. 3 Question no. 40C
The bookwork (ray-tracing equations and Snell's law) was well answered. Routes for finding the upper bound z_m for the path were sometimes long, and only a few candidates made a decent attempt at a local analysis of the ray path near z_m.

Paper no. 4 Question no. 39C
This group velocity/stationary phase question was well answered. Some students did not notice that the frequency ω can be positive or negative. Others tried to put the solution for the initial value problem into a real form immediately, and usually lost a factor of 2.